Debye :

- atoms do not move individually

$\longrightarrow$ quantized wave-like propagation = Sound

$$\omega = \underline{\underline{v}} \, |\vec{k}|$$

For a single mode at frequency ω:

$$\langle E \rangle_{osc} = \hbar\omega \left(n_B(\beta\hbar\omega) + \frac{1}{2} \right)$$

$$\langle E \rangle_{tot} = \sum_{modes} \hbar\omega_{mode} \left(n_B(\beta\hbar\omega_{mode}) + \frac{1}{2} \right)$$

$$n_B = \frac{1}{e^{\beta\hbar\omega} - 1}$$

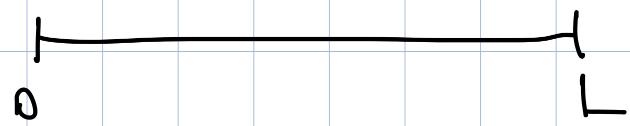
Light	vs	Sound
c		v
2 polarization		3 polarizations

Assume V_{sound} indep. of polarization

" " " " direction

How many k modes do we have?

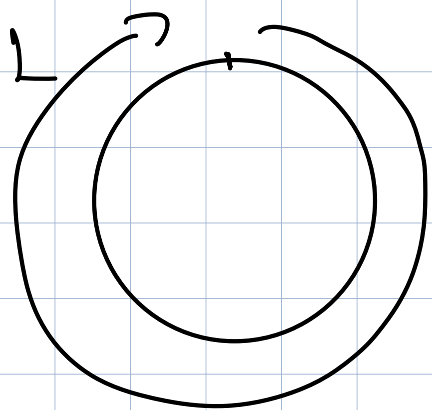
1D box



$$\sin\left(\frac{n\pi x}{L}\right)$$

$$n \in \{1,2,3,\dots\}$$

instead periodic boundary conditions:



"0 = L"

$$e^{ikx} \qquad e^{ikx} = e^{ik(x+L)}$$

$$\to e^{ikL} = 1$$

$$\to kL = 2\pi \times n$$

$$\to k = \frac{2\pi}{L} n \qquad n \in \mathbb{Z}$$

spacing between k's: $\frac{2\pi}{L}$

$$\sum_k \to \int \frac{dk}{2\pi/L}$$

$$\sum_k \to \int dk$$

$$k = \frac{2\pi}{L} n$$

$$\sum_{n=-\infty}^{+\infty} \to \int_{-\infty}^{+\infty} dn = \int \frac{dk}{2\pi/L} \qquad \underline{dk} = \frac{2\pi}{L}\,\underline{dn}$$

$$\sum_{k} \rightarrow \frac{L}{2\pi}\int dk$$

in 3D $\qquad L \times L \times L \qquad$ "periodic box"

$$\vec{k} = \frac{2\pi}{L}\left(n_x , n_y , n_z\right)$$

$$\sum_{n_x=-\infty}^{+\infty}\ \sum_{n_y=-\infty}^{+\infty}\ \sum_{n_z=-\infty}^{+\infty} = \left(\frac{L}{2\pi}\right)^3 \underline{\int dk_x\, dk_y\, dk_z}$$

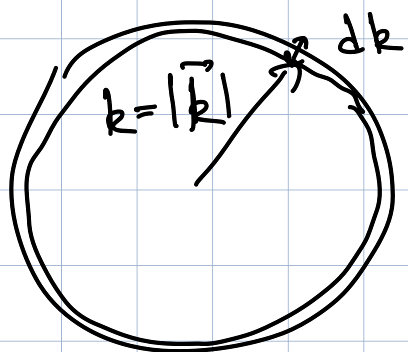
$$= V \boxed{\int \frac{d^3k}{(2\pi)^3}}$$

V = volume = L^3.

Apply to $\langle E \rangle_{tot}$

$$\sum_{modes} \rightarrow 3\sum_{k} \rightarrow 3V \int \frac{d^3k}{(2\pi)^3}$$

$$= \frac{3V}{(2\pi)^3}\int 4\pi k^2\, dk$$



$k = |\vec{k}|$

Sound $\qquad \omega = v\, k$

$$\sum_{\text{modes}} = \frac{3V(4\pi)}{(2\pi)^3 v^3} \int \omega^2 \, d\omega$$

$$\equiv \int g(\omega) \, d\omega$$

$$g(\omega) = \left[\frac{3V(4\pi)}{(2\pi)^3 v^3} \right] \omega^2 \leftarrow$$

$g(\omega)$ = density of states

$g(\omega)\, d\omega$ = # of modes within frequency range ω and $\omega + d\omega$

$$\rightarrow g(\omega) = \underbrace{N}_{\text{total \# of atoms}} \frac{9\omega^2}{\underbrace{\omega_D^3}_{\text{Debye frequency}}}$$

total # of atoms

Debye frequency

$$\omega_D^3 = 6\pi^2 \underbrace{\frac{N}{V}}_{\text{density}} v^3$$

density

$$\langle E \rangle_{tot} = \int_0^\infty \underbrace{d\omega\, g(\omega)}\; \underbrace{\hbar\omega \left[n_B(\beta\hbar\omega) + \frac{1}{2} \right]}_{\text{QM}}$$