

Abstract: Foundations for a revenge free theory of indeterminacy

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A standard strategy for blocking the semantic paradoxes is to identify a class of sentences (or perhaps utterances) as problematic in the hope that restricting attention to sentences outside this class will prevent one from running into trouble. I shall call all such approaches ‘linguistic accounts’ in virtue of their identifying the problematic phenomenon present in the semantic paradoxes with some property of the language or linguistic items used to express the paradoxes. In each case sentences said to be in this problematic class fail to obey the T-schema, are unknowable, are unassertable, and so on, in virtue of having this status.

In this paper I argue that this strategy, although widespread, is fundamentally misguided. There is no classification of sentences into problematic and unproblematic according to which ordinary reasoning can be carried out in cases where we are only concerned with unproblematic sentences. Whatever you take ‘problematic’ to be, there will be some unproblematic sentence that fails the T-schema. This is of wider significance, I think, as a standard strategy for bracketing the semantic paradoxes in other areas of philosophy is to exclude the sentences regarded as problematic from consideration.

The majority of formal and philosophical approaches to the paradoxes attribute the problematic status to sentences and not to what those sentences say. In the second half of this paper I lay the foundations for a non-linguistic theory of the problematic phenomenon responsible for the paradoxes.

1 Linguistic theories

We can formulate very general revenge paradoxes in an abstract setting by introducing a primitive definiteness predicate, $Def(x)$, intended to express whatever it is that is problematic about sentences like the liar which prevents disquotational reasoning. In this framework one can reason about which principles governing Def are possible. Principles of interest include:

1. If $\vdash \phi$ then $\vdash Def(\ulcorner \phi \urcorner)$
2. $\vdash Def(\ulcorner \phi \rightarrow \psi \urcorner) \rightarrow (Def(\ulcorner \phi \urcorner) \rightarrow Def(\ulcorner \psi \urcorner))$
3. $\vdash Def(\ulcorner \phi \urcorner) \rightarrow \phi$
4. $\vdash Def(\ulcorner \neg \phi \urcorner) \rightarrow \neg Def(\ulcorner \phi \urcorner)$
5. $\vdash Def(\ulcorner \phi \urcorner) \rightarrow Def(\ulcorner Def(\ulcorner \phi \urcorner) \urcorner)$
6. $\vdash \forall x Def(\ulcorner \phi \urcorner) \rightarrow Def(\ulcorner \forall x \phi \urcorner)$

It is well known that some of these combinations cannot be consistently maintained. The most surprising is probably Montagues paradox, which prevents one from holding 1 and 3. Gödel's second incompleteness theorem/Löb's theorem prevents 1, 2, 4 and 5. McGee's paradox shows that 1, 2, 4 and 6 are ω -inconsistent. Various linguistic accounts require us to accept some of these principles. For instance, 6 appears to be motivated by the intuitive notion of grounded truth: a universally quantified sentence is grounded by each of its instances, thus if each instance is groundedly true so is it's universal closure.

What I take to be the most far reaching problematic consequence is the following. By the linguistic theorists lights, there will be some property of sentences or utterances of sentences, which is responsible for the semantic paradoxes. So suppose we introduce a predicate G , intended to express whatever the paradoxical phenomenon that is present in the liar that is causing failures of the T-schema. (Or, if we already have Def , let $G(x) = Def(x) \vee Def(\neg x)$.)

Theorem 1.1. *The system consisting of*

(1) *The axiom schema: $G(\ulcorner \phi \urcorner) \rightarrow (Tr(\ulcorner \phi \urcorner) \leftrightarrow \phi)$*

(2) *The rule of proof: if $\vdash \phi$ then $\vdash G(\ulcorner \phi \urcorner)$*

is inconsistent.

Any failure of (1) would indicate a failure of G to express the root of the failures of the disquotational schema. On the other hand, the failure of (2) would violate a norm of assertion central to the role G was supposed to play.

2 Foundations for a non-linguistic theory

It is widely held that there is no fact of the matter whether the truth-teller is true or not. The natural way to represent this is by means of an operator, ∇ , read as 'there's no fact of the matter whether' or 'it's indeterminate whether', that can be grammatically combined with a sentence to make another sentence. Although my example was about a sentence, the truth-teller, the ascription of indeterminacy involved an operator. The distinction is clearer when one ascribes determinacy instead. If I say that there is a fact of the matter that snow is white, I have said something about the world, about snow determinately possessing whiteness, and not about the English sentence "Snow is white", or any way of referring to that fact. The predicate formulation, $Def(\text{"Snow is white"})$, is about language and can only be expressed in another language by explicitly mentioning the English sentence.

One might think there is a natural correspondence between the two ways of talking: given an operator 'it's ... that p ', there is a predicate of sentences 'in English s is used to say that p and it's ... that p ', or 'it's ... that s is true in English.' There are independent reasons to doubt the correspondence. The ancient Greeks believed that Hesperus was a star, but they didn't believe that "Hesperus is a star" is a true sentence of modern English. They did believe what English speakers say using the sentence "Hesperus is a star", but these locutions are clearly not synonymous as what the ancient Greeks believed about Hesperus had nothing to do with modern English. As Prior has pointed out, any sentence can be reformulated to be about language. "I broke a leg" can be "I broke something English speakers normally refer to with the word "leg"".

In the case of the paradoxes there is particular reason to doubt the correspondence; if λ is the liar sentence, then λ and $Tr(\ulcorner \lambda \urcorner)$ are not intersubstitutable in extensional contexts so the second paraphrase does not work. On the other hand the first paraphrase requires the disquotational schema: in English $\ulcorner \phi \urcorner$ says that ϕ . Assuming that every sentence says at

most one thing, this schema fails for a sentence which says of itself that it says something which is not the case.

The distinction between linguistic and non-linguistic accounts of indeterminacy is much more familiar in the literature on vagueness. Although most accounts of vagueness are linguistic, a promising non-linguistic approach is outlined in Hartry Field's paper 'Indeterminacy, Degree of Belief, and Excluded Middle'. In this paper Field argues that even if indeterminacy is a conceptually primitive notion – that no reductive account of it is possible – there might still be a way to outline it's cognitive, normative and inferential role in such a way as to constrain it's interpretation and single out the notion at play to anyone who has it. Field's account relied on a non-standard theory of propositional attitudes which I shall adopt for now.¹ I take the role of the determinacy operator to entail at least that:

- Its inferential role is governed by the modal logic KT.
- It licenses the use of disquotational reasoning: if you know it is determinate whether or not ϕ , you may assume that $\ulcorner \phi \urcorner$ is true if and only if ϕ .

The role should of course include much more than just this. Intuitively one may only permissibly assert that/possibly know whether/sensibly question/wonder/hope/fear/care whether ϕ if it is determinate that ϕ .² Let us call all this the Δ -role.

There is a substantial technical question as to whether any operator can satisfy the first two parts of the Δ -role, which is answered by the fact that you can show the theory DT, described below, is consistent and has standard models (i.e. Kripke models in which every world is a standard model of arithmetic.)

DT consists of the Δ -role plus the Barcan formula:

Nec If $\vdash \phi$ then $\vdash \Delta\phi$

K $\Delta(\phi \rightarrow \psi) \rightarrow (\Delta\phi \rightarrow \Delta\psi)$

T $\Delta\phi \rightarrow \phi$

BF $\forall x\Delta\phi \rightarrow \Delta\forall x\phi$

RT $(\Delta\phi \vee \Delta\neg\phi) \rightarrow (Tr(\ulcorner \phi \urcorner) \leftrightarrow \phi)$

and the truth theory:

- $Tr(\ulcorner \phi \rightarrow \psi \urcorner) \leftrightarrow (Tr(\ulcorner \phi \urcorner) \rightarrow Tr(\ulcorner \psi \urcorner))$
- $Tr(\ulcorner \phi \vee \psi \urcorner) \leftrightarrow Tr(\ulcorner \phi \urcorner) \vee Tr(\ulcorner \psi \urcorner)$
- $Tr(\ulcorner \phi \wedge \psi \urcorner) \leftrightarrow Tr(\ulcorner \phi \urcorner) \wedge Tr(\ulcorner \psi \urcorner)$
- $Tr(\ulcorner \neg\phi \urcorner) \leftrightarrow \neg Tr(\ulcorner \phi \urcorner)$

Nec If $\vdash \phi$ then $\vdash Tr(\ulcorner \phi \urcorner)$

Conec If $\vdash Tr(\ulcorner \phi \urcorner)$ then $\vdash \phi$

¹I have my own conjectures as to what indeterminacy claims amount to; but the non-reductionist approach is a useful placeholder for the purposes of this paper.

²Regarding Field's specific proposal one also has that if one is certain that it's indeterminate whether ϕ then one's credence in ϕ should be 0 and one's credence in $\neg\phi$ should be 0.

3 Higher order indeterminacy, assertion and logic

Given our theory of indeterminacy it is natural to wonder what happens to revenge sentences, like $\delta = \neg\Delta Tr(\ulcorner\delta\urcorner)$. It turns out that δ is second order indeterminate; DT proves $\neg\Delta\Delta\delta$. More generally DT proves $\neg\Delta^{n+1}\delta_n$ where $\delta_n = \neg\Delta^n Tr(\ulcorner\delta\urcorner)$. Thus we have a theory in which the revenge paradoxes give rise to a rich hierarchy of higher order indeterminacy, agreeing with Hartry Field's recent diagnosis of the revenge paradoxes. Indeed, none of the following principles restricting higher order indeterminacy can be consistently added to DT:

$$B \quad \phi \rightarrow \Delta\neg\Delta\neg\phi$$

$$4 \quad \Delta\phi \rightarrow \Delta\Delta\phi$$

$$5 \quad \neg\Delta\phi \rightarrow \Delta\neg\Delta\phi$$

The theory thus gives rise to a hierarchy of stronger and stronger operators.³ A picture suggested by Field is that each operator, Δ_α , in this hierarchy satisfies the Δ -role and does a partial job, although a better job as α increases, of characterising the true notion of defectiveness, or paradoxicality, that is common to all the semantic and revenge paradoxes. This aspect of Field's approach often invites the objection that his theory is expressively limited - that it cannot express the notion which his hierarchy of determinacy predicates is trying to approximate.

In this section of the paper I argue that the conditionals in the Δ -role can be strengthened to biconditionals, and that there is accordingly only one notion of determinacy at play here, Δ , although it is one which iterates. I give some reasons to be sceptical of the claim that the revenge liar, δ , is paradoxical in the same sense the liar is. According to the strengthened Δ -role you must reject the claim that δ is indeterminate. Not because it's false, but for the same kind of reasons one rejects the claim that the liar isn't true: because it's indeterminate. This gives rise to a quite natural picture, in which indeterminacy infects matters concerning what we can in principle know, and what we may permissibly assert. For example, the question of whether we should assert δ (given we know as much as we possibly could) is left indeterminate.

If we have time I will also discuss the nature of the \vdash symbol used in the formalised theory DT. Two linguistic accounts of \vdash are presented, a proof theoretic and a model theoretic account, and it is argued that they can both be brought into the object language in completely orthodox ways. However, an epistemic account of logic is argued for, and it is shown that an operator expressing strong epistemic necessity, L , can be consistently introduced into the language. Essentially metalinguistic claims, like the necessitation rule for truth, can then be stated as ordinary conditionals in the object language like $L\phi \rightarrow LTr(\ulcorner\phi\urcorner)$. It can be shown that DT can be conservatively extended by the axioms $L\alpha$ for each axiom of DT and by a corresponding conditional for each rule of inference.

³There is a disanalogy with Field's project here: to iterate into the transfinite Field uses the truth predicate and quantification, whereas for a classical logician the result of doing this will be very different from using, for example, infinite conjunctions. Note also that the distinction between operators and predicates is not so wide for a non-classical theorist, provide one is careful about context sensitivity.