

# Frege's proof of $\beta$

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$$f\left(\bigwedge_p q\right)$$

$$\vdash \Phi(a) = a \smallfrown \dot{\alpha} \Phi(\alpha) \\ \vdash \lambda \dot{\alpha} \left( \smallfrown^{\mathfrak{g}} \vdash \mathfrak{g}(a) = \alpha \right. \\ \left. \vdash \dot{\epsilon} \Phi(\epsilon) = \dot{\epsilon} \mathfrak{g}(\epsilon) \right) = a \smallfrown \dot{\alpha} \Phi(\alpha)$$

$$\vdash \vdash a \smallfrown Y = (a \smallfrown Y) \smallfrown a \\ \vdash \vdash a = \dot{\epsilon}(\smallfrown \epsilon) \\ \vdash \dot{\alpha}((\dot{\epsilon}(\epsilon \smallfrown (\epsilon \smallfrown \alpha))) \smallfrown (\dot{\epsilon}(\epsilon \smallfrown (\epsilon \smallfrown \alpha)))) = Y$$

$$\smallfrown^{\mathfrak{a}} \smallfrown \mathfrak{a} = \mathfrak{a} \smallfrown \dot{\epsilon}(\smallfrown \epsilon)$$

$$\lambda \dot{\alpha} \left( \smallfrown^{\mathfrak{g}} \vdash \mathfrak{g}(a) = \alpha \right. \\ \left. \vdash \dot{\epsilon} \Phi(\epsilon) = \dot{\epsilon} \mathfrak{g}(\epsilon) \right) = a \smallfrown \dot{\alpha} \Phi(\alpha)$$