HOMEWORK 5: THE RESIDUE THEOREM
DUE: FRIDAY, OCTOBER 18TH

(1) Using Euler's formula, show that the complex zeroes of $\sin(\pi z)$ are simple and exactly at the integers. What is their residue if you consider $\frac{1}{\sin(\pi z)}$?

(2) Evaluate the integral
$$\int_{-\infty}^{\infty} \frac{dx}{1+x^4}.$$

(3) Show that
$$\int_{-\infty}^{\infty} \frac{\cos(x)dx}{x^2+a^2} = \pi\frac{e^{-a}}{a}$$
for any $a > 0$.

(4) Show that
$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^{n+1}} = \pi\frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)}$$

(5) Show that
$$\int_0^{2\pi} \frac{d\theta}{a+b\cos(\theta)} = \frac{2\pi}{\sqrt{a^2-b^2}}$$
for $a > |b| > 0$ real numbers. (**hint:** Use the Euler Relations on cos and then express this as an integral on the unit circle.)