The mapping cylinder is formed by taking a map $f : X \to Y$ and forming

$$M_f = (X \times I \bigcup Y)/\sim$$

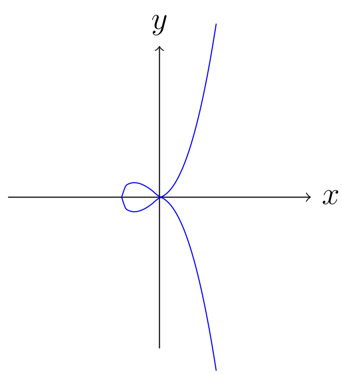
where $(x, 1) \sim y$ if and only if $f(x) = y$. A deformation retraction of M_f to Y is given as follows:

$$F(x, s) = \begin{cases} (x, (1-s)t + s \cdot 1) & z = (x, t) \in X \times I \\ y & z = y \in Y \end{cases}$$

Here the $\cdot 1$ is written for emphasis: Note that a linear homotopy of I to $\{1\}$ is given by $G(t, s) = (1-s)t + s \cdot 1$. This way $G(t, 0) = t$ and $G(t, 1) = 1$. This is exactly what is happening in the second coordinate on X with respect to F (thank you Weitao for pointing this out).

A nice example which I failed to write down today in class today, due to time constraints is as follows:

Example 0.1. Consider the relation $y^2 = x^2(x+1)$. The points satisfying this is $\mathbb{R}^2$ look as follows:



Let $t \in \mathbb{R}$ be a parameter of the curve, so that $f : \mathbb{R} \to \mathbb{R}^2$ has an image looking like the graph above.

Consider the mapping cone M_f. Note that at the 'top' ($t = 0$ in $X \times I$) you will have a copy of $\mathbb{R}$, and at the 'bottom' ($t = 1$) you will have this graph in the space.

Try to envision the resulting space M_f.