COMPLEX ANALYSIS: MIDTERM

1) (10 points) Define the integral of a continuous function $f : \Omega \to \mathbb{C}$ along a piecewise-smooth path $\gamma : [a, b] \to \Omega$.

2) (10 points) Define what it means for a function to be analytic at $z_0 \in \mathbb{C}$.

3) (15 points) State the Cauchy-Riemann equations. When do they ensure holomorphicity?

4) (15 points) State Cauchy's Integral Theorem.

5) (25 points) Let $z = x + iy$ and $f(z) = u(z) + iv(z)$. Suppose $u(z) = 4xy^3 - 4x^3y$. Find a function $v(z)$ that makes $f(z)$ an entire function.

6) (25 points) Define for each $\alpha \in \mathbb{R}$ the quantity

$$I(\alpha) = \int_{-\infty}^{\infty} e^{-(x+i\alpha)^2} dx$$

Show that in fact $I(\alpha)$ is independent of α, and thus equal to $I(0) = \sqrt{\pi}$.

7) (20 points) Prove Liouville's theorem assuming Cauchy's inequality.

8) (30 points) Compute the following integral:

$$\int_{-\infty}^{\infty} \frac{\cos(\pi x)}{(x^2+4)^2} dx$$