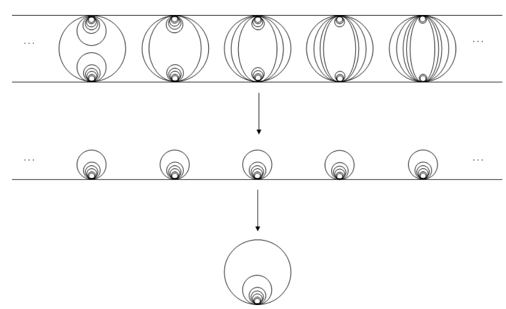
HOMEWORK 7: DECK TRANSFORMATIONS & HOMOLOGY
DUE: MONDAY, NOVEMBER 13

1) We have constructed a 'universal cover' of any path connected, locally path connected, and semilocally simply connected space X; $p : \tilde{X} \to X$. It is called universal because it is a covering space of every other path connected covering space of X.

   We will now construct another type of universal cover. A path connected cover $q : \hat{X} \to X$ is called *Abelian* if it is normal and has an Abelian group of deck transformations $G(\hat{X})$. Show that there is a universal such q. Call it $q : X^{ab} \to X$. Namely, that there exists such a q with the property that if $s : \bar{X} \to X$ is an Abelian cover, then q factors as $q = s \circ q'$:
$$q : X^{ab} \xrightarrow{q'} \bar{X} \xrightarrow{s} X$$
   Describe this cover for $S^1 \vee S^1$.

2) It can be shown that for every covering space $q : Z \to Y$ and finite-sheeted covering space $p : Y \to X$, that $q \circ p : Z \to X$ is a covering space. This allows us to conclude that the universal cover of K, the Klein Bottle, is $\mathbb{R}^2$ since it has a 2-sheeted cover by $\mathbb{T}^2 = S^1 \times S^1$.



   Consider the example to the right of a covering space of the shrinking wedge of circles (the bottom most cover). This cover is infinite sheeted and thus the above statement doesn't apply. Show that the composition of the 2 covering spaces is not a covering space.

3) Run through the computation of $H_i^{\Delta}(K)$, where K is the Klein Bottle. Do the same for the triangular parachute T which is obtained by taking Δ^2 (the triangle) and identifying all 3 vertices.

4) The analogue of real projective space for the complex numbers, $\mathbb{CP}^n$, is a n-dimensional 'complex manifold', or $2n$-dimensional 'real manifold'. It is constructed inductively by adjoining simplices in every other dimension: $\Delta_0, \Delta_2, \ldots, \Delta_{2n}$. Compute $H_i^{\Delta}(\mathbb{CP}^n)$ for every $i \geq 0$.