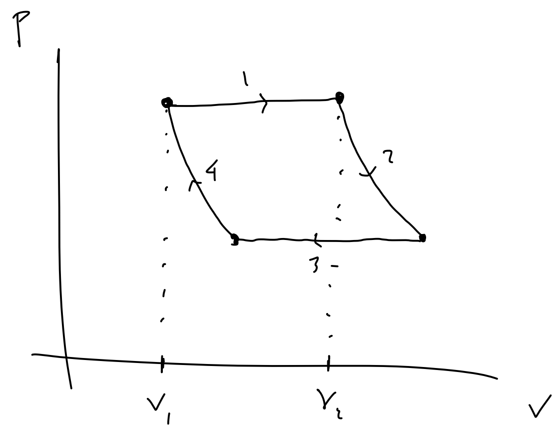
Physics 410 Quiz #6 – Thursday, April 17, 2025

Name:________________________________

1. Consider a photon gas undergoing a reversible Carnot cycle: step 1 is an isothermal expansion at τ_h from V_1 to V_2; step 2 is an isentropic expansion to τ_l, V_3; step 3 is an isothermal compression at τ_l to V_4; and step 4 is an isentropic compression to τ_h, V_1.
   a. [1 point] Sketch a representation of this cycle on a PV-diagram.



* isothermal: p independent of V

* isentropic: $p \propto (V\tau^3)\cdot\frac{\tau}{V}$

$$\propto (V\tau^3)^{4/3}\cdot\frac{1}{V^{2/3}}$$

$$p \propto \frac{1}{V^{2/3}}$$

   b. [5 points] Calculate the work done by the gas, and the heat exchanged with the environment, during step 1.

Isothermal:

$$W = \int_{V_1}^{V_2} p\cdot dV = \int_{V_1}^{V_2} A\tau_h^4\, dV = \boxed{A\tau_h^4(V_2 - V_1)}$$

$$Q = \int \tau_h d\sigma = \tau_h(\sigma_{final} - \sigma_{initial}) = \boxed{4A\tau_h^4(V_2 - V_1)}$$

c. [5 points] Calculate the work done by the gas, and the heat exchanged with the environment, during step 2. (Do not leave V_3 in your answer; you should only have the volumes V_1 and/or V_2 in your result.)

Isentropic: $4AV_2\tau_h^3 = 4AV_3\tau_\ell^3 \rightarrow V_3 = V_2\left(\frac{\tau_h}{\tau_\ell}\right)^3$

$\boxed{Q=0}$ since isentropic.

$$W = \cancel{Q}^{0} - \Delta U = -3A\left(V_3\tau_\ell^4 - V_2\tau_h^4\right)$$

$$= \boxed{3AV_2\tau_h^3\left(\tau_h - \tau_\ell\right)}$$

d. [4 points] What are ΔU, and $\Delta\sigma$, for the entire cycle? Explain.

Entropy & energy are state functions. Since this is a reversible cycle, final & initial states are identical. $\rightarrow$ U, σ return to their original values: $\Delta U = 0$, $\Delta\sigma = 0$

2. Consider a reaction involving the ionization of hydrogen: $H \leftrightarrow e + H^+$ at temperature τ. The ionization energy of hydrogen is I. The net charge of the entire gas is zero.

a. [3 points] Write down the equilibrium constant $K(\tau)$ for this reaction, treating all species as an ideal gas. You may assume that the mass of H and the mass of H^+ are equal. You may leave the result in terms of the electron mass m_e.

$$K(\tau) = \frac{n_{Qe}\cdot n_{QH^+}}{n_{QH}}\, e^{-I/\tau} \simeq n_{Qe}\, e^{-I/\tau} = \boxed{\left(\frac{m_e\tau}{2\pi\hbar^2}\right)^{3/2} e^{-I/\tau}}$$

b. [2 points] Suppose the concentration of H is doubled. By what factor does the concentration of e change? Show your work.

$$\frac{[e][H^+]}{[H]} = K(\tau). \qquad \text{Neutral} \rightarrow [e]=[H^+]$$

$$\rightarrow [e]^2 = [H]K(\tau) \longrightarrow [e] = \sqrt{[H]}\cdot\sqrt{K(\tau)}$$

$\rightarrow$ H doubles $\rightarrow$ $[e]$ changes by factor of $\sqrt{2}$