HOMEWORK 6: VAN KAMPEN AND COVERING SPACES
DUE: MONDAY, OCTOBER 30

1) Show that if S is a finite collection of points in $\mathbb{R}^n$, and $n \geq 3$, that $\mathbb{R}^n \setminus S$ is a simply connected set.

2) Find $\pi_1(\mathbb{R}^2 \setminus S)$ for a finite collection of points S.

3) Show that $\pi_1(\mathbb{R}^2 \setminus \mathbb{Q}^2)$ is an uncountable set.

4) Let X be the quotient of S^2 obtained by identifying 2 antipodal points. Compute $\pi_1(X)$. (**Hint:** It may be beneficial to view it as a cell complex).

5) For a covering space $p : \tilde{X} \to X$, and $A \subseteq X$, show that the induced map $p : p^{-1}(A) = \tilde{A} \to A$ is a covering space.

6) Show that if a path connected, locally path-connected space X has $\pi_1(X)$ finite, then every map $X \to S^1$ is null-homotopic. (**Hint:** Lifting properties)

7) Demonstrate that $\mathbb{T}^2$ is a 2-sheeted cover of the Klein Bottle, but is not a cover of $\mathbb{RP}^2$.