

Minimal Foundations

A Humean Account of Metaphysical Analysis

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Nomenclature

Note to readers

What follows is a very rough draft of a book on what I have dubbed ‘metaphysical foundationalism’. It defends a view about fundamental structure that places emphasis on metaphysical analysis, and the sort of precision afforded by the framework of higher-order logic. The book draws on several of my already published and unpublished works—primarily the first two works listed below—although much of the material is new or has been reworked.

- *A Philosophical Introduction to Higher-order Logics*. chapters 0, 7 and 8. (Routledge 2023).
- *Logical Combinatorialism*. (*Philosophical Review*, 2020).
- *Could the truths of mathematics have been different*. (*Philosophical Review*, forthcoming.)
- *Logics of Metaphysical Definition*. (Currently under review).

Unlike the above papers, which are somewhat technical, my goal has been to write the book in a way that requires as little mathematical sophistication as possible, referring the interested reader to these other papers for details. However, due to the nature of the work it is impossible to avoid technicalities entirely.

The book is currently missing the final two chapters on cheap objects and cheap facts, and there are several incomplete sections. This is intended only to give the readers a sense of the scope of the book and its structure, however the writing is sometimes sketchy and not representative of the final version of the book.

Chapter 1

Introduction

The world is a rich and interesting place. It is rich in terms of the sorts of things that there are: there are movies, kings, shadows, corporations, elementary fields, numbers, money, e-mails, games, chairs, properties, words, statues, and so on. It is also rich in terms of the sorts of facts there are: there are the facts of physics, ethics, mathematics, modal facts, and so on.

Do these things and facts form an unstructured jumble, all on a par metaphysically? Many people have thought not. This has been variously expressed via a number of slogans. Some objects are “more basic”, “come first”, are “more real”, than others. A donut is less basic than the atoms that compose it, and the atoms are less basic than the elementary fields from which they arise. On the other hand, perhaps the donut is more basic than the shadow it casts, and the hole in its centre. Similarly, some facts are more basic, or come *prior* to others. Perhaps the truths of physics are more basic than the mental truths, or the ethical truths.

Those who talk in terms of things being more basic than others typically also think that things eventually bottom out. There are some completely basic individuals, properties, relations, etc. which cannot be further reduced into more basic things: the *fundamental* individuals, properties, and relations.

In broad strokes, this book is a defence of this sort of view. But the view is schematic along in a couple of parameters. It does not tell us which sorts of things lie at the bottom of this hierarchy. And it does not tell us what sort of relationship we have in mind when we talk about one thing being more basic than another, or what the wider theoretical role of that relationship is (philosophical jargon like *more basic than*, *metaphysically prior to*, *more real than* is hopelessly obscure in my view, unless they are related to concepts and projects we already recognize as useful).

1.1 Metaphysical Definition

Let's start with the second question: what is the relationship of metaphysical basicness? I discern three main philosophical traditions in contemporary philosophy concerned with metaphysical foundations, all theorizing in different terms. The oldest employs a purely modal relationship, *supervenience*, that holds between two classes of properties when differences in the instantiation of properties from one class metaphysically necessitate differences in the instantiation of properties in the other class. As Lewis puts it, “a supervenience thesis is, in a broad sense, reductionist. But it is a stripped-down form of reductionism unencumbered by dubious denials of existence, claims of ontological priority, or claims of translatability”.¹ According to another, the key theoretical relationship is the notion of *ground*, a relationship between propositions expressing a kind of metaphysical explanation. *A* grounds *B* when the truth of *A* explains, in the target metaphysical sense, why *B* is true.² The other tradition is based on metaphysical analysis (cf. ?). Here the key theoretical relationship is that of metaphysical definition: one individual, property or relation being definable, in the target metaphysical sense, in terms of some others. A paradigm example would be the definability of *being a bachelor* in terms of *being married* and *being a man*, with the former property thus considered less basic than the latter properties. These traditions are not straightforwardly in competition: the notions of supervenience, ground, and metaphysical definition may all be in good standing. However, this book rests firmly in the last tradition insofar as it is metaphysical analysis that is doing the theoretical heavy lifting.

Unfortunately, the notion of metaphysical definition is somewhat obscure. It is a notion that is supposed to be informed by linguistic definition: the predicate “is a bachelor” is synonymous with a logically complex predicate “is a man and not married”, and so can be defined from “is married” and “is a man” using logical operations alone. But how do we make this precise, if we do not assume that reality is structured as a language? What is the metaphysical analogue of a logical operation? Moreover, what sort of role should claims of metaphysical definition play? Can metaphysical definitions be circular? And are there atoms of analysis—does metaphysical definition eventually bottom out? The primary goal of my work on this topic has been to provide a precise framework in which to raise and settle these sorts of questions.

1.2 Foundational Minimalism

The other question I raised asks what lies at the bottom of the hierarchy of metaphysical analysis. Wittgenstein and Russell, in their logical atomist phases,

¹? p29.

²The literature on grounding is truly vast. A key figure in this tradition is Kit Fine, see [CITE]. Some in this tradition also consider a relation that can hold between individuals, which they also call “ground” (Schaffer [CITE]). However, insofar as individuals are not the subjects or objects of explanation, this relation, whatever it is, seems less well connected with the target notion of metaphysical explanation.

would talk of the *logical atoms*. These are not physical atoms or mereological atoms, but the endpoints of metaphysical analysis: individuals, properties and relations which cannot be further broken down through metaphysical analysis. But what sorts of things are these? Wittgenstein was staunchly neutral: the atoms of the Tractatus were placeholders, perhaps to be revealed by science. Russell suggests that they are momentary sense data, like patches of colour or sounds, as well as simple qualities, like *being red*.³

There is a venerable tradition that looks to physics to provide us with basic properties and relations. Physicalists maintain that this hierarchy bottoms out at the objects, truths, and relations of physics, whatever they turn out to be. I am broadly sympathetic to physicalist sentiments, but I think physicalism will ultimately end up boiling down to a methodological dictum (“the fundamental will be revealed through scientific inquiry!”) than a guide to what is fundamental. It is not possible to rule out *a priori* the possibility that the final theory of physics invokes mental properties, fundamental moral quantities, or what have you. There is nothing inherently unscientific about these properties. What makes a theory scientific or not is a matter of methodology and our epistemic access to it expresses, not the sorts of things that it posits. The division between physical and non-physical propositions is in part a matter of how the languages that express them were arrived at, as well as how we go about investigating their truth.⁴

Nonetheless, it seems plausible to me that the ultimate fundamental entities are not going to be drastically different from the sorts of entities that are posited by our best physical theories. Spacetime points, organized by some fundamental topological and geometric relations, fields with some fundamental relations between them and spacetime points, and things like that. Perhaps the most influential articulation of this kind of fundamental basis is found in David Lewis’s thesis of Humean supervenience. According to Lewis the most basic objects, the logical atoms at the type of individuals, are spacetime points. The spacetime points are connected to one another by certain necessary relationships, which includes logical relationships (this point is distinct from that one, etc.) as well as geometrical relationships (this point lies between these two, etc.). The fundamental properties are unary qualitative properties that are possessed by these spacetime points, and the facts about videogames, economics, ethics, and so on are all completely determined by which spacetime points instantiate which of these unary fundamental properties.

Humean supervenience may not actually be compatible with modern physics. There are physical possibilities that agree entirely about the individual qualitative properties of spacetime points and yet disagree about the quantum state of the whole universe. One might take this to indicate that the logical atoms are not the *mereological* atoms. Jonathan Schaffer [REF] takes the line that there is a single fundamental individual, the entire universe, and it is the unary properties it instantiates that determine all other facts. The logical atom is, strikingly,

³[CITE] Russell ‘The Philosophy of LA’, p3, [CITE] Tractatus.

⁴I am liberal as to what counts as scientific inquiry; it may include inference to the best explanation, armchair judgments, as well as scientific experiments.

the mereological anti-atom. A more modest revision to Humean supervenience, however, keeps spacetime points at the bottom but allows for fundamental relations with higher arities: binary relations, ternary relations, and so on.

For the sake of convenience and concreteness, I will often adopt the picture of spacetime points as the foundational objects. But I do not pretend to know that our ultimate physical theory will turn out this way. Some physical theories—string theory and loop quantum gravity among them—take spacetime to be a non-fundamental entity, arising from some more basic physical primitives still. Whatever the ultimate theory is, however, I take it that it is *minimal* in ways that are well modeled by a fundamental ontology of spacetime points. The inventory of fundamental things will not include the vast majority of the things listed in the first paragraph: economies, universals, social objects, most material objects, So spacetime points and their properties and relations are excellent choice of thing to insert into the Tractarian placeholder of a “logical atom” to probe its viability. This is Lewis’s attitude about Humean supervenience ([REF] Humean Supervenience debugged p26):

The point of defending Humean Supervenience is not to support reactionary physics, but rather to resist philosophical arguments that there are more things in heaven and earth than physics has dreamt of. Therefore if I defend the philosophical tenability of Humean Supervenience, that defence can doubtless be adapted to whatever better supervenience thesis may emerge from better physics.

This book is a development and defense of this kind foundational minimalism, using spacetime physics as a placeholder for the minimal fundamental basis. There are a number of challenges to the view that this book addresses.

One has to do with how modal facts fit into the picture. If all the fundamental properties and relations concern spacetime points and fields, there is a puzzle about how facts about what could and couldn’t have been true come about. How does the fact that spacetime *could* have had a Gödelian geometry, or a Euclidean geometry, or what have you, arise from facts about the actual configuration of the spacetime points and fields. As it happens, they are not arranged in a Gödelian, or Euclidean structure, and moreover, there is nothing in their actual configuration that would seem to contain information about which alternate geometries are possible or not.

My answer to this is the execution of the project I have elsewhere called *modal logicism*. According to this project, all the key notions in the philosophy of modality can be reduced to pure logic: the notion of a necessity, the notion of a possible world, the notion of necessity in the highest-degree, propositional entailment, and so on. Of course, like the mathematical logicians, I take this logic to include higher-order logic.

The subject matter of mathematics is also challenging to account for given a minimalist worldview. Clearly there are certain general patterns in nature to do with number, shape, quantity and so on. If I have twelve pebbles, I can arrange them into a uniform rectangle that isn’t just a line, whereas if I have seven I can’t. Moreover, pebbles that have the latter property can be

arbitrarily numerous. These sorts of facts do not obviously arise out of the patterns in the fundamental objects—spacetime points, fields, etc. The platonist posits abstract mathematical objects, and will attempt (somehow) to explain the natural patterns in terms of the facts about the abstract objects: e.g. that the abstract number 7 is prime, and that there are infinitely many prime abstract numbers. The minimalist cannot appeal to these sorts of facts. My strategy is also a form of mathematical logicism.

Of course, these challenges barely scrape the surface. If the fundamental is exhausted by, say, spacetime points, fields, and relations between them, how do microwaves, economies, holes, properties, and videogames fit into the picture? How do facts about ethics, chance, physical necessity, and so on arise out of these fundamental facts? These are hard questions, and my treatment here, while less detailed, is contained in the final two chapters of this book.

The reader will have noticed logicism as a common theme here. Many of the facts that don't seem to appear in the minimal worldview turn out to be truths of logic. But the word “logic” has messy baggage, and I do not care about the label too much. Some protectors of the word vigilantly maintain that the truths of logic should be self-evident, or should be necessarily true, or can't have substantive metaphysical or mathematical implications. I personally don't hold out much hope that there is an interesting set of truths that satisfy any non-trivial subset of these roles. But this issue is tangential, because whatever we call them, there is a special collection of truths that can be stated using only logical words: truth functional connectives, and devices for making generalizations capturing certain sorts of schematic patterns. These are truths clearly worthy of singling out in virtue of their ideological parsimony, topic neutrality and the generality of the facts they express. I am arguing that many substantive truths of metaphysics and mathematics are truths that can be stated using only these words, and are thus ‘logical’ in this modest sense.

Similarly, some philosophers maintain that there is a sharp division between metaphysics, mathematics and logic, or that these disciplines shouldn't have implications for one another. This seems like a relatively recent idea, however, and not one that is applied to other disciplinary boundaries.⁵ If you read Cantor, Dedekind, Frege, Russell, and so on, the questions of these disciplines weave seamlessly into one another. I see the present work as a part of the recent revival of “higher-order metaphysics” which takes many of the key questions of metaphysics to be purely logical questions, in the sense that they can be stated in the language of higher-order logic.

1.3 No Necessary Connections

David Hume famously denied the existence of necessary connections between distinct entities. The denial of necessary connections, I maintain, is actually a

⁵Physicists and chemists, for instance, do not think that their disciplines should have no implications for the other.

very powerful principle with far reaching, and sometime unrecognized, consequences. Many of the key parts of the minimal metaphysical foundationalism defended in this book follow from it: metaphysical parsimony, intensionalism, modal combinatorialism, mathematical nominalism, the existence of unique decompositions of entities into their fundamental constituents, and the idea that all necessities follow from identities. To set the stage, then, I devote this section to informally drawing out some of these consequences.

The general thought behind Hume’s dictum, in contraposed form, is that all necessities come from identities somehow (i.e. logically follow from them). The principle has found various different articulations and is widely applied in metaphysics and the sciences. It is typically understood to be broad enough so as to concern necessary connections between propositions, properties and relations as well as individuals; but also to except, as Hume presumably did, the necessary connections given by logic.

As I will use the dictum, it is partly a stipulation about the theoretical role of a particular modality which is central to my project. I refer to it, suggestively, as *logical necessity* and identify with the broadest kind of necessity there is (chapter [REF]). But it is partly substantive, requiring reality to be fine-grained to support a modality satisfying this role. Various post-Kripkean theses taken to be distinctive of “metaphysical necessity” are not part of its theoretical role.⁶

We can illustrate the principle with some applications. Suppose we are attempting to formulate our fundamental inventory of things according to a Newtonian world view: we have spacetime points, particles, primitive properties and relations determining the masses and charges of the particles, and a location relation between particles and spacetime points that determines the spatiotemporal path that each particle traces out.

Take two uncontroversially different quantities, charge and speed, and consider the (false, as it happens) physical hypothesis that charge and speed covary:

The charge of a particle is always the same its speed.

If someone came along and told you that, while charge and speed are *different* quantities, this claim was necessary, it would be most charitable to understand them as building some restriction into their notion of necessity. They are different quantities, but they could correlate if there was a law of nature connecting them. Thus, perhaps they are claiming that it is necessary *given the laws of nature* that they covary. Indeed for any pair of distinct quantities there are consistent hypotheses connecting them which would necessitate their coincidence, and consistent hypotheses that would necessitate their coming apart. But conditional on no such connecting hypotheses there aren’t any logical correlations between different quantities. So it is not *logically* necessary, necessary in the broadest sense whatever that might be, that these two quantities coincide. Lewis [REF] (“New work” p366 journal version, “against structural universals” p41) goes as far as to say that positing logically necessary correlations like this is un-

⁶See chapter [REF].

intelligible; it involves a misapplication the concept of logical necessity.⁷ This is an instance of Hume’s dictum. Since charge and speed are different properties, and while there can be physically necessary connections between them, there can’t be a logically necessary connection between them.

Hume’s dictum is also, I claim, a powerful principle of *parsimony*. Consider another application. Particles also have positions at each time, so there is a well-defined notion rate of change of a particle’s position with respect to time. Suppose we have posited our primitive properties and relations determining the positions of particles at each time, and we are considering whether we need to posit a further class of properties corresponding to speeds—*moving at $\alpha m/s$* —in addition. Clearly our picture of reality would be incomplete if we didn’t posit charges in addition to the positional primitives; is it the same with speed? Parsimony requires that we don’t posit a *further* quantity: we already have the speed properties once we have the spacetime paths of the particles. The speed of a particle *just is* the magnitude of its positional derivative with respect to time, the latter is something we can already define from our basic properties and relations. Speed properties are *identical* to positional derivatives.

This attractive thought follows from Hume’s dictum. Consider the alternative: that there are *two* properties, the speed property *having speed $\alpha m/s$* , and then there is a different property *having a positional derivative with magnitude $\alpha m/s$* . But of course these two quantities, speed and magnitude of positional derivative, vary in perfect tandem. So, like the hypothesis connecting charge with speed, we have a thesis positing a connection between two quantities:

The speed of a particle is always the same as the magnitude of its positional derivative.

But unlike the case of charge, this is not a mere physical hypothesis. These are necessarily the same, not just with respect to what is physically necessary: they are equivalent with respect to logical necessity, necessity in the broadest sense whatever that might be.⁸ Hume’s dictum says that they can’t really be different quantities after all. When you see a connection between two different quantities, properties, etc. it is always legitimate to ask “why do they covary in this way?” Such an explanation might appeal to the laws of physics, the laws of economics, the laws of metaphysics, or some other logically contingent regularity. To posit a necessary connection between different quantities that isn’t premised on any such hypotheses is to posit a magical unexplained necessity. An unexplained necessity that logic alone can’t guarantee. Lewis refers to these variously as

⁷ “If you said that wherever carbon is instantiated, bromine must necessarily be instantiated next to it, that would make good enough sense as a matter of nomological necessity. There is no such law of nature , but there could have been. But suppose you said that it was a matter of necessity simpliciter-absolute ‘logical’ or ‘metaphysical’ necessity. Then what you say is not only false; it is entirely unintelligible how it could be true.” [CITE] Lewis Against structural universals, p41.

⁸ Perhaps there is a quantity that coincidentally coincides with positional derivative magnitude, but comes apart in physically impossible worlds. But I think we have no right to call any property like this “speed”.

brute modal facts (see also Dorr [CITE]) which he denounces as a form of “modal magic”.

Hume’s dictum can also be viewed as a principle of granularity. For it says that there cannot be logically necessary connections between distinct propositions, properties, and relations.

Propositional Intensionalism If it is logically necessary that P if and only if Q , then for P to be true *just is* for Q to be true.

Property Intensionalism If it is logically necessary that something has F if and only if it has G , then to be F *just is* to be G .

The statement of Intensionalism for relations of higher arity generalize these in the obvious way.

The laws of propositional logic are the paradigm examples of logical necessities. Since it is logically necessary that P if and only if it’s not the case that it’s not the case that P ($\neg\neg P$), it follows that for it to be the case that P is for it to not be the case that it is not the case that P . We obtain, in this same way, many propositional identities, such as the identity between $P \wedge Q$ and $Q \wedge P$, $\neg\forall x.Fx$ and $\exists x.\neg Fx$ and so on. In ? I introduced a theory, formulated in a suitable higher-order language, that was intended to capture all identities like these. Cian Dorr and I subsequently dubbed it *Classicism*, for it upholds all the identities guaranteed by equivalence in classical logic.⁹ It is discussed and defined formally in chapter [REF] section [REF]. ?, echoing similar sentiments in ? 5.43, justifies what I am calling Classicism using a similar line of thought as Lewis:

I find it very unsatisfactory to be left with no explanation of formal logic except that it is a collection of “necessary facts”. The conclusion of a formal inference must, I feel, be in some sense contained in the premisses and not something new; I cannot believe that from one fact, e.g. that a thing is red, it should be possible to infer an infinite number of different facts, such as that it is not not-red, and that it is, both red and not not-red. These, I should say, are simply the same fact expressed by other words. (p.42)

If there were two facts, P and $\neg\neg P$, for instance, then the necessary correlation between their truth-values would have to be another brute modal fact.

We have so far focused on necessary equivalences. Not all necessary connections are equivalences, however. Some are necessary implications in one direction only: it is broadly necessary that all vixens are foxes, but not conversely. However, this implication is still a logical consequence of a higher-order identity between predicates, namely that vixens *just are* female foxes. For it is a logical necessity that all female foxes are foxes, because the prejacent is an instance of the logical law of conjunction elimination when formulated precisely. But by replacing *female foxes* with *vixens* in this statement, using Leibniz’s law,

⁹See [CITE] TBN, Classicism

we obtain the desired conclusion. Actually, *all* strict implications follow from identities: the claim that necessarily every F is G follows from the identity *being F and G is being F* .¹⁰ I believe this captures what Ramsey means when he says that “the conclusion of a formal inference must [...] be in some sense contained in the premisses and not something new”. (Indeed, given Intensionalism this can be reversed: the implication also entails the identity, so necessary implications are necessarily equivalent with (and thus identical to) identities.)

Applying Hume’s dictum to the letter also leads one naturally to a form of mathematical nominalism. According to the standard platonist account, mathematics is both necessary, according the broadest kind of necessity, and concerned with particular abstract objects not directly given to the senses; numbers, vectors, sets, and so forth. But such an account seems to posit necessary relationships between distinct individuals that go beyond what is given by logic. Consider how Hume’s dictum implies that I could have been a Roman emperor: if it were logically impossible that would be a brute modal fact. If the number 7 is just another individual, why couldn’t it also have been a Roman emperor, and thus have failed to be a prime number? Perhaps, in order to retain Hume’s dictum and platonism, we should maintain that mathematics consists of logically contingent facts concerning abstract objects that are nonetheless *mathematically* or *metaphysically* necessary. But this strikes me as an unattractive account of the contents of mathematical claims. For as well as the platonic facts, facts about abstract numbers, there are also claims about mathematical patterns in nature. For example, the following ought to be tightly connected to the primeness of seven:

It is not possible to arrange seven different pebbles into a uniform rectangle except as a line of pebbles.

This mathematical pattern is *logically* necessary, I claim, for it can be formalized with suitable predicates expressing spatial relations and derived in first-order logic. Even if the platonic number 7 had become a Roman emperor, and had stopped being a prime number, it still wouldn’t be possible to take seven pebbles and arrange them in a non-trivial rectangle. So the platonic facts are not doing a great job at capturing the mathematical patterns we find in nature; they are not even modally tracking the mathematical patterns. The real contents of mathematics are more tightly connected to these implications. The account of mathematics I given in chapter [REF], a form of logicism, simply identifies mathematical claims with very general mathematical patterns. And, like the pattern indicated above, patterns can obtain or not independently of whether there are any abstract objects. Higher-order generalizations are formulated in completely logical terms, without reference to the special mathematical vocabulary of sets and numbers, and can directly imply particular claims such as our claim about pebbles.¹¹

¹⁰It is clearly logically necessary that everything that’s both F and G is G , and so using the property identity and Leibniz’s law in the antecedent we get that it’s logically necessary that everything that’s F is G .

¹¹Given my account, there are still some challenges. For instance, given certain metaphysical

The most pure form of a necessary relationship between distinct entities is the relation of being distinct itself. Applying Hume’s dictum strictly, then, also requires one to relinquish the logical necessity of distinctness. An argument due to Arthur Prior establishes that relinquishing the necessity of distinctness also requires us to give up the Brouwerian axiom for logical necessity: that what is possibly necessary is true.¹² Many philosophers who are used to theorizing in terms of “metaphysical necessity”, understood broadly along the lines of ?, will therefore find this modality to be somewhat alien to their way of thinking. It’s important to emphasize that while these consequences reveal that we are dealing with an unfamiliar modality, they do not reveal it to be an ill-conceived or incoherent one.¹³ Firstly, nothing compels us to identify the notion of logical necessity I am positing with metaphysical necessity, conceived in broadly Kripkean terms. It is entirely consistent with the metaphysical necessity of distinctness, and with the usual principles of S5 as a logic of metaphysical necessity. Secondly, it’s worth reiterating that we have introduced this modality by a particular theoretical role. It was not a concept we necessarily had antecedently, and any pretheoretic judgments we have about how it should behave ought to be taken with that in mind. My attitude is that it is better to set these untutored intuitions aside and follow the theory where it leads, judging the result by its fruits, familiarizing oneself with the unfamiliar if necessary.

The final and most important role Hume’s dictum plays in this book has to do with its interactions with metaphysical foundationalism. Historically, the principle has maintained a close connection with metaphysical foundationalism, in the form of the claim that the fundamental entities are modally “freely recombinable”. For the logical atomists the elementary propositions—statements concerning the features of the atoms of logical analysis—were all logically independent of one another. Any combination of the elementary propositions can be true, while the remaining are false. The logical atoms are thus freely modally recombinable. According to Armstrong’s combinatorial theory of possibility the atomic statements applying fundamental relations (a “universal”) to some arguments are all logically independent.¹⁴ This is also the driving idea behind Lewis’s thesis of Humean supervenience, “named in honor of the great denier of necessary connections”.¹⁵ Spacetime points and unary qualitative properties of them are the logical atoms. The elementary propositions ascribe to each point one of these properties: any point can have any of these properties while any other point can have any other properties. There are no necessary connections between these statements. (There are ways, however, in which Lewis’s view falls short of its namesake. For it looks like there *are* necessary connections between the spacetime points: namely the geometric relationships. If x lies on a line

assumptions one can prove that the mathematical patterns corresponding to arithmetical truths are logically necessary or logically necessarily false. Yet the laws of pure logic, if recursively axiomatizable, do not settle all patterns corresponding to arithmetical truths (?).

¹²[CITE] Prior .

¹³Some allegations of incoherence in relation to contingent distinctness and failures of the Brouwerian axiom are addressed in ?.

¹⁴[Cite].

¹⁵[CITE] Lewis philosophical papers vol 2, p ix.

between y and z , then it does so necessarily. For these are not purely logical relationships; such facts do not seem to be logically necessary. In a truly Humean picture, one might argue, the spacetime points should be able to instantiate any configuration of geometric relations, even those that contradict geometric laws.)

So another way of understanding Hume's dictum, in the context of metaphysical foundationalism, is as a principle of modal recombination concerning the fundamental.¹⁶ Of all the ways that Hume's dictum is understood, this one receives perhaps the most resistance (Jessica Wilson [CITE]). The strong version of Hume's dictum I defend in this book has some *prima facie* strange implications. If *being more massive than* is a fundamental relation, then it is logically possible that it be non-transitive; for logic does not require any primitive relation to be transitive. If *physical necessity* is a fundamental operator, it is logically possible that it be non-factive. Logic doesn't even require that there be more than two propositions, a truth and a falsehood, so even this should be logically possible. The reader should, of course, bear in mind my remarks about logical modality being a theoretical notion rather than a pretheoretic one when interpreting these theses.

On the other hand, I will argue, the recombinability of the fundamental entities is not merely an optional addition to metaphysical foundationalism, motivated solely by the desire to eliminate unexplained necessary connections: it is completely central to metaphysical foundationalism. This point, I think, is not widely recognized. Throughout the book, I will argue that the recombinability of the fundamental is a consequence of (and in some cases equivalent to) other key claims about fundamentality: that the fundamental form a minimal supervenience base, that they are atoms of metaphysical analysis (they not definable in simpler terms), and that everything has at most one metaphysical analysis in terms of the fundamental properties and relations.

Recombination The fundamental properties and relations can stand in any logically consistent relationship with one another.

Minimal Supervenience The fundamental properties and relations form a *minimal* supervenience base.

Logical Atoms The fundamental properties and relations are metaphysically basic and cannot be metaphysically defined out of one another.

Unique Decomposition The metaphysical analysis of an entity as a logical combination of fundamental entities is unique.

These other ideas should be more familiar to those sympathetic to the foundationalist project; Hume's dictum is thus more central to the view that it might at first seem.

¹⁶[CITE] ?, Russell Hawthorne, Schaffer, Wang...