

POSSIBLE PRESENTATION TOPICS

- 1) The test ideal: This ideal of R is a measure of how far the ring is from being F -regular. For this talk, I would like it to be defined, show its existence, and relate it back to (local) F -regularity.
- 2) F -signature of a ring: A more numeric invariant measuring whether a ring is F -regular or not. For a given ring, the F -signature lies between 0 and 1. If it is 1, the ring is regular. If it is positive, the ring is F -regular.
- 3) F -pure threshold: Extending our notion of F -splitting to pairs (R, \mathfrak{a}^t) , this measure how F -split R and \mathfrak{a} are simultaneously.
- 4) Symbolic Powers: A very important alternative to the classical or Frobenius powers of an ideal are the symbolic powers. A survey of what they are, how they are computed, as well as what containments can be derived (with the aforementioned powers) would suffice. Reference Briancon-Skoda Theorem.
- 5) Ext_R^i : Giving the same homological treatment that $-\otimes_R M$ got to $\text{Hom}_R(-, M)$. The interest would be in proving some basic properties of Ext and how it behaves with respect to a flat homomorphism (e.g. flat base change).
- 6) Local Cohomology: Measuring the complexity of a ring at its points has been the focus of the course. Here is another measure. We should see a definition (or 2), and an example of how it can be used in practice. In particular, to show that F -regular rings are Cohen-Macaulay.
- 7) Tight Closure: One of the classical theories of positive characteristic commutative algebra, tight closure has been indispensable. Seeing the definition of tight closure, some computations, and showing that F -regular rings are rings which have every ideal tightly closed would suffice.
- 8) The original proof of Kunz Theorem: Did not use perfection of rings or fancy base change arguments. A presentation of this would be nice to see for historical purposes.
- 9) Prove the version of the Principal Ideal theorem for $I = \langle x_1, \dots, x_c \rangle$, yielding a height/codimension bounded above by c .
- 10) Other proposals are acceptable...