Higher order vagueness

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Classical solutions: the dialectic so far

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- Therefore: there is a last small number. (a number n, such that n is small and $n + 1$ isn't.)

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- If a classical approach to the Sorites which rejects sharp boundaries is even to get off the ground it shouldn't find there being a last small number sufficient for there being a sharp boundary.
- The standard view: there is a last number but it's vague which number that is.

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  - n is a vague boundary for 'small' iff it's vague whether n is small. ("It's vague whether n is the last small number.")
- Classical logicians accept the existence of boundaries. But it is existence of sharp boundaries not vague ones that should bother us.

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- Clearly you can iterate these questions any number of times.

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- That is to say: p is determinate, it's determinately determinate, it's determinately determinately determinate... and so on and so forth for any (finite!) number of iterations.
- The central question of this paper: is there a sharp boundary between the determinately* small numbers and the rest?

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- Determinacy comes with the possibility of knowledge: if it's determinate which the last determinate* small number is we shouldn't be surprised to hear things like 'Fred stopped being a determinate* child at 1238907487.190872109 nanoseconds of age'.

Argument that ‘determinately* small’ is precise version I

“Start off with the set of small numbers, remove the borderline cases, then remove the borderline borderline cases, then the borderline borderline borderline cases... and so on forever. Initially this process will shrink the set of numbers we are left with. But it must stop shrinking at some point as there are only finitely many small natural numbers. However, the reasoning goes, we are then left with the precise set of numbers which are determinately^n small for every n.”

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  - **(DIST)** $\bigwedge_{i<\omega} \Delta\phi_i \rightarrow \Delta \bigwedge_{i<\omega} \phi_i$.
- 'A conjunction of determinate truths is determinate'.
- From this it follows that $\Delta^* p \rightarrow \Delta\Delta^* p$.

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  3. B^*: $\Delta(p \rightarrow \Delta p) \rightarrow (\neg p \rightarrow \Delta\neg p)$
- Here $\phi_1 := \neg\Delta\neg p$; $\phi_{n+1} := \neg\Delta\neg(q \wedge \phi_n)$

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- The approach: use techniques from modal logic.
- If we can give the meaning of Δ in terms of points and accessibility, and we can motivate constraints on these models, then we can use methods from modal logic to determine their truth.

Modal logic and vagueness

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- Technical point: a modal logic is usually characterised by a class of frames. In the context of modelling vagueness this would be a bad assumption: we ought to characterise things by classes of models - I'm going to pass over this subtlety for now.
- Explain frame conditions for B^n, B^* etc...

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- In fact any classical theory can be. (And many non-classical theories.)
- If you define 'admissible' correctly then pretty much every theory (including non-classical theories) holds that determinacy is truth in all admissible precisifications.

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- Definition: an interpretation is **admissible** iff it's not determinately incorrect.

A few comments

- What we've said doesn't guarantee that it's determinate that ϕ iff ϕ is true on every admissible interpretation: it might be the case that it's not determinate that $\neg\phi$ but global penumbral connections ensure that there's no precisification of the entire language that makes ϕ true. We can add this as an additional claim about vagueness.

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- That said, the claim that determinate truth is truth in all admissible precisifications is not particularly substantive.
- Classical views such as epistemicism and contextualism accept this claim.
- Even non-classical views accept it! (E.g. Łukasiewicz)

How to get a Kripke frame (cutting out some details)

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- Let v^* be a member of W.
- Note: for the intended interpretation we let v^* be the correct interpretation in the sense defined above. (Which W, R to take follow from this.) v^* will be a vague name.

Recap

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- It relied on DIST and at least one of the weakenings of the Brouwerian B principle.

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  "If you construct a sentence using only precise vocabulary you get a precise sentence."'
- If we reject DIST that means we can have situations where ϕ_n is precise for each n, but the conjunction of the ϕ_n is vague. By our principle this means that infinitary conjunction is vague which is just implausible.

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- This is where the argument that classical accounts of vagueness can be represented by Kripke frames comes in useful.

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A case for B

- Williamson provides a class of Kripke frames for modelling vagueness.
- $\langle W, R \rangle$ is a fixed margin frame iff there is some metric d over W and $\alpha \in \mathbb{R}$ such that Rxy iff $d(x, y) \leq \alpha$.
- These frames are clearly symmetric: if the distance between x and y is less than α the distance between y and x is less than α.

Explanation

- The distance $d(x, y)$ between two interpretations represents how similar they are. If x and y agree about the cutoff points of every predicate except 'small' where they differ by one number, then x and y are considered to be very close. (So $d(x, y) \approx 0$.) If they differ radically in their assignment of cutoff points they are very distant.

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- Δp is true at a point x iff p is true in the ball of radius α around x.
- So α represents how close an interpretation has to be to x to be admissible according to x.

Mahtani's objection

- Williamson's models allow for higher order vagueness: the extension of 'admissible' varies from precisification to precisification. But they don't allow it to be vague how *close* x and y have to be for y to count as admissible according to x.

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Generalising the argument

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- Note that close points are points that interpret similarly, so we shouldn't have close points have significantly different accessibility ranges.
- Put the constraint that for every x and y, $|f(x) - f(y)| \leq d(x, y)$.

The logic

- Call any Kripke frame that can be generated from a metric d and an assignment of accessibility ranges f, a v-frame. What is the logic of v-frames?

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Theorem

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Theorem

The modal logic KT is sound and complete with respect to the class of v-frames.

- This means that none of the weakenings of B, including B^* are invalid on these frames.

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- This result relies on some pretty implausible models based on ad hoc metrics.
- Draw on board.
- We might want to restrict attention to v-frames that are dense and where the admissibility range is always non-zero.
- Let's restrict attention to v-frames where the metric space is $\mathbb{R}^n$ and $f(x) > 0$ for every $x \in \mathbb{R}$.

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- BUT: not B^*

Some technical results

Theorem

KTB^* is neither compact, canonical nor strongly complete with respect to any class of frames.

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KTB is neither compact, canonical nor strongly complete with respect to any class of frames.*

Proof.

Let $\Sigma := \{p, \neg\Delta\neg q\} \cup \{\Delta(q \to \Delta^n \neg p) \mid n \in \omega\}$. It is easy to see that every finite subset of Σ is KTB* consistent. Simple model.

However, if $\mathcal{F}$ validates KTB* then $\mathcal{F}$ has the backtrack property so at no point of $\mathcal{F}$ is every member of Σ true: roughly Σ says that p and that q is possible, but that any q world is necessarily necessarily ... a $\neg p$ world. $\square$

Some technical results

Theorem

KTB^* has the finite model property with respect to the class of finite frames with the backtrack property.

Some technical results

Theorem

KTB has the finite model property with respect to the class of finite frames with the backtrack property.*

- This means that KTB* is sound and complete with respect to the class of frames with the backtrack property.

Open question

- Is KTB* sound and complete with respect to the class of v-frames over $\mathbb{R}^n$?

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- Is KTB* sound and complete with respect to the class of v-frames over $\mathbb{R}^n$?
- I'd love to know!

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The big picture

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- We discovered we can't give up DIST.
- Following Mahtani we found we should give up B^n and $B^{n\prime}$ for each n.
- Unclear whether we can give up on B^*: $\Delta(p \rightarrow \Delta p) \rightarrow (\neg p \rightarrow \Delta\neg p)$.

B^*

- However, if we do give up B^* then it's consistent to say that it's (determinately) vague where the boundary for the determinately* small numbers lies.

- THE END