Problem Set #5

Back to Week 5

5/5 points earned (100%)

Quiz passed!

1 / 1 points

1.
Consider a directed graph with distinct and nonnegative edge lengths and a source vertex s. Fix a destination vertex t, and assume that the graph contains at least one s-t path. Which of the following statements are true? [Check all that apply.]

- [x] The shortest (i.e., minimum-length) s-t path might have as many as $n-1$ edges, where n is the number of vertices.

Correct Response

- [x] There is a shortest s-t path with no repeated vertices (i.e., a "simple" or "loopless" such path).

Correct Response

- [ ] The shortest s-t path must include the minimum-length edge of G.

Correct Response

- [ ] The shortest s-t path must exclude the maximum-length edge of G.

Correct Response

1 / 1 points

2.
Consider a directed graph $G = (V, E)$ and a source vertex s with the following properties: edges that leave the source vertex s have arbitrary (possibly negative) lengths; all other edge lengths are nonnegative; and there are no edges from any other vertex to the source s. Does Dijkstra's shortest-path algorithm correctly compute shortest-path distances (from s) in this graph?

- Maybe, maybe not (depends on the graph)
- Always

Correct Response
One approach is to see that the proof of correctness from the videos still works. A slicker solution is to notice that adding a positive constant M to all edges incident to s increases the length of every s-v path by exactly M, and thus preserves the shortest path.

- Never
- Only if we add the assumption that G contains no directed cycles with negative total weight.

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1 / 1 points

3.
Suppose you implement the functionality of a priority queue using a *sorted* array (e.g., from biggest to smallest). What is the worst-case running time of Insert and Extract-Min, respectively? (Assume that you have a large enough array to accommodate the Insertions that you face.)

- $\Theta(n)$ and $\Theta(n)$
- $\Theta(n)$ and $\Theta(1)$

Correct Response

- $\Theta(\log n)$ and $\Theta(1)$
- $\Theta(1)$ and $\Theta(n)$

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1 / 1 points

4.
Suppose you implement the functionality of a priority queue using an *unsorted* array. What is the worst-case running time of Insert and Extract-Min, respectively? (Assume that you have a large enough array to accommodate the Insertions that you face.)

- $\Theta(n)$ and $\Theta(n)$
- $\Theta(1)$ and $\Theta(n)$

Correct Response

- $\Theta(n)$ and $\Theta(1)$
- $\Theta(1)$ and $\Theta(\log n)$

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1 / 1 points

5.
You are given a heap with n elements that supports Insert and Extract-Min. Which of the following tasks can you achieve in $O(\log n)$ time?

- Find the largest element stored in the heap.
- None of these.
- Find the fifth-smallest element stored in the heap.

Correct Response

- Find the median of the elements stored in the heap.
