COURSE NOTES MATH 368: COMMUTATIVE ALGEBRA IN *char* $p > 0$
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$$\operatorname{Hom}_{S/I}(F^e_* S/I, S/I) \cong \frac{F^e_*(I^{[p^e]} : I)}{F^e_* I^{[p^e]}} \operatorname{Hom}_S(F^e_* S, S)$$

$$\begin{array}{ccc} 0 & & 0 \\ \downarrow & & \downarrow \\ I & \xrightarrow{\psi|_I} & I \\ \downarrow & & \downarrow \\ S & \xrightarrow{\psi} & S \\ \downarrow & & \downarrow \\ S/I & \xrightarrow{\bar{\psi}} & S/I \\ \downarrow & & \downarrow \\ 0 & & 0 \end{array}$$

$$\Phi^e_S : F^e_* S \cong S^{[F^e_* K : K] p^{ed}} \to S$$

$$\Phi^e_S(F^e_* c x_1^{\alpha_1} \cdots x_x^{\alpha_n}) = \begin{cases} 1 & c = d^{p^e} \in K,\ \alpha_i = p^e - 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{F^e_*(I^{[p^e]} : I)}{F^e_* I^{[p^e]}} \cdot \operatorname{Hom}_S(F^e_* S, S) \to \operatorname{Hom}_{S/I}(F^e_* S/I, S/I)$$

$$f \cdot \psi \mapsto \psi(F^e_* f \cdot -)$$

$$R^{perf} = \varprojlim_F R^{p^e} \to \cdots \xrightarrow{F} R^{p^2} \xrightarrow{F} R^p \xrightarrow{F} R \xrightarrow{F} R^{\frac{1}{p}} \xrightarrow{F} \cdots \to \varinjlim_F R^{\frac{1}{p^e}} = R_{perf}$$

$$K[x_1, \ldots, x_n]^{perf} = \varprojlim_F K^{perf} \to \cdots \xrightarrow{F} K^p[x_1^p, \ldots, x_n^p] \xrightarrow{F} K[x_1, \ldots, x_n]$$
$$\xrightarrow{F} K[x_1^{\frac{1}{p}}, \ldots, x_n^{\frac{1}{p}}] \xrightarrow{F} \cdots \to K_{perf}[x_1^{\frac{1}{p^\infty}}, \ldots, x_n^{\frac{1}{p^\infty}}] = K[x_1^{\frac{1}{p}}, \ldots, x_n^{\frac{1}{p}}]_{perf}$$

$$\left(F_p[x^{\frac{1}{p^\infty}}]/(x)\right)^{perf} = F_p[\widehat{x^{\frac{1}{p^\infty}}]/}(x) \to \cdots \xrightarrow{F} F_p[x^{\frac{1}{p^\infty}}]/(x) \xrightarrow{F} F_p[x^{\frac{1}{p^\infty}}]/(x)$$