Consider $\mathbb{R}^2$ with the Euclidean Topology vs the Product Topology from 2 copies $\mathbb{R}$. Call the spaces $\mathbb{R}^2_e$ and $\mathbb{R}^2_p$ respectively for brevity.

I claim these topologies are equivalent (i.e. a set is open in one if and only if it is open in the other). First note that bases for each topology can be given as follows:

$$\mathscr{B}_e = \{B_\epsilon(x) \ : \ x \in \mathbb{R}^2, \ \epsilon > 0\}$$

$$\mathscr{B}_p = \{(a,b) \times (c,d) \ : \ a < b, \ c < d\}$$

Let $U \in \mathscr{B}_e$ be open in the Euclidean Topology. Then U can be expressed as a union of basis elements

$$U = \bigcup_{x \in U} B_{\epsilon_x}(x)$$

for various $\epsilon_x > 0$ depending on the choice of x. But for every ϵ_x, one can inscribe an open square

$$S_x := (x_1 - \sqrt{2}\epsilon_x, x_1 - \sqrt{2}\epsilon_x) \times (x_2 - \sqrt{2}\epsilon_x, x_2 - \sqrt{2}\epsilon_x)$$

You can check this with some basic trigonometry :) Therefore, one can express

$$U = \bigcup_x S_x$$

This shows that U is open in the product topology.

On the flip side, given a rectangle $(a,b) \times (c,d) \subseteq \mathbb{R}^2$. Suppose WLOG $b - a \leq d - c$. Then one can express inscribe a ball inside of this square as

$$B_\epsilon\left((\frac{b+a}{2}, \frac{d+c}{2}) \right) \subseteq (a,b) \times (c,d)$$

Performing the same trick as above, writing any open set as a union of squares around every point, shows that U is also open in the Euclidean topology.