Announcements

First **quiz** today, at the end of class

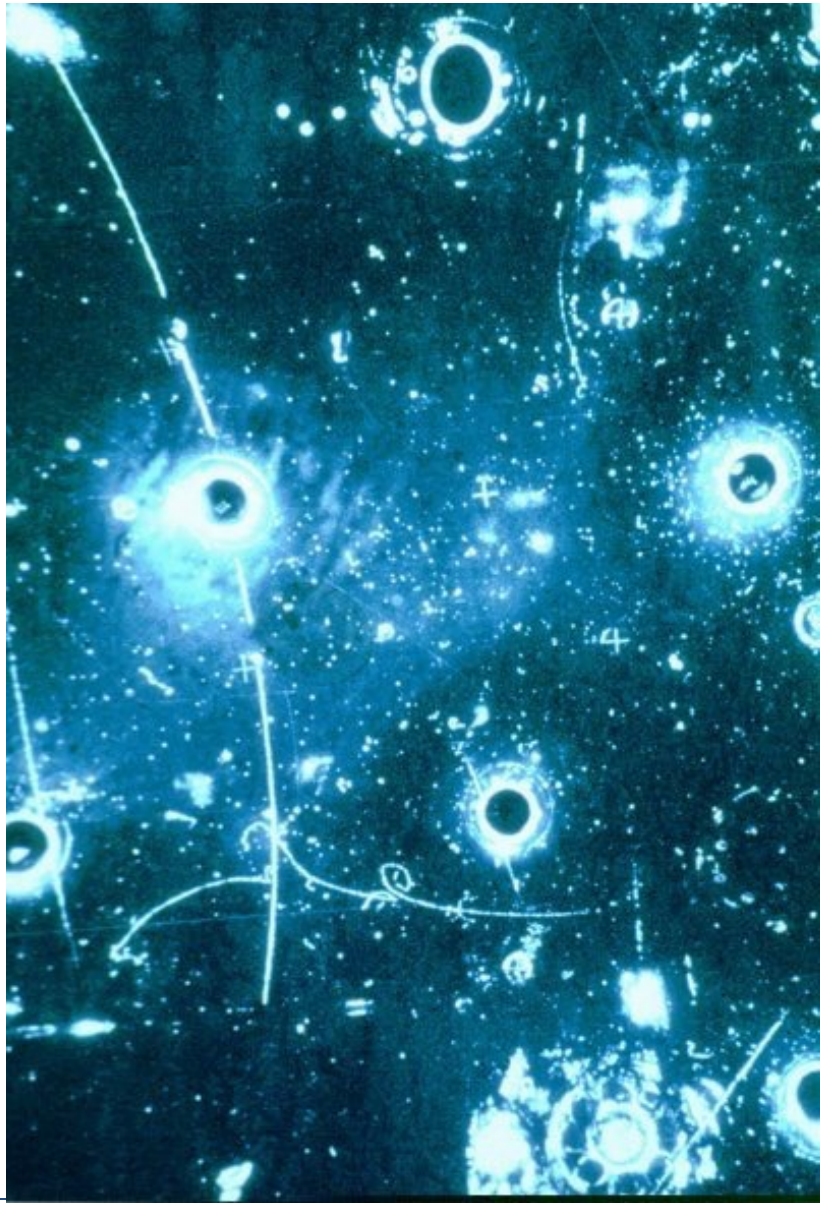
Homework due Monday at the start of class (3pm)

Please submit homework on gradescope: linked to the D2L page, and you were added to the course according to the D2L roster

Upload either pictures of handwritten work or typeset work

There are two versions of the assignment on gradescope, you will submit to the one for your section (803 or 493) – this is also the only one that should be visible to you

If you have not used gradescope before you can make an account with your MSU email address

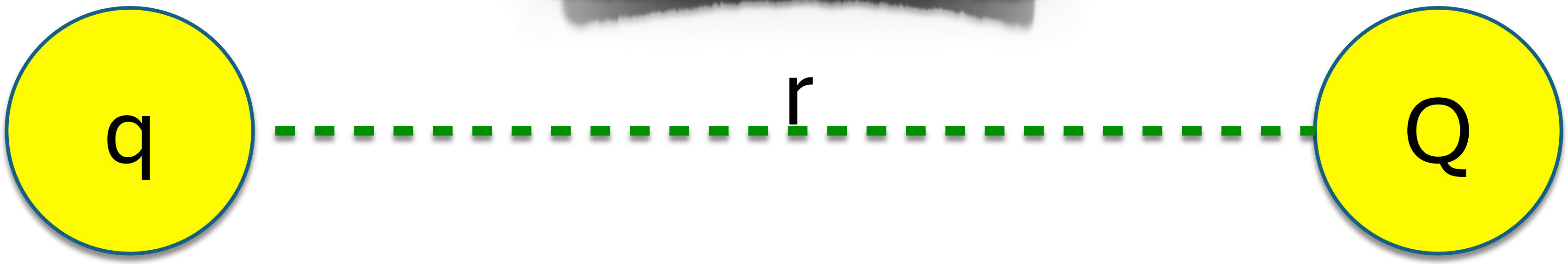


Electromagnetic Interactions

A thought experiment:

- Consider two electrons in free space. Classical E&M tells us that there is a force on the first by the second, and vice versa.
- There must be some mechanism to describe this quantum-mechanically:
  Exchange of photons

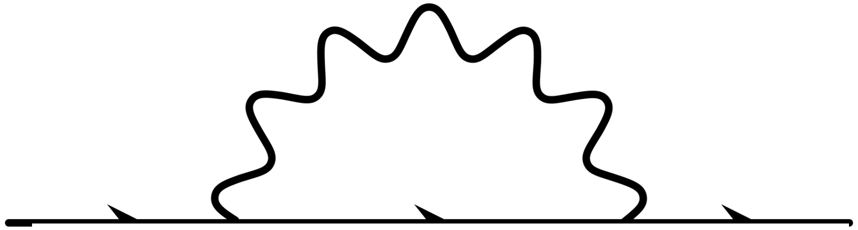
$$F = \frac{kqQ}{r^2}$$



Loops!

Consider an electron traveling along through space

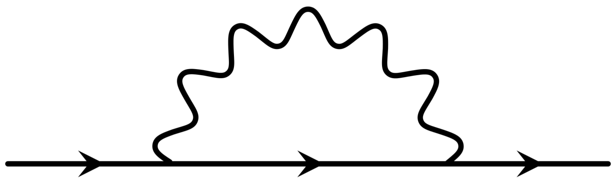
- This electron is always emitting and reabsorbing photons.
- These photons are quantum fluctuations, bound by Heisenberg's Uncertainty Principle



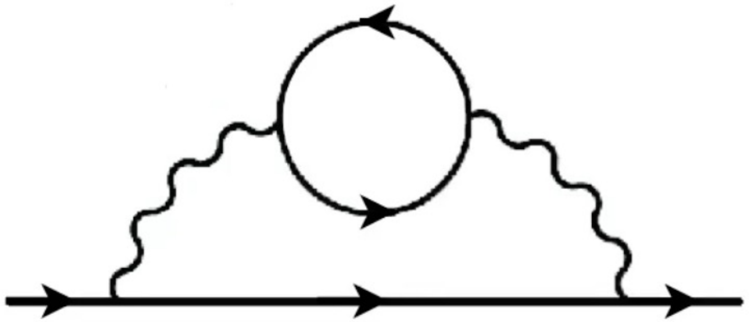
Loops make things more interesting!

Loops!

Interactions occur when another particle happens to "run into" the virtual photon (or gluon, or W, etc).



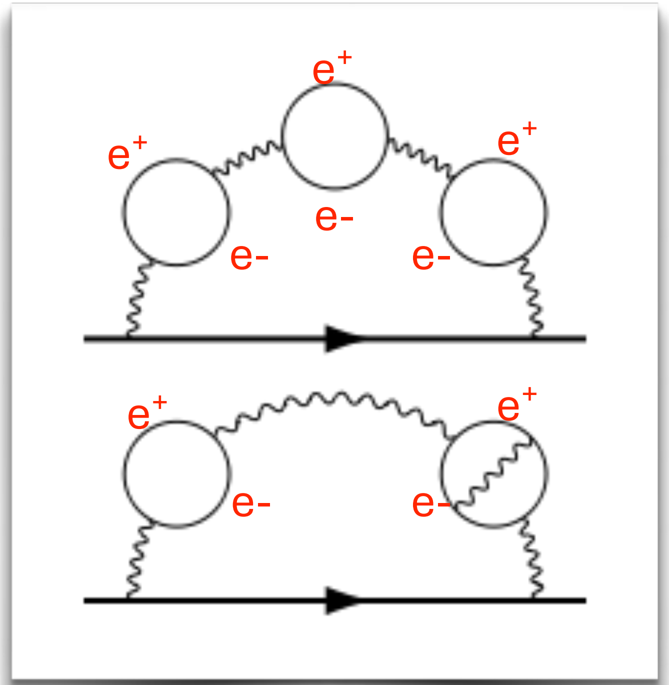
Even more complex: virtual particles can have their own fluctuations.
Eg, a photon decaying to an electron/positron pair.

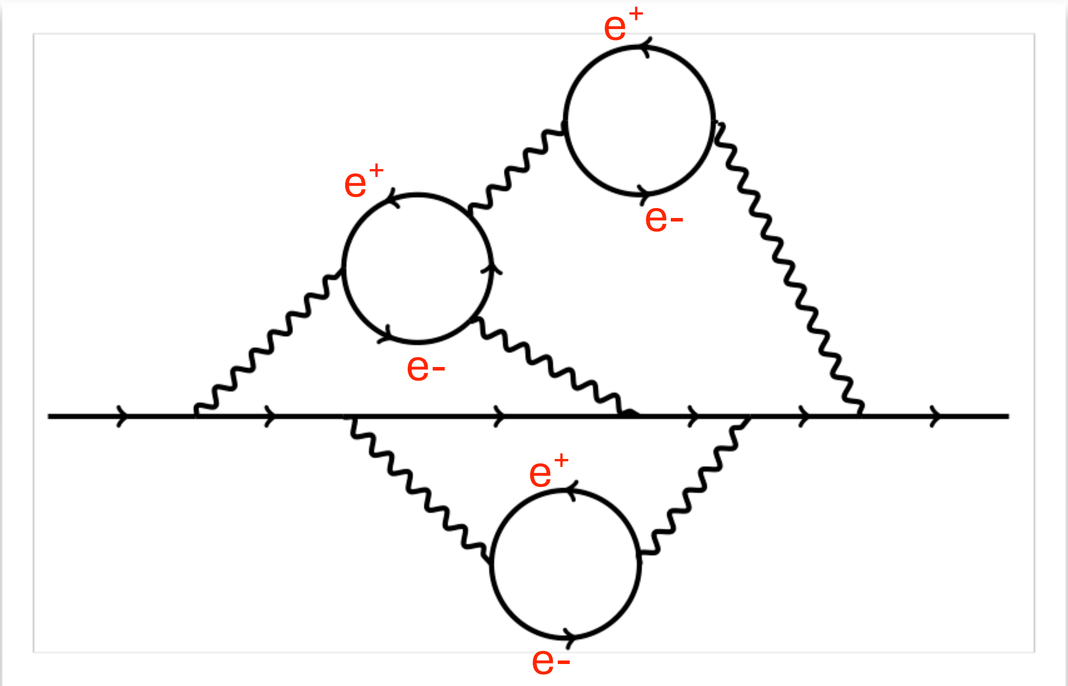


Vacuum Polarization

These quantum fluctuations are random and continuous

- Charged particles are thus effectively surrounded by a “cloud” of virtual particles.
- In addition to allowing interactions, these particles give rise to a critical phenomena: Vacuum Polarization

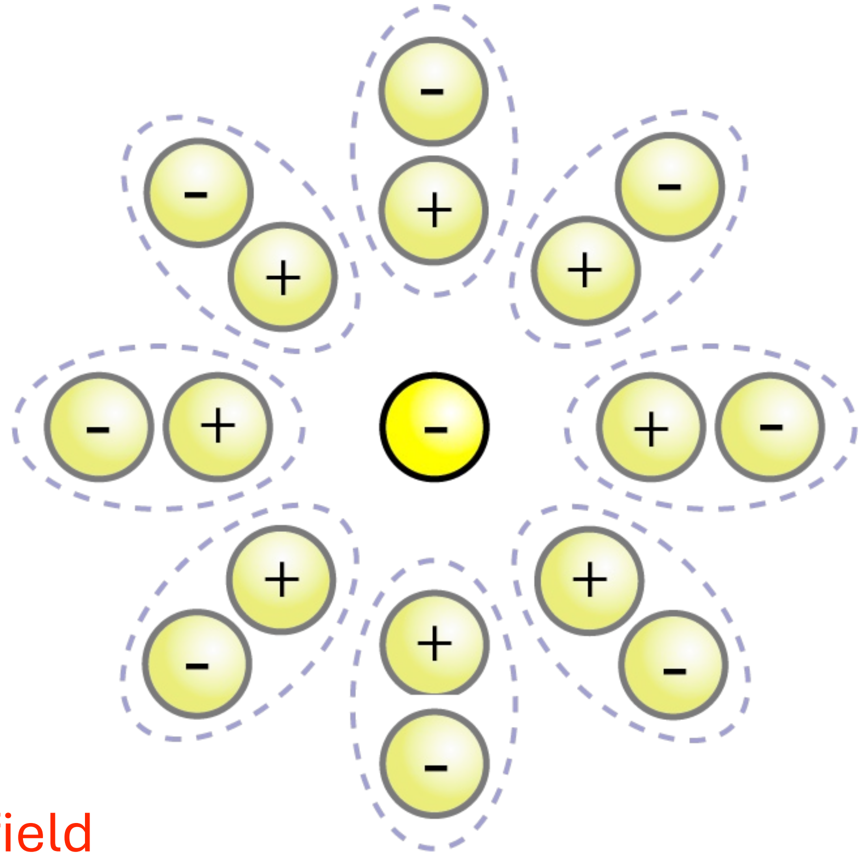




Vacuum Polarization: Electromagnetic Force

Just as within a dielectric medium, the original charge polarizes the virtual charges around it.

Consider a charged probe moving into this cloud. What does it "see"?



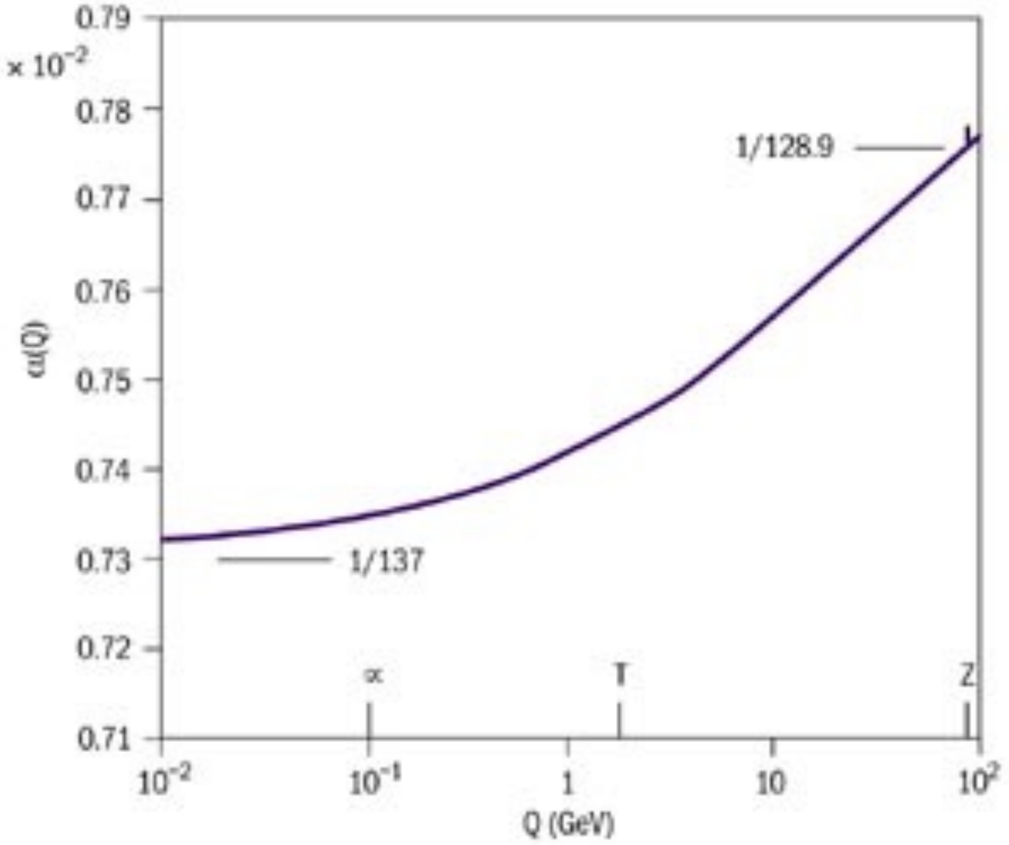
The charge moving into the cloud sees the diffuse charge, not just the other electron

1) The diffuse charge "looks" like it has a smaller electric field
2) The field increases as the probe moves towards the cloud

Coupling Strength

Interaction strength vs Probe energy

Electromagnetic Interaction Strength



Probe Energy

useful relation (fine structure constant)

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}(\mathrm{SI}) = \frac{e^2}{\hbar c}(\mathrm{cgs}) = \frac{1}{137}$$

(text book)

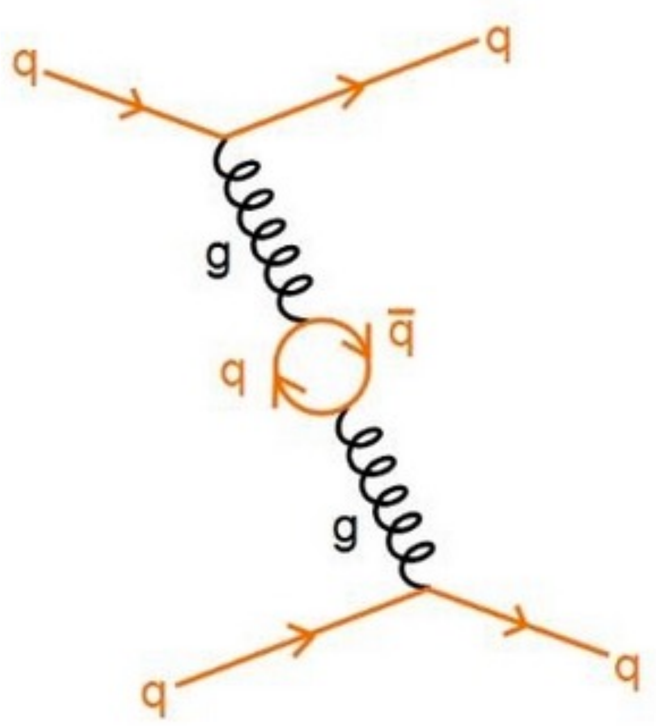
Increasing probe energy = gets farther into the cloud

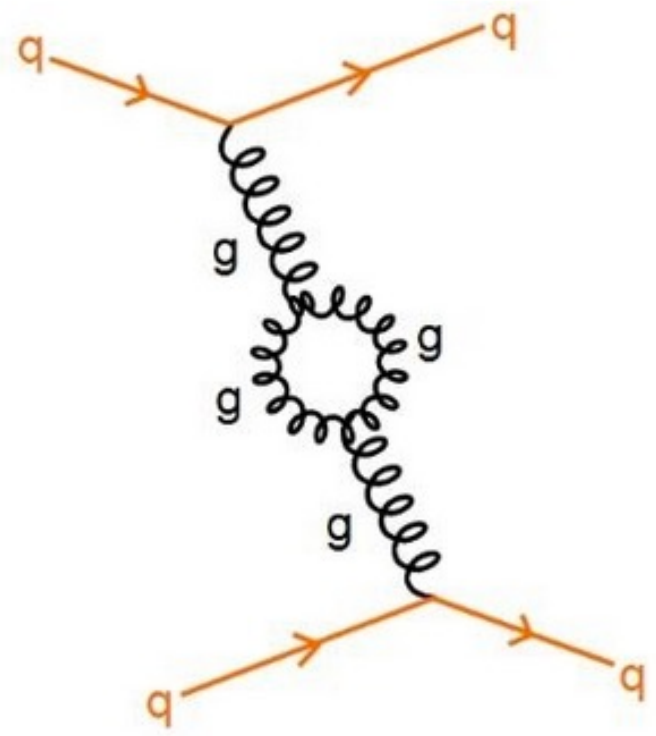
= larger effective charge/field

Vacuum Polarization: Strong Force

There is a very similar physical behavior for the strong force & gluons.

But the result is very different!

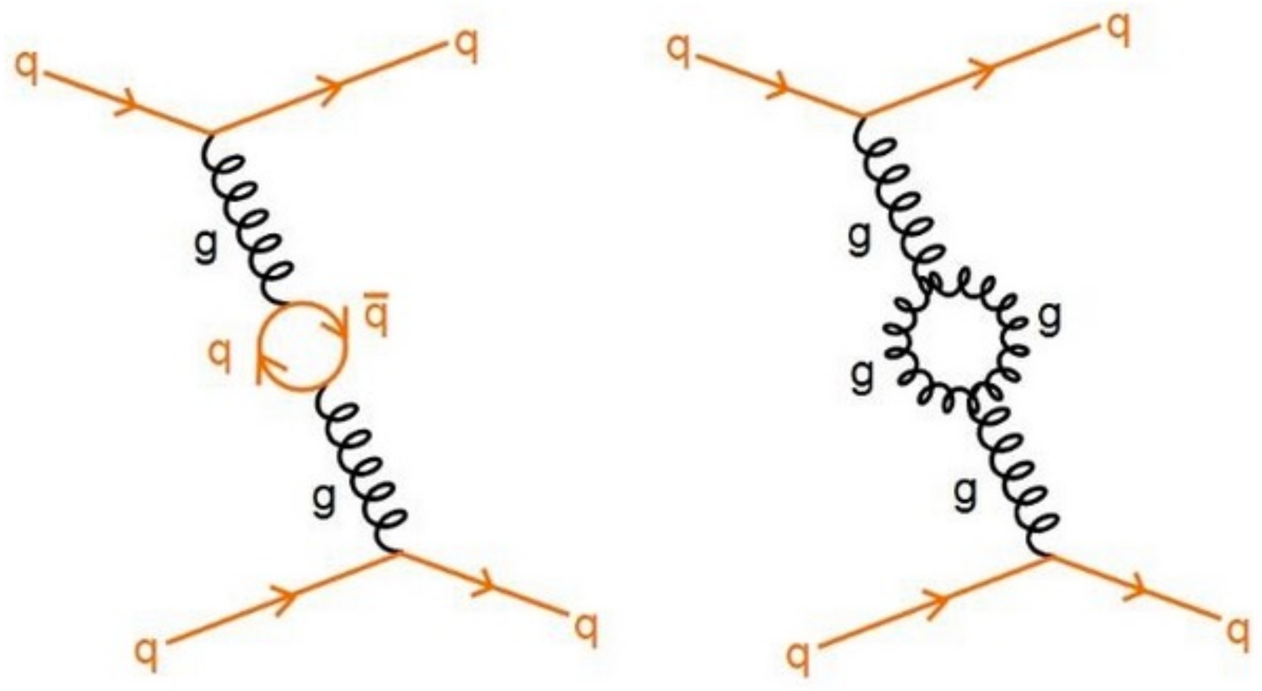




Vacuum Polarization: Strong Force

The differences arise from the fact that gluons carry color charge

- Photons have no electric charge: no photon-photon vertices
- Gluons can create **quark** AND **gluon** loops
- So the vacuum color charge polarization effect has contributions from **quarks** AND **gluons**.



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- Photons have no electric charge: no photon-photon vertices
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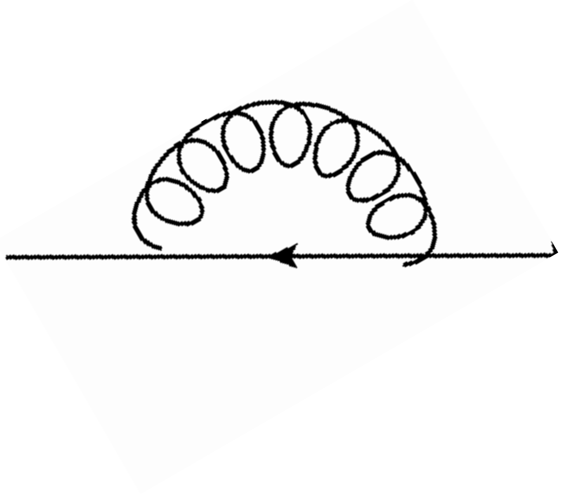
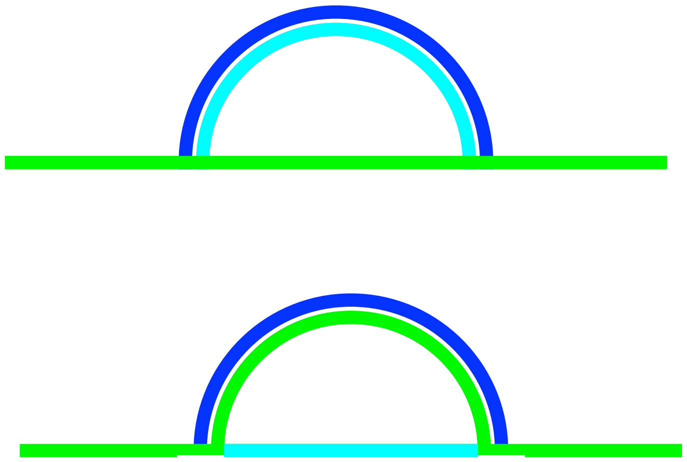
Color charge leads to lots of combinations:

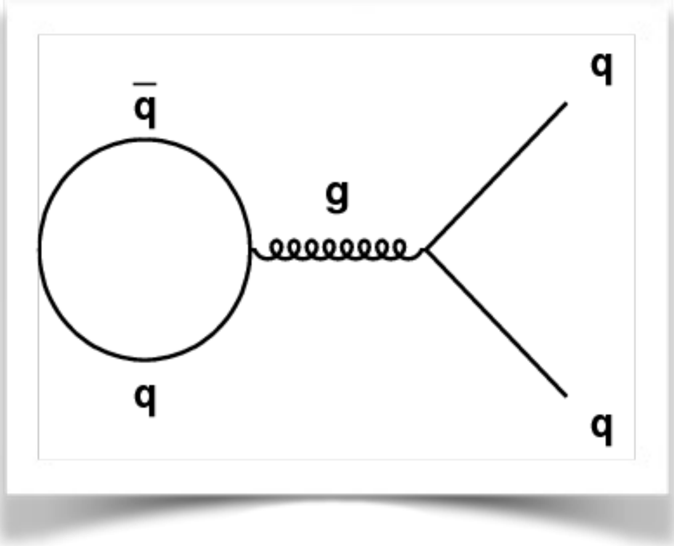


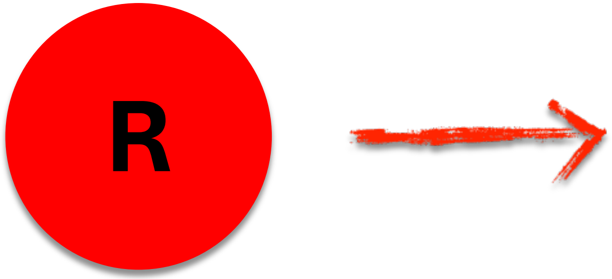


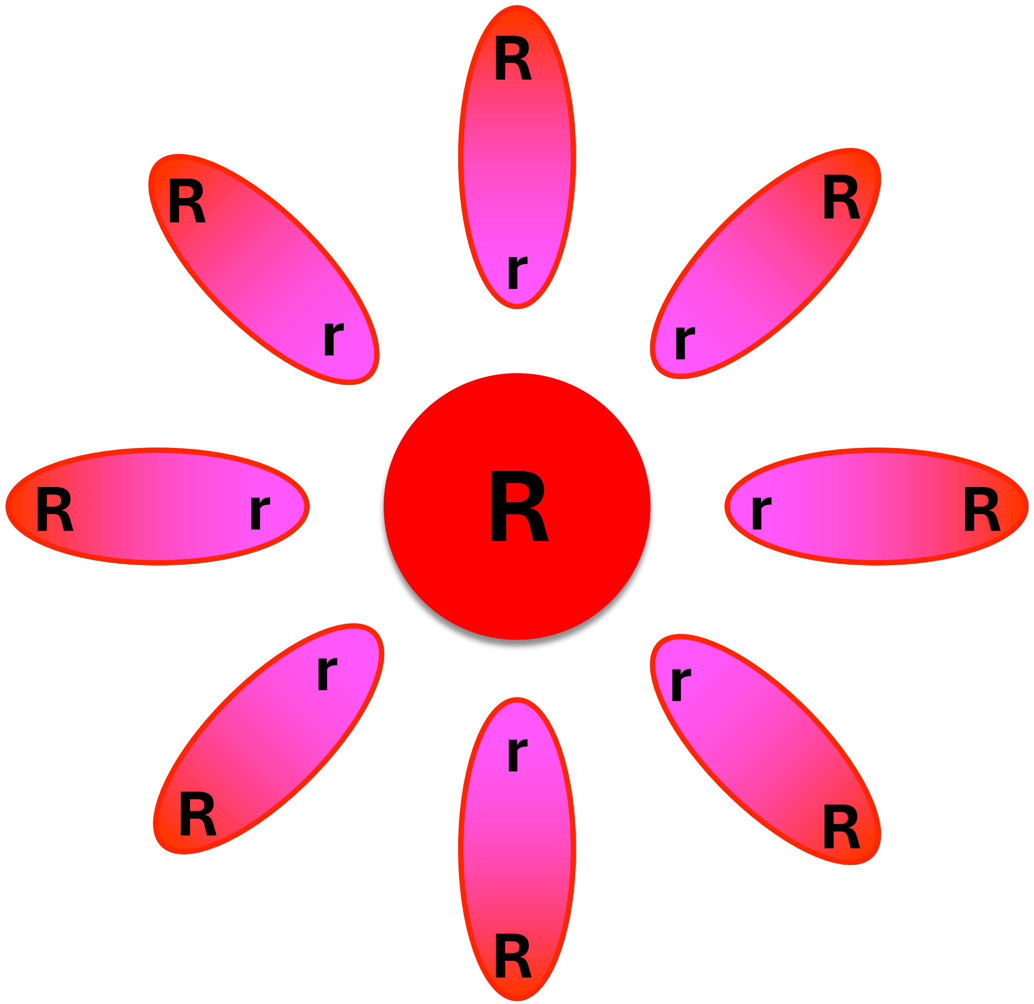
+ Many others!

Vacuum Polarization: Strong Force

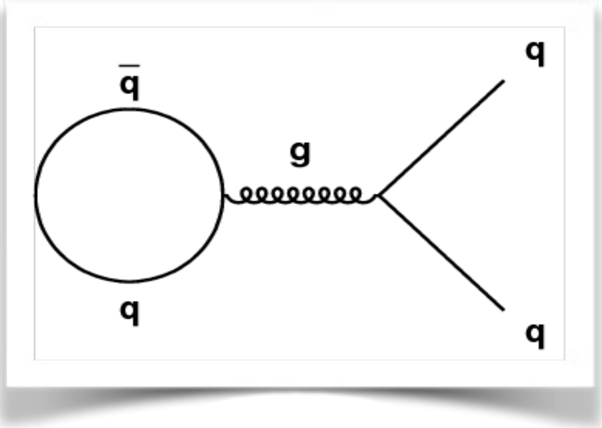
First, think about only the quarks:





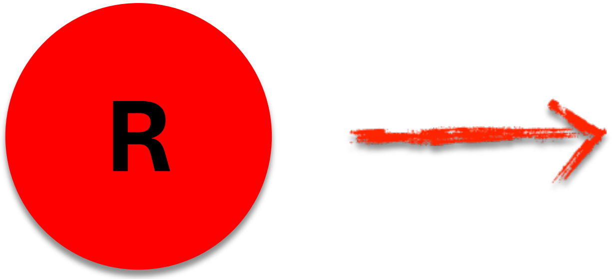


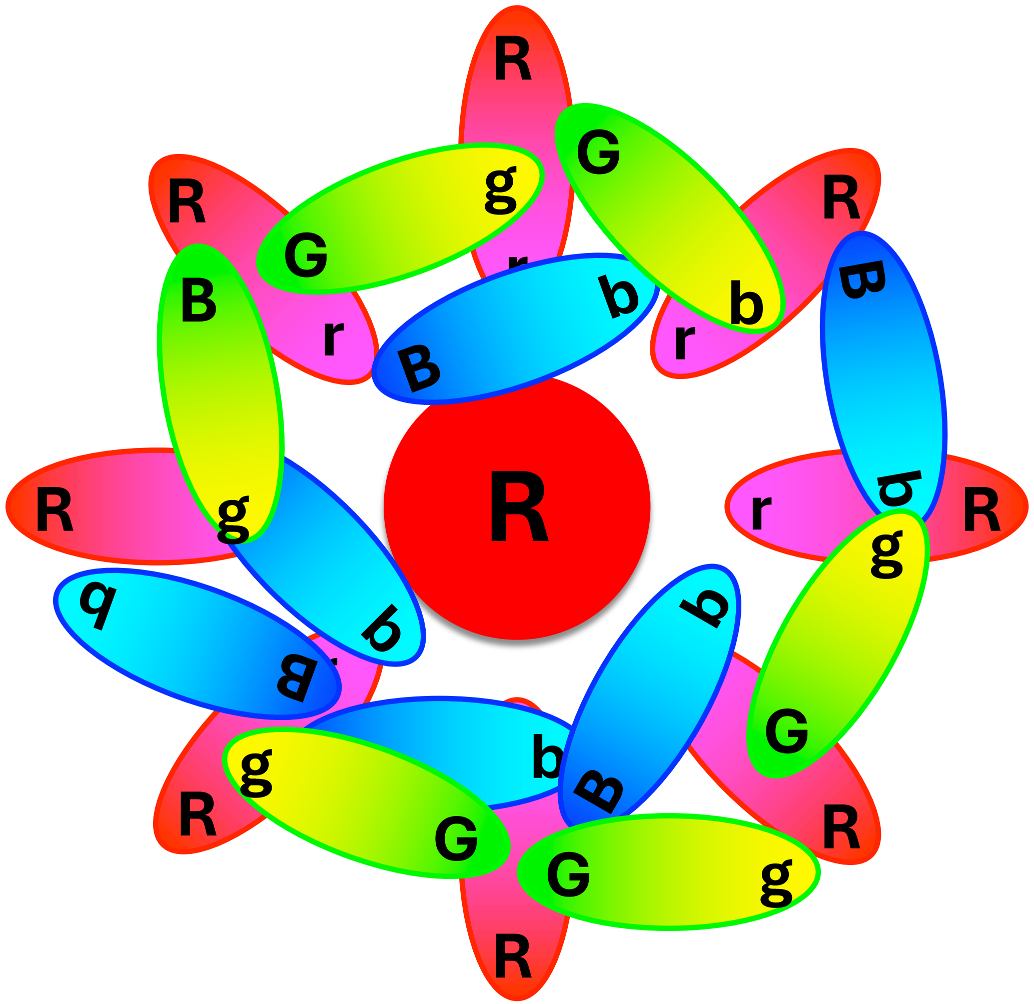
Vacuum Polarization: Strong Force



But: gluons can also interact here

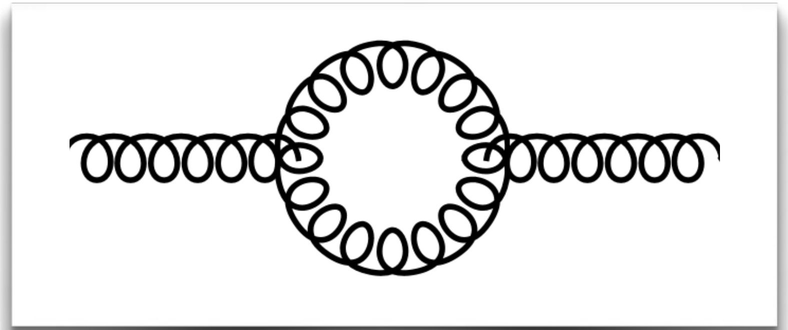
So, between virtual quarks, have lots of virtual gluons

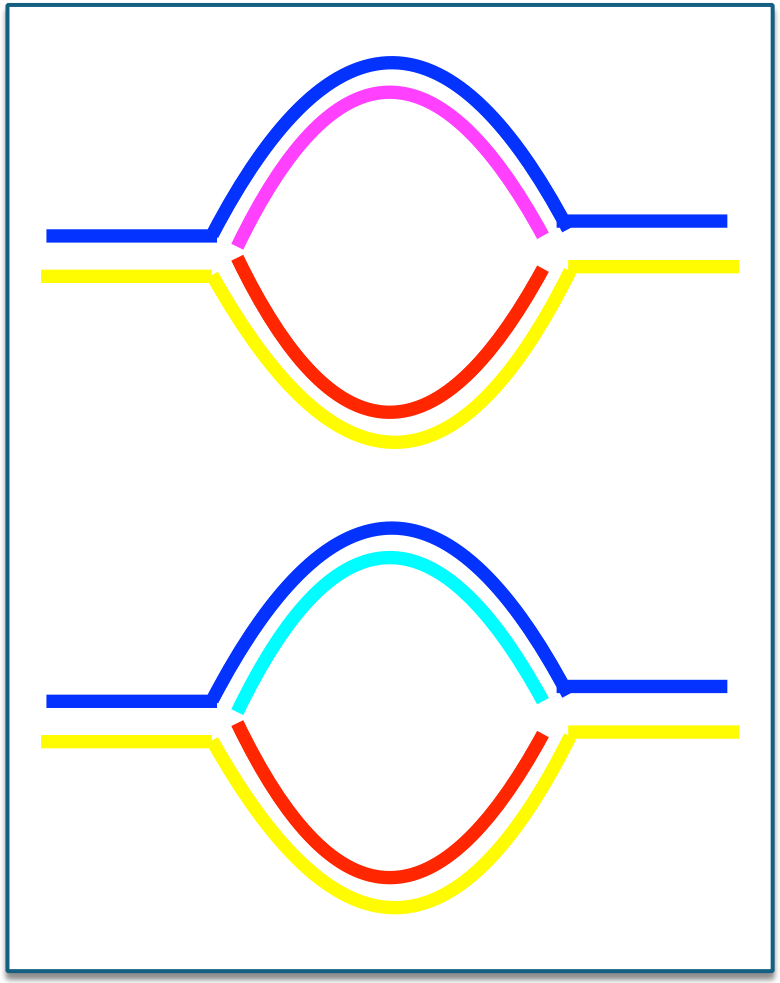




Vacuum Polarization: Strong Force

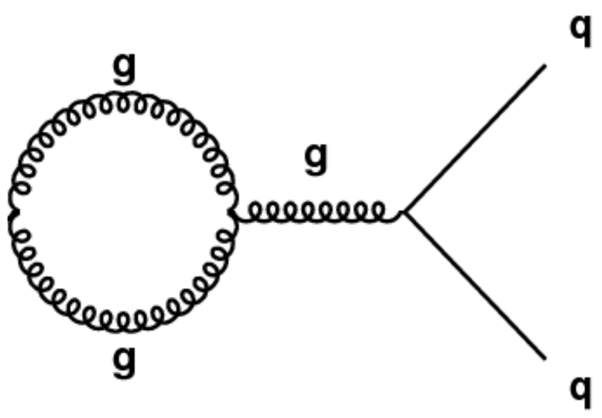
Gluons can also interact in many color combinations:

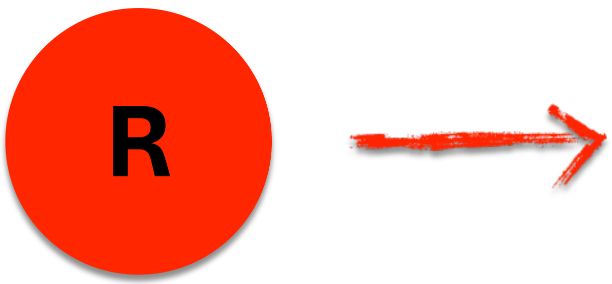


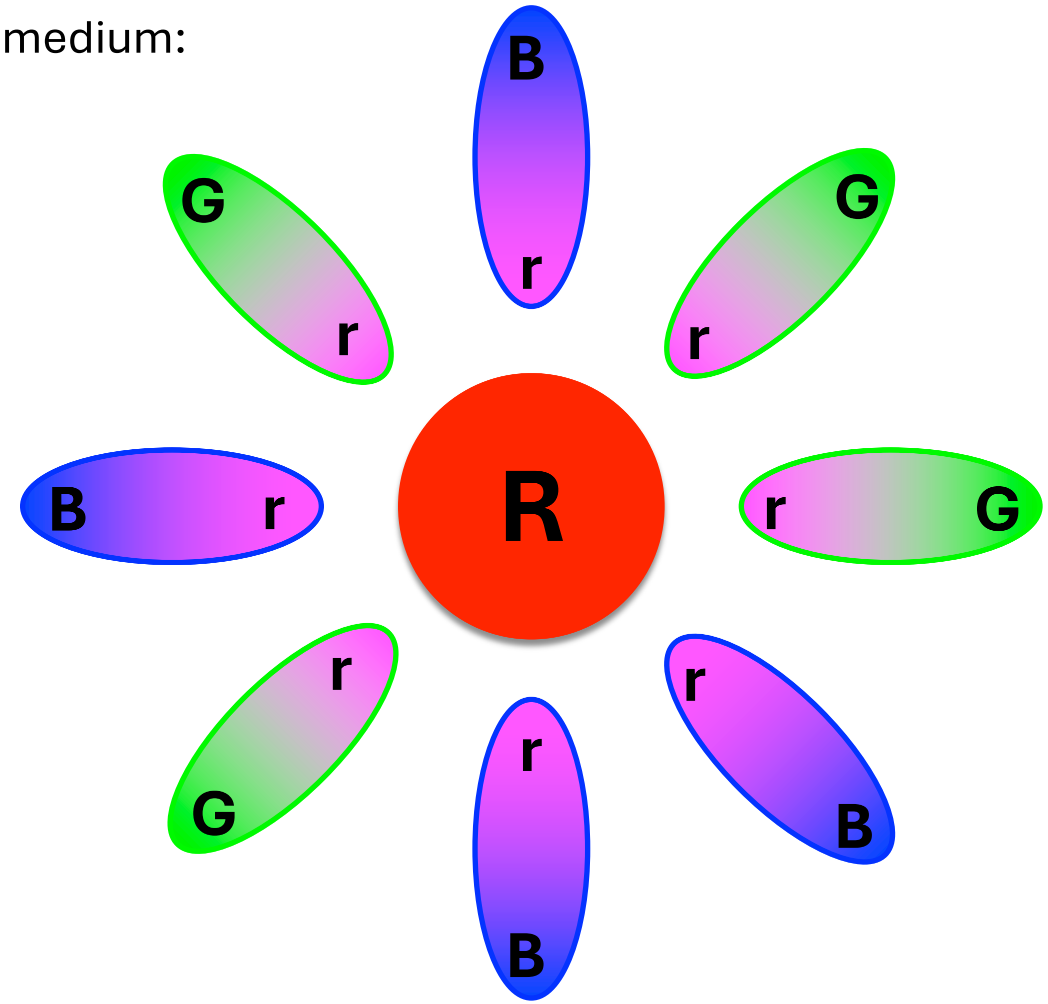


Vacuum Polarization

In addition, gluons are polarized in the surrounding medium:

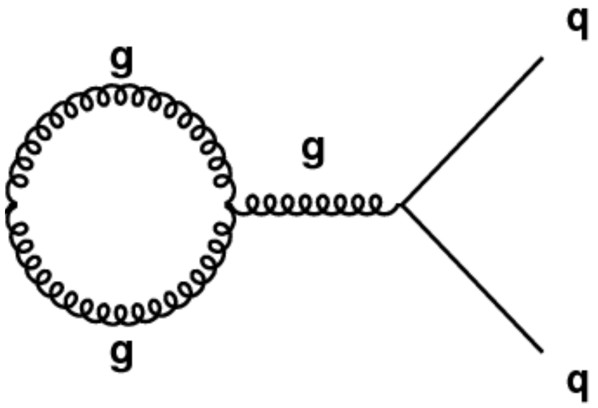


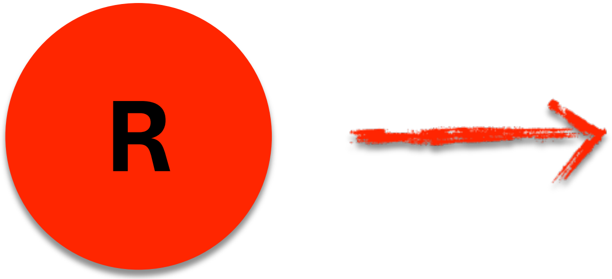


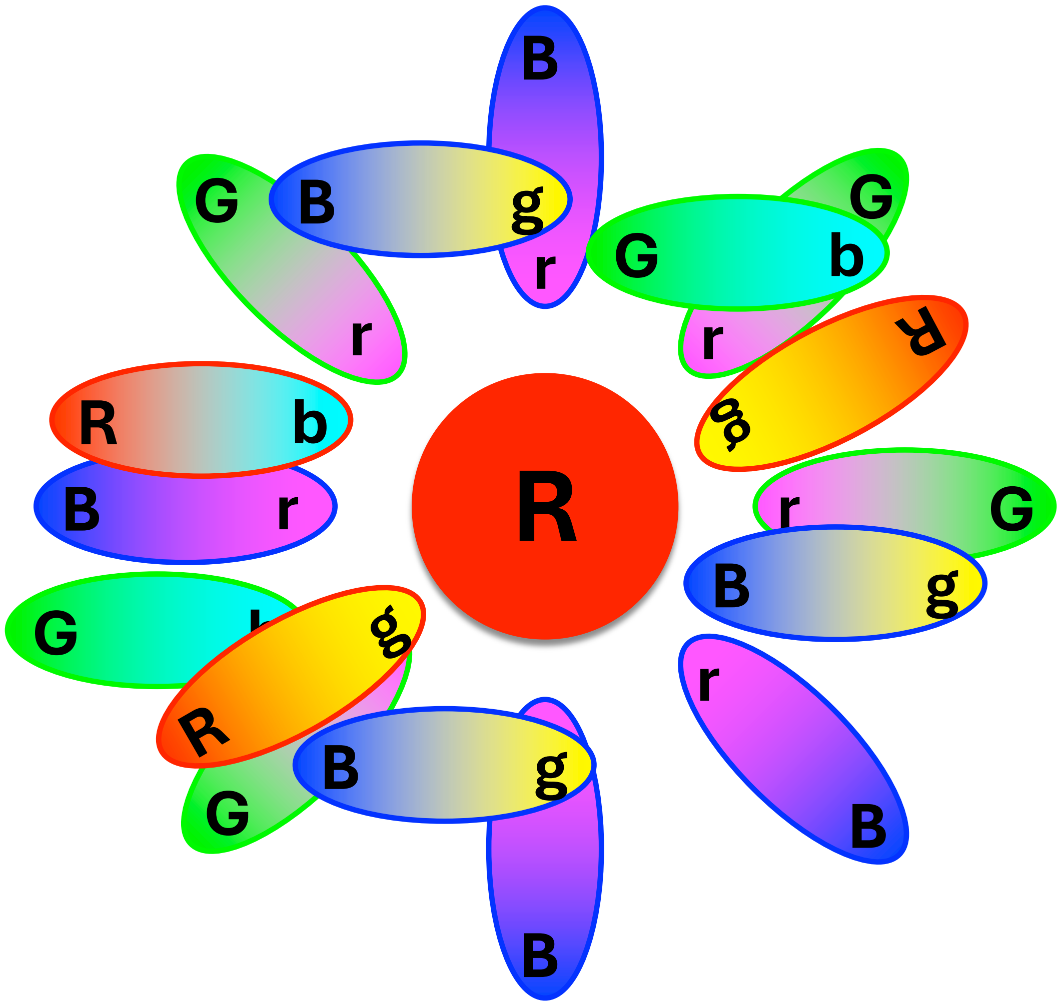


Vacuum Polarization

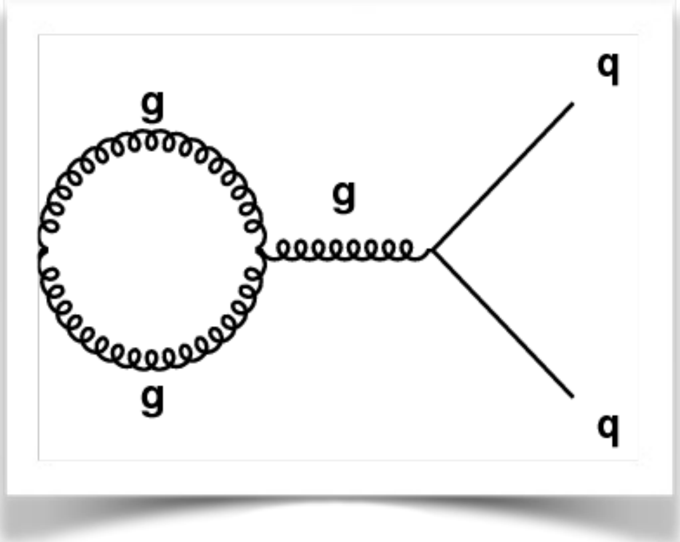
And also all the interactions around those:

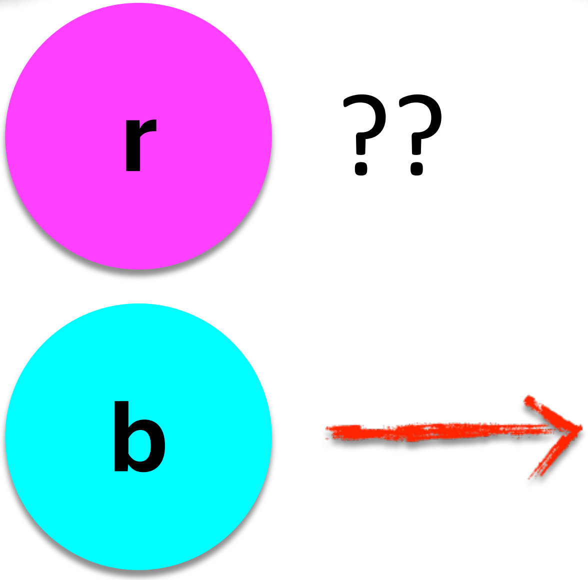






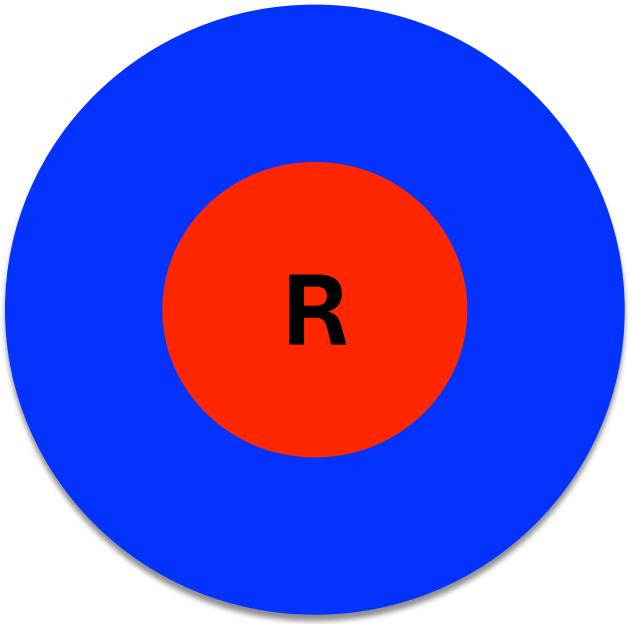
Vacuum Polarization “Color Camouflage”:





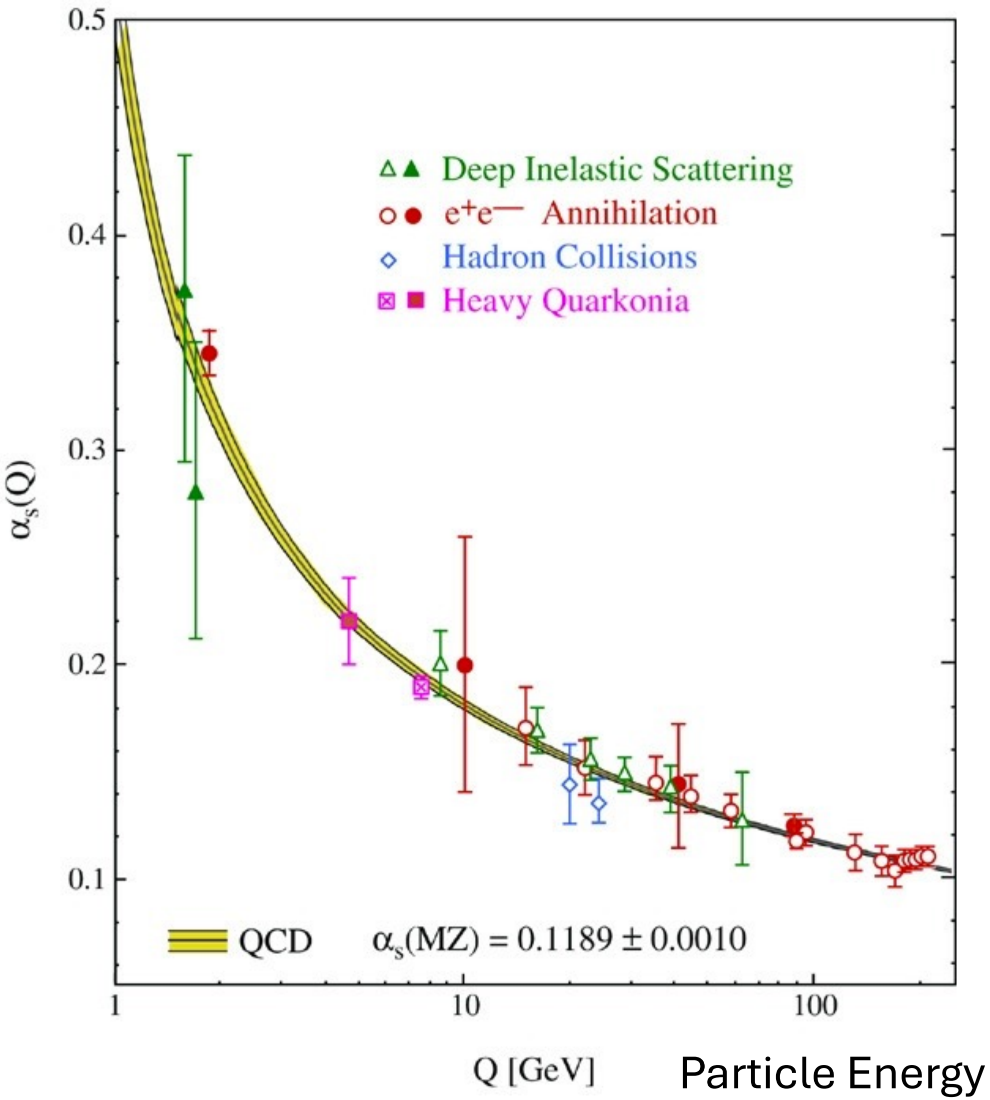
Maybe you don’t know what is in the center based on the interactions alone; makes predictions very complicated

- ➔ more complicated diagrams contribute more and more to the process!
- ➔ Strong coupling is >1



Running Color Couplings

Strong Interaction Strength

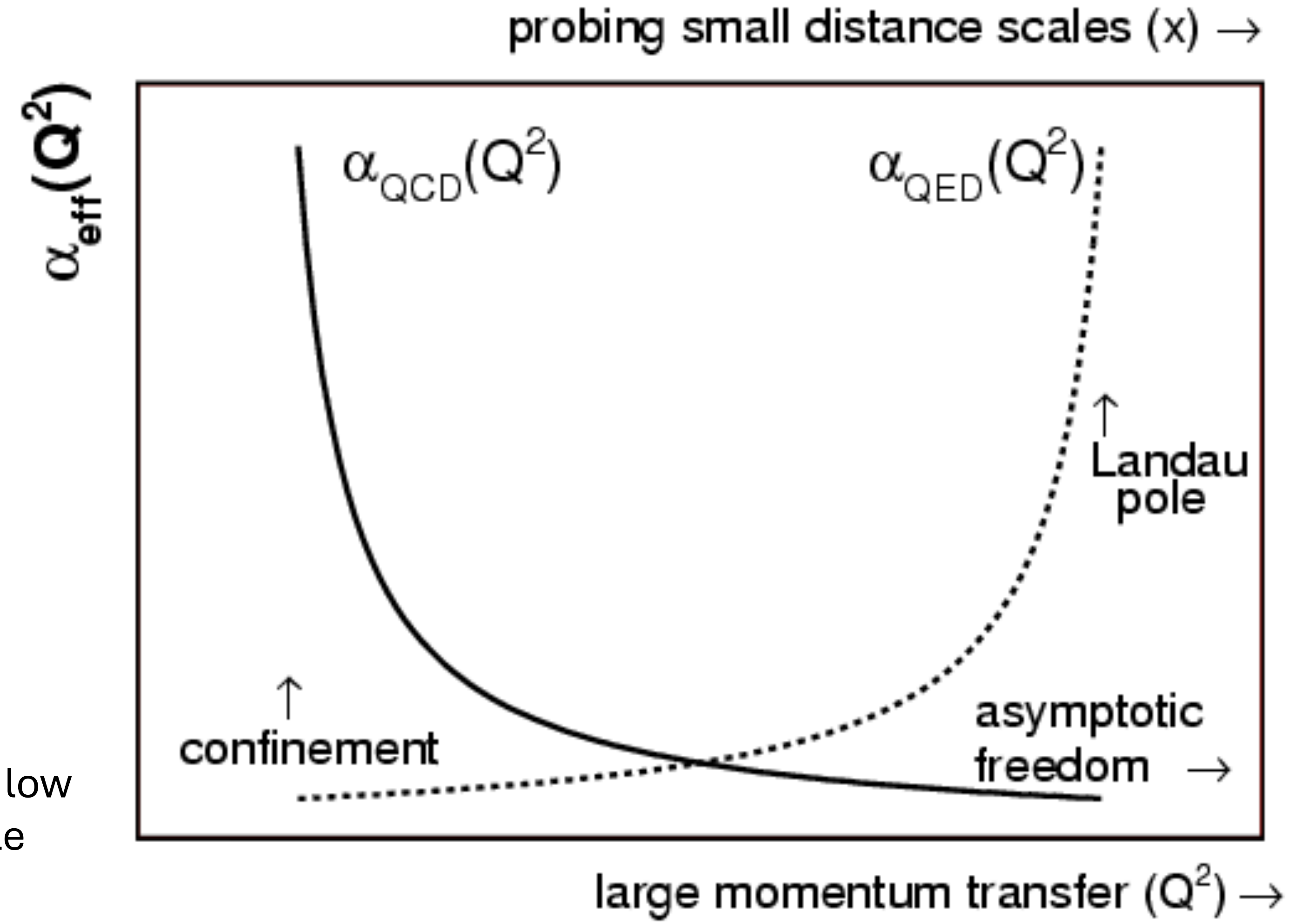


Particle Energy

Lower energy particle that is farther away from the center of the field will see many interactions

Deeper into field, fewer interactions (ie. only sees red quark at center)

Running of Color vs Electric Charges



Quark confinement at low energy: don't see single quarks in nature

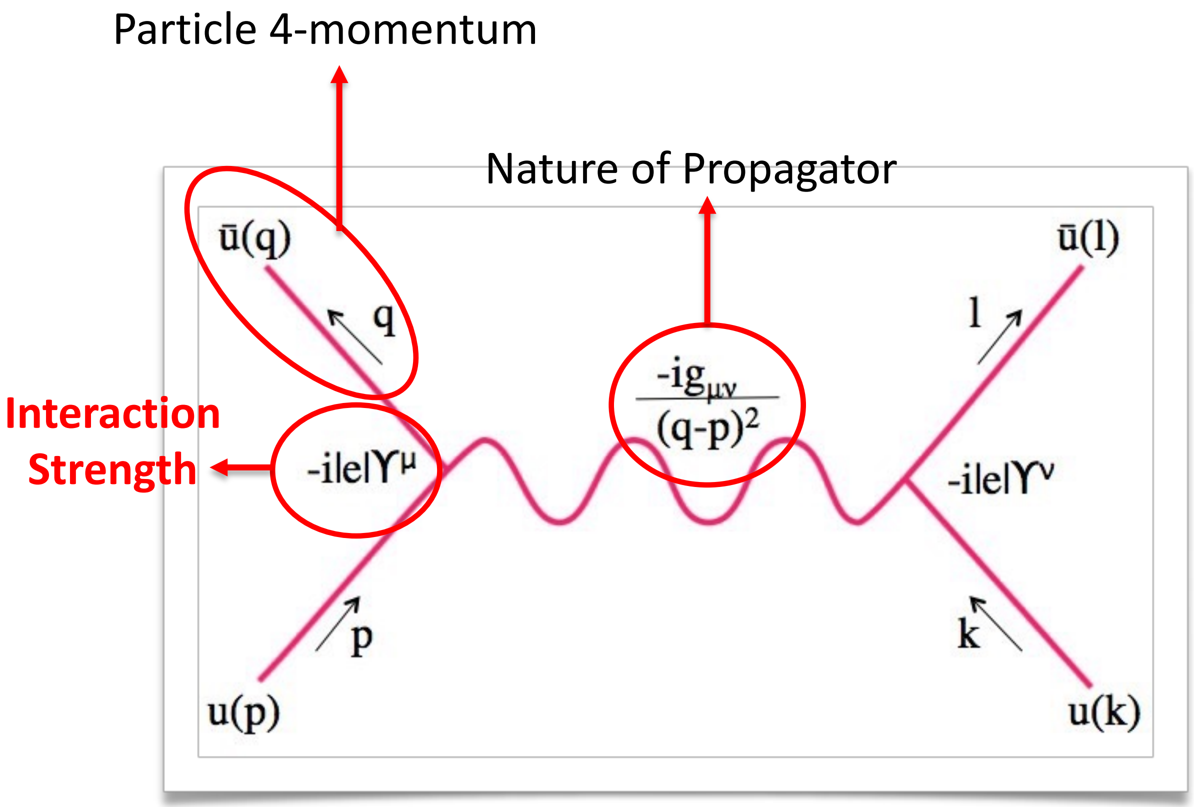
Asymptotic freedom: More energy you give protons/quarks, the more the interaction strength goes down

This will become important as we get to the next topic: calculating with Feynman diagrams!

Calculations

Feynman diagrams are useful visualization tools, but they were invented to help perform calculations!

Feynman diagrams encode the information needed to calculate things like interaction probabilities, differential kinematic distributions, etc.

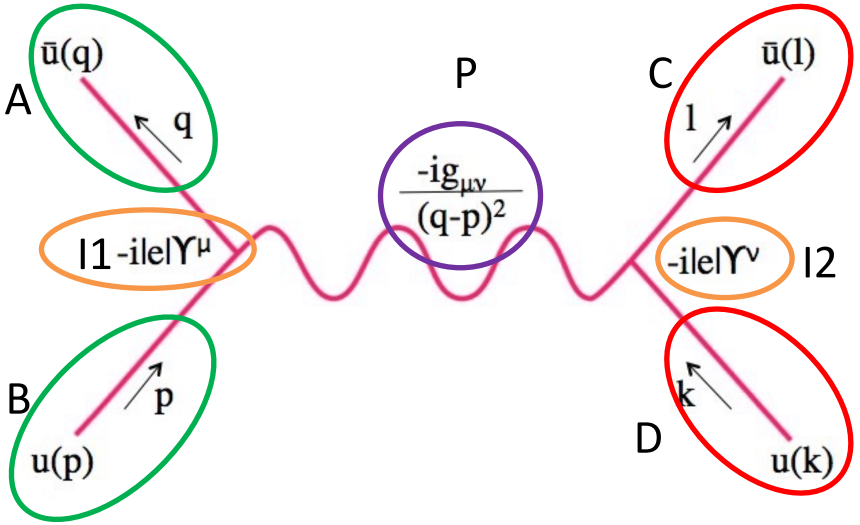


The Matrix Element

We can use the information encoded in the Feynman diagram to calculate things like the reaction cross section, the properties of the decay of particles and resonance phenomena.

A & B: Incoming particle wavefunctions.
P: Propagator wavefunction
C & D: Outgoing particle wavefunctions.
I1 & I2: Interaction strengths

$$\mathcal{M} \propto (\,A \times I1 \times B\,)\,(\,P\,)\,(\,C \times I2 \times D\,)$$



The Matrix Element

We can use the information encoded in the Feynman diagram to calculate things like the reaction cross section, the properties of the decay of particles and resonance phenomena.

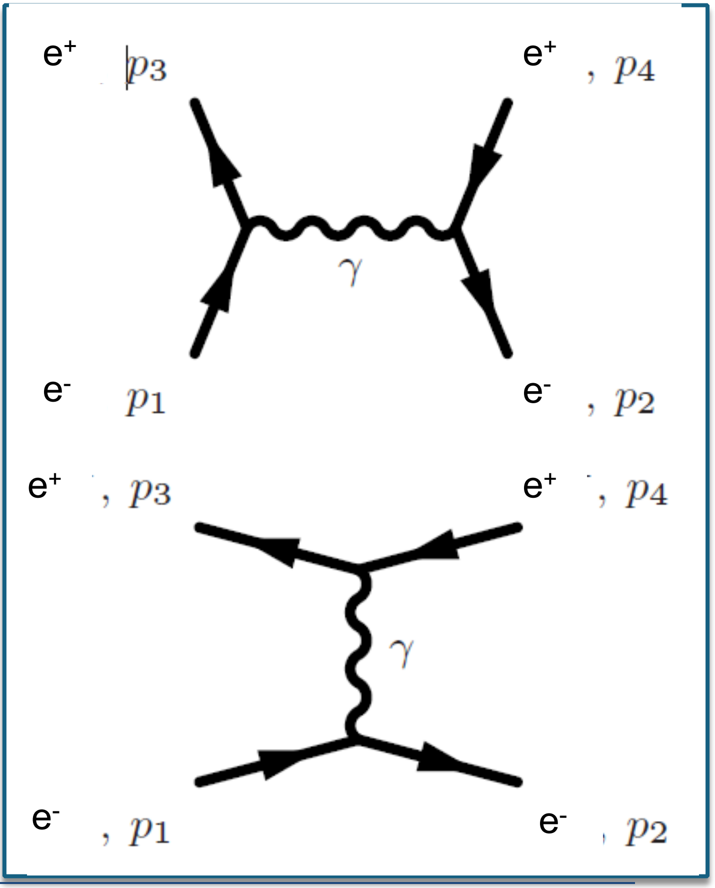
If there are N Feynman diagrams for the same final state, then the matrix elements sum linearly:

$$\mathcal{M}_{tot} = \mathcal{M}_1 + \mathcal{M}_2 + \ldots. + \mathcal{M}_n$$

The matrix element can be negative, giving rise to interference effects. For example:

$$\frac{d\sigma}{d\Omega} \propto \left|M_{tot}\right|^2 \propto \left|M_1\right|^2 + \left|M_2\right|^2 \pm 2\left|M_1 M_2\right|$$

Interference can be constructive or destructive

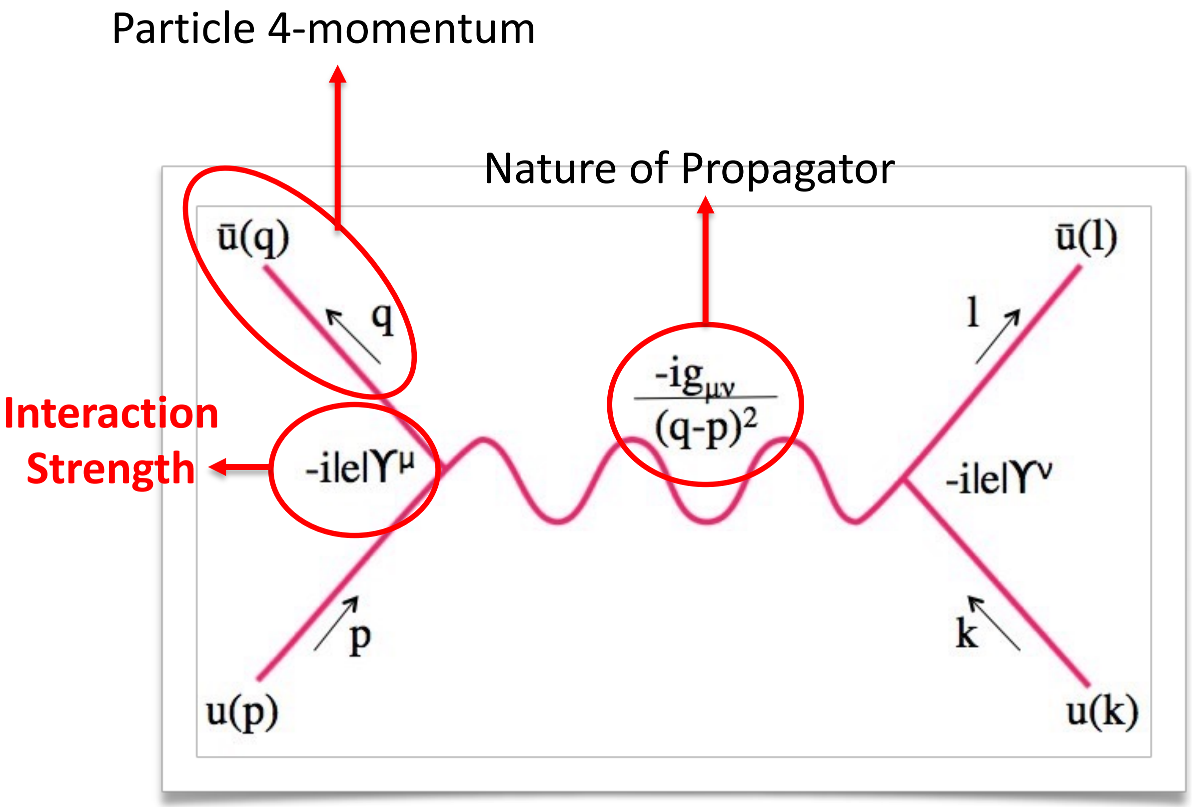


Calculations

How do you know how many Feynman diagrams to use?

One of the pieces that comes into play is the relative interaction strength

Force	Relative Strength
Strong	1
Electromagnetic	1/137
Weak	10^{-6}



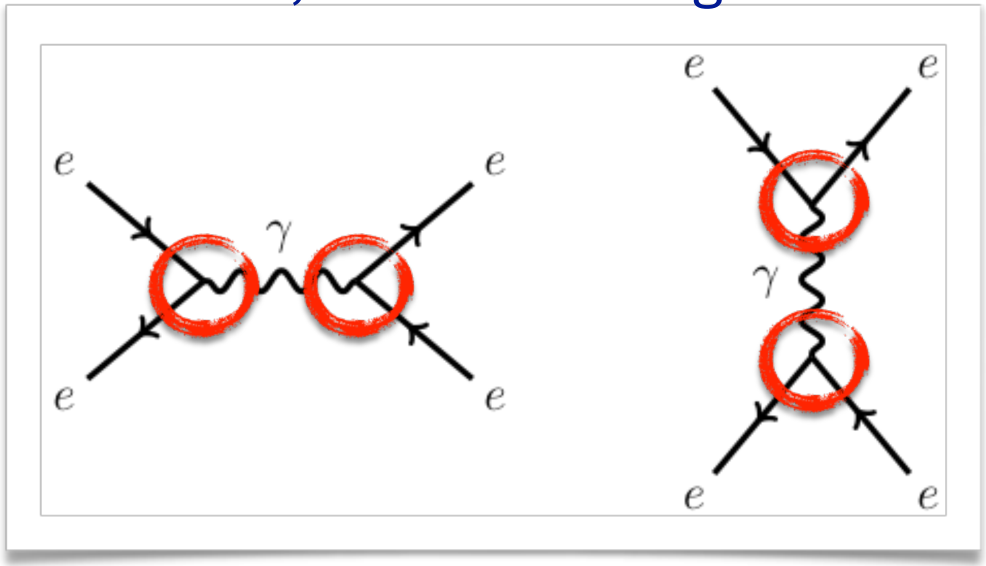
Order of Diagrams

For fixed particle momenta, QED interactions have a fixed interaction strength.

- Thus for a given diagram, we can count up the number of vertex factors.
- We refer to this as the order of the diagram.

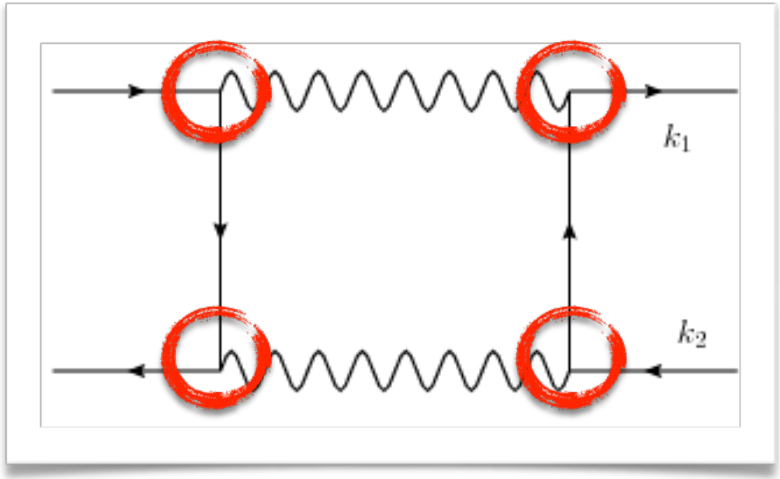
"Leading Order" diagrams

- 2 Vertices
- aka, Tree Level diagrams



"Second Order" diagrams

- 4 Vertices
- aka, Next-to-leading diagrams (NLO)

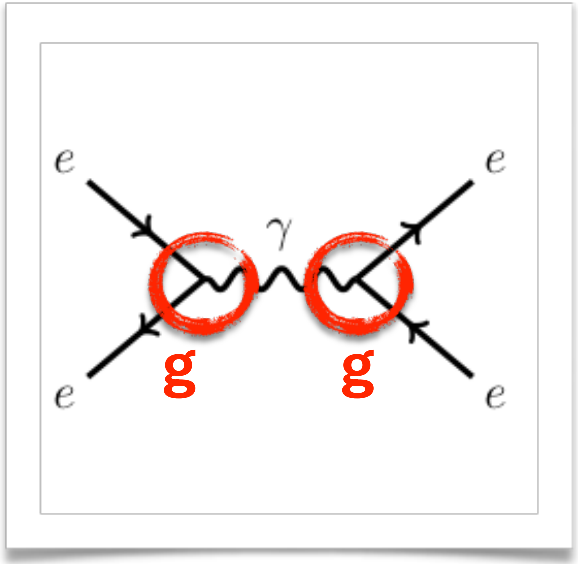


Vertex Factors & Probability

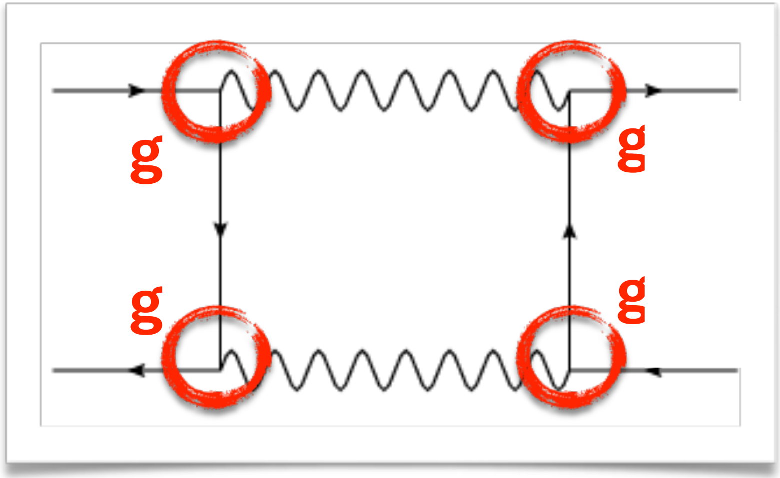
For fixed particle momenta, QED interactions have a fixed interaction strength.

- Thus for a given diagram, we can count up the number of vertex factors.
- We refer to this as the order of the diagram.

$$\sigma \propto |\mathcal{M}_1|^2 \propto g^2$$



$$\sigma \propto |\mathcal{M}_2|^2 \propto g^4$$

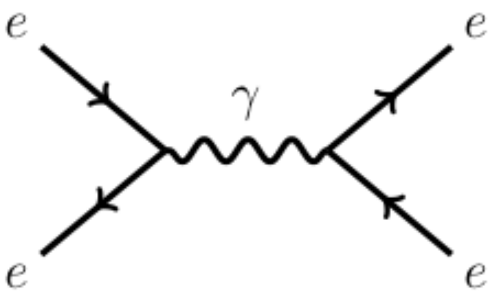


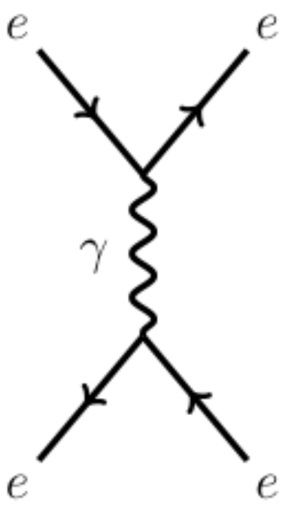
Suppressed by factor g^2.
For QED, $g^2 \sim 1/137$

Coupling Strength: QED

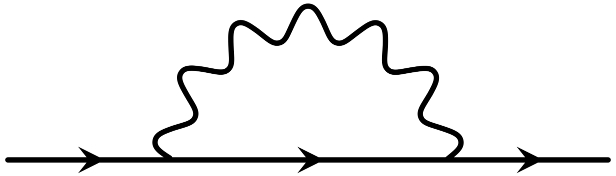
Each QED vertex introduces a factor of 1/137 -> don’t need to consider more diagrams to very well predict the process

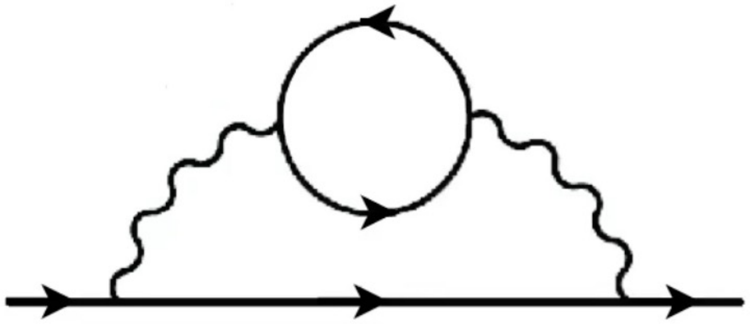
Base process:





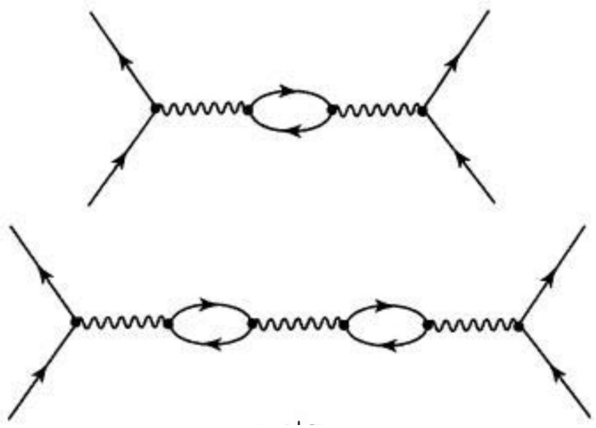
Can mostly ignore:





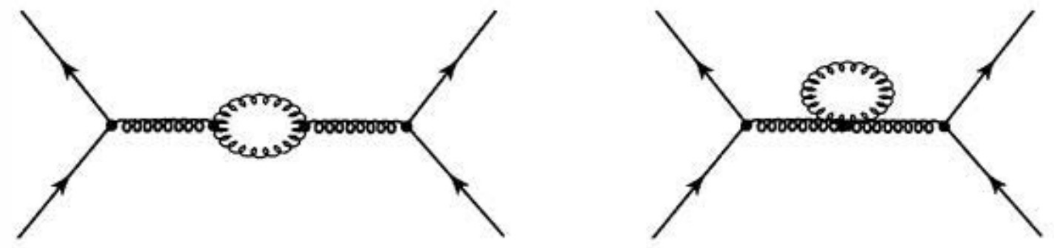
Coupling Strength: QCD

Quark Loops:



-> increase coupling

Gluon loops:



-> decrease coupling

Critical point: $a = 2f - 11n$

$f = 6$ = number of quarks
$n = 3$ = number of colors

➔ $a = -21$, so coupling decreases at short distance
➔ Allows us to do calculations at these small distances

The Matrix Element

We can use the information encoded in the Feynman diagram to calculate things like the reaction cross section, the properties of the decay of particles and resonance phenomena.

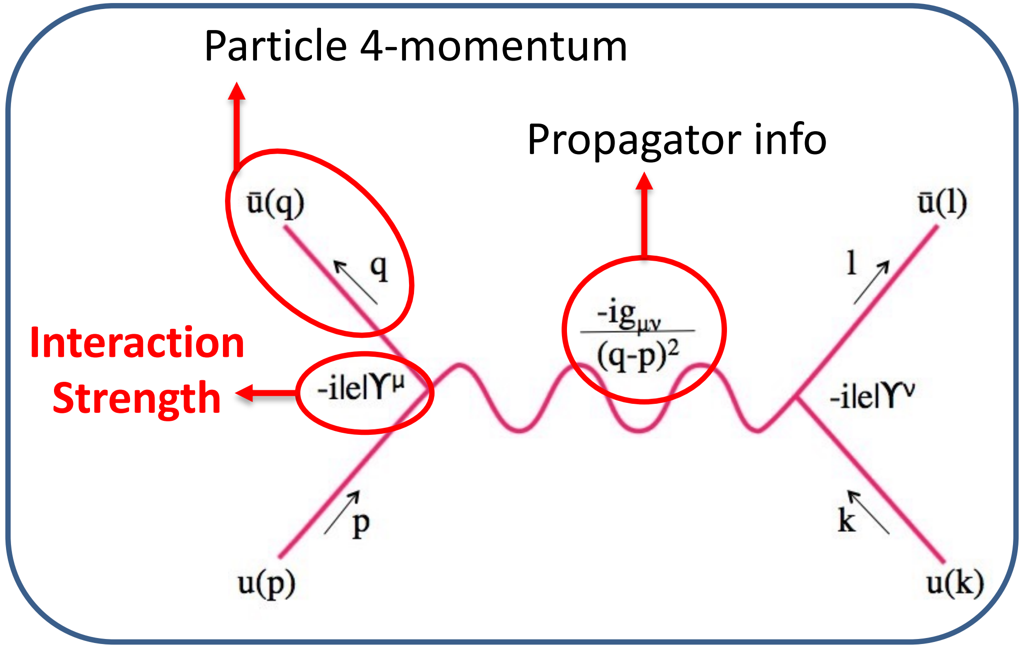
Cross section depends on the square of $\mathcal{M}$

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2}\frac{q_f^2}{v_i v_f}|M|^2$$

Decay rates depend on the square of $\mathcal{M}$

$$d\Gamma(A \rightarrow a + \cdots n) = \frac{1}{2m_A}|M|^2\, dQ$$

Matrix Element for a given reaction ($\mathcal{M}$) is encoded in the Feynman diagram. Also known as the **Scattering Amplitude**.



Particle Decays

In general, all particles are unstable and can decay to other particles. Rough rules:

1) You cannot decay to a higher energy (mass) final state. A W boson cannot create real (*ie, not virtual*) W→tb decays because $M_W < M_{Top} + M_{Bottom}$
2) If there is not a lower energy/mass state, the particle cannot decay. Electrons do not decay because there is no lighter charged lepton.
3) Must satisfy conservation rules. Eg, charge conservation, lepton/quark number, etc.

Each particle decay is associated with a lifetime τ and a "natural decay width" Γ

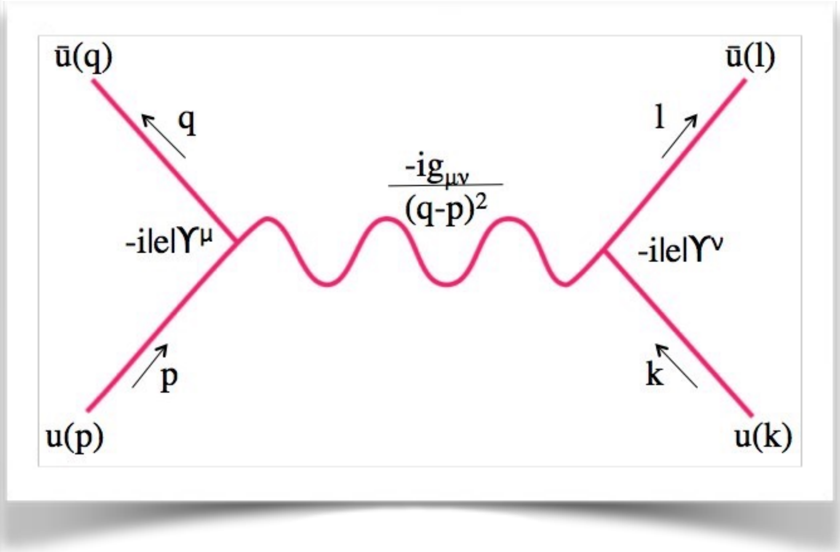
$$\Gamma = 1/\tau$$

Particle abundance has exponential behavior:

$$N(t) = N_0 e^{-\Gamma t}$$

Decay rates depend on the square of $\mathcal{M}$

$$d\Gamma(A \to a + \cdots n) = \frac{1}{2m_A} |M|^2 \, dQ$$



Natural Width & Lifetime

In general, all particles are unstable and can decay to other particles. Rough rules:

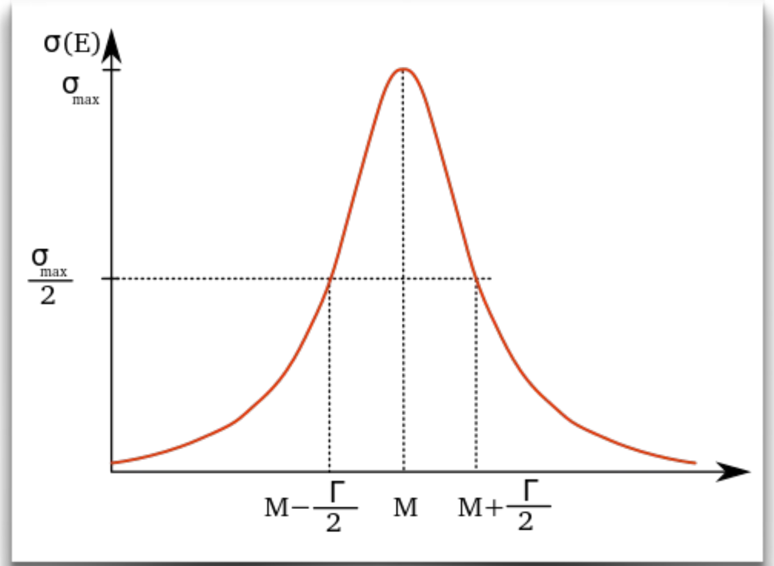
1) You cannot decay to a higher energy (mass) final state. A W boson cannot create real (*ie, not virtual*) $W \rightarrow tb$ decays because $M_W < M_{Top} + M_{Bottom}$
2) If there is not a lower energy/mass state, the particle cannot decay. Electrons do not decay because there is no lighter charged lepton.
3) Must satisfy conservation rules. Eg, charge conservation, lepton/quark number, etc.

If a particle can exhibit many different decays (eg, $Z \rightarrow e^+e^-$, $Z \rightarrow \mu^+\mu^-$, etc) then the width is the sum of the partial widths.

$$\Gamma = \sum \Gamma_i$$

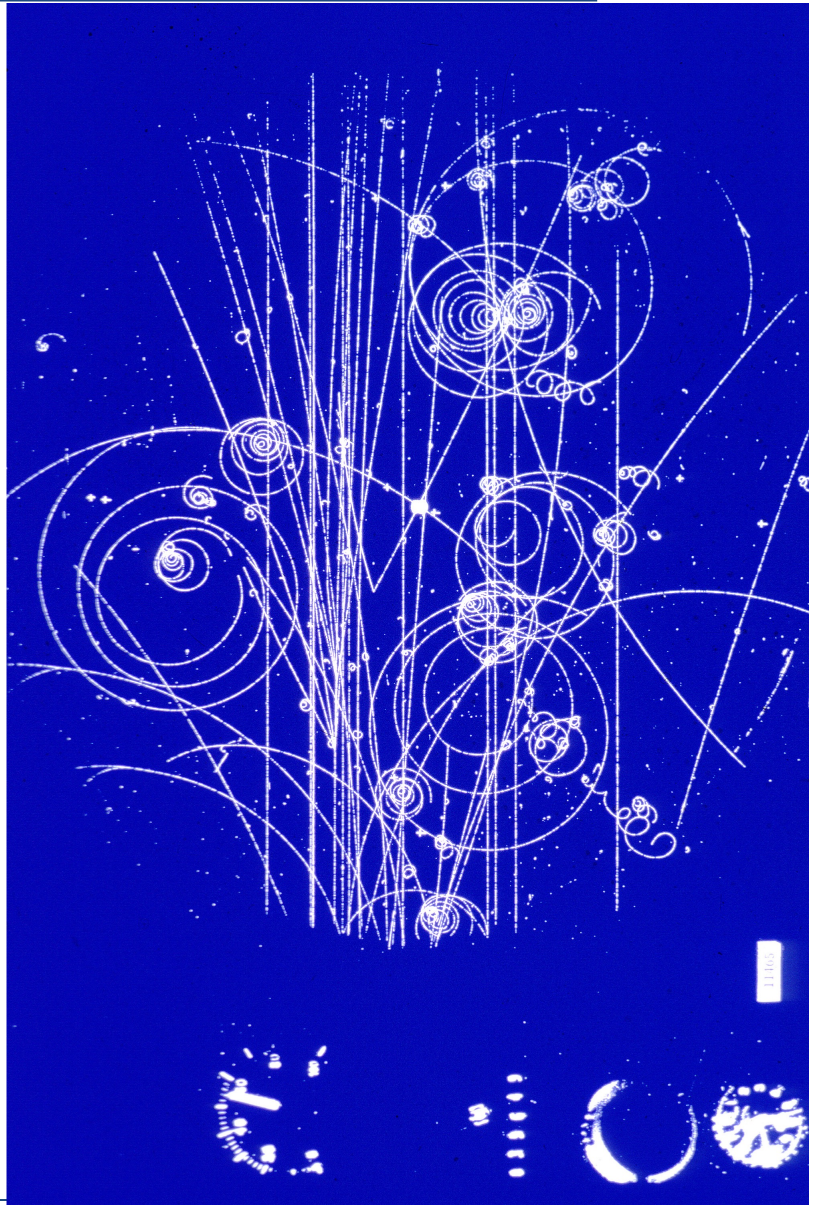
This natural width can be observed in experiments that produce particles "on resonance". Breit-Wigner distribution has a width related to the natural width:

$$P(E) = \frac{1}{2\pi} \frac{\Gamma}{(E-M)^2 + \Gamma^2/4}$$



P: probability to produce the particle
E: center-of-mass energy of the collision
M: mass of the particle being produced

Recap / Up Next



This time:

Particle dynamics & interactions
Exchange forces
Conservation laws
Feynman diagram basics

Next time:

Relativity!