Homotopy is a general word meant to determine a type of continuous deformation. Explicity, we say the following things:

- We say $f, g : X \to Y$ are homotopic, written $f \simeq g$, if there exists $F : X \times I \to Y$ a continuous map (where $I \subset \mathbb{R}$ topologically) with the property that $F(x, 0) = f(x)$ and $F(x, 1) = g(x)$.

  We can then think of varying $t \in I$ to form a family of maps $F(x, t) = f_t(x)$ with $f_t : X \to Y$. So, we are continuously deforming f to g as maps.

- A second notion we have spoke about is a relative homotopy, which is a general way to phrase a whole lot of other definitions quickly.

  We say $f \simeq g$ rel A if $f \simeq g$, and the F that does it has the property that $F(a, t) = a$ for every $a \in A$ and $t \in I$. So F fixes A for all of time.

  Thus a swift way to describe a deformation retraction is $Id_x \simeq r$ rel A.

- Two spaces X, Y are homotopy equivalent if $\exists f : X \to Y$ and $g : Y \to X$ such that $f \circ g \simeq Id_Y$ and $g \circ d \simeq Id_X$.

  This says essentially that X and Y are topologically equivalent. We will see tomorrow that $\pi_1(X) = \pi_1(Y)$ in this setting.

- Lastly, we have our notion of equivalence that defines the fundamental group: Two paths $\gamma_0, \gamma_1 : I \to X$ connecting x_0 to x_1 are homotopic (as paths) if $\gamma_0 \simeq \gamma_1$ rel $0, 1$. This again is the swift way to say that we can continuously deform γ_0 to γ_1 while keeping the endpoints x_0 and x_1 fixed. Equationally, $\exists \Gamma : I \times I \to X$ such that:

  - $\Gamma(t, 0) = \gamma_0(t)$
  - $\Gamma(t, 1) = \gamma_1(t)$
  - $\Gamma(0, s) = x_0$
  - $\Gamma(1, s) = x_1$