Suppose that X is covered by closed sets X_α which are locally finite: for any point $x \in X$, there is a U open containing x such that $U \cap X_\alpha \neq \emptyset$ for at most finitely many α.

Let U_x denote the open set with this property for each $x \in X$. Then X is certainly covered by the U_x, and additionally $U_x \subset X_{\alpha_1} \cup X_{\alpha_2} \cup \cdots \cup X_{\alpha_{n_x}}$ (these α_i are the ones for which $X_{\alpha_i} \cap U_x \neq \emptyset$). By part (1), the function $f|_{X_\alpha}$ is continuous for each α, so since there are only finitely many, is continuous on $X_{\alpha_1} \cup X_{\alpha_2} \cup \cdots \cup X_{\alpha_{n_x}}$. Restricting f to U_x also gives you a continuous function with even better properties in the subspace topology than $X_{\alpha_1} \cup X_{\alpha_2} \cup \cdots \cup X_{\alpha_{n_x}}$. In particular, what does it mean to be open in U_x?

This should give you enough information to complete the exercise.