HOMEWORK 3: CONTRACTIBILITY
DUE: MONDAY SEPTEMBER 29

1) Show the following are equivalent:
   i. X is contractible
   ii. Every map $f : X \to Y$ is null-homotopic
   iii. Every map $g : Y \to X$ is null-homotopic.

   Note: null-homotopic means homotopic to a constant map.

2) Show that S^∞ (given by continually attaching 2 disks $\mathbb{D}^n$ to S^{n-1} for all $n \geq 1$) is contractible.

   Note: No finite dimensional sphere is contractible!
   Hint: A fabulous property when working in $\mathbb{R}^\infty$, or S^∞, is that the map translating each coordinate 1 to the right is an injective map (not surjective) preserving metrics.

3) If X is connected and a union of S^2, with any 2 intersecting in at most 1 point, then show that X is homotopy equivalent to a wedge sum of S^1's and S^2's.

   Hint: Try doing it for 2 and 3 S^2's intersecting in various ways. Deduce the general method from this. Example 0.9 in Hatcher may give some assistance.

4) Show that a CW complex X is contractible if it is a union of 2 contractible cell complexes A, B, whose intersection is contractible.

5) Find a 2-dimensional cell complex which contains both $S^1 \times I$ and the mobius band M as deformation retracts.

   Hint: Note that both spaces deformation retract to a copy of $S^1 \times \{\frac{1}{2}\}$. Additionally you may freely use the following:

   Theorem 0.1. *Given any continuous map $f : S^1 \to S^1$, the mapping cylinder M_f is a CW complex.*