Representing Counterparts

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  5. Is reductionist about de re possibility.

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Introduction

Dividing the Issues

The Logic of Essential Properties

Modals in English

Actuality Operators

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- This distinction is also important for other metaphysical views.
- The correct semantics for modals in English should not turn on metaphysical issues.

Chisholms paradox

Imagine you have a bicycle, B_0. B_0 could be made from slightly different parts - it could have had different brake pads for example, call this bicycle B_1 - intuitively this is not a different bicycle, but a the same bicycle made differently: $B_0 = B_1$. However if we run a sequence of bicycles, $B_0, B_1, B_2, \ldots$ across worlds, each differing from the last by only a nut or a bolt, we should eventually arrive at a bicycle, B_N differing in all its parts from B_0. This last bicycle surely cannot be the same bicycle as B_0, for it shares nothing in common with it, save being a bicycle, which certainly isn't enough to secure identity. Thus, for any step in the sequence, $B_n = B_{n+1}$ yet $B_0 \neq B_N$ contradicting transitivity of identity.

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- (Not exhaustive.)

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- But you can if you're a counterpart theorist.

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- Note that similar things can be said about Gibbards paradox of statue and the lump of clay.

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- Salmon draws the conclusion that the true logic of possibility and necessity is not S5.
- I think the correct conclusion is that the logic of *essential properties* is not S5.

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- So let $S := \{\langle x, w\rangle \mid x \text{ exists in world } w\}$
- And let $\mathcal{C}$ be the counterpart relation.
- $\langle S, \mathcal{C}\rangle$ is a Kripke frame for a propositional modal logic.

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  - $\Box\phi$: this is the property of having ϕ essentially.

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- It seems the upshot of Chisholms paradox is just the failure of this principle.
- Each consecutive pair in the sequence $\langle B_0, w_0\rangle, \langle B_1, w_1\rangle, \langle B_2, w_2\rangle \ldots \langle B_N, w_N\rangle$ is accessible to the other, yet $\langle B_0, w_0\rangle$ is not accessible to $\langle B_N, w_N\rangle$, contradicting transitivity.

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- Gibbards paradox seems to involve a violation of this principle, since it involves the counterpart relation being non-euclidean.

Counterpart relation

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- This gives us S5.

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- What are the prospects for doing things this way?

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- Lewis mentions some slightly strange ones:
- "I could have been Frank Sinatra [while everything else remained as it actually is.]"

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f. "For all we know Cicero might not be the great writer Tully after all."

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- The counterpart relation that is used to validate the essentialist intuition in the first conjunct is not the same as that found in the second conjunct.
- Each different kind of modality comes with its own counterpart relation, just as each comes with its own accessibility relation.

Epistemic counterpart relations

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- (We can define doxastic counterpart relations similarly.)

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  1. It's syntax matches the logical forms of English sentences more closely.
  2. It's a language containing primitive modal operators. (No quantifying over worlds, or possibilia.)

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- The basic aim: to provide a counterpart theoretic semantics for QML@.

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- For example, one prominent example was:
  - $\Diamond\exists x(@Fx \leftrightarrow @\neg Fx)$
- This, and similar inconsistent formulae, translate to true CPT sentences when you have individuals with no (or several) counterparts in the actual world, and you interpret @ in a variety of plausible ways.

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- In admitting multiple counterparts in a single world, Lewis says, we are cutting possibilities finer than worlds

Lewis

"To illustrate, consider these two possibilities for me. I might have been one of a pair of twins. I might have been the first-born one, or the second-born one. These two possibilities invelve no qualitative difference in the way the world is. Imagine them specified more fully: there is the possibility of being the first-born twin in a world of such-and-such maximally specific qualitative character. And there is the possibility of being the second-born twin in exactly such a world. The haecceitist says: two possibilities, two worlds. They *seem* just alike, but they must differ somehow. They represent, *de re*, concerning someone. Hence they must differ with respect to the determinants of the representation *de re*; and these must be non-qualitative, since there are no qualitative differences to be had. [...]

Lewis

[...] I say: two possibilities, sure enough. And they do indeed differ in representation *de re*: according to one I am the first-born twin, according to the other I am the second-born. But they are not two worlds. They are two possibilities within a single world. The world in question contains twin counterparts of me, under a counterpart relation determined by intrinsic and extrinsic qualitative similarities (especially, match of origins.) Each twin is a possible way for a person to be, and in fact is a possible way for me to be. I might have been one, or I might have been the other. There are two distinct possibilities for me. But they involve only one such possibility for the world: it might have been the world inhabited by two such twins."
– David Lewis, 'On the Plurality of Worlds', p231

Possibilities

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- So possibilities are just world function pairs on this set up.

Example

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  - $\langle w, \sigma \rangle$ where σ is a function taking me to the first born twin. That is, in this possibility I'm represented by the first born twin.
  - $\langle w, \sigma' \rangle$ where σ' is a function taking me to the second born twin, but otherwise takes the same values as σ. In this possibility I'm represented by the second born twin.

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- With this in place we can interpret $@\phi$ w.r.t. some sequence of individuals and a world.
- We interpret it as just truth in the actual world, where each individual is represented by its canonical counterpart in the actual world.

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  - If S satisfies the above conditions, then $\mathit{Form}(\mathcal{L}) \subseteq S$.

Counterpart Structures

Definition

A **counterpart structure** is a quintuple $\langle \mathcal{W}, \mathcal{D}, Ind(\cdot), \mathcal{C}, w^* \rangle$ satisfying the following conditions:

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4. $w^* \in \mathcal{W}$ and $Ind(w^*) \neq \emptyset$

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Counterpart Models

Definition

A **counterpart model** is a sextuple $\langle \mathcal{W}, \mathcal{D}, Ind(\cdot), \mathcal{C}, w^*, [\![\cdot]\!]^{\cdot} \rangle$ where $\langle \mathcal{W}, \mathcal{D}, Ind(\cdot), \mathcal{C}, w^* \rangle$ is a counterpart structure, and

- $[\![\cdot]\!]^{\cdot} : [Pred^n \to [\mathcal{W} \to \mathcal{P}(\mathcal{D}^n)]]$

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- $[\![\cdot]\!]^{\cdot} : [Pred^n \to [\mathcal{W} \to \mathcal{P}(\mathcal{D}^n)]]$
- For $w \in \mathcal{W}$, $P_i^n \in Pred^n$, $[\![P_i^n]\!]^w \subseteq Ind(w)^n$

Intuitively, we may think of $[\![P_i^n]\!] : \mathcal{W} \to \mathcal{P}(\mathcal{D}^n)$ as the intension of P_i^n.

Counterpart Functions

Definition

Given a counterpart structure, $\langle \mathcal{W}, \mathcal{D}, Ind(\cdot), \mathcal{C}, w^* \rangle$, and a world, $w \in \mathcal{W}$, we say that σ is a **counterpart function** for w iff σ is a (possibly) partial function from $\mathcal{D}$ into $Ind(w)$ which is a subset $\mathcal{C}$. We write it as follows:

- $CF(\sigma, w) \Leftrightarrow \sigma : \mathcal{D} \rightharpoonup Ind(w), \sigma \subseteq C$

A **maximal counterpart function** is a counterpart function for w that is maximal in $\mathcal{C}$

- $MCF(\sigma, w) \Leftrightarrow \forall \tau (CF(\tau, w) \wedge \sigma \subseteq \tau \rightarrow \sigma = \tau)$

Possibilities

Definition

Given a counterpart structure, $\mathfrak{A} = \langle \mathcal{W}, \mathcal{D}, Ind(\cdot), \mathcal{C}, w^* \rangle$, we may define the set of **possibilities**, S, with respect to the structure:

- $S(\mathfrak{A}) := \{\langle w, \sigma \rangle \mid w \in W, MCF(\sigma, w)\}$

Each world is paired with a maximal counterpart function for that world, which provides the extra information concerning de re representation.

The maximality condition on counterpart functions ensures that, if you have some counterpart or other in a world, you are represented in the world in one way or another by one of these counterparts.

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Definition

An admissible actuality, for a counterpart structure $\langle \mathcal{W}, \mathcal{D}, Ind(\cdot), \mathcal{C}, w^* \rangle$, and a world, $w \in \mathcal{W}$ is a possibility of the form $\langle w^*, \tau \rangle$ such that $\forall x \in Ind(w^*)(\tau(x) = x)$

Truth clauses

- $\langle w, \sigma, v \rangle \models P_i^n x_1, \ldots, x_n \Leftrightarrow \langle \sigma(v(x_1)), \ldots, \sigma(v(x_n)) \rangle \in [\![P_i^n]\!]^w$

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- $\langle w, \sigma, v\rangle \models \Diamond\phi \Leftrightarrow \langle w', \sigma', v\rangle \models \phi$ for some $\langle w', \sigma'\rangle \in S$
- $\langle w, \sigma, v\rangle \models \exists x_i \phi \Leftrightarrow \langle w, \sigma, v'\rangle \models \phi$ for some $v'[i]v$ such that $v'(x) \in Ind(w)$

Truth and Validity

- **true** *in the model* $\mathcal{M} = \langle \mathfrak{A}, [\![\cdot]\!] \rangle$ *with respect to an admissible actuality* $s = \langle w^*, \sigma^* \rangle$ *and a valuation* v *iff* $\langle \mathcal{M}, s \rangle, \langle w^*, \sigma^*, v \rangle \models \phi$.

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- **s-valid** *in* $\mathfrak{A}$ *with respect to* $s = \langle w^*, \sigma^* \rangle$ *iff* $\langle \mathcal{M}', s \rangle, \langle w^*, \sigma^*, u \rangle \models \phi$ *for every model,* $\mathcal{M}' = \langle \mathfrak{A}, [\![\cdot]\!]' \rangle$, *based on* $\mathfrak{A}$ *and every valuation* u.

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- **valid** *in* $\mathfrak{A}$ *iff it is s-valid in* $\mathfrak{A}$ *with respect to every admissible actuality.*

Consequence

- **an s-consequence** *of* Γ *in* $\mathfrak{A}$ *with respect to an admissible actuality,* s*, iff for any model,* $\mathcal{M}' = \langle \mathfrak{A}, [\![\cdot]\!]' \rangle$*, based on* $\mathfrak{A}$ *and any valuation* v*, if* $\langle \mathcal{M}', s \rangle, \langle w^*, \sigma^*, v \rangle \models \psi, \forall \psi \in \Gamma$*, then* $\langle \mathcal{M}', s \rangle, \langle w^*, \sigma^*, v \rangle \models \phi$*.*

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Kripke validity

Theorem

Let $\mathcal{M} = \langle \mathcal{W}, \mathcal{D}, Ind(\cdot), \mathcal{C}, w^*, [\![\cdot]\!] \rangle = \langle \mathfrak{A}, [\![\cdot]\!] \rangle$ be a counterpart model, and $s = \langle w^*, \sigma^* \rangle$ be an admissible actuality for $\mathfrak{A}$. Then there is a Kripke model $\mathcal{M}' = \langle \mathcal{W}', \mathcal{D}', Ind'(\cdot), w^{*\prime}, [\![\cdot]\!]' \rangle$ such that the following are equivalent for any formula ϕ:

- $\langle \mathcal{M}, s \rangle, \langle w, \sigma, v \rangle \models \phi$ *for every* $\langle w, \sigma \rangle \in S(\mathfrak{A})$ *and every valuation* v
- $\mathcal{M}', \langle w, v \rangle \models \phi$ *for every* $w \in \mathcal{W}'$ *and valuations* v.

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- $\mathcal{M}, \langle w, v \rangle \models \phi$ *for every* $w \in \mathcal{W}$ *and valuations* v.

Corollary

A formula, ϕ, in the language of QML@ is valid in every counterpart structure if and only if it is valid in every Kripke structure.

Further Work

- The framework for dealing with actuality operators I've just outlined can be used by the first order counterpart theorists too.

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- The equivalence of the counterpart theoretic semantics with the Kripke semantics shows that this translation schema will never translate inconsistent sentences to consistent sentences, or vice versa.

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