Problem 1

Kittel & Kroemer, Chapter 3, problem 1 [Free energy of a two state system]

(a): 1 point

Find an expression for the free energy as a function of τ of a system with two states, one at energy 0 and one at energy ϵ.

(b): 1 point

From the free energy, find expressions for the energy and entropy of the system. The entropy is plotted in Figure 3.11.

Problem 2

Adapted from: Kittel & Kroemer, Chapter 3, problem 3 [Free energy of a harmonic oscillator]

A one-dimensional harmonic oscillator has an infinite series of equally spaced energy states, with $\epsilon_s = s\hbar\omega$, where s is a positive integer or zero, and ω is the classical frequency of the oscillator. We have chosen the zero of the energy at the state $s = 0$.

(a): 1 point

Find the partition function for this system. The answers for (b), (c), and (d) in this problem will follow from this starting point. Note: do not leave your answer as an infinite sum! See if you can work out an exact expression for Z.

(b): 1 point

Show that for a harmonic oscillator the free energy is

$$F = \tau \ln[1 - e^{-\hbar\omega/\tau}]$$

Note that at high temperatures such that $\tau \gg \hbar\omega$ we may expand the argument of the logarithm to obtain $F \simeq \tau \ln(\hbar\omega/\tau)$.

(c): 1 point

Find the internal energy U.

(d): 1 point

Show that the entropy is

$$\sigma = \frac{\hbar\omega/\tau}{e^{\hbar\omega/\tau} - 1} - \ln[1 - e^{-\hbar\omega/\tau}].$$

Problem 2 continued on next page. . .

(e): 1 point

Compare your answer for U to what you obtained in problem 3 of Homework 2, where you calculated the energy of N harmonic oscillators. Is the answer you obtained there equal to N times the answer you obtained for U in this problem? Write it down, and explain how they are related.

(f): 1 point

Compare your answer for σ to what you obtained in problem 3 of Homework 2. They look different, because one is a function of U while the other is a function of τ. Show that they are the same (except for the factor of N) by substituting $U(\tau)$ into the expression for σ you obtained in Homework 2.

Problem 3

In this problem you will connect the "fundamental" quantities σ, τ we have been working with, with the "conventional" quantities S and T.

Consider a block of copper at a temperature $T = 300$ K.

(a): 1 point

How much heat energy (in Joules) would you have to add to the block to double the number of accessible quantum states in the copper? (Assume that the block is thermally isolated from its surroundings, except for the heat you are adding. Also assume that the amount of heat you are adding is small enough that the temperature of the copper remains constant throughout the process.)

(b): 1 point

By how much will the block's entropy S increase when the heat is added? (Note that your answer should have units of J/K.)

Problem 4

Consider a system with 3 quantum states, labeled by index $n = 0, 1, 2$, with respective energies $\epsilon_0 = 0$, $\epsilon_1 = a$, and $\epsilon_2 = 3a$. The system is in thermal equilibrium with a reservoir at temperature τ.

(a): 1 point

What is the probability of finding the system in the $n = 1$ state?

(b): 1 point

What is the average energy of the system? (You could use a general formula, but it is easier to work directly from the Canonical distribution function.)

(c): 1 point

Make a table showing the numerical values of the probabilities for the system to be in each of the 3 states for the following specific values of the temperature: $\tau = 0$, $\tau = a$, and $\tau = 100a$. (Your table should have $3 \times 3 = 9$ entries.) Notice what happens when the temperature is very high.

Problem 5

Consider an N-particle system thermally isolated from its surroundings. The multiplicity g depends on the energy U as $g(U) = e^{\sqrt{NU/U_0}}$, where U_0 is a constant with dimensions of energy.

(a): 1 point

Find expressions for the entropy σ and the temperature τ in terms of N, U, and U_0.

(b): 1 point

Find expressions for the energy U and entropy σ in terms of τ, N, and U_0.

(c): 1 point

Let $N = 6.02 \times 10^{23}$ and $U_0 = 10^{-20}$ J. If the temperature is $T = 300$ K, calculate numerical values (with correct units) for the energy U, entropy S, and free energy F of the system.