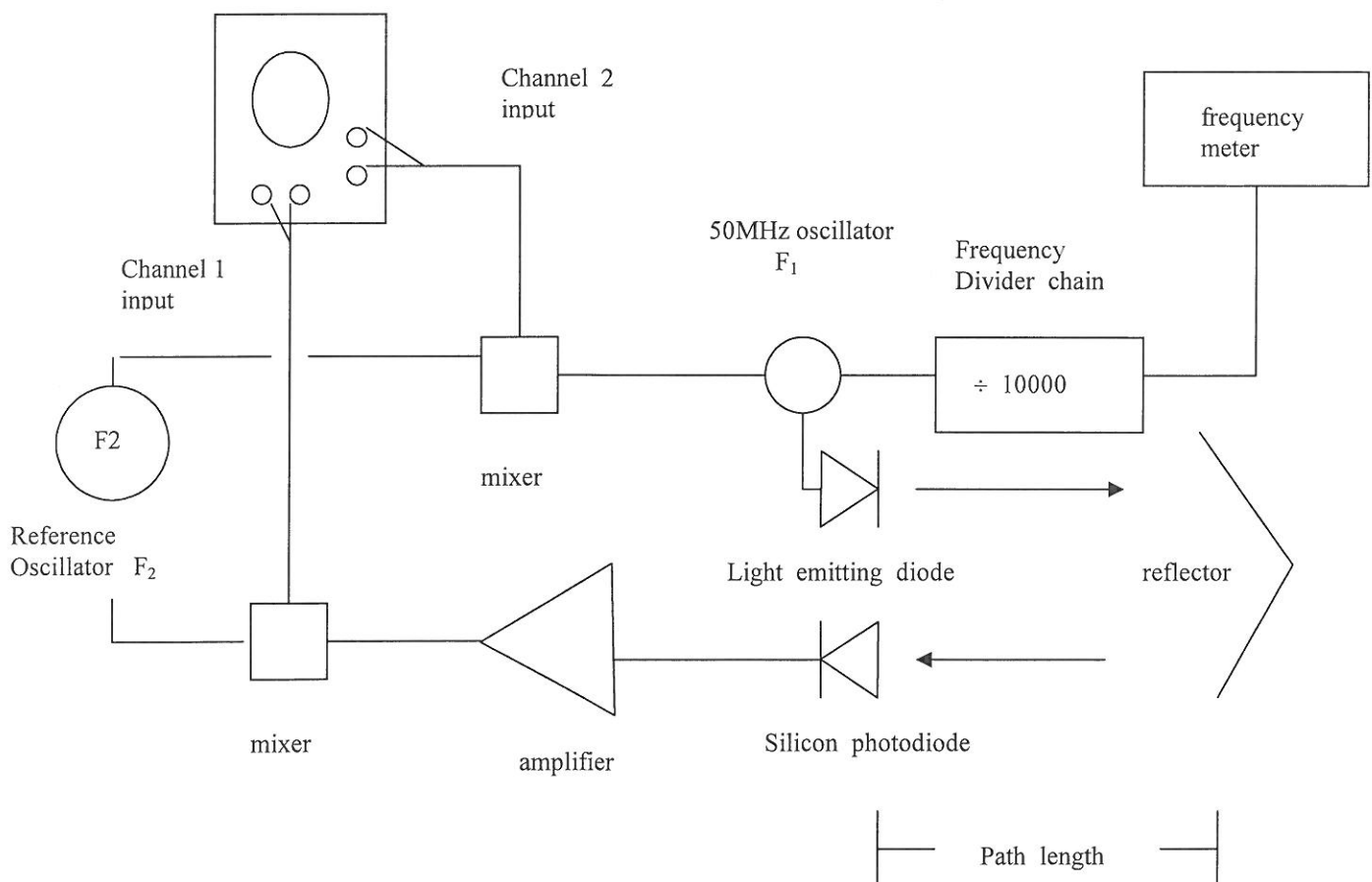


## TO MEASURE THE SPEED OF LIGHT IN AIR AND OTHER MEDIA

### PRINCIPLE OF OPERATION

The principle of operation is as follows. A light emitting diode is used as a source of light modulated at a frequency of approximately 50MHz. The light travels down an optical bench and is reflected back to a photodiode. The phases of the 50MHz signals transmitted and received are displayed on an oscilloscope as an ellipse. The reflecting mirror is moved close to a position where the two signals are in phase, and the position against the scale noted. The mirror is then moved to the position where the signals are in antiphase, and the position again noted. The increase in path length represents a  $180^\circ$  phase shift at the modulation frequency. If the modulating frequency is measured, using for example a 237 frequency or a 245 scaler timer and a stopwatch, then the velocity can be calculated. The following schematic diagram gives the basic principle used in this method of measuring the speed of light.



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The reference oscillator has a frequency,  $f_2$ , where  $f_2 = f_1 - 50\text{kHz}$  and  $f_1 = 50\text{MHz}$ . By beating the transmitted and received signals ( frequency  $f_1$  ) separately against the reference oscillator and using a suitable mixer circuit it can be shown that the phase relationships in the transmitted and received signals are transferred to the beat frequency signals at  $50\text{kHz}$ . Students should prove this result by assuming that the output voltage from each mixer is  $a_1V + a_2V^2$  where  $V$  = sum of the input voltages and  $a_1$  and  $a_2$  are constant coefficients. (You can also assume that each mixer circuit has a filter on the output which only transmits signals with frequency  $< 100\text{ kHz}$  ).

Using this result it is possible to determine the phase relationship changes between the transmitted and received signals by observation of the phase changes in the lower frequency beat signals. The analysis required to calculate the speed of light is outlined in Appendix A given on a separate sheet,

N.B.  $d_1 - d_2 = 2 \times$  ( separation distance of the two mirror positions corresponding  
To a phase change of  $\pi$  )

NOTE – PLEASE THINK CAREFULLY ABOUT HOW THIS EXPERIMENT WORKS – WHY ARE YOU USING A MODULATED LIGHT SOURCE?

(make sure you understand the explanation above – if in doubt ask a demonstrator)

**Also think very carefully about your uncertainties.**

### PROCEDURE

- (1) Check the alignment of the light beam as described in separate sheet
- (2) Position the mirrors at an arbitrary position and display both channels on the oscilloscope. Move mirrors along track and observe change in phase between the two sinusoidal signals
- (3) Switch the oscilloscope to x-y operation. In x-y mode you are observing Lissajou figures – see Appendix B.
- (4) Position mirrors near one end of track and adjust phase control on electronics box to produce a straight line trace [ either at  $\sim 45^\circ$  or at  $135^\circ$  )  
The actual slope of the line is not important as it depends on the relative amplitude of the x and y signals.  
Check that a straight line is obtained by increasing both ch1 and ch2 sensitivities.
- (5) Note position of mirror carriage on scale.
- (6) Move mirror carriage until a straight line trace is again obtained and note the position

- (7) The distance moved by the carriage then gives a direct measure of  $\left( \frac{d_1 - d_2}{2} \right)$

*You may want to check the change in phase between the emitted and received signal by switching the scope back to normal operation. In particular you should check that you are actually measuring a received signal and not just observing noise.*

- (8) Repeat the measurement at least five times and obtain an average value of  $\left( \frac{d_1 - d_2}{2} \right)$

- (9) Note the value of  $f_1$ .  
This frequency can be read from the Racal timer unit.  
The displayed frequency is the L.E.D. modulation frequency ( $\sim 50\text{MHz}$ ) divided by 10,000.

- (10) Calculate the speed of light in air.

- (11) Measure the value of the speed of light in Perspex and water using the procedure described in the separate sheet.

- (12) Calculate the refractive index for Perspex and water at a wavelength of approximately  $6570\text{\AA}$

## APPENDIX A

### *Principle of Operation*

A high frequency voltage  $V^E$  of the form

$$V^E = V_0^E \sin 2\pi f t \quad (1)$$

(where  $f \sim 50$  MHz) Is used to amplitude modulate the light output from a forward biased light emitting diode. The amplitude  $A^E$  of the light emitted by the diode is given by the following equation

$$A^E = A_0^E \cos(2\pi f t - \beta^E) \quad (2)$$

where  $\beta^E$  represents the phase lag in the diode.

The amplitude  $A^D$  of the light at a detector diode a distance  $d$  from the emitting diode can be represented by the expression

$$A^D = A_0^D \cos\left\{2\pi f \left(t - \frac{d}{U}\right) - \beta^E\right\} \quad (3)$$

where  $U$  is the group velocity of light in the medium through which it is travelling.

The voltage  $V^D$  developed at the output of the detector diode,  $\propto$  intensity  $I \propto$  amplitude<sup>2</sup>. Thus

$$V^D = \cos^2\left\{2\pi f \left(t - \frac{d}{U}\right) - \beta^E - \beta^D\right\} = V_0^D [1 + \cos 2X]/2 \quad (4)$$

where  $\beta^D$  represents the phase lag in the detector diode and its associated circuitry, and

$$X = 2\pi f \left(t - \frac{d}{U}\right) - \beta^E - \beta^D = 2\pi f t - \phi - \beta^E - \beta^D$$

From equations (1) and (4) it can be seen that the phase difference between the voltage applied to the emitting diode and the voltage obtained from the detecting diode is given by

$$\phi = 2\pi f d / U + \beta^E + \beta^D \quad (5)$$

Thus by measuring the phase differences  $\phi_1$  and  $\phi_2$  between the emitter and detector voltages, for two different emitter-detector separations  $d_1$  and  $d_2$ , it is possible to derive the group velocity of light in the medium

$$U = 2\pi f (d_1 - d_2) / (\phi_1 - \phi_2) \quad (6)$$

### *Alignment of the Light Beam*

This is best done for the first time in a darkened room. The apparatus is stable enough that it requires only a minimum of realignment after being moved from place to place. Mark on a piece of white card a horizontal line the height of the LED above the bench. Place one of the lens units on the track in front of the LED, and adjust the separation to give a parallel beam of light approximately 40mm in diameter on the card at the further end of the bench.

*Frequency Measurement.* The modulation frequency is in the region of 50MHz, and as such is too high to be measured by readily available equipment. A digital divider by  $10^4$  is included in the electronics box, and the output frequency at the sockets  $f \times 10^{-4}$  can be measured on a 237 frequency meter, or a 245 scaler used with a stopwatch.

### *Measurement of the velocity of light in solids and liquids*

To measure the velocity of light in solids or liquids relative to the velocity in air, it is only necessary to place a rod of the solid material, or a cell containing the liquid, in the optical path of the apparatus and note the resulting phase change produced on the CRT display.

Since better accuracy is achieved when using Lissajous figures if the phase angle is in the region of  $0^\circ$  or  $180^\circ$ , a better experimental procedure is as follows. With the rod (cell) in position in the light path and the lens and rod (cell) positions adjusted for maximum signal from the detector, the phase control is adjusted until a straight line Lissajous figure is obtained. The mirror position is noted. The rod (cell) is now removed from the light path and the mirrors displaced away from the emitter and detector boxes until the same straight line Lissajous figure is again obtained.

If the length of the rod (cell) is  $L$  and extra distance moved by the mirror to regain the straight line display, is  $t$ , then you should show that:

$$\frac{\text{group velocity of light in medium}}{\text{group velocity of light in air}} = \frac{L}{L + 2t}$$

### **Results**

These measurements should be repeated and mean values of the relative group velocities of light in various substances obtained. Hence obtain estimates of the group refractive index of these materials at the wavelength of the red light (- 657 nm) emitted by the diode. Where the group refractive index  $n_g$  is defined as

$$n_g L = \frac{\text{group velocity of light in vacuum}}{\text{group velocity of light in medium}}$$

It may be assumed that within the limited accuracy of this experiment, the group velocity of air  $\approx$  group velocity of light in vacuum.

## APPENDIX B

### Phase Shift Measurements

The horizontal control section may have an XY mode that lets you display an input signal rather than the time base on the horizontal axis. This mode of operation opens up a whole new area of phase-shift measurement techniques.

The phase of a wave is the amount of time that passes from the beginning of a cycle to the beginning of the next cycle, measured in degrees. Phase shift describes the difference in timing between two otherwise identical periodic signals.

One method for measuring phase shift is to use XY mode. This involves connecting one signal to the vertical system as usual and then another signal to the horizontal system. (This method only works if both signals are sinusoidal.) This set up is called an XY measurement because both the X and Y axis are

tracing voltages. The waveform resulting from this arrangement is called a Lissajous pattern (named for French physicist Jules Antoine Lissajous and pronounced LEE-sa-zhoo). From the shape of the Lissajous pattern, you can tell the phase difference between the two signals. You can also tell their frequency ratio. Figure 43 shows Lissajous patterns for various frequency ratios and phase shifts.

The XY measurement mode originated with analog oscilloscopes. Due to their relatively low sample density, DSOs may have difficulty creating real-time XY displays. Some DSOs create an XY image by accumulating data points over time, then displaying the composite. Digital Phosphor Oscilloscopes, on the other hand, are able to acquire and display a genuine XY mode image in real-time, using a continuous stream of digitized data. DPOs can also display an XYZ image with intensified areas.

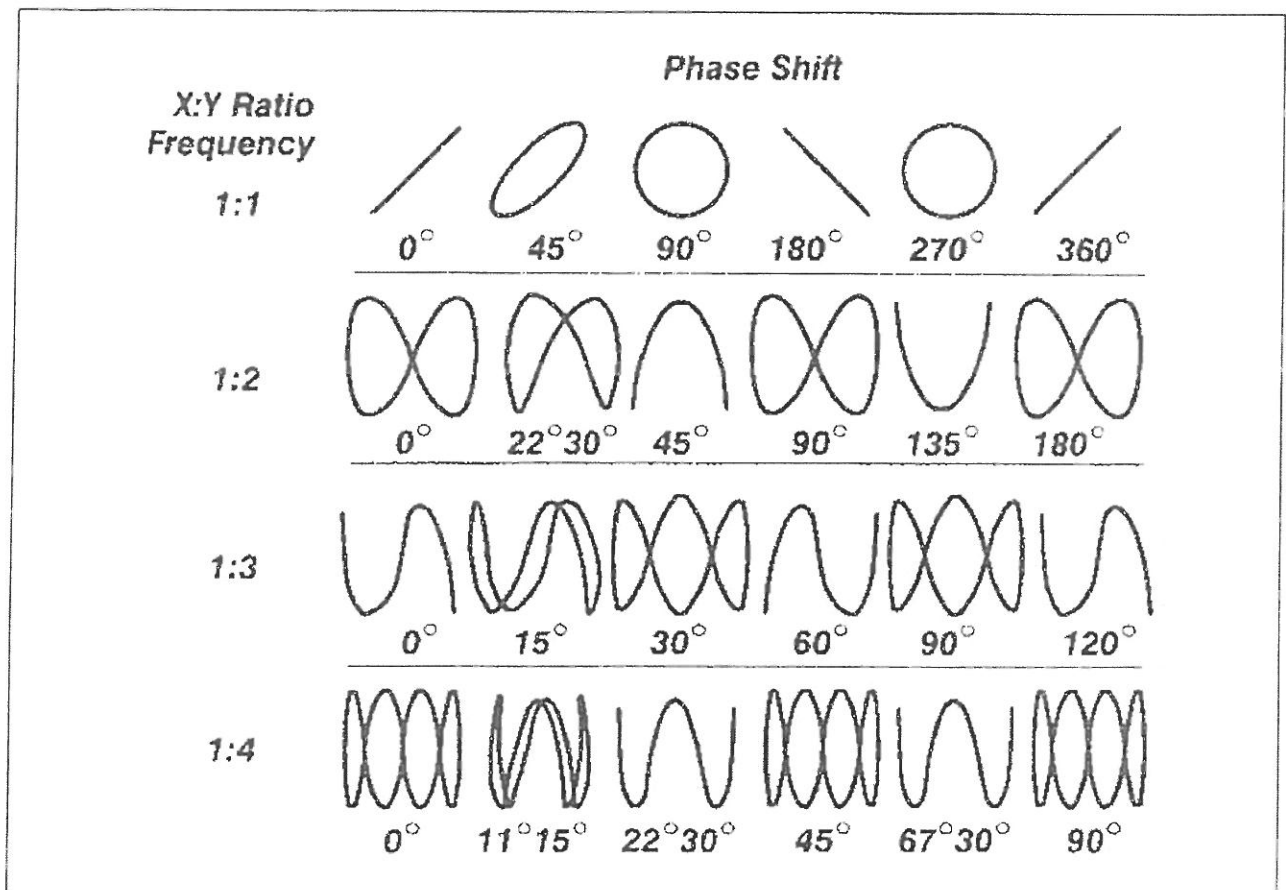


Figure 43. Lissajous patterns.