Time-varying Correlation of South African Property Stocks and the ALSI

Andrew Hyde[a]

[a] *Stellenbosch University, South Africa*

Abstract

This analysis of the current yield spreads in the local bond market places the current high spreads into historical context.

1. Introduction

2. Data and Methodology

2.1. Data

The dataset used in this study is the ALSI returns data from 2005 to 2022, which included both traditional equities and REITs. Investigation into the data reveals that there are four unique sectors in the data, namely; Financials, Industrials, Resources and Property. There are missing values present in the dataset.

Given that the objective of this study is to explore the time varying correlation between the ALSI equities and REITs missing data poses a problem. To address this concern, the dataset is separated into two groups, one group for ALSI equities and the other for REITs. Next, the problem of missing values is dealt with by imputing values for both groups separately.

Separating the data into two groups, before imputing values, is done to preserve the properties and distribution elements of the ALSI returns and REIT returns. With that in mind, missing values for the ALSI equities are imputed using returns distribution draw from the collective ALSI group less REITs. Missing values for each REIT equity are imputed using the returns distribution of that equity. This is done to enable analysis of correlation between REITS, as well as with the rest of the ALSI equities. The code used in this section follows a practical covered in Financial Econometrics 871 (Katzke, 2022b).

Email address: 23365935@sun.ac.za (Andrew Hyde)

Figure 2.1 displays the available data for each REIT. Based on availability of data the following REITs are selected: Capital & Counties Properties PLC, Emira Property Fund Limited, Hyprop Investments Limited, Growthpoint Properties Limited, Resilient Reit Limited, Redefine Properties Limited and SA Corporate Real Estate Limited.

Given that this analysis is conducted to explore the time-varying correlation relationships of equity pairs, returns data is log transformed and mean scaled so that the data used conforms to a normal distribution centered around its mean. This is especially important as this allows one to make inferences based on statistical assumptions.

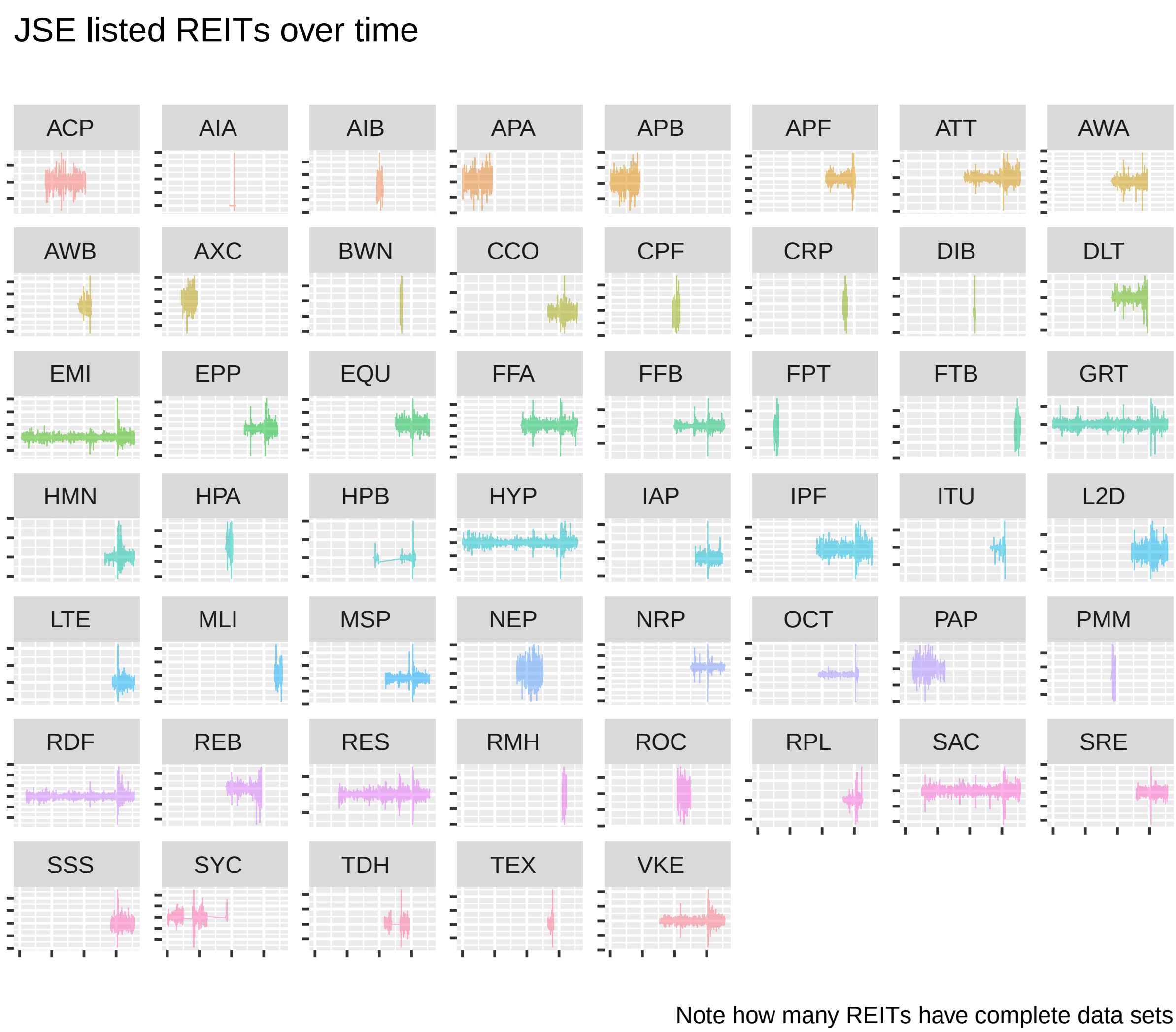


Figure 2.1: REITs Dataset

2.2. Methodology

A Dynamic Conditional Correlation Generalized AutoRegressive Conditional Heteroskedasticity (DCC GARCH) model is used to perform this analysis. This model allows for the estimation of time-varying conditional correlation structures that are noise reduced, taking the GARCH(1,1) model further by allowing for multivariate volatility modeling (Engle, 2002; Katzke, 2022d).

The DCC model makes use of non-Linear combinations of univariate GARCH models to directly model the correlations as a dynamic time-varying process i.e. estimating the conditional correlation matrix directly (Engle, 2002)

The DCC GARCH model follows a two-step approach.

2.2.1. Step One

Estimates are obtained by fitting a univariate GARCH(1,1) model to the residuals of the vector autoregression (VAR) using the combined imputed data. The VAR allows for the examination of relationships between series over time and the residuals it produces α_t can be broken down into the structural volatility component z_t and the noise component μ_t, provided α_t are white noise errors/residuals Katzke, 2022c).

The DCC GARCH model is defined as follows:

$$H_t = D_t.R_t.D_t,$$

where H_t is positive definite variance-covariance matrix which is splits into identical diagonal matrices D_t and R_t, the time-varying correlation estimates. The estimation of R_T requires it to be inverted at each estimated period, therefore a proxy similar to a GARCH(1,1), denoted by $Q_{ij,t}$, is to be used (Engle, 2002).

$$\begin{aligned} Q_{ij,t} &= \bar{Q} + a\left(z_{t-1}z'_{t-1} - \bar{Q}\right) + b\left(Q_{ij,t-1} - \bar{Q}\right) \\ &= (1-a-b)\bar{Q} + az_{t-1}z'_{t-1} + b.Q_{ij,t-1} \end{aligned}$$

Where $Q_{ij,t}$ the unconditional (sample) variance estimate between series i and j, and $\bar{Q}$ is the unconditional matrix of standardized residuals from each univariate pair estimate.

The following equation is used to estimate R_t:

$$R_t = \text{diag}\left(Q_t\right)^{-1/2} Q_t.\, \text{diag}\left(Q_t\right)^{-1/2}.$$

Which has bivariate elements:

$$R_t = \rho_{ij,t} = \frac{q_{i,j,t}}{\sqrt{q_{ii,t} \cdot q_{jj,t}}}$$

The resulting DCC model is then formulated as:

$$\begin{aligned}
\varepsilon_t &\sim N\left(0, D_t \cdot R_t \cdot D_t\right) \\
D_t^2 &\sim \text{ Univariate GARCH } (1,1) \text{ processes } \forall(\mathrm{i}, \mathrm{j}), \mathrm{i} \neq \mathrm{j} \\
z_t &= D_t^{-1} \cdot \varepsilon_t \\
Q_t &= \bar{Q}(1-a-b) + a\left(z_t' z_t\right) + b\left(Q_{t-1}\right) \\
R_t &= \text{Diag}\left(Q_t^{-1}\right) \cdot Q_t.\, \text{Diag}\left(Q_t{}^{-1}\right)
\end{aligned}$$

2.2.2. Step Two

Using the standardized residuals from step one, the dynamic, time-varying conditional correlations estimates can be obtained using a log-likelihood approach.

The volatility approximation series that is estimated H_t, can then be standardized and used in fitting a DCC model for η_t (Katzke, 2022c).

$$\eta_{i,t} = \frac{\hat{\alpha_{i,t}}}{\hat{\sigma_{i,t}}}$$

The DDC GARCH model is run twice. The first iteration models the time-varying conditional correlation structure between the ALSI and REITs, as well as the time-varying correlation structure between the seven individual REITs included in the study.The second applies a stratification method to the data before the DCC GARCH model is re-run. The stratification methods enables one to examine how these time-varying conditional correlation structure change in periods of low and high volatility.

The stratification technique is used to isolate return dates when South African markets experienced high levels of volatility. To do this, the South African Rand is used as a benchmark index and is filtered for its own top and bottom decile quantile (10%) by monthly standard deviation of Rand volatility. These dates are then used to filter the ALSI_returns into dates with low and high volatility. The code used in this section follows a practical covered in Financial Econometrics 871 (Katzke, 2022a).

Following the stratification, the time-varying conditional correlation structure is mapped between Capital & Counties Properties PLC (Capco/CCO) and both the ALSI and other SA listed REITs. This section further explores the relationship between Capital & Counties Properties PLC, a UK based REIT and Redefine Properties Limited, an SA based REIT, whom are both listed on the JSE.

3. Results and Discussion

3.1. Time-varying Correlation: REITs and ALSI

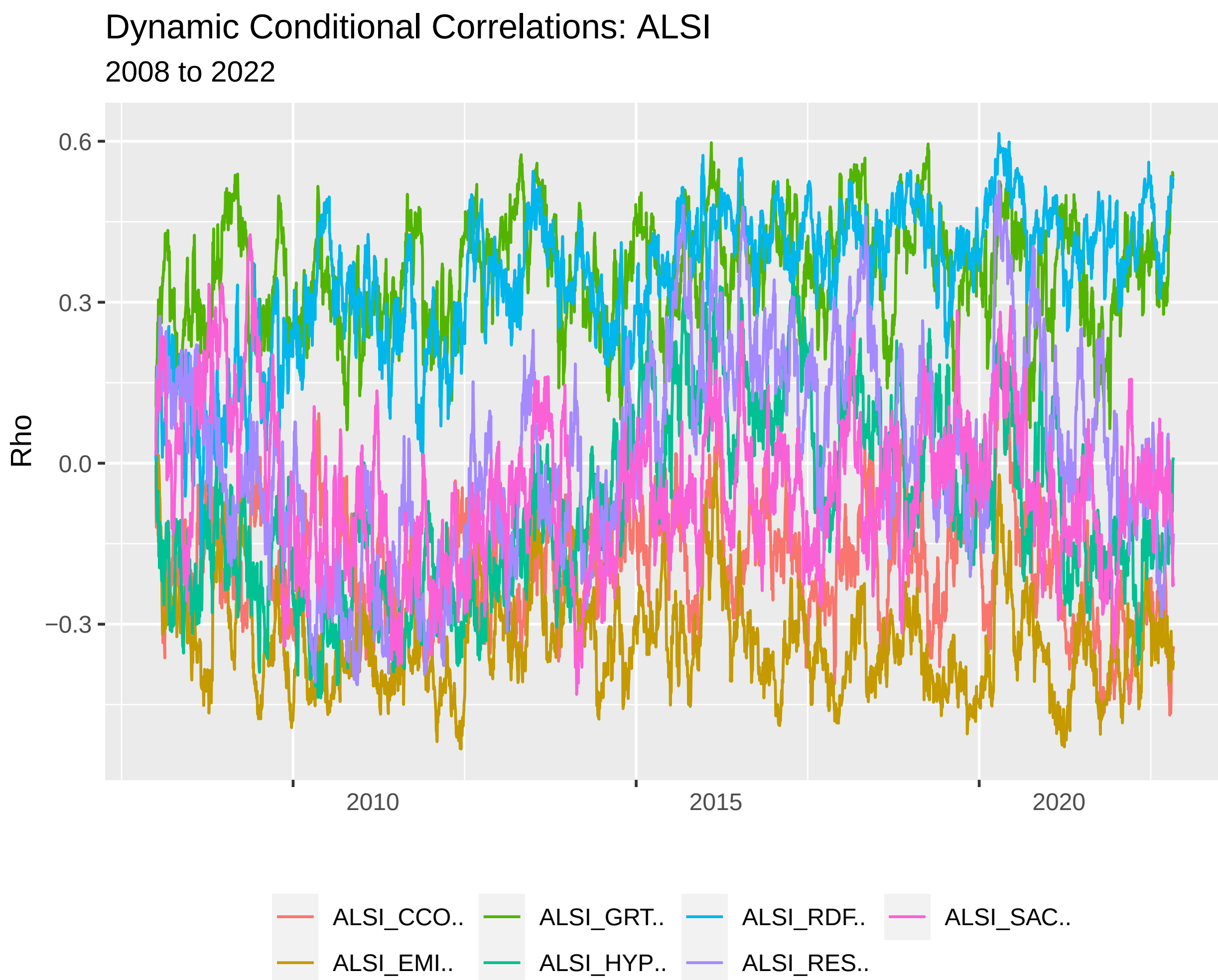
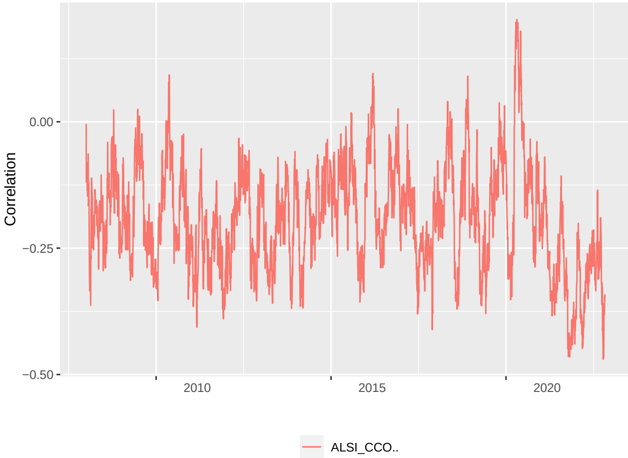
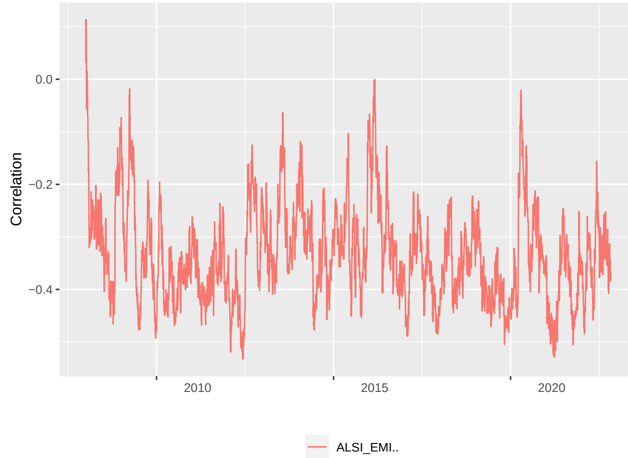


Figure 3.1: Dynamic Conditional Correlations Graph

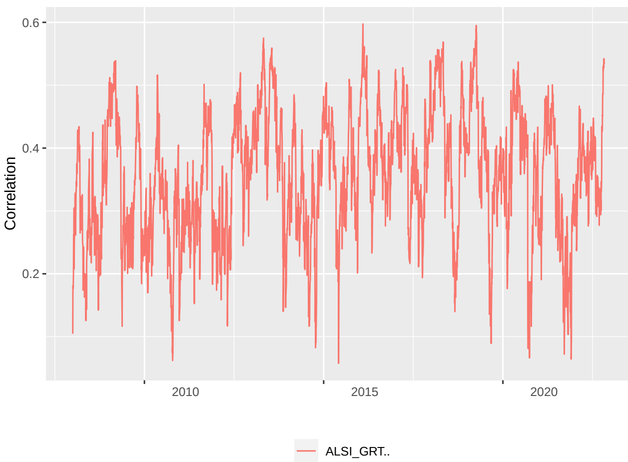
Dynamic Conditional Correlations: ALSI and CCO
2008 to 2022



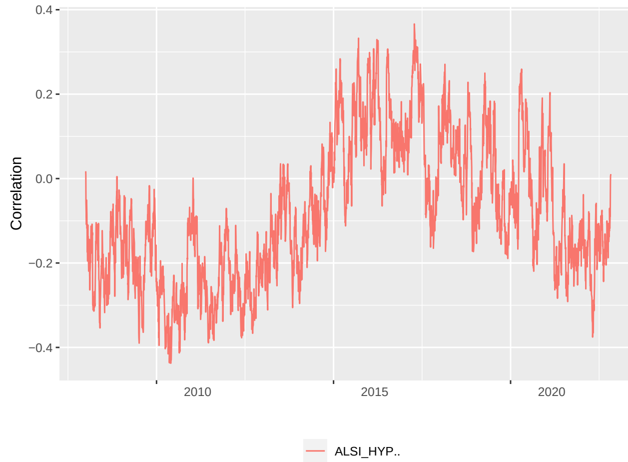
Dynamic Conditional Correlations: ALSI and EMI
2008 to 2022



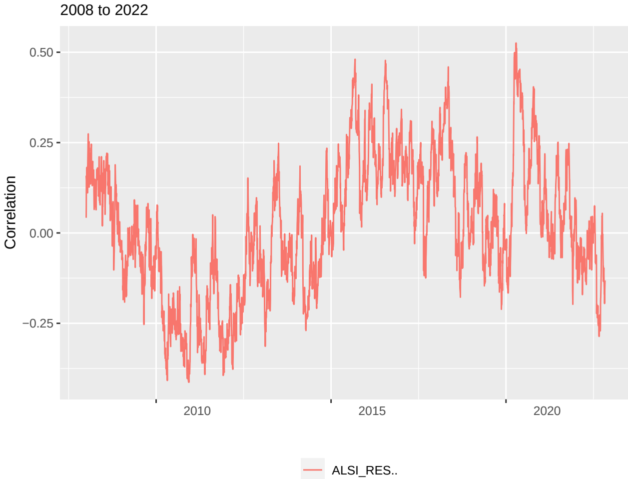
Dynamic Conditional Correlations: ALSI and GRT
2008 to 2022



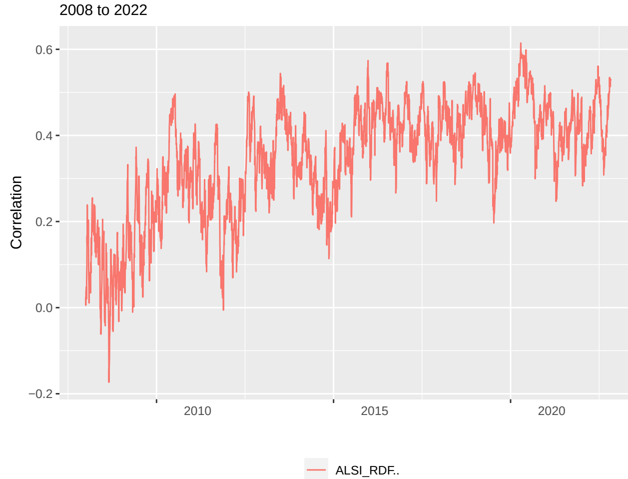
Dynamic Conditional Correlations: ALSI and HYP
2008 to 2022

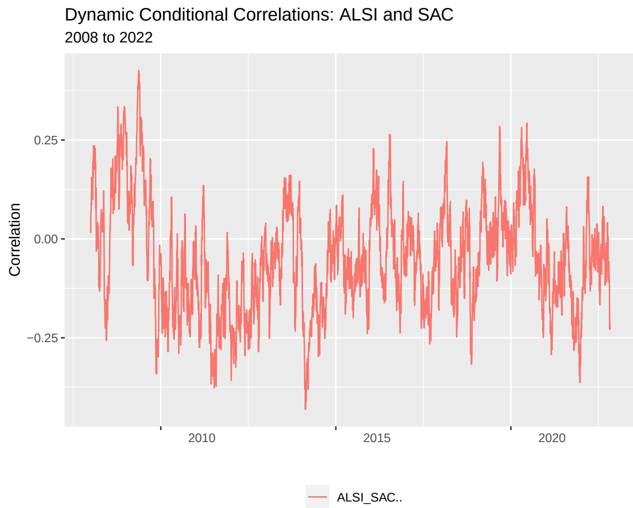


Dynamic Conditional Correlations: ALSI and RES
2008 to 2022



Dynamic Conditional Correlations: ALSI and RDF
2008 to 2022





3.2. Time-varying Correlation: Periods of High and Low Volatility

In this section, periods of high USD/ZAR (Dollar Rand) volatility are isolated and used as to filter for the ALSI combined and imputed data. The premise being that periods of high Rand volatility can act as an indicator for high levels of volatility in South Africa financial markets and other asset classes. These highly volatile periods are then used as an index to filter the returns data for periods where South African markets were volatile.

Given that the high volatility combine imputed ALSI returns data will have large missing gaps due to periods of moderate or low volatility, dynamic correlations between equity pairs will have to be charted for short periods of a time. This is due to the fact that the graphing function used will not skip whole year periods.

Following this methodology of running multiple DCC models on smaller periods of high volatility decreases the run time of the model.

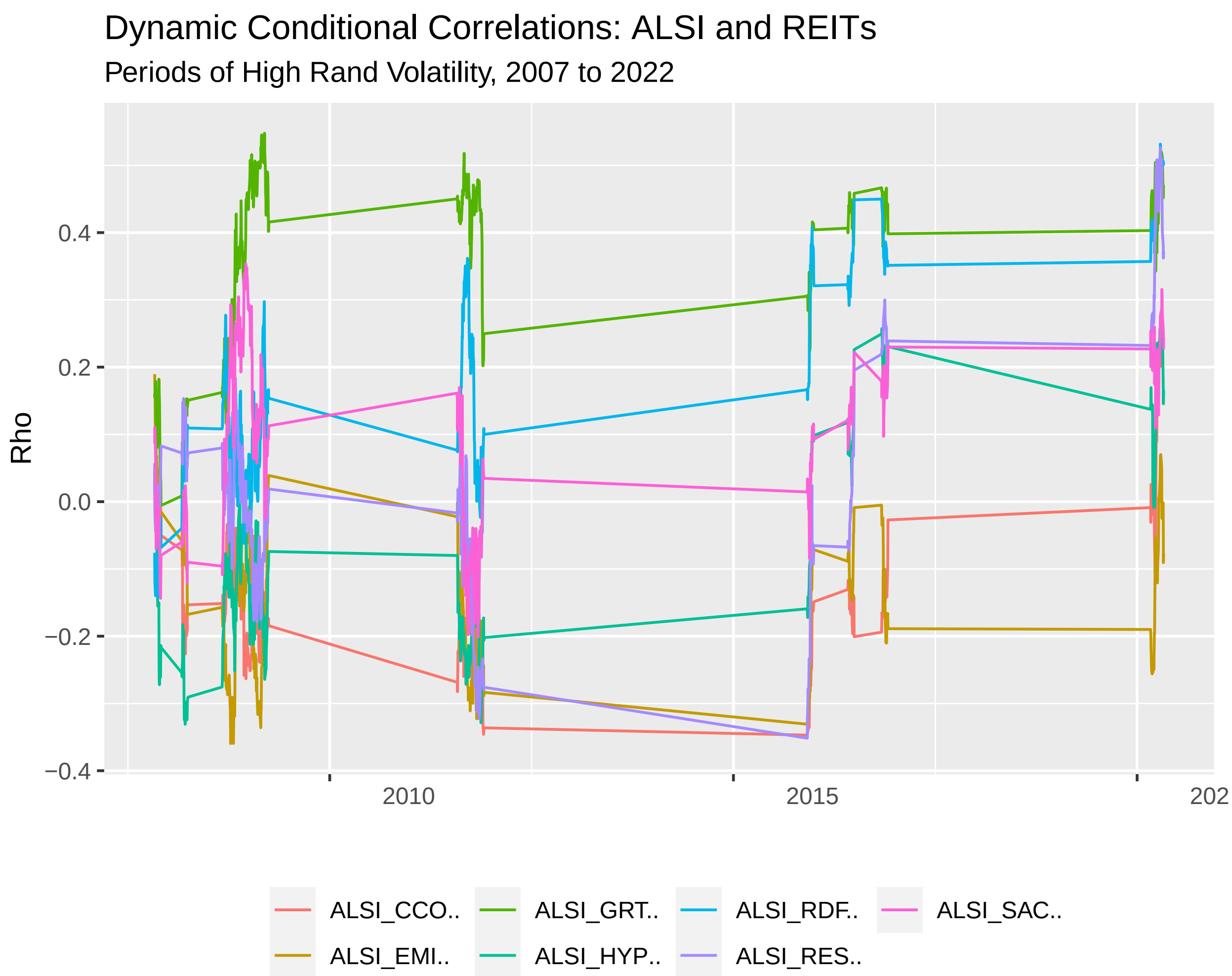


Figure 3.2: Dynamic Conditional Correlations Graph

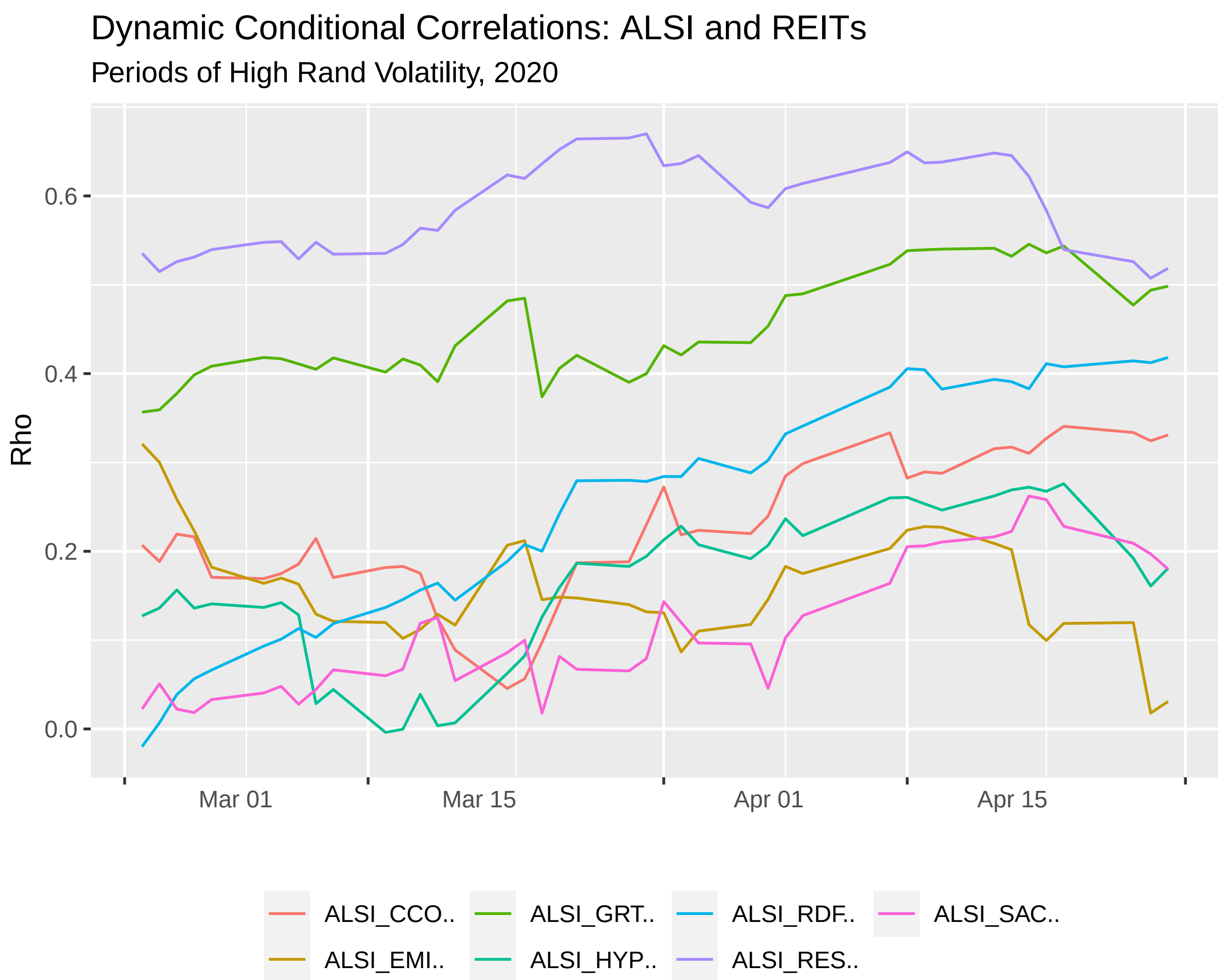


Figure 3.3: Dynamic Conditional Correlations Graph

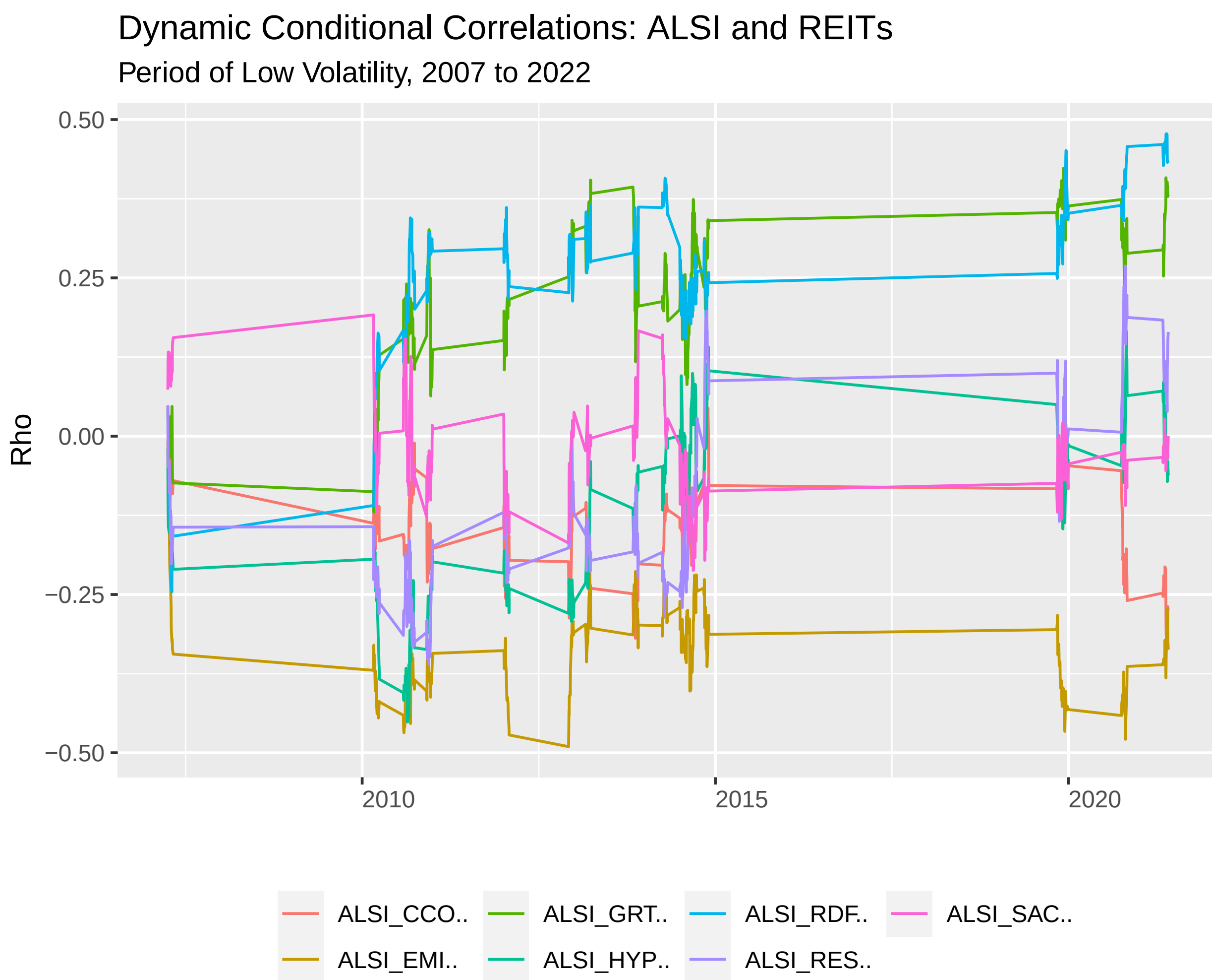


Figure 3.4: Dynamic Conditional Correlations Graph

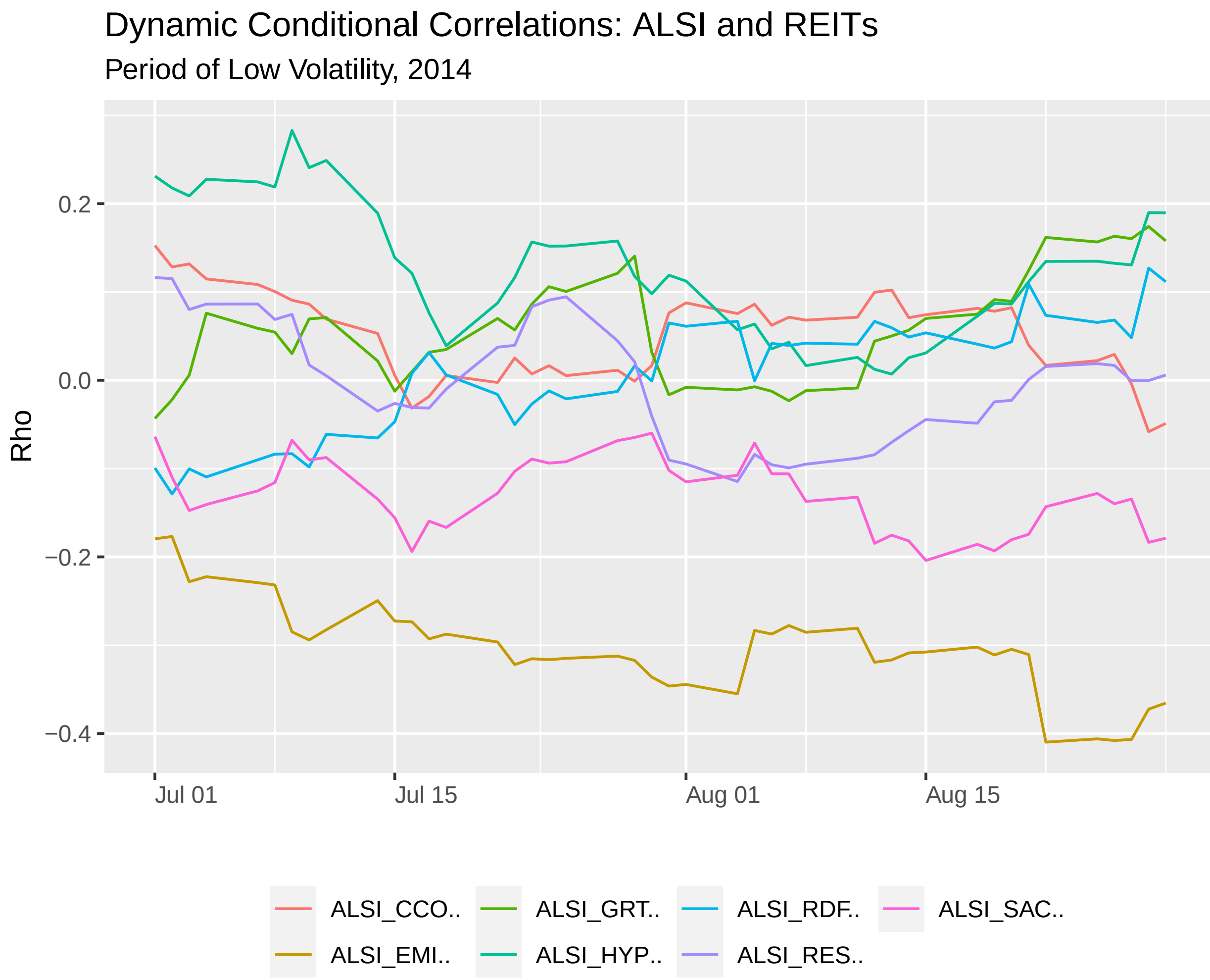


Figure 3.5: Dynamic Conditional Correlations Graph

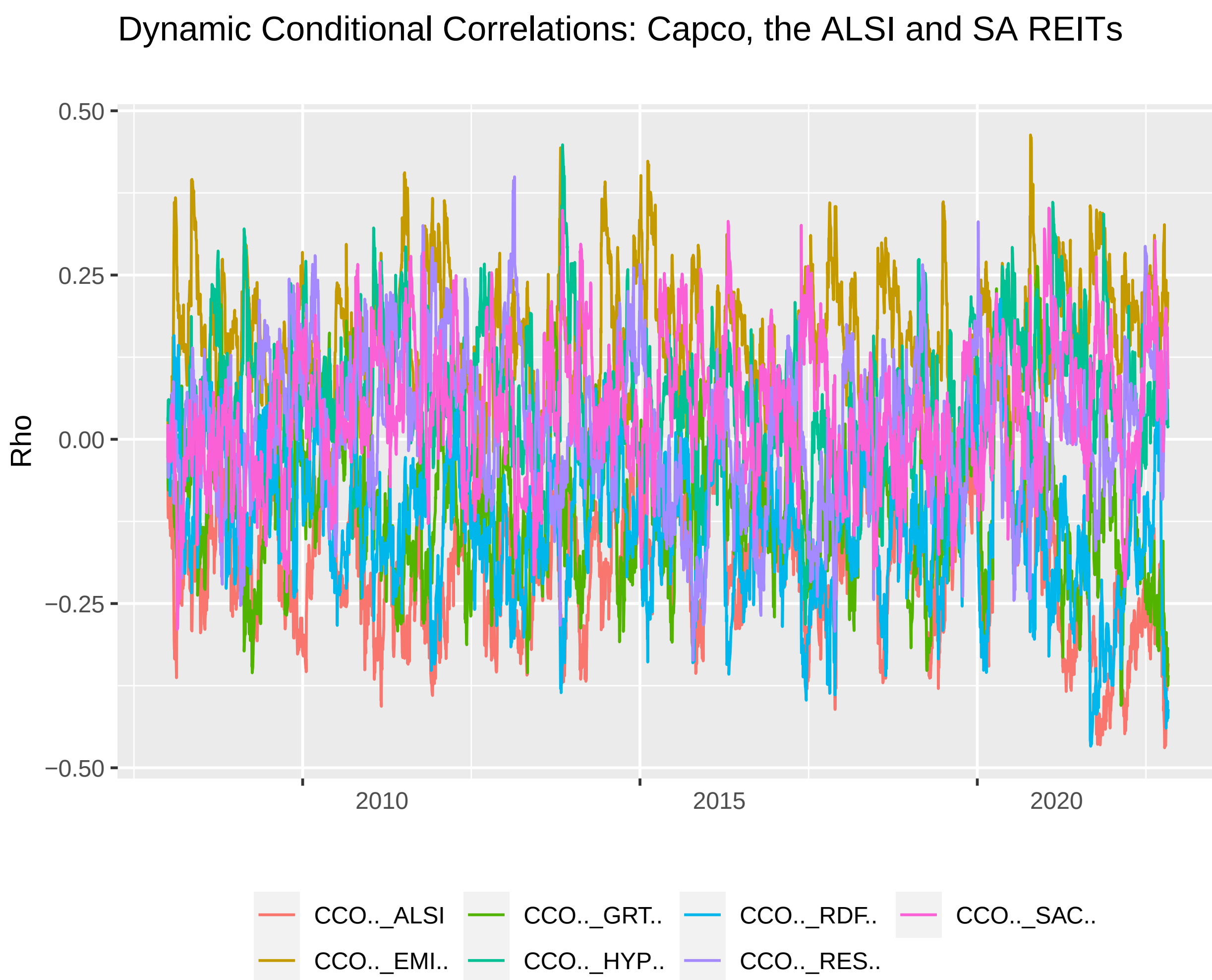


Figure 3.6: Dynamic Conditional Correlations Graph

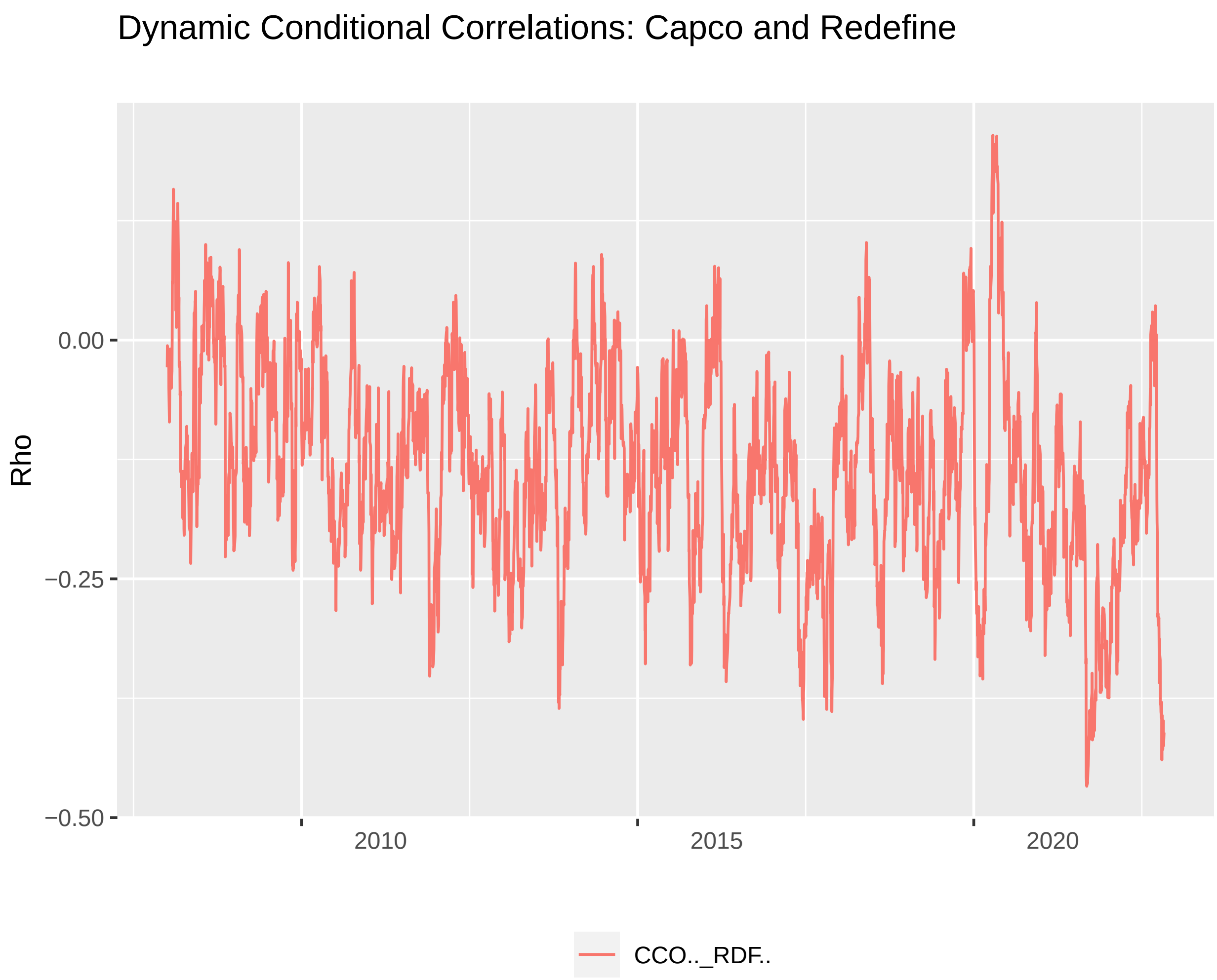


Figure 3.7: Dynamic Conditional Correlations Graph

4. Conclusion

References

Engle, R. (2002) Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models. Journal of Business & Economic Statistics, 20, 339-350.