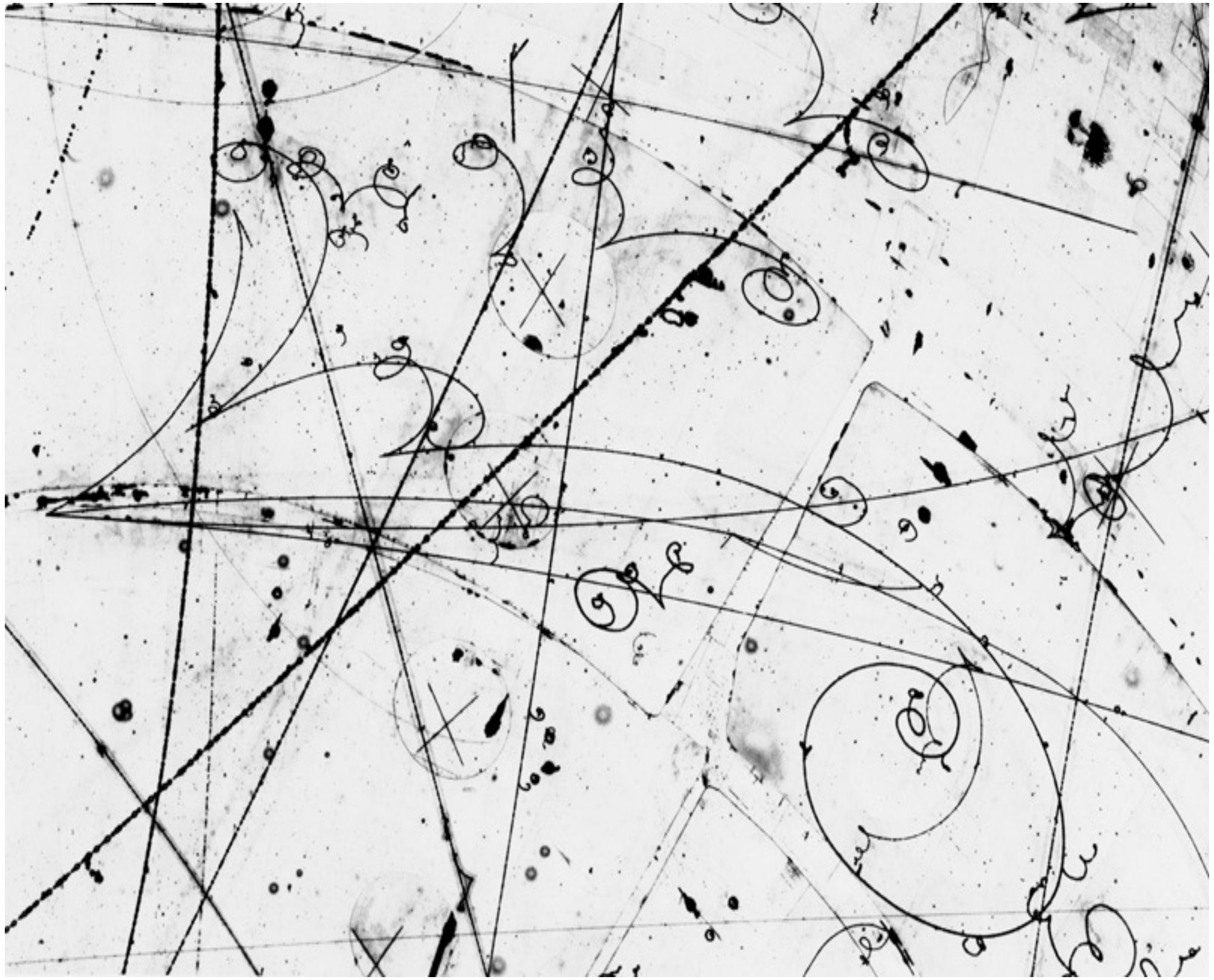
Announcements

Homework:

-HW2 posted on D2L.
-Due on gradescope Monday Feb. 10 at 3pm
-Submit on gradescope

Quizzes:

-Next quiz today
-Graded quizzes at the front of class if you haven't gotten yours yet



Kinematics of Collisions

Some basic rules about relativistic collisions:

- Total energy is always conserved
- Momentum is conserved

Einstein has already showed us how to reconcile the issue of energy & mass conservation

- Kinetic energy might not be conserved *This is also true of classical collisions*
- Mass may not be conserved *This never happens in classical collisions*

Types of Collisions

Some basic rules about relativistic collisions:

- Total energy is always conserved
- Momentum is conserved
- Kinetic energy might not be conserved
- Mass may not be conserved

1. **Elastic collisions:** Kinetic energy is conserved. Rest energy and mass are conserved
2. **Inelastic collisions:**
   1. Kinetic energy decreases: rest energy and mass increase
   2. Kinetic energy increases: rest energy and mass decrease

Collisions

Particle physics frequently relies on high-energy collisions of particles to learn about nature.

Fixed Target Collision:

Colliding Beams:

Lorentz Invariance in Collisions

Remember Lorentz invariance when you think about collisions!

Example collision:

Laboratory frame:

"Boost" along common axis until total momentum is zero

Center-of-Momentum frame:

Note: The Lab frame can BE the CoM frame!

C-o-Mass vs C-o-Momentum?

What is the difference between the Center-of-Mass and Center-of-Momentum inertial frames?

$$v_{CM} = \frac{\sum m_i v_i}{\sum m_i} = \frac{p_{total}}{\sum m_i} = 0$$

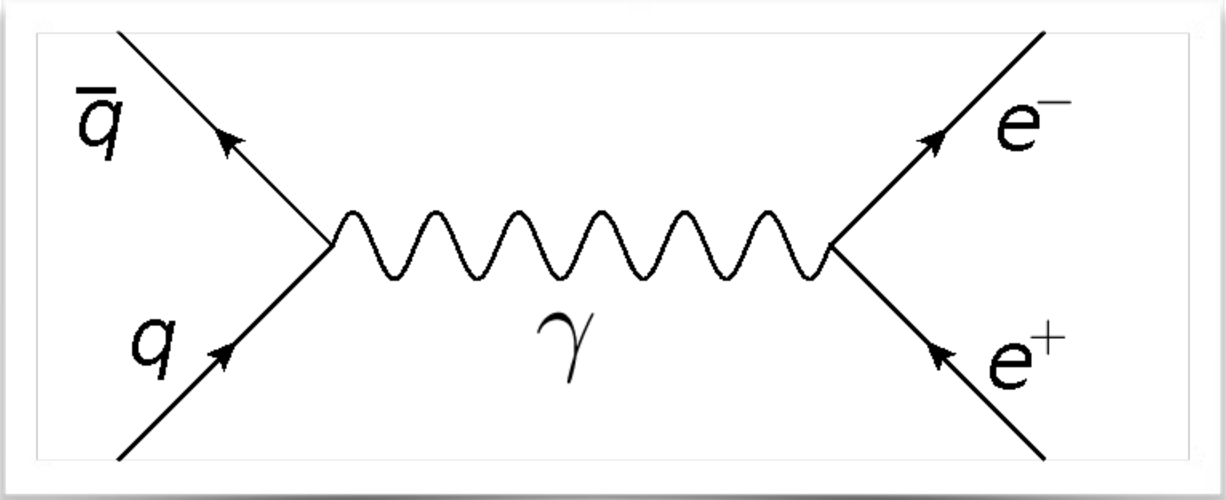
When the center-of-mass isn't moving, the total momentum is 0.

However, the center-of-mass frame technically means an inertial frame where the center-of-mass is at the origin of the coordinate system.

For the calculations in this class, the difference doesn't really matter much either way!

More on Collisions

A relativistic collision



Momentum & Energy Conservation ⟶ $$p^\mu_{\bar{q}} + p^\mu_q = p^\mu_{e^-} + p^\mu_{e^+}$$

Lorentz Invariance ⟶ $$s^2 = E_i^2 - p_i^2 c^2 = E_f^2 - p_f^2 c^2$$

Collisions

<u>Fixed Target Collision:</u>

Fixed Target Collisions

Fixed Target Collision:

$$p_b^\mu = \begin{pmatrix} E_b \\ p_{x,b} \\ p_{y,b} \\ p_{z,b} \end{pmatrix} \qquad p_t^\mu = \begin{pmatrix} m_t \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$p_{\text{tot}}^\mu = p_b^\mu + p_t^\mu = \begin{pmatrix} E_b + m_t \\ p_{x,b} \\ p_{y,b} \\ p_{z,b} \end{pmatrix}$$

-> Similar to decays, where initial particle is at rest

Example:

- A pion decays into a muon and a neutrino. What is the energy of the muon?

- 1. Picture

2. Conservation of momentum:

$$p_\pi = p_\mu + p_\nu$$ -> pion at rest initially, no momentum

$$p_\mu = -p_\nu$$

Example:

- A pion decays into a muon and a neutrino. What is the energy of the muon in terms of the particle masses?
- 3. Conservation of energy -> Use the invariant $E^2 - \mathbf{p}^2c^2 = m^2c^4$

$$E_\pi = E_\mu + E_\nu$$

where

$$E_\pi = m_\pi c^2$$

$$E_\mu = c\sqrt{m_\mu^2 c^2 + \mathbf{p}_\mu^2}$$

$$E_\nu = |\mathbf{p}_\nu| c = |\mathbf{p}_\mu| c$$

$$E^2 - \mathbf{p}^2c^2 = m^2c^4$$

$$m_\pi c^2 = c\sqrt{m_\mu^2 c^2 + \mathbf{p}_\mu^2} + |\mathbf{p}_\mu| c \longrightarrow |\mathbf{p}_\mu| = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} c \longrightarrow E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} c^2$$

With 4-Vectors

- A pion decays into a muon and a neutrino. What is the energy of the muon?
- Previous solution treated energy and momentum separately

$$\begin{pmatrix} E_\pi \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} E_\mu \\ px_\mu \\ py_\mu \\ pz_\mu \end{pmatrix} + \begin{pmatrix} E_\nu \\ px_\nu \\ py_\nu \\ pz_\nu \end{pmatrix}$$

With 4-Vectors

- A pion decays into a muon and a neutrino. What is the energy of the muon?
- Can also solve without breaking into components -> less algebra

1. Conservation of 4-momentum: $p_\pi = p_\mu + p_\nu \longrightarrow p_\nu = p_\pi - p_\mu$

2. Scalar Product of each side with itself: $p_\nu^2 = p_\pi^2 + p_\mu^2 - 2p_\pi \cdot p_\mu$

3. Apply the invariant: $p_\mu p^\mu = \frac{E^2}{c^2} - \mathbf{p}^2 = m^2c^2$ -> for any real particle

So:

$$p_\nu^2 = 0; \quad p_\pi^2 = m_\pi^2 c^2, \quad p_\mu^2 = m_\mu^2 c^2;$$

$$p_\pi \cdot p_\mu = \frac{E_\pi}{c}\frac{E_\mu}{c} = m_\pi E_\mu$$

$$\longrightarrow \quad E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} c^2$$

-> three momentum components are 0 because π momentum is 0

Collisions

Colliding Beams:

Colliding Beams

Colliding Beams:

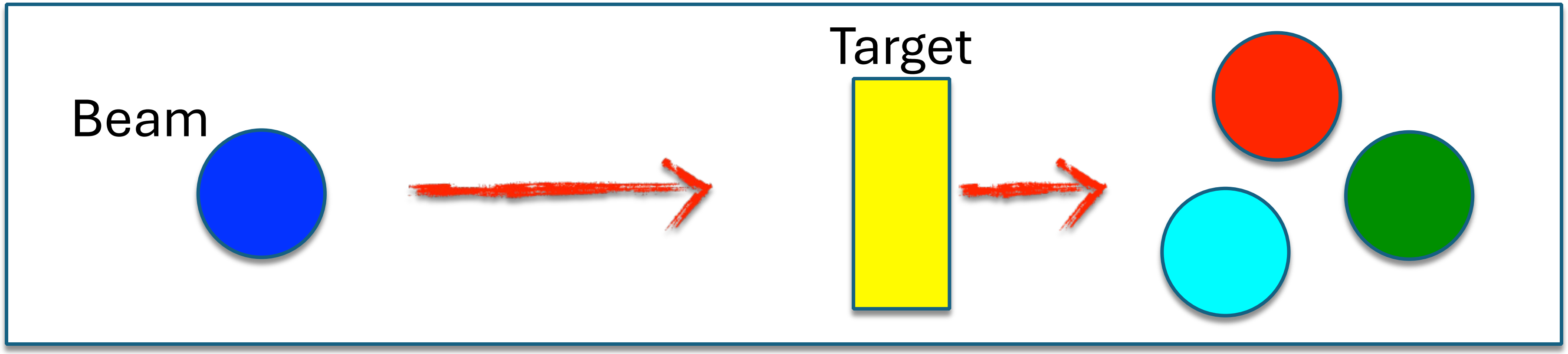
$$p^{\mu}_{\rm tot} = p^{\mu}_{b,1} + p^{\mu}_{b,2}$$

$$p^{\mu}_{\rm tot} = \begin{pmatrix} 2E_b \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Momentums cancel out

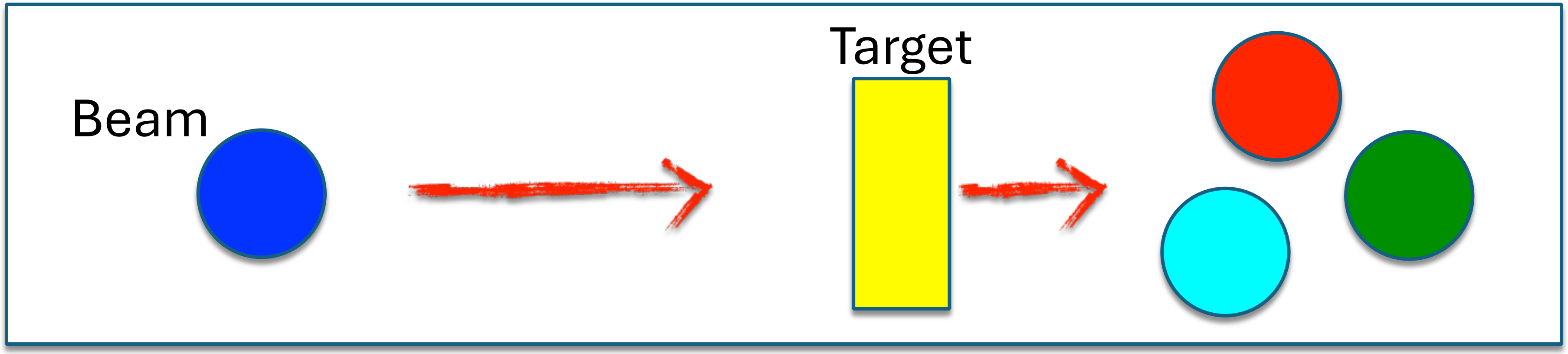
Energy Thresholds

What is the minimum energy required for a beam particle to produce a specific final state reaction?



Energy Thresholds

What is the minimum energy required for a beam particle to produce a specific final state reaction?



Short, simple answer:

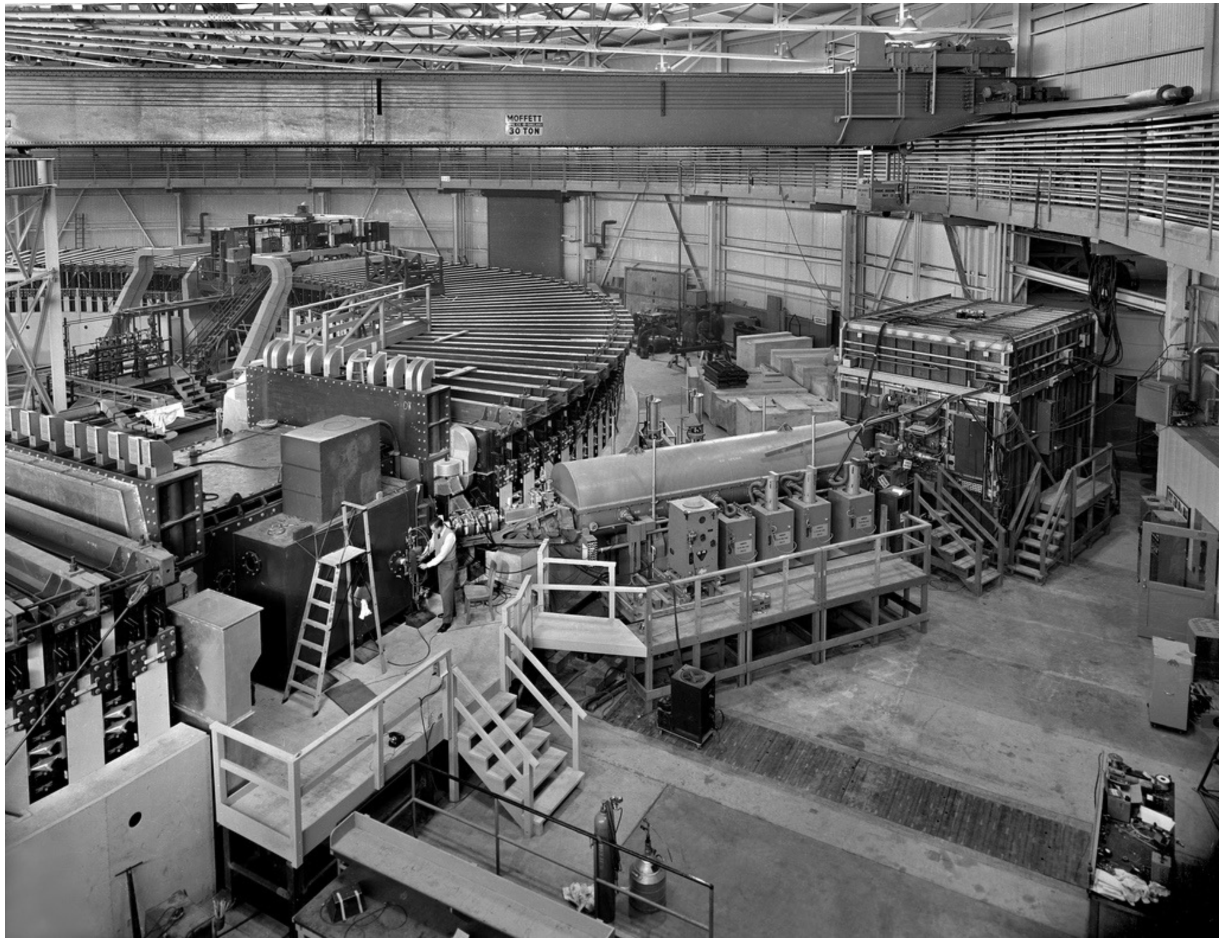
- The minimum (or threshold) energy for a reaction to occur creates a final state with ZERO MOMENTUM.
- All the energy supplied goes into mass, not kinetic energy.

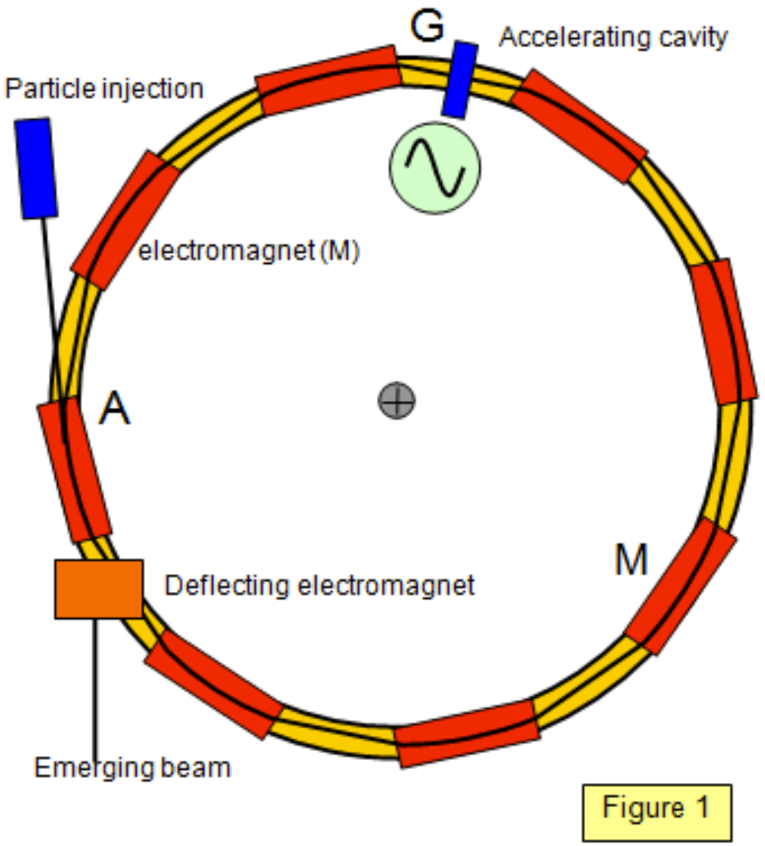
Bevatron (1954)

Built to discover anti-protons through proton-proton collisions

What is the threshold energy for this interaction?

$$p + p \rightarrow p + p + p + \bar{p}$$





Bevatron (1954)

Built to discover anti-protons $$p + p \rightarrow p + p + p + \bar{p}$$

What is the threshold energy for this interaction?

Lab frame:

Center of mass frame:

Bevatron (1954)

Built to discover anti-protons $$p + p \to p + p + p + \bar{p}$$

What is the threshold energy for this interaction?

Total 4-momentum in lab frame :
(before)

$$p^{\mu}_{TOT} = \left(\frac{E + mc^2}{c}, |\mathbf{p}|, 0, 0 \right)$$

E, **p** = incident proton energy, momentum
m = proton mass

Total 4-momentum in CM frame:
(after)

$$p^{\mu\,\prime}_{TOT} = (4mc, 0, 0, 0)$$

-> cannot directly compare these, but can compare the invariants!

$$p_\mu p^\mu = \frac{E^2}{c^2} - \mathbf{p}^2 = m^2c^2$$

$$\left(\frac{E}{c} + mc \right)^2 - \mathbf{p}^2 = (4mc)^2$$

-> write **p** in terms of E and m and solve $E = 7mc^2$

Threshold energy is, with proton mass = 1 GeV:
E_b =7 GeV and E_{kin} = 6 GeV

Bevatron (1954)

Built to discover anti-protons $$p + p \rightarrow p + p + p + \bar{p}$$

-> cannot directly compare these, but can compare the invariants! $$p_\mu p^\mu = \frac{E^2}{c^2} - \mathbf{p}^2 = m^2c^2$$

$$\left(\frac{E}{c} + mc\right)^2 - \mathbf{p}^2 = (4mc)^2$$

-> write **p** in terms of E and m and solve $E = 7mc^2$

-> this is total beam energy, subtract m to get KE: $$E_{kin} = 6mc^2$$

Proton mass ~ 1 GeV:

E_b =7 GeV and $E_{kin} = 6$ GeV

-> Anti protons were discovered at about this threshold! First observation of anti-matter.

Threshold Energy

- **Fixed target:** A beam with total energy E_b hits a fixed target. What is the most massive particle that can be produced?
  - The COM energy is the Mass of the heaviest particle that can be produced, i.e. $M = \sqrt{s^2} = \sqrt{2m_p^2 + 2m_p E_b}$
  - For large beam energies E_1, it goes as $\sqrt{E_b}$
- **Colliding:** Two proton beams colliding, each with energy E_b. What is the most massive particle that can be produced?
  - Mass $M = \sqrt{s^2} = 2E_b$
  - For large beam energies E_b, it goes as E_b, i.e. linear

Recap / Up Next

This time:

Special Relativity

Relativistic Collisions

Next time:

Symmetries

Group Theory

Conservation Laws