HINT FOR HOMEWORK 4: QUESTION 1 HINT

The question is as follow: Use Corollary .20, stated below for your convenience, to show that if (X, A) has the homotopy extension property, then $X \times I$ **deformation** retracts onto $X \times \{0\} \cup A \times I$.

Corollary 0.1. *If (X, A) has the homotopy extension property and the inclusion $\iota : A \to X$ is a homotopy equivalence, then A is a deformation retract of X.*

Deduce that the following theorem (.18) holds for any pair (X, A) with the homotopy extension property:

Theorem 0.2. *If (X, A) is a CW pair and we have attaching maps $f, g : A \to Y$ that are homotopic, then $X \coprod_f Y \simeq X \coprod_g Y$ rel X.*

Here is a suggested list of steps for solving this problem.

1) Reduce the problem (by tracking definitions (this is the point of this problem at some level)) by Corollary .20 to the following claim: If (X, A) be pair with the HEP, then $i : X \times \{0\} \cup A \times I \to X \times I$ is a homotopy equivalence.

2) There are many ways to show this, but one way is to demonstrate that both are spaces are deformation retracts of the same space, as in the previous homework.

3) For the second part, extending Theorem .18, follow the proof of Theorem .18 from the sentence about a deformation retraction.