HOMEWORK 8: FINITENESS & NAGATA-SMIRNOFF
DUE: NOVEMBER 16

1) An open covering is **point-finite** if any point is in at most finitely many elements. Find an example of a point finite ppen covering which is not locally finite.

2) Show that if X is second-countable, the $\mathcal{A}$ is a countably locally finite set if and only if $\mathcal{A}$ is countable.

3) In the uniform topology, let $\mathfrak{B}_n$ is the collection of all subsets $\prod_{i \in \mathbb{N}} X_i \subseteq \mathbb{R}^{\mathbb{N}}$ with $X_1 = \ldots = X_n = \mathbb{R}$, and $X_m = \{0\}$ or $X_m = \{1\}$ for $m > n$. Show $\mathfrak{B} = \cup_{i=1}^{\infty} \mathfrak{B}_i$ is countably locally finite but not countable or locally finite.

4) Show that a T1 space has a locally finite basis if and only if it is discrete.

5) Find a non-discrete space which has a countably locally finite basis, but is not second-countable.

6) Find an example to show that a paracompact space can have an open cover $X = \bigcup_{\alpha \in \Lambda} U_\alpha$ which doesn't have a locally finite *subcover* $X = \bigcup_{\alpha \subseteq \Lambda'} U_\alpha$ with $\Lambda' \subseteq \Lambda$.

7) Find an example of a locally compact Hausdorff space which is not normal.[1] You may use any asserted results from Class 16.

[1] Therefore locally compact and Hausdorff do not imply paracompact.