So it turns out question 5 is a bit difficult to formulate a solution to. Here is the question:

Show that two deformation retractions r_t^0 and r_t^1 of a space X onto a subspace A can be joined by a **continuous** family of deformation retractions $r_t^s : X \times I \times I \to X$ from X to A. That is to say that r_t^s is a deformation retraction for each s.

So what is meant here is as follows: Can you find a map r_t^s such that for any fixed $s \in I$, $r_s^t : X \times I \to X$ is a deformation retraction, with r_t^0 and r_t^1 the desired deformation retractions. With the help of Molly during office hours, I found a map (there are many possibilities) that does this. It is written as follows:

$$r(x,t,s) = r_t^s(x) = \begin{cases} r_t^0 \circ r_{2st}^1(x) & s \leq \frac{1}{2} \\ r_{2t(1-s)}^0 \circ r_t^1(x) & s \geq \frac{1}{2} \end{cases}$$

Show that this map has the desired properties:

- $r(x,t,0) = r_t^0(x)$
- $r(x,t,1) = r_t^1(x)$
- At $s = \frac{1}{2}$, both definitions of the function can be used.
- For fixed $s \in I$, $r(x,t,s)$ is a deformation retraction.

I will append this to the homework file.

Note that this also shows $r_t^0 \simeq r_t^1$ for any 2 deformation retractions!