Frege's proof of β

Andrew Bacon

$$f\left(\begin{array}{l}\top\\ \llcorner\, q\\ \quad p\end{array}\right)$$

$$\begin{array}{l}\vdash\!\!\sqcap \Phi(a) = a \frown \dot\alpha \Phi(\alpha)\\ \quad \llcorner\, \backslash\dot\alpha \left(\begin{array}{l}\top \overset{\mathfrak{g}}{\smile} \top \quad \mathfrak{g}(a) = \alpha\\ \qquad \llcorner\, \dot\epsilon \Phi(\epsilon) = \dot\epsilon \mathfrak{g}(\epsilon\end{array}\right) = a \frown \dot\alpha \Phi(\alpha)\end{array}$$

$$\begin{array}{l}\vdash\!\!\sqcap\!\!\sqcap a \frown Y = (a \frown Y) \frown a\\ \quad \llcorner\, a = \dot\epsilon(_\top\, \epsilon)\\ \quad \llcorner\, \dot\alpha((\dot\epsilon(\epsilon \frown (\epsilon \frown \alpha))) \frown (\dot\epsilon(\epsilon \frown (\epsilon \frown \alpha)))) = Y\end{array}$$

$$\top \overset{\mathfrak{a}}{\smile} \top \ \mathfrak{a} = \mathfrak{a} \frown \dot\epsilon(_\top\, \epsilon)$$

$$\backslash\dot\alpha \left(\begin{array}{l}\top \overset{\mathfrak{g}}{\smile} \top \quad \mathfrak{g}(a) = \alpha\\ \qquad \llcorner\, \dot\epsilon \Phi(\epsilon) = \dot\epsilon \mathfrak{g}(\epsilon\end{array}\right) = a \frown \dot\alpha \Phi(\alpha)$$