MATH 368: Commutative Algebra in Characteristic $p > 0$, Spring 2018

Class Meetings: MWF at 9:00-9:50 in **Stetson Court 103**
Instructor: Andrew Bydlon
Email: atb4@williams.edu (usually, I won't respond $\geq$9 pm)
Office Hours: Tuesday 10:30-Noon, Thursday 2:30-4 in **Bascom 303**.

Text: Though the material for this course will be taken from many sources, the primary resource for the course will be **Abstract Algebra, by Dummit and Foote**. Of particular interest are Chapters 7, 9, 10, 13, and 15.

In addition, I will refer to the proof of Kunz Theorem on regularity of rings in positive characteristic and some course notes to be made available.

Prerequisite Information: One should have completed a basic course on *Abstract Algebra*, with particular focus on groups and rings.

Course Description: We will study the structure of rings in characteristic $p > 0$. This includes studying the singularities of such rings and the implications thereof. We will also describe how some things about characteristic 0 rings can be proved by characteristic $p > 0$ reductions.

Here is a broad list of topics which we will cover based on the backgrounds of the class.

1. Ring Theory: Definitions, examples, homomorphisms, quotient rings, ideals, and the field/ring of fractions.
2. Modules: Definition, examples, as a generalization of ideals, as a generalization of vector spaces, homomorphisms and quotients, generation, direct sums & free modules.
3. Fields: Field extensions, algebraic extensions, separability, and classification of finite fields.
4. Spectrum of a ring: Noetherian property, dimension, containments in Spec, Hilbert Nullstellensatz, and localization.
5. Regular Rings: Examples and non-examples, other characterizations, implications, localizations at primes.
6. The Frobenius Homomorphism
7. Kunz's Theorem: A positive characteristic test for regularity.
8. Fedder's Criterion: Classifying all of the homomorphisms from $R^{\frac{1}{p^e}}$ back to R.
9. F-split and F-regularity: Weakenings of the notion of regular, to classify 'mild' irregularities of rings.

In addition, we may have some class time devoted to Macaulay2 code.

Glow: I will use GLOW as a method of communication, as well as a gradebook.

Grading: The following are the grade components and the percentage each contributes to a student's final grade:

- **Homework Assignments (50%)**- Approximately once every 2 weeks, a homework assignment will be due.
- **In Class Presentations (20%)** We will devote one class out of every 2 weeks to presenting some solutions from the homework sets to the class.

- **Final Presentation (30%)**- On the final class periods of the semester, the Students will present a result from a research paper regarding information from the course. Students can determine which subject matter they find most appealing, and I will assign a paper/theorem to them to prepare.

Note: The instructor retains the right to modify this grading scheme during the course of the semester; students will, of course, be well notified of any adjustments.