Problem Set #6

Back to Week 6

5/5 points earned (100%)

Quiz passed!

1 / 1 points

1.
Suppose we use a hash function h to hash n distinct keys into an array T of length m. Assuming simple uniform hashing --- that is, with each key mapped independently and uniformly to a random bucket --- what is the expected number of keys that get mapped to the first bucket? More precisely, what is the expected cardinality of the set $\{k : h(k) = 1\}$.

- m/n
- $n/(2m)$
- $1/m$
- $m/(2n)$
- $1/n$
- n/m

Correct Response
Use linearity of expectation, with one indicator variable for each key. The probability that one key hashes to the first bucket is $1/m$, and by linearity of expectation the total expected number of keys that hash to the first bucket is just n/m.

1 / 1 points

2.

You are given a binary tree (via a pointer to its root) with n nodes, which may or may not be a binary search tree. How much time is necessary and sufficient to check whether or not the tree satisfies the search tree property?

- [x] $\Theta(n)$

Correct Response

For the lower bound, if there is a violation of the search tree property, you might need to examine all of the nodes to find it (in the worst case).

- [ ] $\Theta(n \log n)$
- [ ] $\Theta(height)$
- [ ] $\Theta(\log n)$

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1 / 1
points

3.

You are given a binary tree (via a pointer to its root) with n nodes. As in lecture, let size(x) denote the number of nodes in the subtree rooted at the node x. How much time is necessary and sufficient to compute size(x) for every node x of the tree?

- [x] $\Theta(n)$

Correct Response

For the lower bound, note that a linear number of quantities need to be computed. For the upper bound, recursively compute the sizes of the left and right subtrees, and use the formula size(x) = 1 + size(y) + size(z) from lecture.

- [ ] $\Theta(n^2)$
- [ ] $\Theta(n \log n)$
- [ ] $\Theta(height)$

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1 / 1
points

4.
Which of the following is *not* a property that you expect a well-designed hash function to have?

- The hash function should be easy to store (constant space or close to it).
- The hash function should be easy to compute (constant time or close to it).
- The hash function should "spread out" most (i.e., "non-pathological") data sets (across the buckets/slots of the hash table).
- The hash function should "spread out" every data set (across the buckets/slots of the hash table).

Correct Response
As discussed in lecture, unfortunately, there is no such hash function.

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1 / 1
points

5.
Suppose we relax the third invariant of red-black trees to the property that there are no *three* reds in a row. That is, if a node and its parent are both red, then both of its children must be black. Call these *relaxed* red-black trees. Which of the following statements is *not* true?

- Every binary search tree can be turned into a relaxed red-black tree (via some coloring of the nodes as black or red).

Correct Response
A chain with four nodes is a counterexample.

- The height of every relaxed red-black tree with n nodes is $O(\log n)$.
- Every red-black tree is also a relaxed red-black tree.
- There is a relaxed red-black tree that is not also a red-black tree.