Announcements

Homework:

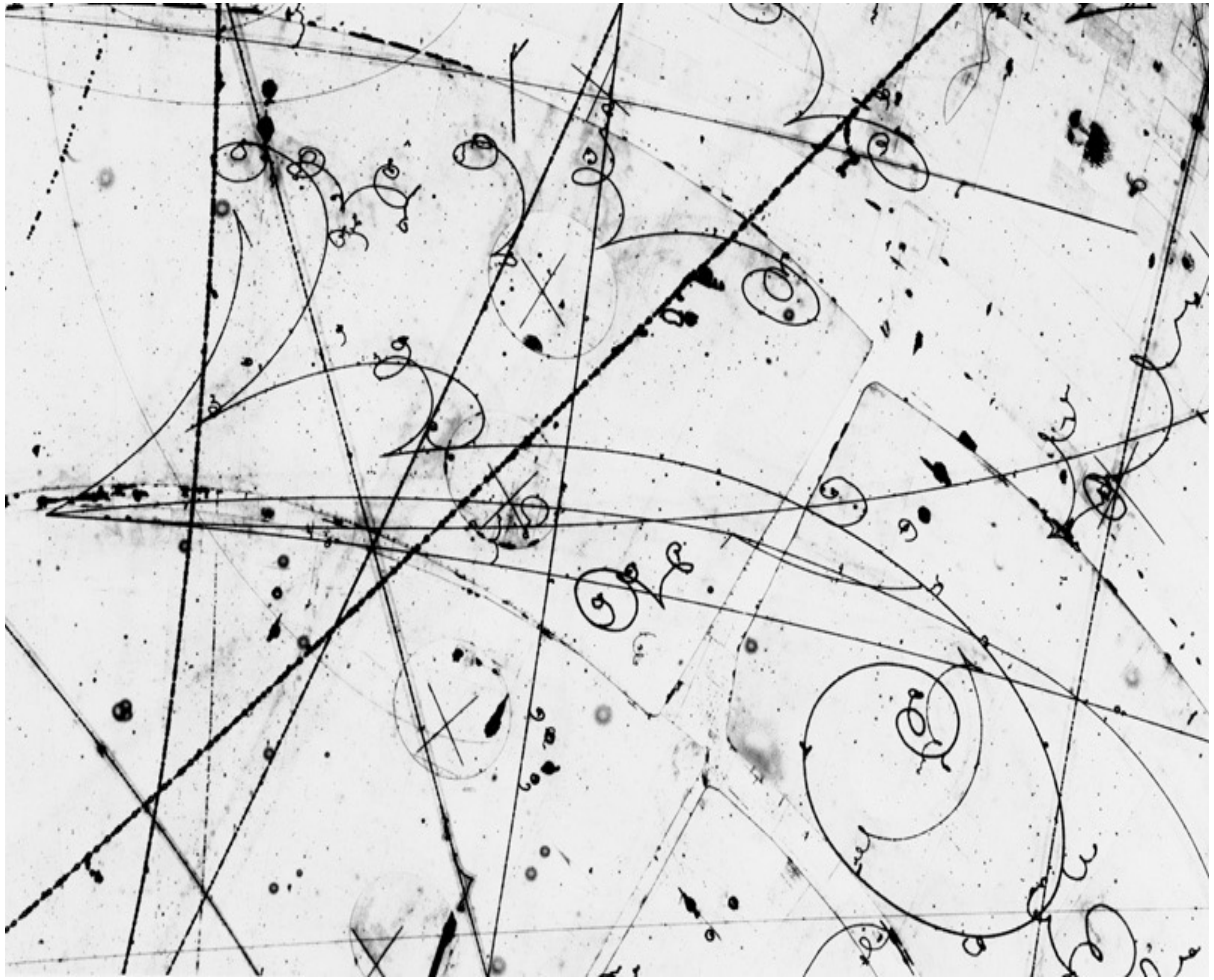
Submit on gradescope; due at the start of class today

Next assignment posted before next class; due two weeks from today

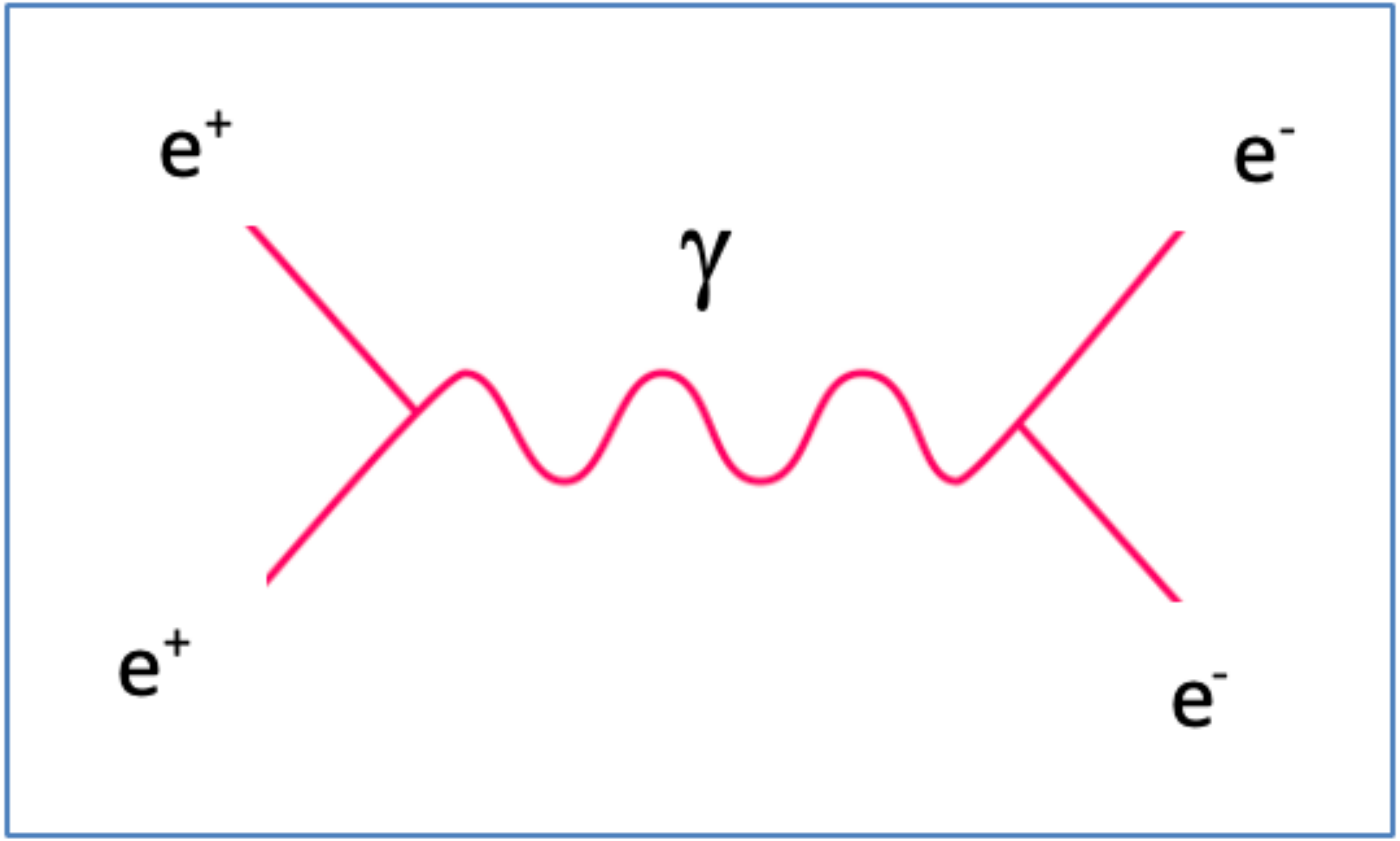
Quizzes:

Pick up quizzes after class

Next quiz on Friday



Is this process allowed?



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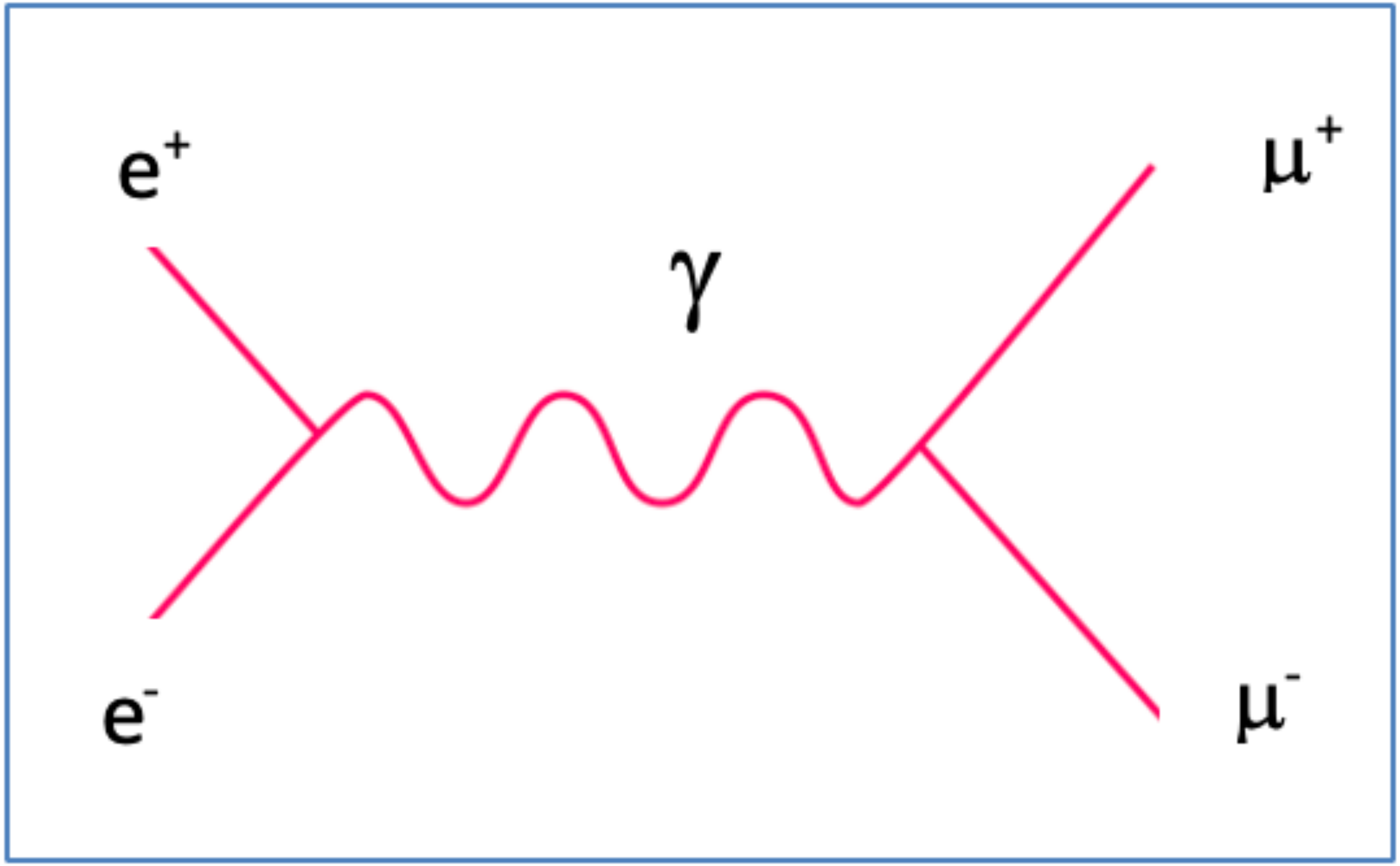
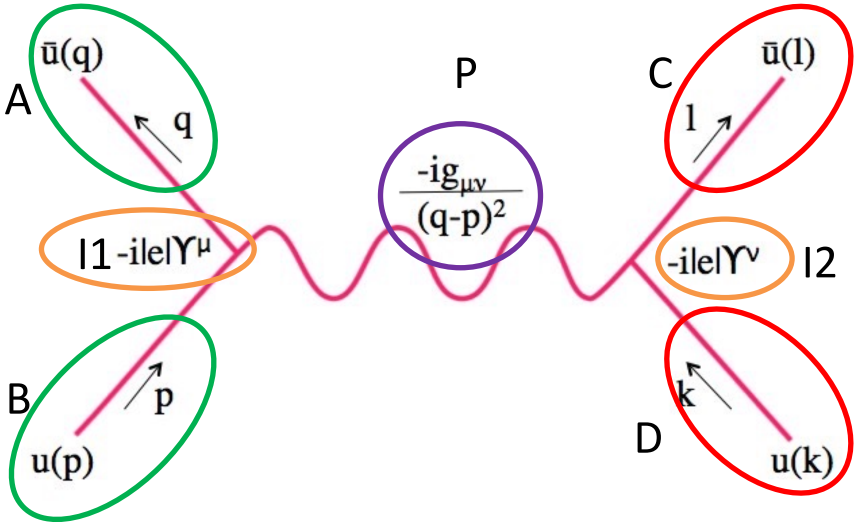


The Matrix Element

We can use the information encoded in the Feynman diagram to calculate things like the reaction cross section, the properties of the decay of particles and resonance phenomena.

A & B: Incoming particle wavefunctions.
P: Propagator wavefunction
C & D: Outgoing particle wavefunctions.
I1 & I2: Interaction strengths

$$\mathcal{M} \propto (\, A \times I1 \times B \,)(\, P \,)(\, C \times I2 \times D \,)$$



The Matrix Element

We can use the information encoded in the Feynman diagram to calculate things like the reaction cross section, the properties of the decay of particles and resonance phenomena.

Main two quantities calculated depend on the square of $\mathcal{M}$:

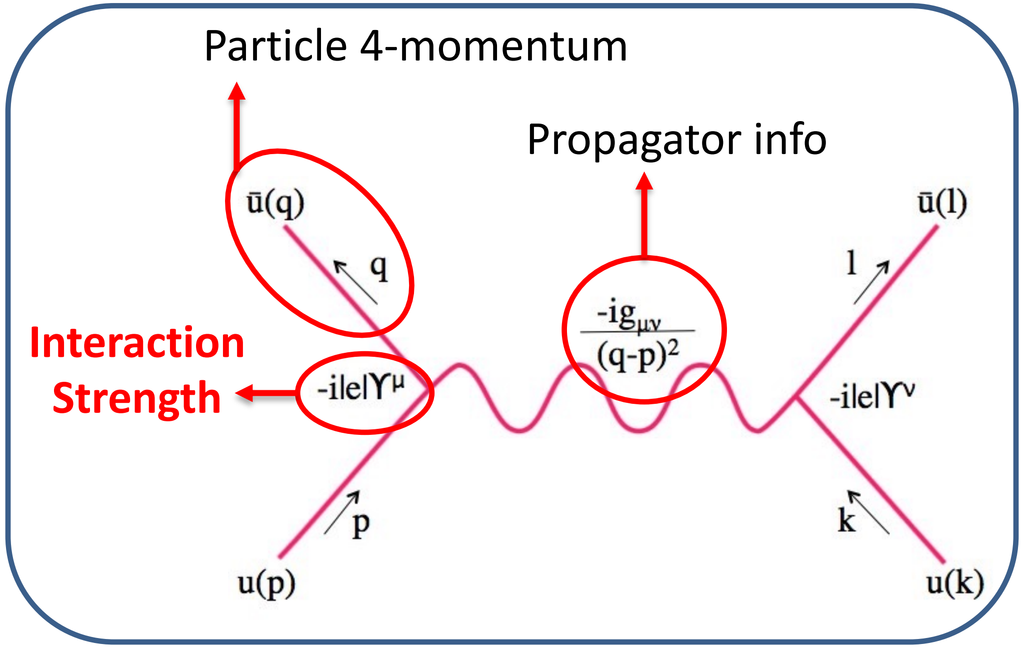
Cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2} \frac{q_f^2}{v_i v_f} |M|^2$$

Decay rates

$$d\Gamma(A \to a + \cdots n) = \frac{1}{2m_A} |M|^2 \, dQ$$

Matrix Element for a given reaction ($\mathcal{M}$) is encoded in the Feynman diagram. Also known as the **Scattering Amplitude**.



Particle Decays

In general, all particles are unstable and can decay to other particles. Rough rules:

1) Cannot decay to a higher energy (mass) final state. For example, a W boson cannot create a real (*ie, not virtual*) W→tb decay because $M_W < M_{Top} + M_{Bottom}$
2) If there is not a lower energy/mass state, the particle cannot decay. Electrons do not decay because there is no lighter charged lepton.
3) Must satisfy all conservation rules. Eg, charge, lepton/quark number, etc.

Each particle decay is associated with a lifetime τ and a "natural decay width" Γ

$$\Gamma = 1/\tau$$

Particle abundance has exponential behavior:

$$N(t) = N_0 e^{-\Gamma t}$$

Decay rates depend on the square of $\mathcal{M}$

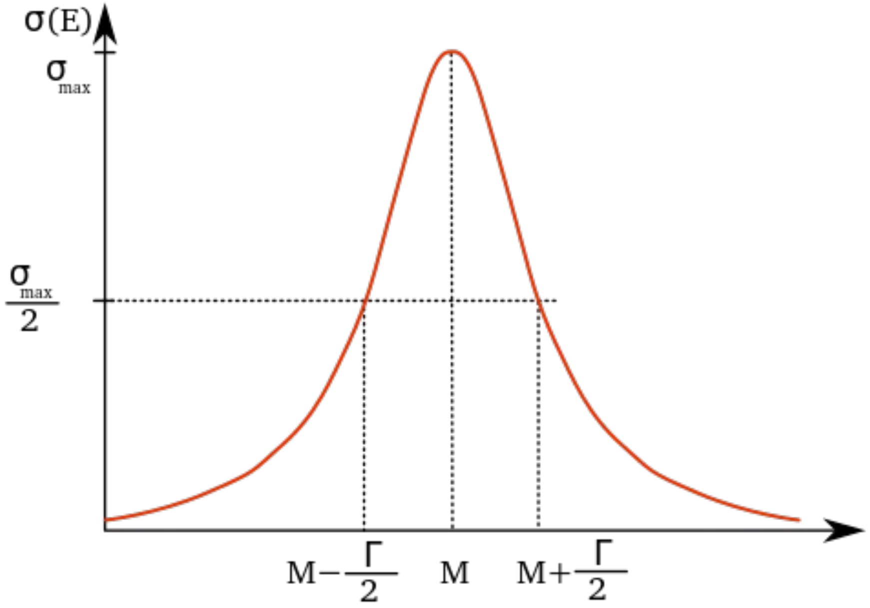
$$d\Gamma(A \to a + \cdots n) = \frac{1}{2m_A}|M|^2 dQ$$

Natural Width & Lifetime

If a particle can exhibit many different decays (eg, $Z \rightarrow e^+e^-$, $Z \rightarrow \mu^+\mu^-$, etc) then the width is the sum of the partial widths.

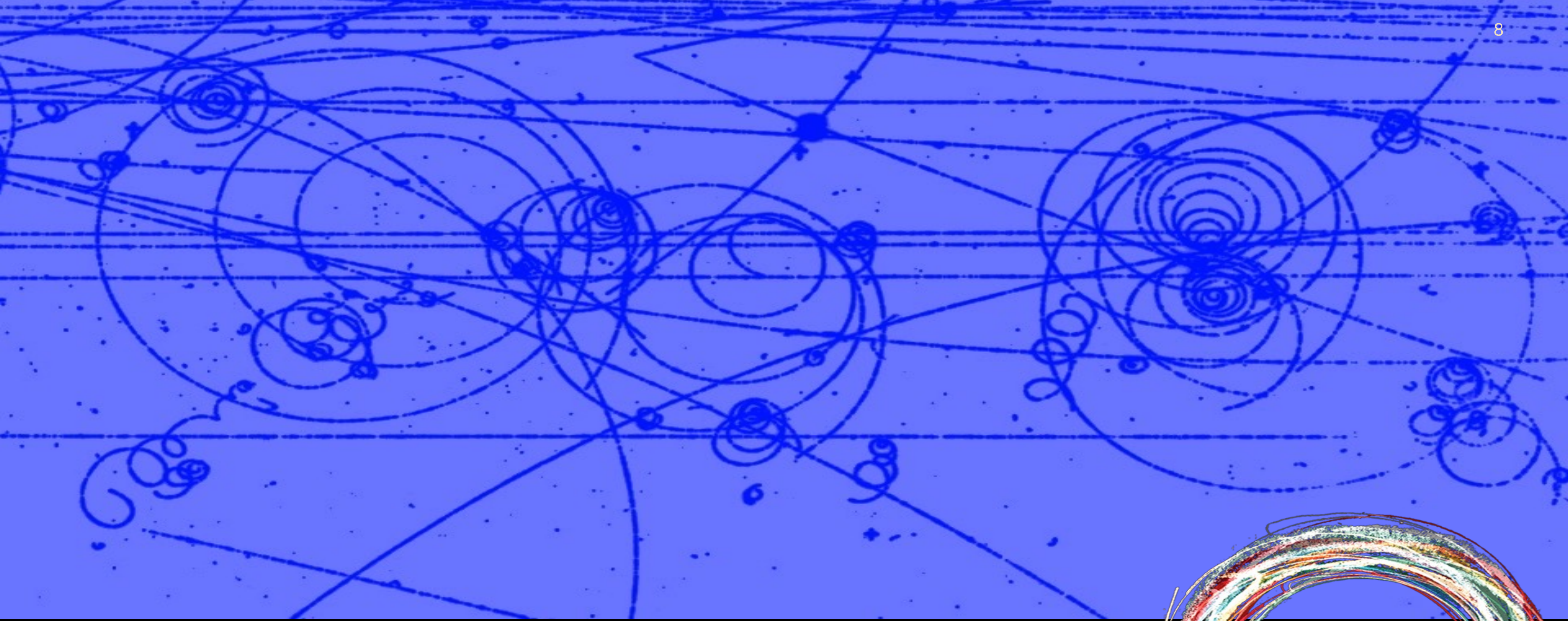
$$\Gamma = \sum \Gamma_i$$

This natural width can be observed in experiments that produce particles "on resonance". Breit-Wigner distribution has a width related to the natural width:

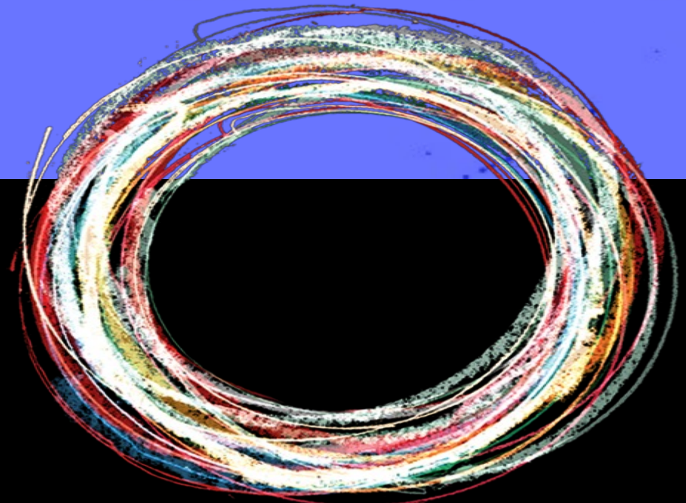


P: probability to produce the particle
E: center of mass energy of the collision
M: mass of the particle being produced

$$P(E) = \frac{1}{2\pi} \frac{\Gamma}{(E-M)^2 + \Gamma^2/4}$$



LECTURE 4
SPECIAL RELATIVITY



PHY 493 / 803

Recap / Up Next

<u>Last time:</u>

Particle Dynamics

- Exchange of force carriers
- Conserved quantities

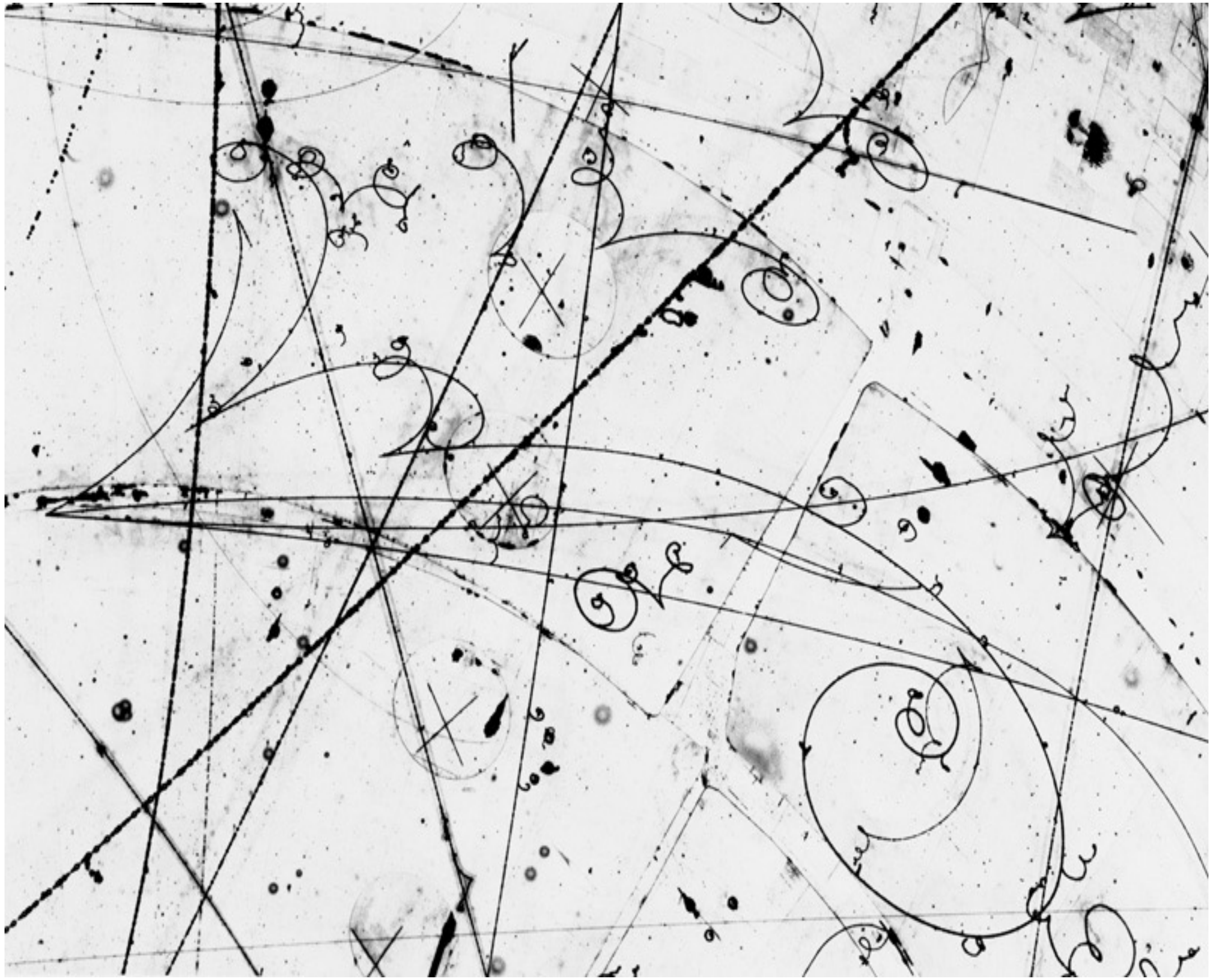
Vacuum polarization

- Asymptotic Freedom

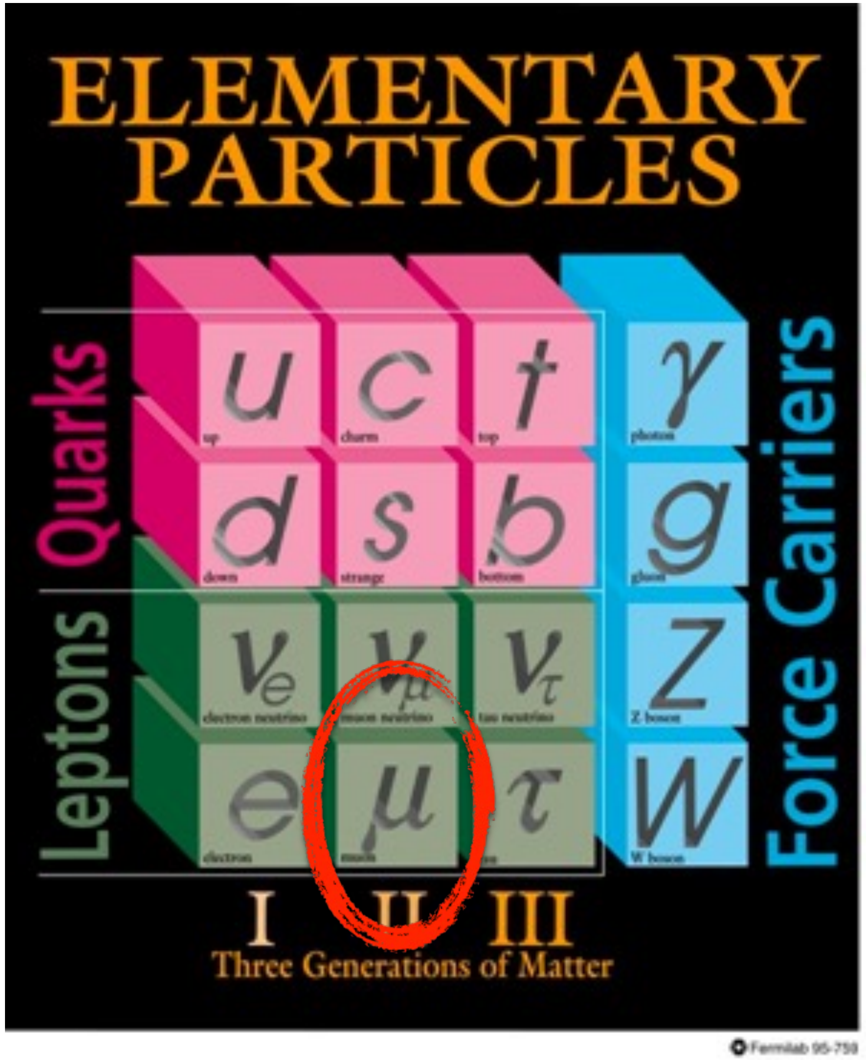
<u>This time:</u>

Special Relativity

Relativistic Collisions

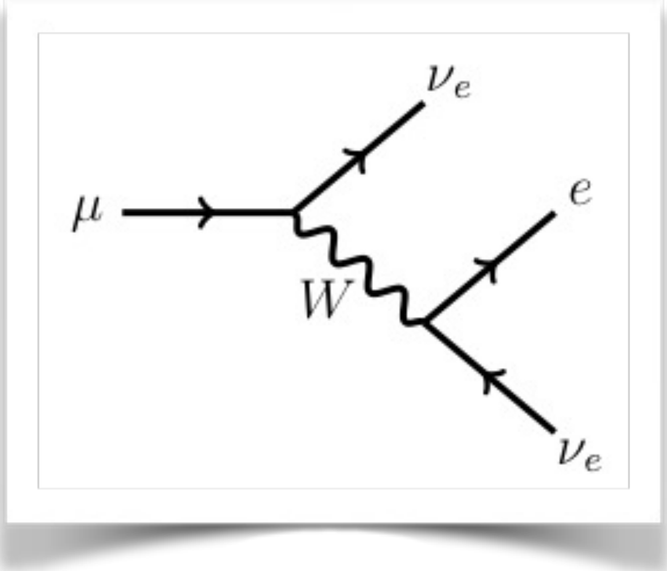


The Muon



Mass: 105.66 MeV/c^2 (~212xM_e)

Not the lightest lepton, so we know it decays



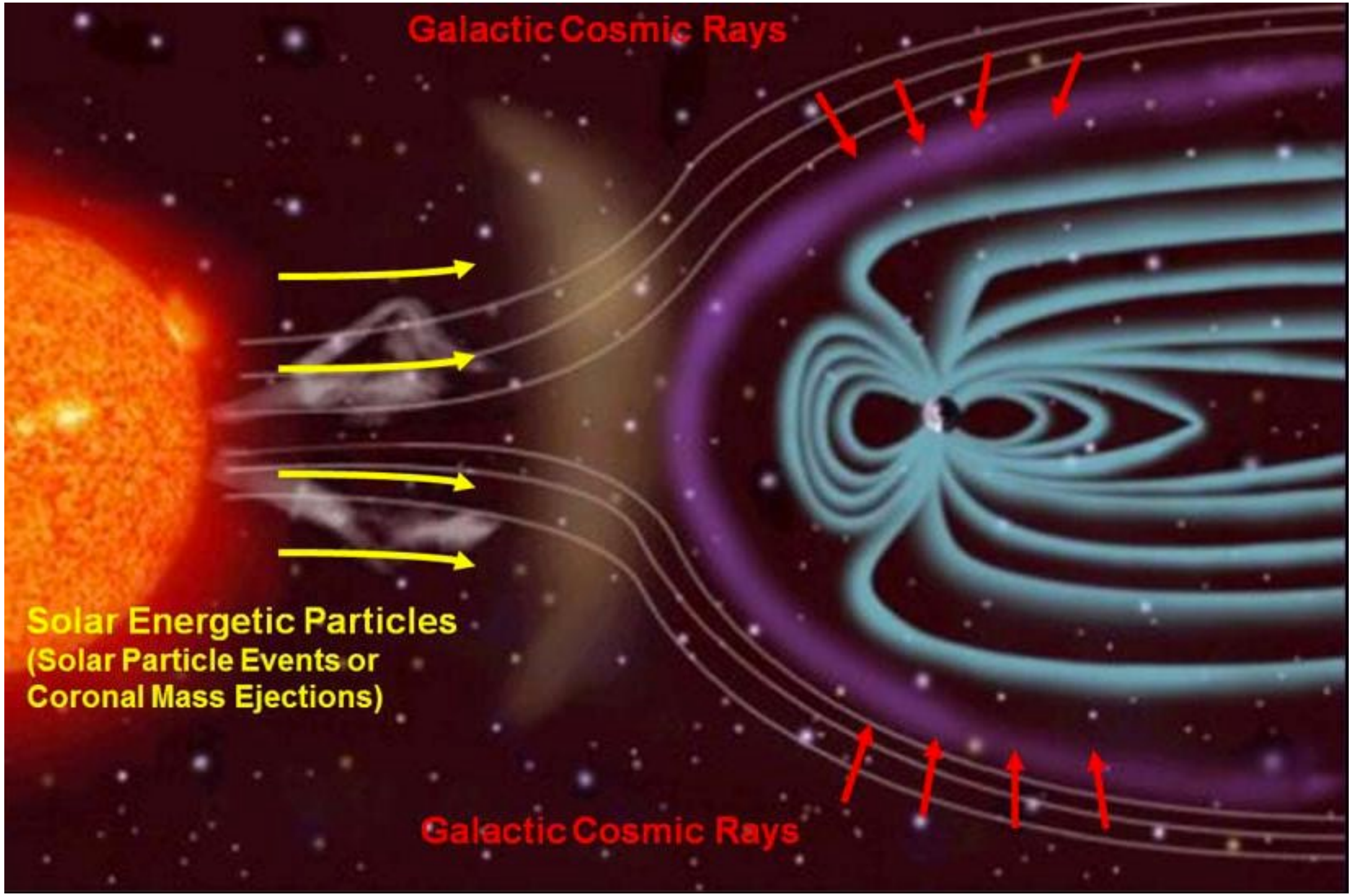
Mean Lifetime: 2.2×10^{-6} sec

Where do we see muons? Cosmic Rays

Earth is constantly being showered with particles

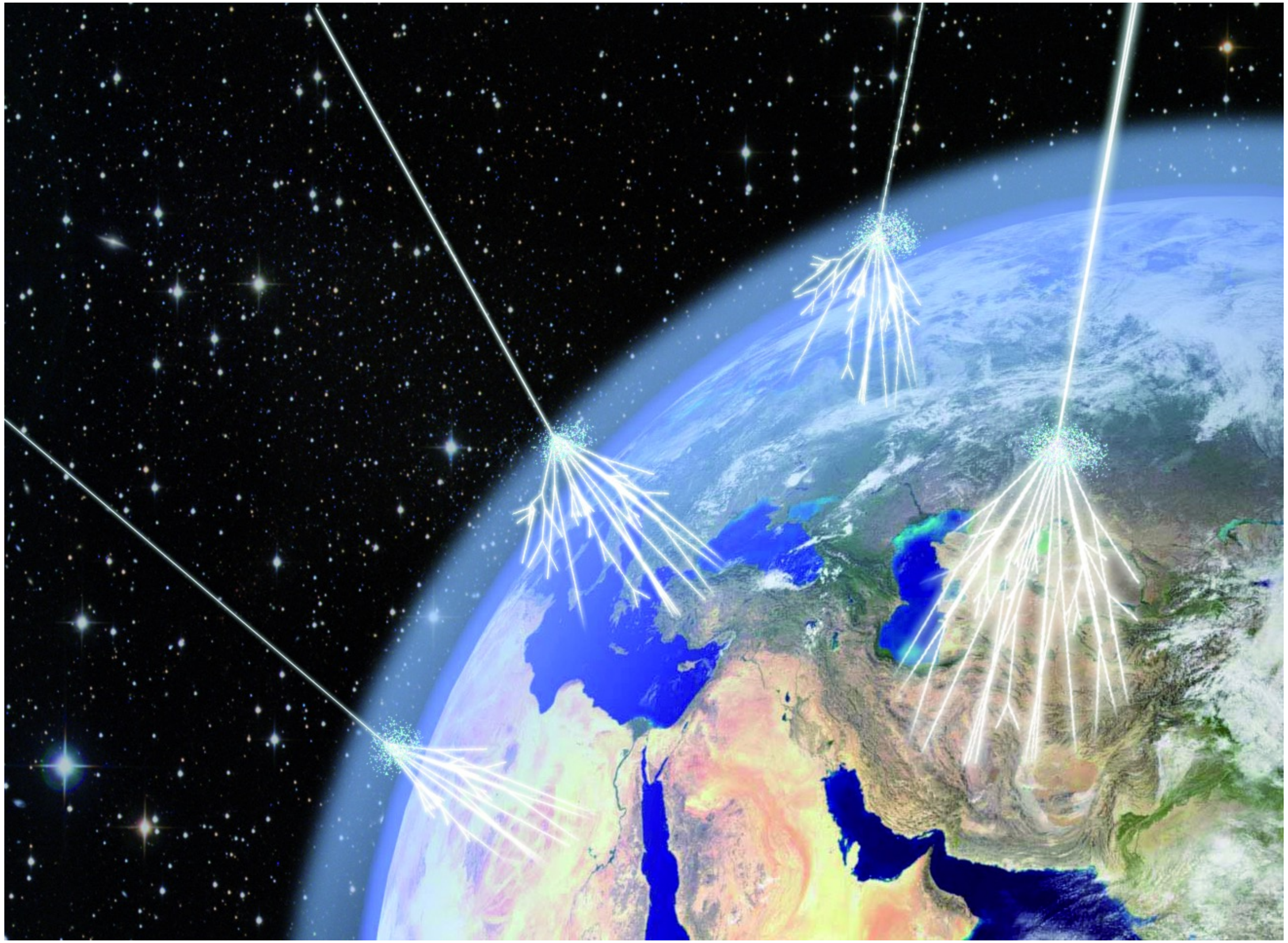
H, He, p accelerated out from the sun

Black holes, quasars, etc.



Charged particles deflected by Earth's magnetic field

Cosmic Ray Showers



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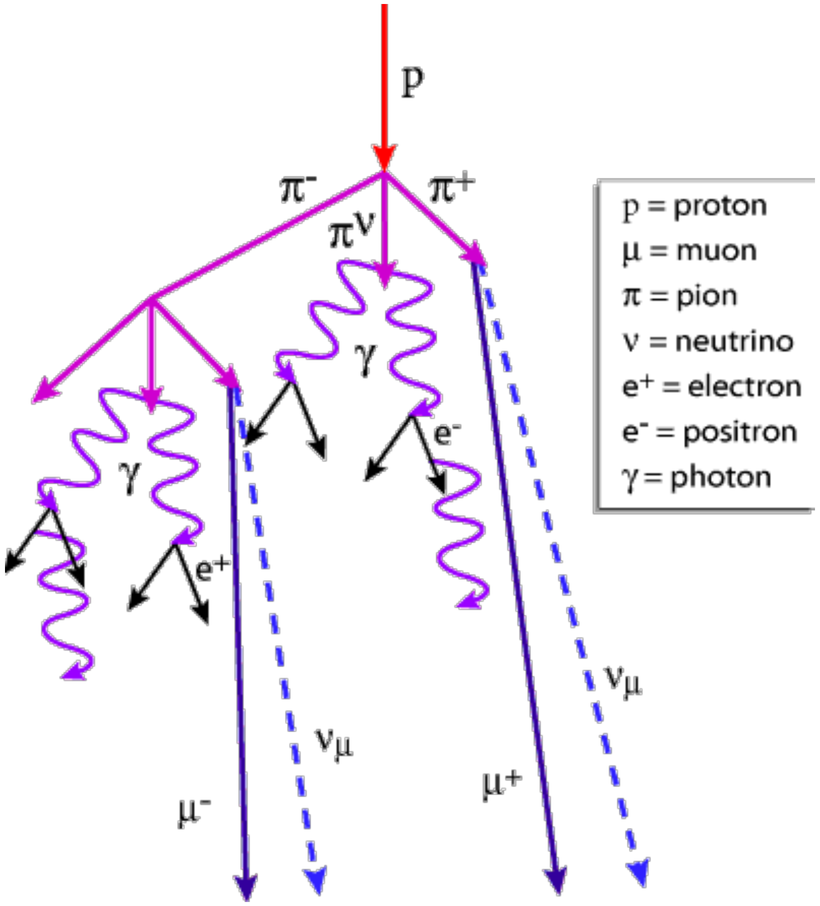
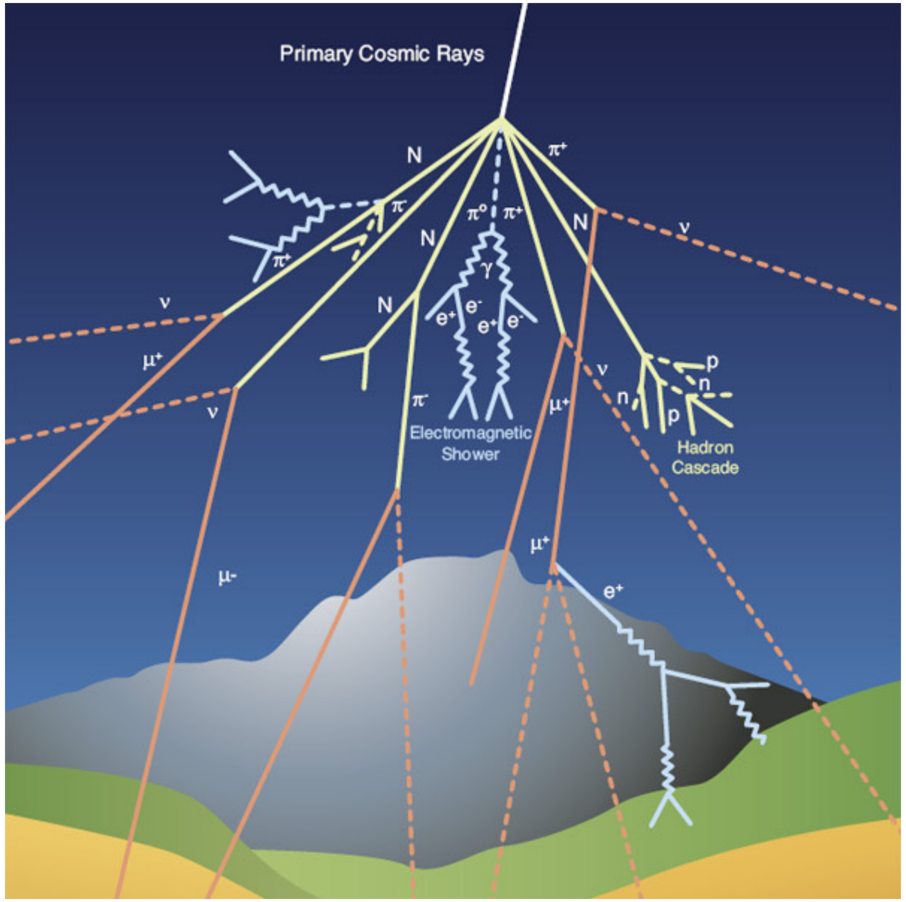




Detecting Cosmic Ray Muons

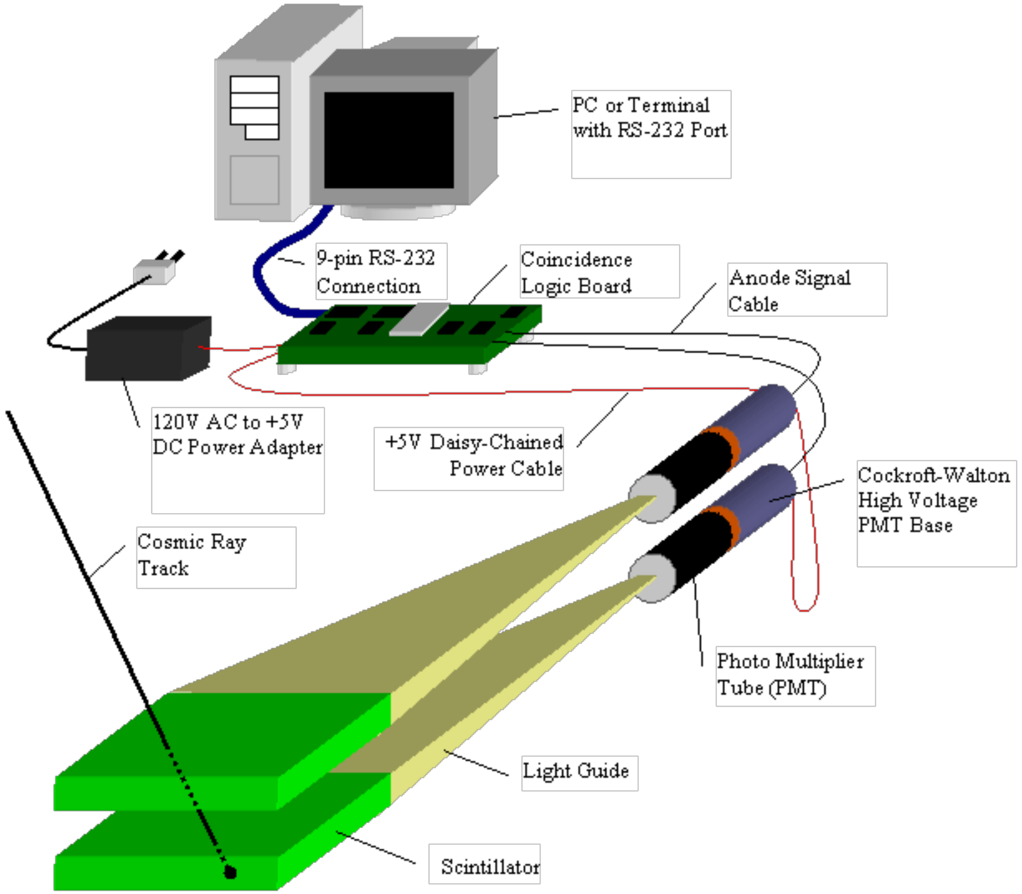
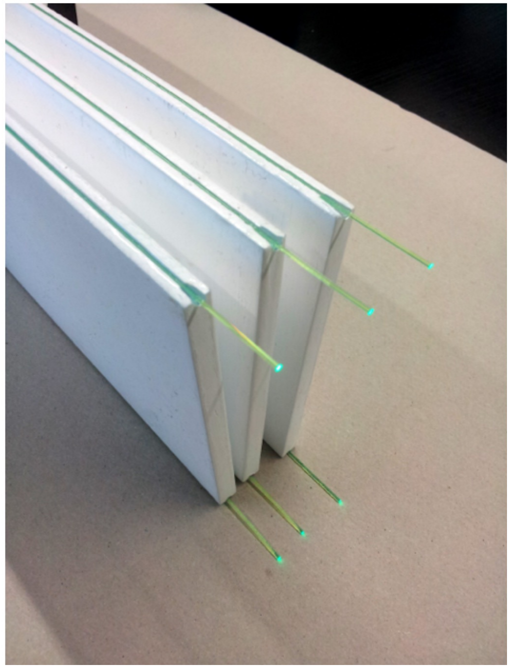


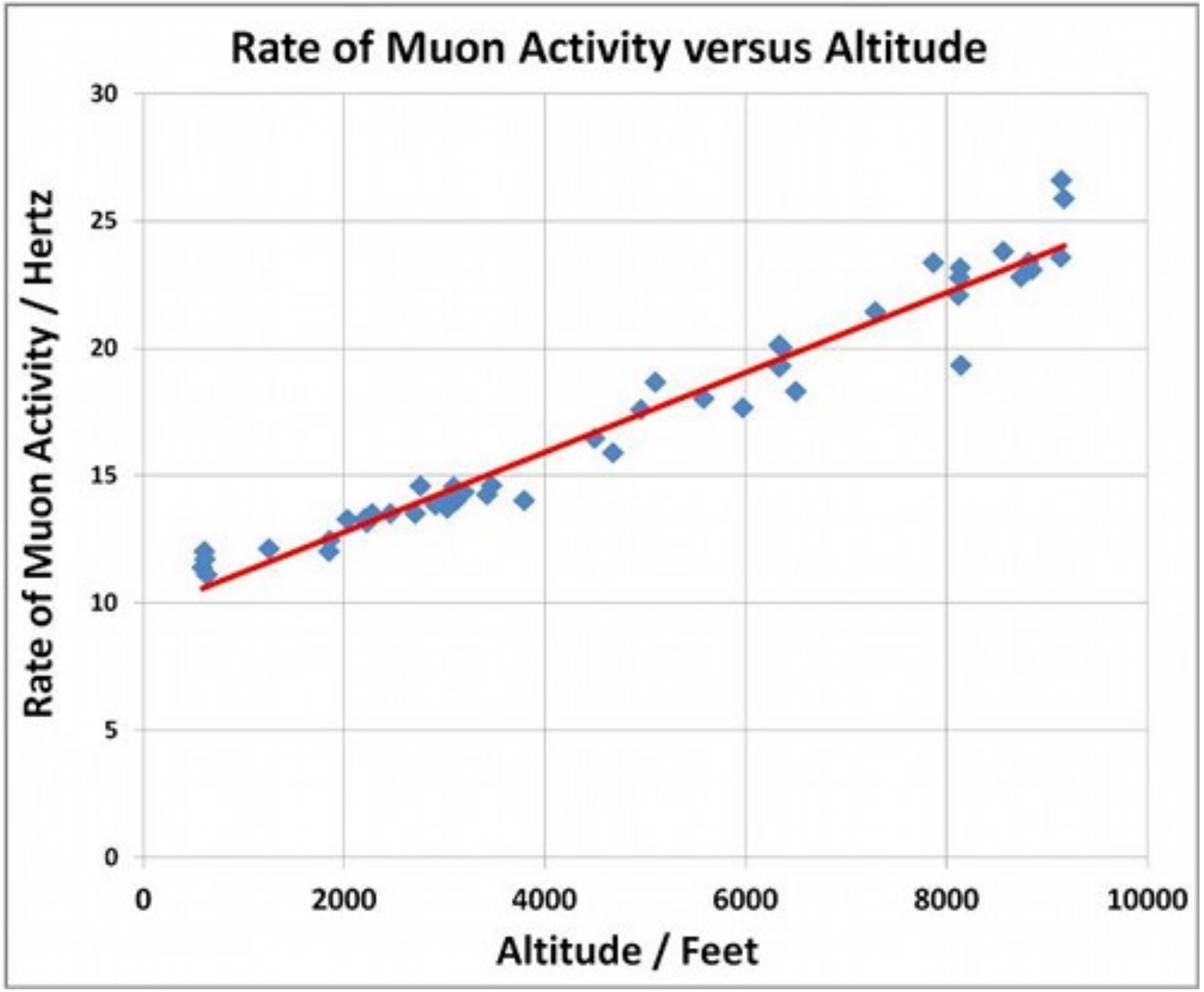
Figure 1. QuarkNet Cosmic Ray Detector System

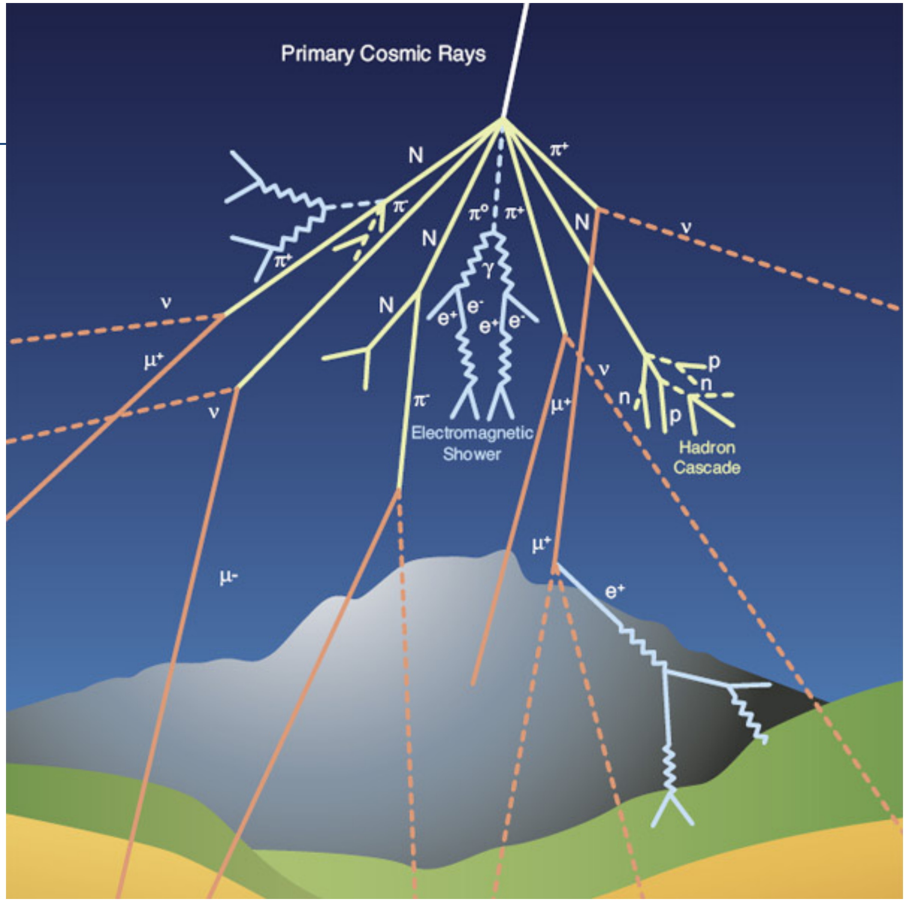
MicroBooNE Cosmic Ray Tagger



Instruments **2017**, *1*(1), 2

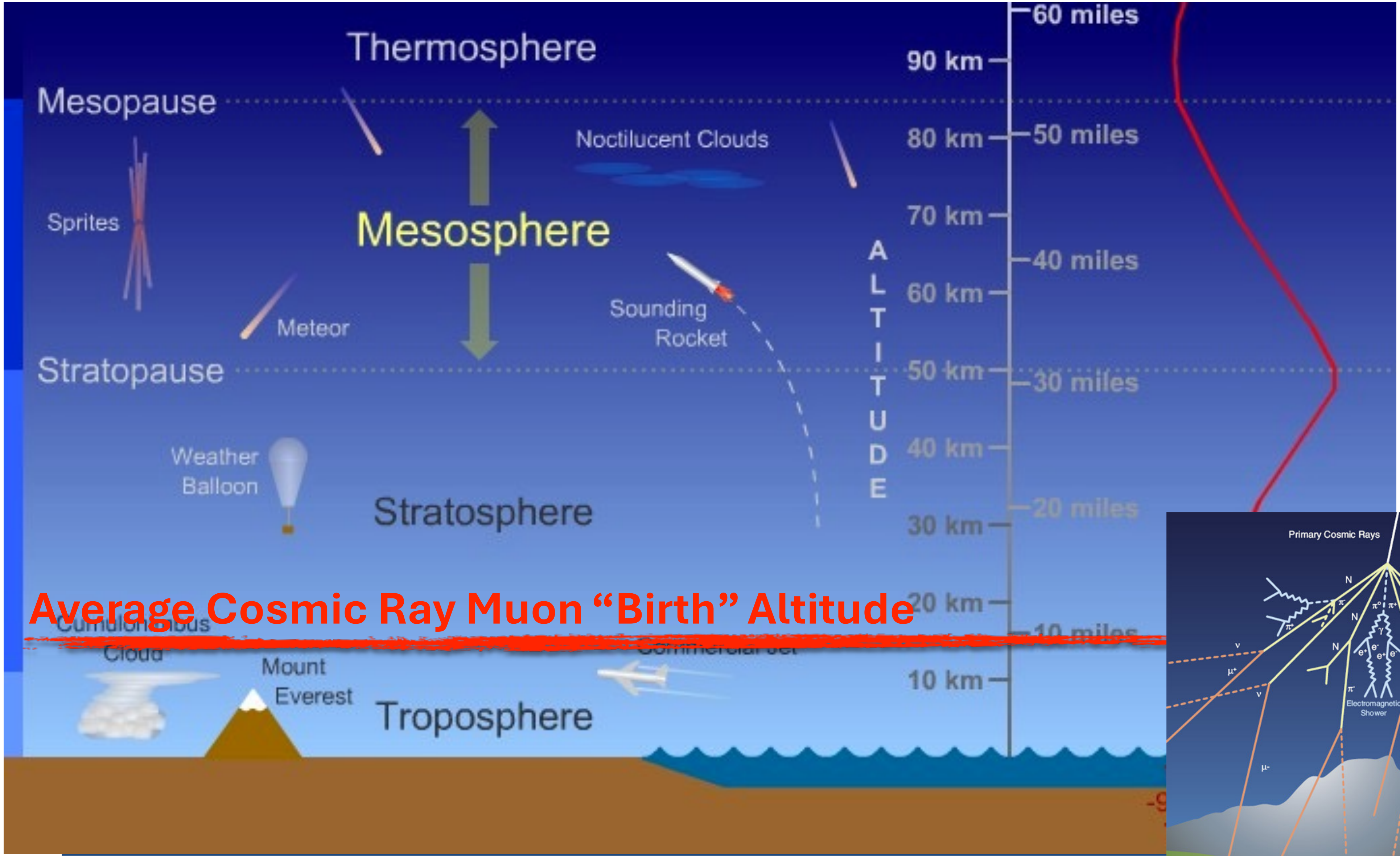
Cosmic Ray Muons



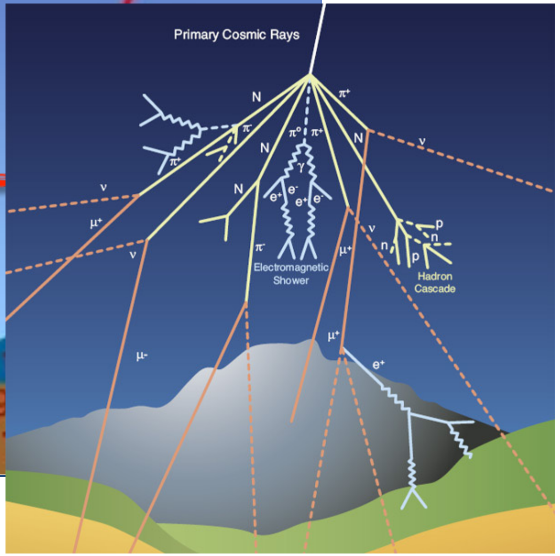


Higher up you go: more muons you see

Where are muons created?



Average Cosmic Ray Muon “Birth” Altitude



A Sanity Check

Does it make sense that muons travel about 15 km to reach us at sea level?

Muon Mean Lifetime: 2.2×10^{-6} sec

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Distance muons traveling at nearly the speed of light will go before decaying:

$$
\begin{aligned}
d &= v \times \tau \\
&= (2.99 \times 10^{8}\,\mathrm{m/s})(2.2 \times 10^{-6}\,\mathrm{s}) \\
&= 660m
\end{aligned}
$$

Or, to reach sea level, 15km from their production, muons must travel for:

$$
\begin{aligned}
t &= d/c \\
&= (15 \times 10^{3}\,\mathrm{m})/(2.99 \times 10^{8}\,\mathrm{m/s}) \\
&= 5 \times 10^{-5}\mathrm{s}
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All muons should decay before reaching us, but we observe: $\sim 1/\mathrm{cm}^2/\mathrm{min}$

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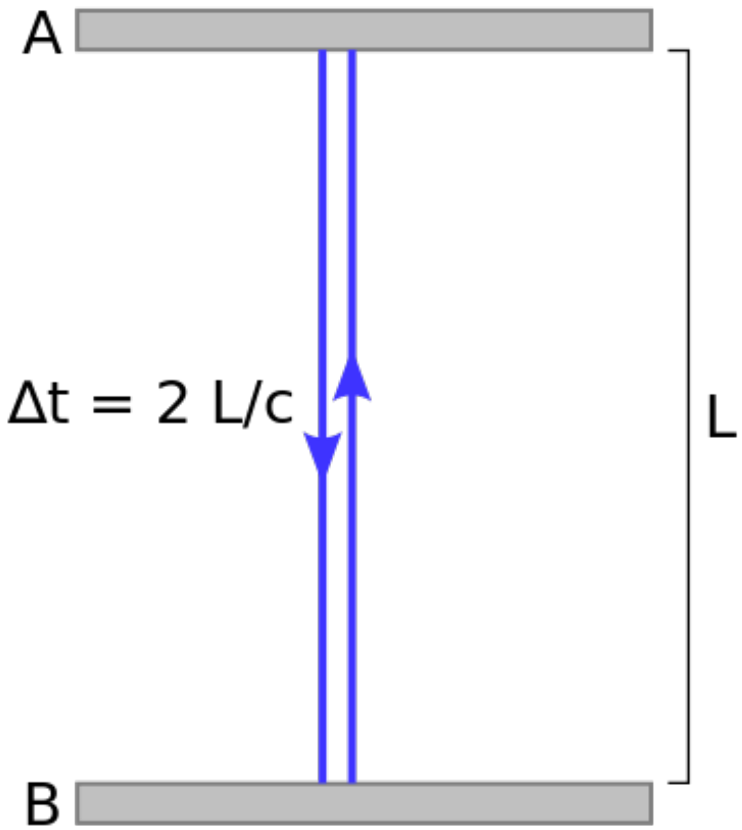
-> Need relativity to explain muons!

Time

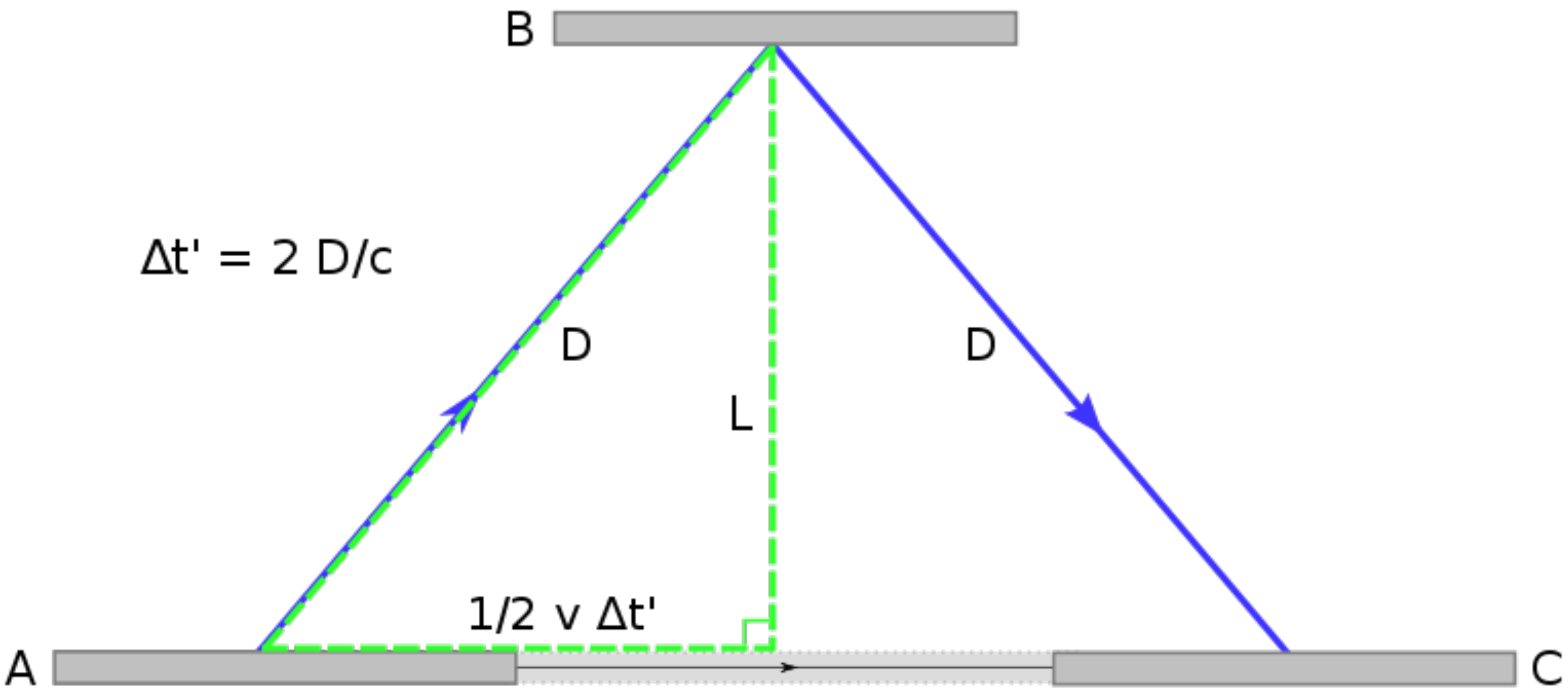
Consider a clock made of two mirrors and a light beam



Time Dilation: Moving Clocks are Slow

Consider a clock made of two mirrors and a light beam

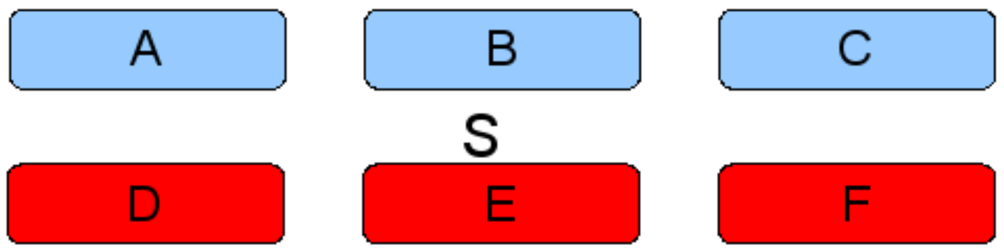
Light moves at a fixed speed so the moving frame appears to have a longer path for the light to follow!
->Clocks in a moving frame appear to move slower than in the lab frame



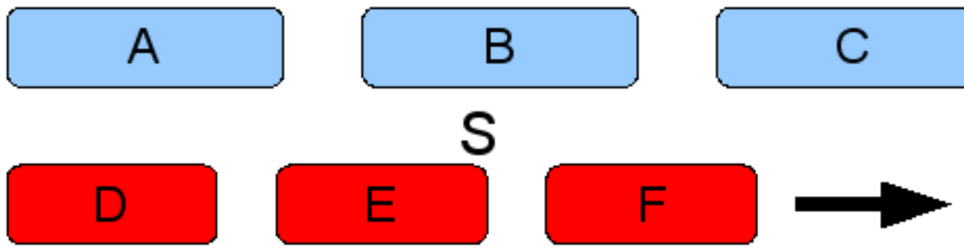
Length Contraction

An object moving close to the speed of light appears to be contracted along the direction of motion.

1. At rest rods are the same length:



2. When the red rods are moving relative to the blue rods, they appear to be shorter to an observer in the blue frame:

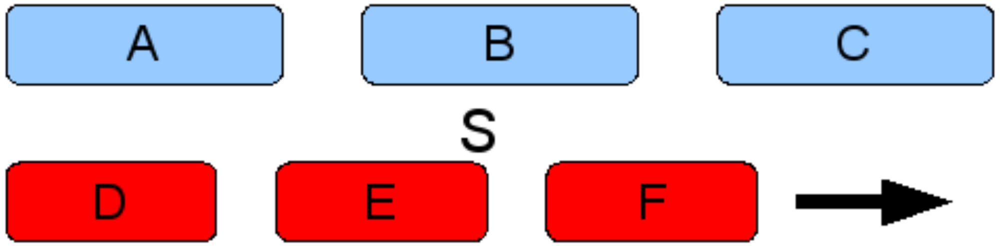


Relativistic Implications

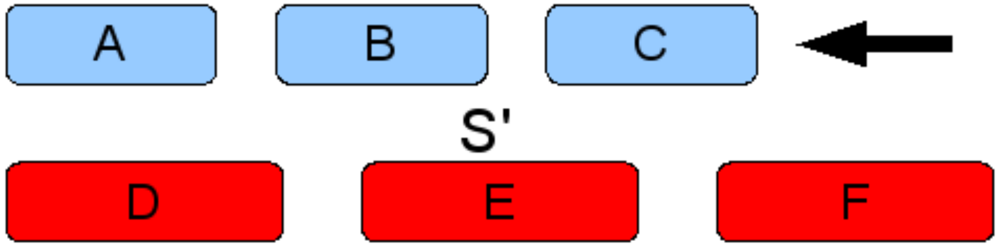
An object moving at a high rate of speed appears to be contracted along the direction of motion.

Consider two sets of rods in two inertial frames.

Observer in frame S (blue rods), at rest relative to S' (red rods).



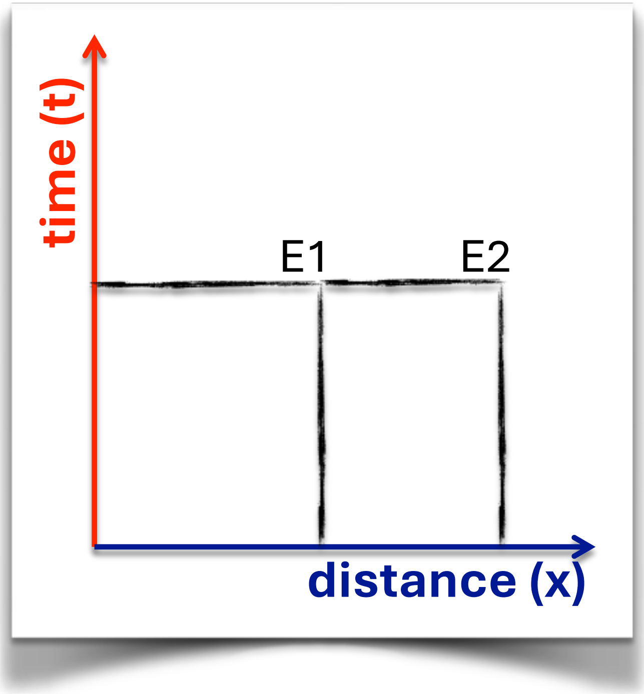
Observer in frame S' (red rods), at rest relative to from S (blue rods).



Implications

Simultaneity

Events that are simultaneous in one frame (t1=t2) are not necessarily simultaneous in a co-moving inertial frame. (Simultaneity is not a conserved quantity)



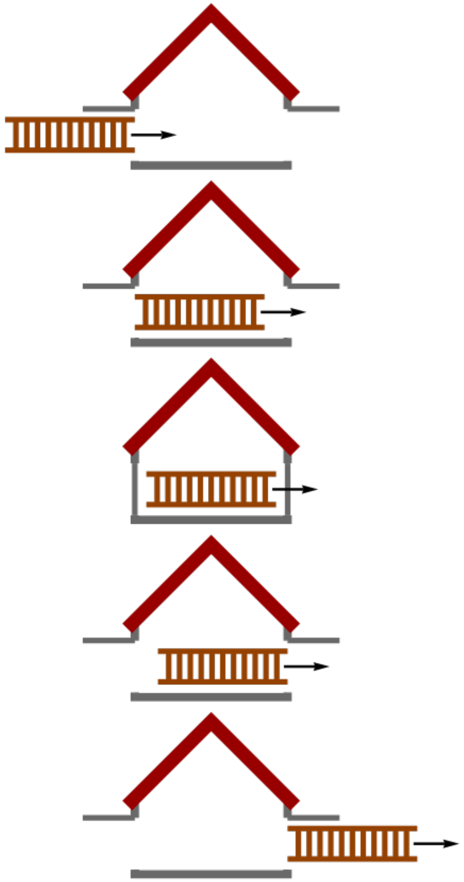
Causality

The order in which events happen depends on the observer's frame. But it is not possible to reverse the order for events that are causally connected.

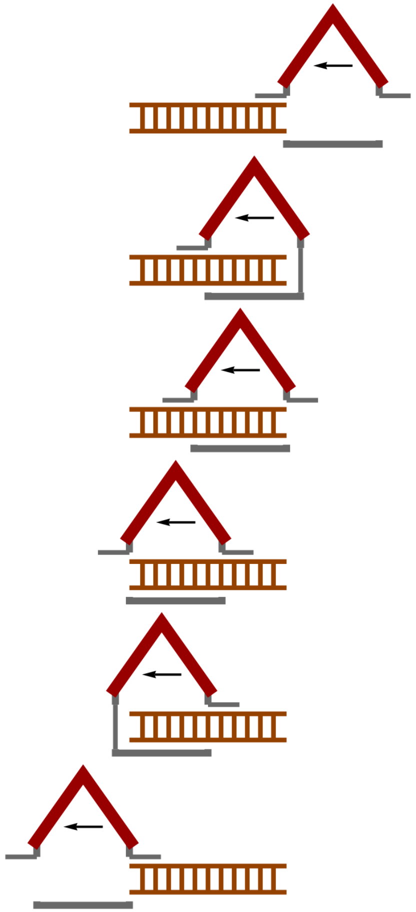
Ladder in the garage paradox

There is a garage with a door in the front and the back, and a long ladder. The ladder is too big to fit into the garage … but what if we move very fast?

Garage rest frame:
The ladder is length-contracted, so it fits inside the garage



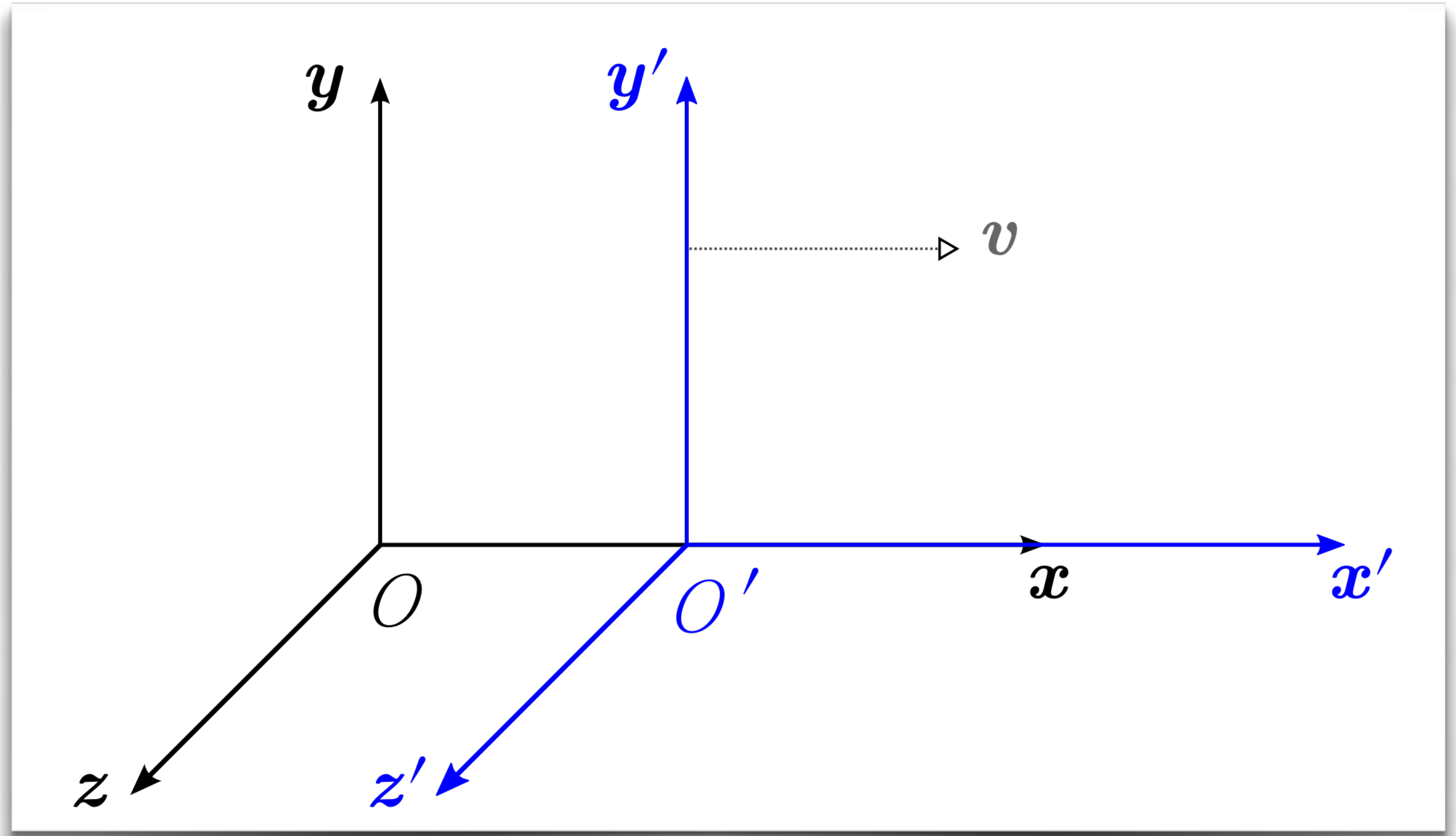
Ladder rest frame:
The garage is length-contracted, so the ladder never fits in the garage



Inertial Frame

<u>Inertial frame:</u> Any system that satisfies Newton's 1st law

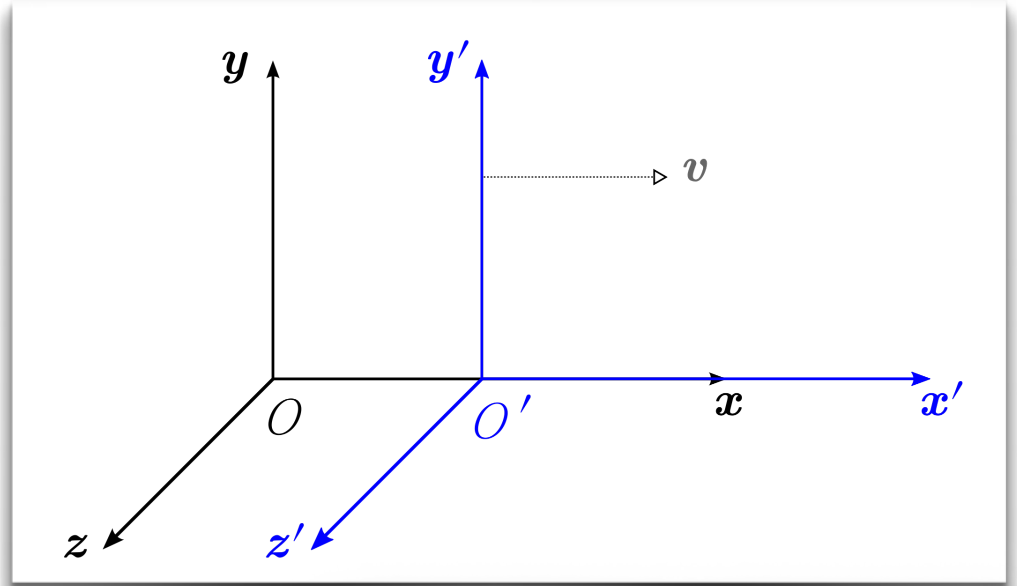
Conservation of momentum implies that physics must be the same in all inertial frames



Velocity Transformations

Galilean Transformations: first, the ones you're used to

Think of velocity transformations as a translation in space-time



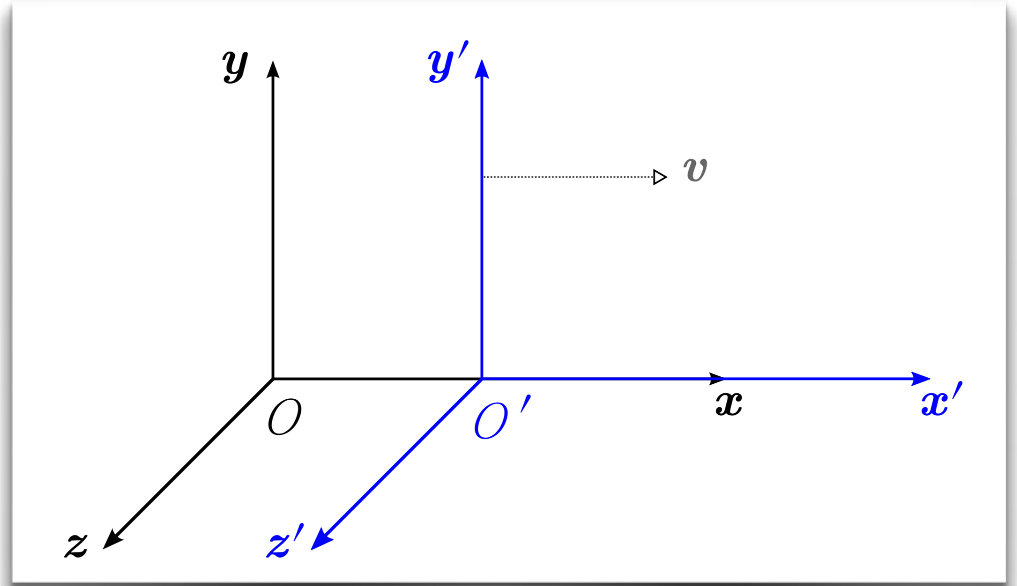
O' perspective: x is falling farther and farther behind us. $x' = x - vt$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -v/c & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Velocity Transformations

<u>Lorentz Transformations:</u> Consider 4-D space-time fabric of motion

Galilean transformations are the low-velocity approximations of these



Displacement occurs in both spatial and time coordinates

$$x' = -\beta\gamma\, c\, t + \gamma\, x$$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$\beta = \frac{v}{c}$$

$$\gamma^2 = \frac{1}{1-\beta^2}$$

Comparisons

	Non-Relativistic	Relativistic
ct	ct	γ(ct - βz)
x	x	X
y	y	y
z	z	γ(z - βct)

*For a velocity along the z-axis

<u>Length Contraction</u> $L' = L_0/\gamma$

<u>Time Dilation</u> $t' = \gamma\, t_0$

<u>Velocity Addition</u> $v_{AB} = (v_A + v_B)/(1 + v_A v_B/c^2)$

A Sanity Check, follow up

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Muon Mean Lifetime: 2.2×10^{-6} sec

Distance muons traveling at nearly the speed of light will go before decaying:

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All muons should decay before reaching us, but we observe: $\sim 1/\mathrm{cm}^2/\mathrm{min}$

Relativistic Implications

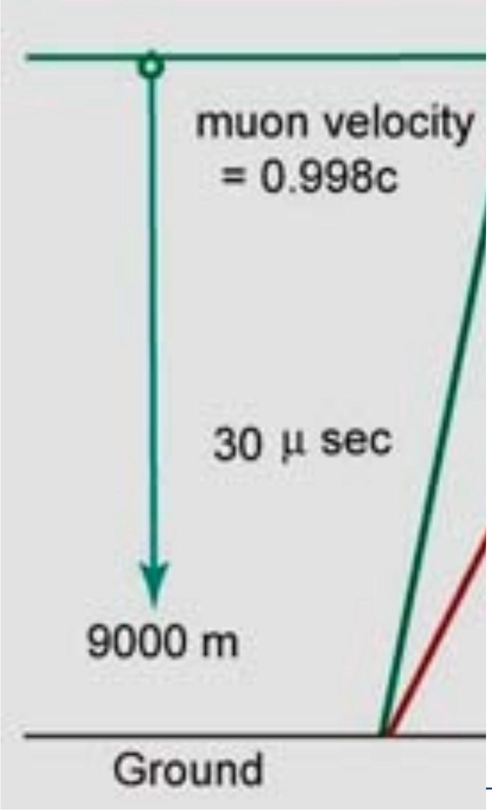
<u>Time dilation:</u>

Clocks in a moving frame appear to move slower than in the lab frame

<u>Length contraction:</u>

Distance in a moving frame appears shorter than in the lab frame

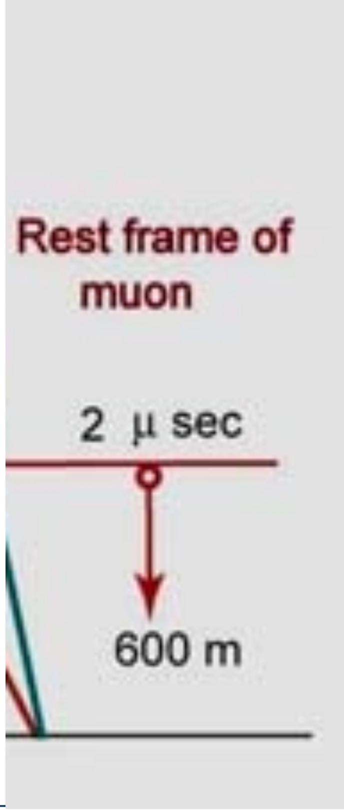
Example: Cosmic ray muon with velocity 0.998 times the speed of light



Lab frame:

Time dilation: muon lifetime ~30μs

- With this lifetime, has to travel a distance of ~9km to reach the ground



Muon rest frame:

- Muon lifetime 2 μs (PDG)
- Distance **length contracted** to need to be only ~600 m