# *Lewis on too many worlds*

Forrest and Armstrong, Kaplan and various others have argued that Lewis' modal realism is inconsistent[1]. They argue that it is possible to embed sets of worlds into single worlds, thus showing that the number of worlds is the same as the number of sets of worlds. This is impossible by Cantors Theorem.

My initial thoughts on hearing this paradox, was that it was no surprise. I had always assumed that the collection of possible worlds was too numerous to be gathered into one set. For example, David Lewis's book, 'Parts of Classes', shows us how to construct, uniquely, for each set, a possible world with the appropriate mereological structure. Thus there must be at least as many possible worlds as sets. Since it would appear that we are committed to proper class many worlds from the outset, it is no surprise that the assumption that we can form a set of worlds leads to contradiction.

So what about a positive account? It is all very well saying there are proper class many worlds, but it is not clear that the notion of proper class is even consistent despite its wide use in set theory[2]. One account of classes, suggested by Uzquiano, is that they are multiplicities. That is, a class is not a thing like a set is, but an irreducibly plural collection of things. While we can talk about classes in the singular, underneath this convenient vocabulary we are talking about pluralities. So perhaps the way to explain this problem is by talking about a plurality of worlds. After all, that *is* the name of the damn book, and with Lewis's own constant use of plural quantification, the notion was hardly unfamiliar to him.

However, the paradox runs deeper.

Suppose, using the pairing methods in the appendix to 'Parts of Classes', and treating possible worlds as if they were atoms, we construct a class of ordered pairs. Each pairs' second member consists of the mereological fusion of a class of worlds and first member the world that embeds that class, given by, say, Armstrong and Forrest's method of recombination. Let W be the class of worlds, let $\uparrow$W be the class of fusions of classes of worlds and let f be the class of pairs from W and $\uparrow$W just defined. Intuitively f is to be thought of as a partial function between classes, so we shall write $f(x) = y$ to mean $\langle x, y\rangle \in f$. According to Armstrong and Forrest's construction f should be well defined so this notation is appropriate. It should also be surjective, so for each y in $\uparrow$W there is an x in W such that $f(x) = y$. Now consider the class:

- $C = \{ x \mid \neg x \leq f(x)\}$

where $\leq$ is the parthood relation. Let $\sigma C \in \uparrow W$ be the fusion of C, then by surjectivity of f there is a w in W such that $f(w) = \sigma C$. Now it quickly follows that

- $w \leq f(w) \Leftrightarrow w \leq \sigma C \Leftrightarrow w \in C \Leftrightarrow \neg w \leq f(w)$

[1] Some of these arguments apply to actualist theories as well. This paper considers Lewis' theory only.
[2] Although, much, if not all class talk can be reduced to talk about formulas with one free variable.

Which is a contradiction.

I do not think it is right to say that this shows the paradox is rooted in the notion of a plurality of worlds. It is not clear to me whether the paradox will disappear if we remove certain mereological principles such as unrestricted composition (of which my argument makes essential use). In fact, this might be the saving grace for other theories of possible worlds. For philosophers who reject such mereological principles outright or those who merely claim they only apply to concrete objects and hence do not apply to the abstracta playing the role of worlds in their theory, such a conclusion would be welcome. For Lewis, who is a strong proponent of unrestricted comprehension, and who treats worlds as concrete objects, this is a serious problem.

It is perhaps for these reasons that Lewis instead restricted his notion of recombination. Lewis makes the existential claim that there is *some* limit on the size of worlds, although he does not claim to know which. "Among the mathematical structures that might be offered as isomorphs of possible space-time's, some would be admitted, and others would be rejected as oversized."[3] Armstrong and Forrest's argument is simply taken to be a proof to the effect that some limit exists, rather than the inconsistency of his theory. When put in this way his response seems less ad hoc. However, it is ad hoc none the less.

I wish to suggest a cut off point which may serve for a restricted principle of recombination which does not share this problem. Suppose we treat the totality of worlds as a proper class interpreted plurally. To apply Lewis's principle of recombination as in the argument above, we assume we can take a class of worlds, proper or not, and form a single world containing them all. Whether or not you think there should be a cut off point in the world of set theoretic mathematical structures for candidate space-times, surely it is not too far fetched to suppose that some arbitrary collections of things (proper classes interpreted plurally) are too numerous to be gathered into one world[4]. I thus suggest as a restriction on the principle of recombination that only sets of things can be gathered into one world. This way, although we end up with proper class many worlds, we do not end up in paradox.

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[3] *On the Plurality of Worlds*, pp103.
[4] For further discussion of this point see D. Nolan "*Classes Worlds and Hypergunk*" The Monist 2004, 87.3 : 3-21 and in the same issue A. P. Hazen "*Hypergunk*" for a response.