

PX920 Workshop:
Bounds on the Effective Properties

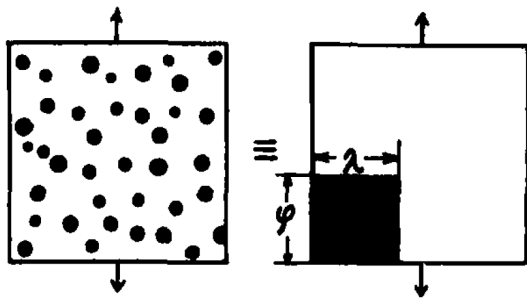
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Problem 1

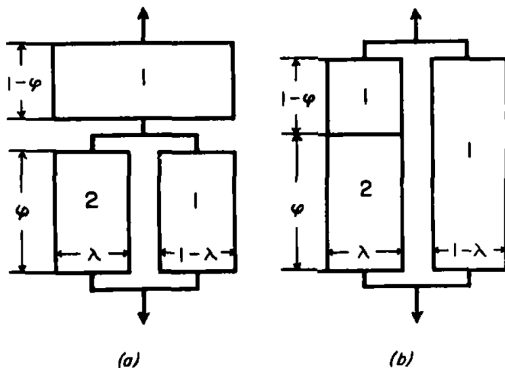
For well-dispersed two-phase systems, a model was proposed by Takanayagi et al. (1964). When a sample of such a system is subjected to tension part of the force pass only through the phase 1 and the others through both the phases in series - both phases are well connected with each, so no interfacial damage is allowed; λ and ϕ are parameters related to the mixing state and the composition of the sample. For spherical particles these two parameters related to the volume fraction of phase 2, V_2 as

$$\lambda = (2 + 3V_2)/5 \quad \text{and} \quad \phi = 5V_2/(2 + 3V_2)$$



Problem 1

Two simple mechanical models (a) and (b) are considered for calculating the effective modulus. It has been proven by model experiments that the **Model (a)** is more suitable.

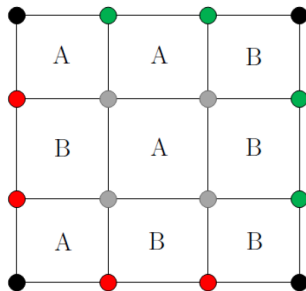


Problem 1

- Derive a formula for an effective Young's modulus in the direction of the load using the **Model (a)**
- Compare it with the Voigt and Reuss averages using a simple Python script for volume fractions V_2 from 0 to $V_2 = 1$.
Assume the following elastic properties for the phases $E_1 = 100\text{GPa}$, $E_2 = 300\text{GPa}$.

Problem 2

Consider an *RVE* domain where constituent materials A and B are randomly distributed. Assume that each material phase is embedded within a single finite-element (FE) of unit dimensions.



Implement the generalized *Voigt and Reuss bounds* into a FE-like solution procedure, given the properties of A and B:

$$E_A = 100 \text{ GPa}, \nu_A = 0.2 \quad \text{and} \quad E_B = 300 \text{ GPa}, \nu_B = 0.1$$

and evaluate the corresponding effective elasticity matrices for the system.