Gus asked the following question in class corresponding to the following result:

Proposition 0.1. *Let A_1, A_2, A_3 be 3 closed sets that cover S^2. Then at least one of them contains a pair of antipodal points $x, -x$.*

The question was can you say the same thing with 4 open sets? I gave the answer no but could not quickly come up with the counterexample. Here it is.

It is easier to demonstrate with a picture. Therefore I encourage you to check out the next page.

The sets are constructed as follows: Fix a small positive ϵ, say $\frac{1}{10}$.

$$A_1 = \overline{B_{2\epsilon}(\sqrt{1-\epsilon^2}, 0, \epsilon)} \cup \{(x, y, z) \mid z \geq \epsilon\} \setminus B_{\epsilon/10}(\sqrt{1-\epsilon^2}, 0, -\epsilon))$$

$$A_2 = \{(x, y, z) \mid (x, y, -z) \in A_1\}$$

Then let A_3 and A_4 be the closure of the remaining 2 connected components of S^2.

These sets do the trick. In particular they are designed so that they contain no antipodal points.