HOMEWORK 3: CAUCHY'S THEORY
DUE: WEDNESDAY, OCTOBER 2ND

1) Show that
$$\int_0^\infty \sin(x^2)dx = \int_0^\infty \cos(x^2)dx = \frac{\sqrt{2\pi}}{4}$$
(**hint:** The pie shaped wedge from $\theta = 0$ to $\theta = \frac{\pi}{4}$ may be a useful path to consider. You may assume $\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$.)

2) Evaluate the integral $\int_0^\infty \frac{\sin(x)}{x}$. It may be useful to show it is equal to $\frac{1}{2i}\int_{-\infty}^{\infty} \frac{e^{ix}-1}{x}dx$ and use the upper semi-circle with 0 removed.

3) If $f(z)$ is continuously complex differentiable in Ω, and $T \subseteq \Omega$ is a triangle, then use Green's Theorem to show that
$$\int_T f(z)dz = 0$$
This proves Goursat's Theorem with stronger assumptions.

4) Let f be a function which is complex differentiable in Ω except possibly at one point w. Let T be a triangle with w in its interior. Show that if f is bounded in a neighborhood of w, then we get the same conclusion from Goursat:
$$\int_T f(z)dz = 0$$

5) Following the ideas of example 9.1 from our notes, show that for $\xi \in \mathbb{R}$, we have
$$e^{-\pi\xi^2} = \int_{-\infty}^{\infty} e^{-\pi x^2} e^{2\pi i x\xi} dx$$

6) If $f : \mathbb{D} \to \mathbb{C}$ is a holomorphic function, show that $d = \sup_{z,w\in\mathbb{D}} |f(z) - f(w)|$ satisfies
$$2|f'(0)| \le d.$$
Moreover, equality holds if and only if f is linear.