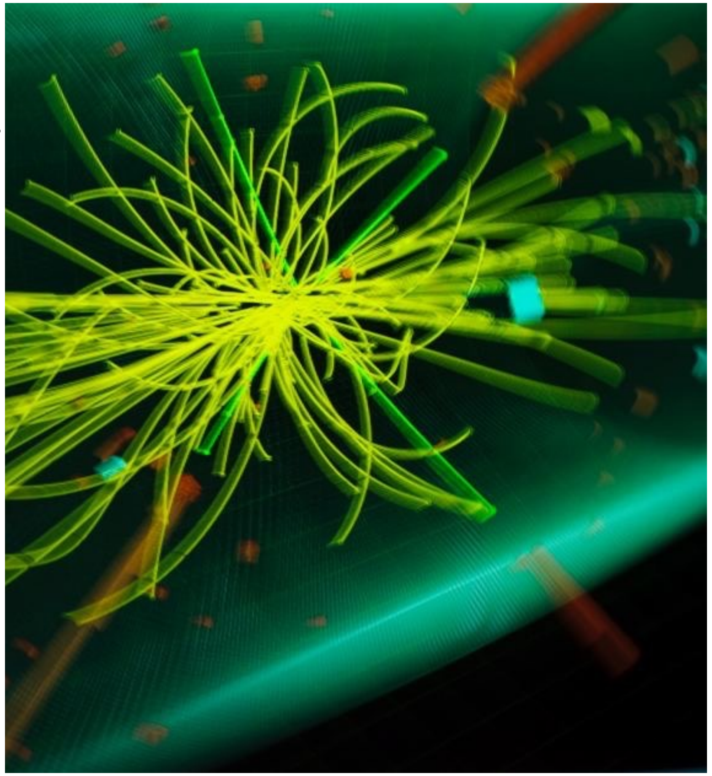
Announcements

Quiz:
- Pickup Friday's quiz after class
- Next quiz on Friday

Homework:
- Second homework due Feb 10 at 3pm. Submit on gradescope.



Paper: Topic due Monday, Feb. 16th at 3pm
Please reply to this google form before then:
https://forms.gle/MmCk8NtrMm7RdfLC7

Midterm: **Friday, Feb 21**.

Paper – PHY493: Tier II Requirement

Goal: Learn more about particle physics topic of your choice. Fulfill physics major writing requirement.

Options:

1. Original review of an important measurement/ topic (10-12 pages)
   - Include: introduction of the topic, summary of the historical context in which the concept was developed, or the measurement was made (e.g. the state of particle physics at the time of the discovery of the muon), importance of the topic and how it applies to our modern understanding. If the topic involves experimental measurements, the paper should include a discussion of the existing measurements and their uncertainties.

2. Original proposal to improve an existing experimental measurement or theoretical technique (6-8 pages)
   - In the style of a proposal to a funding agency. Motivation for the improved measurement clearly defined. Include: description of the related physics, a review of the existing measurements, the proposal for the new measurement and a description of the impact of the improved measurement on our understanding. At least one page must be devoted to a description of the costs associated with the improved measurement (i.e. the funds which need to be awarded to perform the measurement). This estimate of the costs should be thoroughly researched and documented (i.e., it should attempt to be realistic and not imaginary or a “guess”). For example, if you use a number to support costs, you should reference it.

Scientifically literate, but not too technical. Should include necessary figures, equations and a full list of citations

Paper – PHY493: Topic Options

Narrow enough to focus on relevant material within the length limits. ”History of Particle Physics” is too broad, for example

Example Research Paper Topics (PHY 493):

1. Strong CP violation, axions
2. Dark matter and/or dark energy
3. New detector developments
4. Future accelerators and accelerator technology
5. Linear colliders
6. Neutrino-less double beta decay
7. The discovery of the bottom/top/strange/charm quark
8. The discovery of the positron
9. The discovery of the Higgs boson
10. The discovery of beta decay
11. The discovery of electroweak gauge bosons
12. The discovery of the neutrino
13. The observation of neutrinos from supernova 1987a
14. Neutrino masses and neutrino oscillations
15. Neutrino mixing measurements
16. The “solar neutrino puzzle” and its impact on understanding neutrino oscillations
17. Measurements of the strong coupling constant
18. Tests of parity violation in atomic systems
19. The quark-gluon plasma
20. The search for proton decay
21. The discovery of parity violating weak charged and neutral current interactions
22. The discovery of time reversal violation and CP violation in neutral kaon systems
23. The discovery of the J/Y vector particle and its impact on the quark model
24. Is the neutrino a Majorana particle?
25. Is the Higgs boson responsible for mass generation?

Paper – PHY803

Goals:

1. Engage with particle physics literature on a topic of your choice.
2. Practice scientific writing and synthesizing information for publication

Paper requirements: Original synopsis of research paper in particle physics

- Choose a paper of at least 20 pages from an approved peer-reviewed physics journal, some options below:
  - Physical review D (prd.aps.org)
  - Physics letters B (http://www.journals.elsevier.com/physics-letters-b/)
  - Journal of high energy physics (http://www.springer.com/physics/particle+and+nuclear+physics/journal/13130)
- Summarize this paper in a letter-format paper of 5-7 pages
  - Example letter-format papers can be found in Physical Review Letters (prl.aps.org)

Paper Topic Submission

Paper: Topic due Monday: Feb. 16th
Please reply to this google form before then:
https://forms.gle/MmCk8NtrMm7RdfLC7

What is your name? *

Your answer

What is the proposed topic for your paper? *

Your answer

Please provide a link to a paper or reference on your topic. If you are in PHY803 this should be the link to the paper that you will review. *

Your answer

What is the title of the paper or reference in the previous question? *

Your answer

If you are in PHY493 please indicate if you plan to write a review or a proposal. Please see the syllabus for details on these options. *

- Review
- Proposal
- I am in physics 803, not 493

Submit

Clear form

Angular Momentum

Let's use angular momentum as an example for a group representation of a conserved quantity.

J = L + S

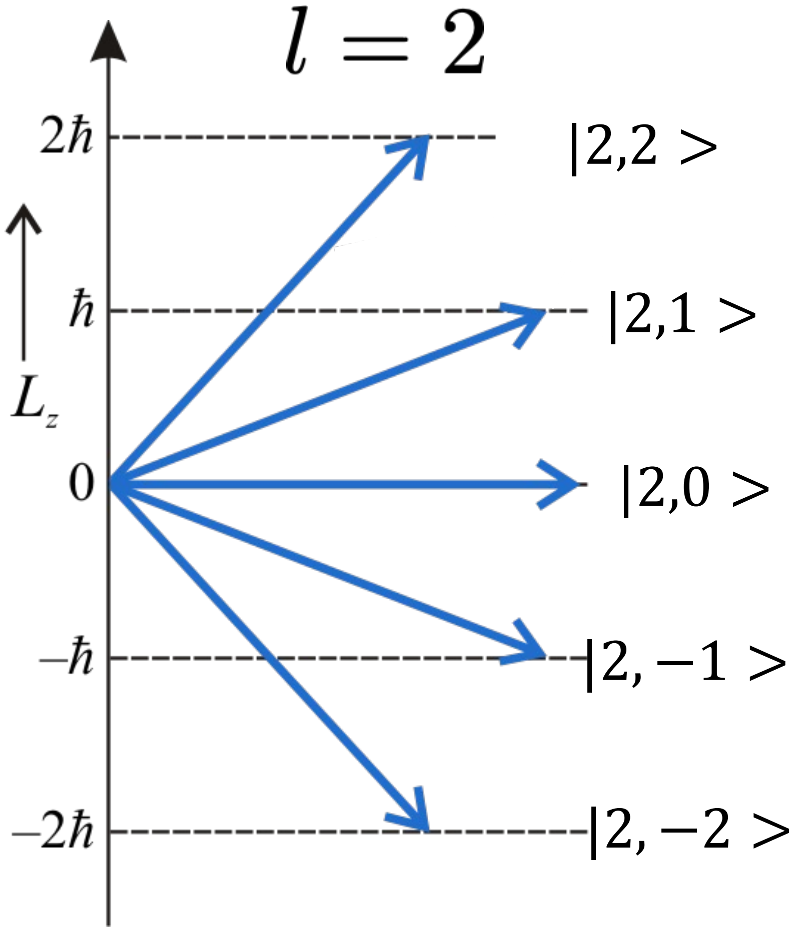
J = total angular momentum, S = spin, L = orbital angular momentum

Quantum mechanically allowed to probe two quantities of the total angular momentum vector (J, L or S…or any admixture):

For orbital angular momentum, quantum mechanically get to measure:

magnitude: $L^2 = \mathbf{L} \cdot \mathbf{L} = l(l+1)\hbar^2$ $\quad l = 0, 1, 2, 3, \ldots.$

z component: $L_z = m_l \hbar$ $\quad m_l = -l, -l+1, \ldots -1, 0, 1, \ldots, l-1, l$



Present the angular momentum wave function as $|l, m>$

l is the orbital quantum number
m is the magnetic quantum number

Angular Momentum

For orbital angular momentum, quantum mechanically get to measure:

magnitude: $L^2 = \mathbf{L} \cdot \mathbf{L} = l(l+1)\hbar^2$ l = 0, 1, 2, 3, ...

z component: $L_z = m_l \hbar$ m_l = -l, -l +1, ...-1, 0, 1, ..., l-1, l

Present the angular momentum wave function as $|l, ml >$

For spin, quantum mechanically get to measure:

$$S^2 = \mathbf{S} \cdot \mathbf{S} = s(s+1)\hbar^2$$ s = 0, ½, 1, 3/2, 2, 5/2, 3, ... -> half integer steps

$$S_z = m_s \hbar$$ m_s = -l, -l +1, ...-1, 0, 1, ..., l-1, l ->integer or half integer, depending on s

Present the spin wave function as $|s, ms >$

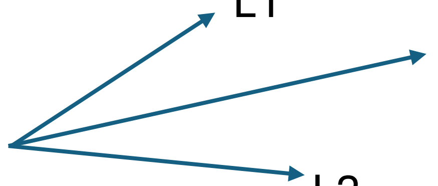
-> spin quantity is fundamental to particle, unlike orbital angular momentum

Addition of Angular Momentum

Angular momenta sum as vectors, but we only have one component and the magnitude:

$$|l_1\ m_1 > |l_2\ m_2 >= |l_{\mathrm{tot}}\ (m_1 + m_2) >$$

-> z components add, mtot = m1+m2



Addition of magnitude depends on the relative orientation:

Parallel: add, Anti-parallel: subtract

$$l_{\mathrm{tot}} = |l_1 - l_2|, |l_1 - l_2| + 1, \ldots, (l_1 + l_2) - 1, l_1 + l_2$$

-> get every l between l1+l2 all the way to l1-l2 !

Addition of Angular Momentum: Mesons

$$|l_1\ m_1 > |l_2\ m_2 >= |l_{\text{tot}}\ (m_1 + m_2) >$$

$$l_{\text{tot}} = |l_1 - l_2|, |l_1 - l_2| + 1, \ldots, (l_1 + l_2) - 1, l_1 + l_2$$

Meson bound state (quark-anti-quark) with zero orbital angular momentum:

What are the possible spin states?

Four possible states:

$$|1/2,\ 1/2 > |1/2,\ 1/2 >= |1,\ 1 >$$
$$|1/2,\ 1/2 > |1/2,\ -1/2 >= |0,\ 0 >$$
$$|1/2,\ 1/2 > |1/2,\ -1/2 >= |1,\ 0 >$$
$$|1/2,\ -1/2 > |1/2,\ -1/2 >= |1,\ -1 >$$

Addition of Angular Momentum: baryons

Example: Three quarks bound in a hadron, zero tot orbital angular momentum:

Add another quark to each of the 2-quark combinations:

Add a quark to the spin-1 meson (lz=+1)

$$|1,\ 1>|1/2,\ 1/2>=|3/2,\ 3/2>$$

$$|1,\ 1>|1/2,\ -1/2>=|1/2,\ 1/2>$$

$$|1,\ 1>|1/2,\ -1/2>=|3/2,\ 1/2>$$

Similar when starting from |1,-1>

Add a quark to the spin-0 meson

$$|0,\ 0>|1/2,\ 1/2>=|1/2,\ 1/2>$$

$$|0,\ 0>|1/2,\ -1/2>=|1/2,\ -1/2>$$

Similar when starting from |1,0>

Addition of Angular Momentum

The **relative likelihood of each final state** can be calculated from spherical harmonics

Example: The angular momentum wave-function for the quark-antiquark pair is a superposition of the possible final states.
For the $m_1 = +1/2$, $m_2 = -1/2$ combination, we get

$$|1/2,\ 1/2 > |1/2,\ -1/2 >= A_1\,|1,\ 0 > + A_2\,|0,\ 0 >$$

The amplitudes are conveniently found in look-up tables, for example in the PDG book
https://pdg.lbl.gov/2021/reviews/rpp2021-rev-clebsch-gordan-coefs.pdf

Clebsch-Gordon Coefficients (*from PDG*)

All numbers in table have an implied square root

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

	J	J	...
	M	M	...
m_1 m_2	Coefficients		
m_1 m_2			
. .			

1/2×1/2

m_1 m_2	1 +1
+1/2 +1/2	1

m_1 m_2	1 0	0 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	1 −1
−1/2 −1/2	1

1×1/2

m_1 m_2	3/2 +3/2
+1 +1/2	1

m_1 m_2	3/2 +1/2	1/2 +1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3

m_1 m_2	3/2 −1/2	1/2 −1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3

m_1 m_2	3/2 −3/2
−1 −1/2	1

2×1

m_1 m_2	3 +3
+2 +1	1

m_1 m_2	3 +2	2 +2
+2 0	1/3	2/3
+1 +1	2/3	−1/3

m_1 m_2	3 +1	2 +1	1 +1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	2/5	−1/2	1/10

m_1 m_2	3 0	2 0	1 0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10

m_1 m_2	3 −1	2 −1	1 −1
0 −1	2/5	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5

m_1 m_2	3 −2	2 −2
−1 −1	2/3	1/3
−2 0	1/3	−2/3

m_1 m_2	3 −3
−2 −1	1

1×1

m_1 m_2	2 +2
+1 +1	1

m_1 m_2	2 +1	1 +1
+1 0	1/2	1/2
0 +1	1/2	−1/2

m_1 m_2	2 0	1 0	0 0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3

m_1 m_2	2 −1	1 −1
0 −1	1/2	1/2
−1 0	1/2	−1/2

m_1 m_2	2 −2
−1 −1	1

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

2×1/2

m_1 m_2	5/2 +5/2
+2 +1/2	1

m_1 m_2	5/2 +3/2	3/2 +3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5

m_1 m_2	5/2 +1/2	3/2 +1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5

m_1 m_2	5/2 −1/2	3/2 −1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5

m_1 m_2	5/2 −3/2	3/2 −3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5

m_1 m_2	5/2 −5/2
−2 −1/2	1

3/2×1/2

m_1 m_2	2 +2
+3/2 +1/2	1

m_1 m_2	2 +1	1 +1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4

m_1 m_2	2 0	1 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	2 −1	1 −1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4

m_1 m_2	2 −2
−3/2 −1/2	1

3/2×1

m_1 m_2	5/2 +5/2
+3/2 +1	1

m_1 m_2	5/2 +3/2	3/2 +3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	−2/5

m_1 m_2	5/2 +1/2	3/2 +1/2	1/2 +1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6

m_1 m_2	5/2 −1/2	3/2 −1/2	1/2 −1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2

m_1 m_2	5/2 −3/2	3/2 −3/2
−1/2 −1	3/5	2/5
−3/2 0	2/5	−3/5

m_1 m_2	5/2 −5/2
−3/2 −1	1

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

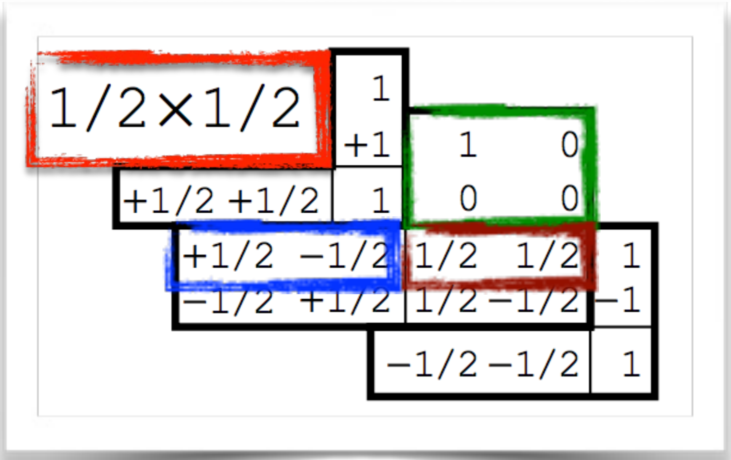
$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

Decomposition of Final States

The Clebsch-Gordon coefficients give us the relative admixtures.

$$|1/2,\ 1/2>|1/2,\ -1/2>= A_1|1,\ 0> + A_2|0,\ 0>$$



$l_1 \times l_2$

$m_1,\ m_2$

$J,\ M$

Amplitudes.
Square root implied

$$|1/2,\ 1/2>|1/2,\ -1/2>= \sqrt{\frac{1}{2}}|1,\ 0> + \sqrt{\frac{1}{2}}|0,\ 0>$$

$$|1/2,\ -1/2>|1/2,\ 1/2>= \sqrt{\frac{1}{2}}|1,\ 0> - \sqrt{\frac{1}{2}}|0,\ 0>$$

Decomposition of Final States

The Clebsch-Gordon coefficients give us the relative admixtures.

$$|1,\ 1 > |1/2,\ -1/2 >= A_3\,|3/2,\ 1/2 > +A_4\,|1/2,\ 1/2 >$$

$1\times 1/2$	3/2 +3/2	3/2 1/2	1/2 1/2	3/2 −1/2	1/2 −1/2	3/2 −3/2
+1 +1/2	1					
+1 −1/2		1/3	2/3			
0 +1/2		2/3	−1/3			
0 −1/2				2/3	1/3	
−1 +1/2				1/3	−2/3	
−1 −1/2						1

$l_1\times l_2$

m_1, m_2

J, M

$$|1,\ 1 > |1/2,\ -1/2 >= \sqrt{\frac{1}{3}}\,|3/2,\ 1/2 > +\sqrt{\frac{2}{3}}\,|1/2,\ 1/2 >$$

Spin-½ Particles

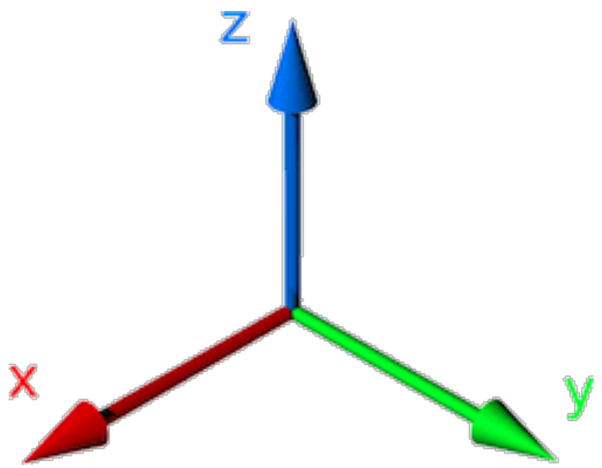
The total spin angular momentum of a spin-½ particle is :

$$S = \sqrt{S^2} = \sqrt{s(s+1)}\,\frac{\hbar}{2} = \frac{\sqrt{3}}{2}\hbar$$

s= spin quantum number

This is different from the quantized projection along a given "spatial" axis, or the particle's spin quantum number:

$$S_z = \pm\frac{\hbar}{2}$$



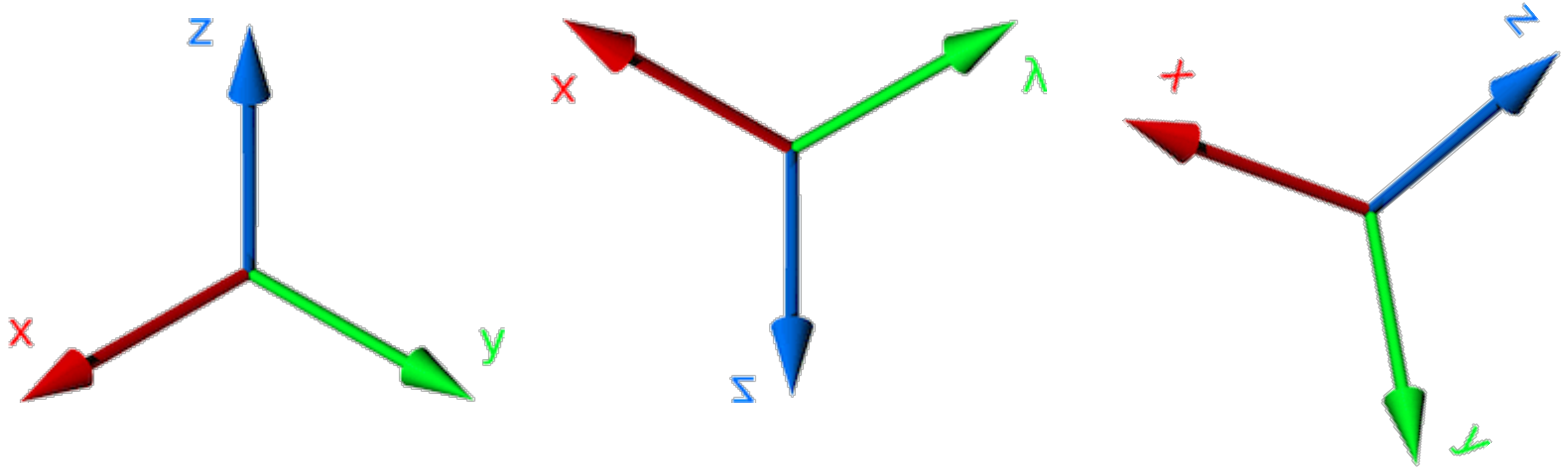
Spin-½ Particles

But the orientation of my axes is completely arbitrary.

So if rotate my coordinate axes, would like to be able to express how particle spin projections transform.

As you may now have guessed, the answer is that we need to introduce a group

The group’s operators will represent spin projection observables



Spin-½ Representations/ Notation

The angular momentum notation from the past several slides has been inefficient.

We will talk about spin-½ particles a lot, so we will introduce a more compact nomenclature.

Instead of:

$$|1/2,\, 1/2>$$

$$|1/2,\, -1/2>$$

We'll use:

"spin up" $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ "spin down"

And so we can use vectors as spin-½ representations

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

normalization requires:

$$|\alpha|^2 + |\beta|^2 = 1$$

Spin-½ Representations

Now that we're using a vector to represent spin 1/2, we must have a different form for spin observable operators

$$\chi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The operator must transform a vector into a vector
The obvious (only?) choice is a matrix to keep the vector a vector

Spin vectors transform via the 2-D representation of the SU(2) group
(The Lie algebra is spanned by 3, 2×2 Hermitian, unitary, complex matrices.)

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Or, more compactly, as the Pauli spin matrices:

$$\hat{\mathbf{S}} = \frac{\hbar}{2}(\sigma_x \; \sigma_y \; \sigma_z) \longrightarrow \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Spin-½ Eigenspinors

Eigenspinors of the spin operators: $$\hat{S}_k\, \chi^{\pm} = \frac{\hbar}{2}\, \sigma_k\, \chi^{\pm} = \lambda\, \chi^{\pm}$$

Eigenvalues are:

$$\lambda = \pm\frac{\hbar}{2}$$

Eigenspinors in x, y, z follow :

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \chi_x^{\pm} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}$$

$$\hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \chi_y^{\pm} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$

$$\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \chi_z^{+} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_z^{-} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Spin-½ Rotations

Using this machinery, we can describe infinitesimal spin rotations:

$$\begin{pmatrix} \alpha' \\ \beta' \end{pmatrix} = U(\theta) \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

where $$U(\theta) = \mathrm{e}^{-i(\theta \cdot \sigma)/2}$$

-> theta vector points along axis of rotation

Shorthand for power series:

$$\mathrm{e}^{A} \equiv 1 + A + \tfrac{1}{2}A^2 + \tfrac{1}{3!}A^3 + \cdots$$

Spin rotations can be represented with the group SU(2)

-> SU(2) is in many ways analagous to SO(3) that represents rotations in 3D Euclidian space

Isospin

Physicists noticed early on that the proton and neutron had apparent symmetries.

The only known constituents of the nucleus.

Nearly degenerate in mass (ΔM/M ~ 1.3/940)

Heisenberg suggested they may be the same particle in two different states of "isospin":

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Isospin

Physicists noticed early on that the proton and neutron had apparent symmetries.

The only known constituents of the nucleus.

Nearly degenerate in mass ($\Delta M/M \sim 1.3/940$)

Heisenberg suggested they may be the same particle in two different states of "isospin":

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

In the isospin formalism, the proton and neutron are isospin "up" and "down"

$$p = |1/2,\, 1/2>$$
$$n = |1/2,\, -1/2>$$

The pi mesons form an isospin triplet:

$$\pi^+ = |1,\, 1>$$
$$\pi^0 = |1,\, 0>$$
$$\pi^- = |1,\, -1>$$

Isospin

Ultimately comes from underlying quarks

But some conclusions still hold: approximate symmetry between the up and down quarks

Driven by close mass degeneracy between up/down quarks

Isospin operators change up to down quarks

$$u = |1/2,\ 1/2 >$$
$$d = |1/2,\ -1/2 >$$

$$p = (u, u, d) = |1/2, +1/2 >$$
$$n = (u, d, d) = |1/2, -1/2 >$$

$$\pi^+ = (u\bar{d}) = |1, +1 >$$
$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) = |1, 0 >$$
$$\pi^- = (\bar{u}d) = |1, -1 >$$

Up and down quarks aren't identical, but extending it to other quarks gives insight to the quark structure.

Isospin: Pions and Eta meson

- Pions have isospin 1
- The neutral pion is |1,0>, which is $u\overline{u} - d\overline{d}$

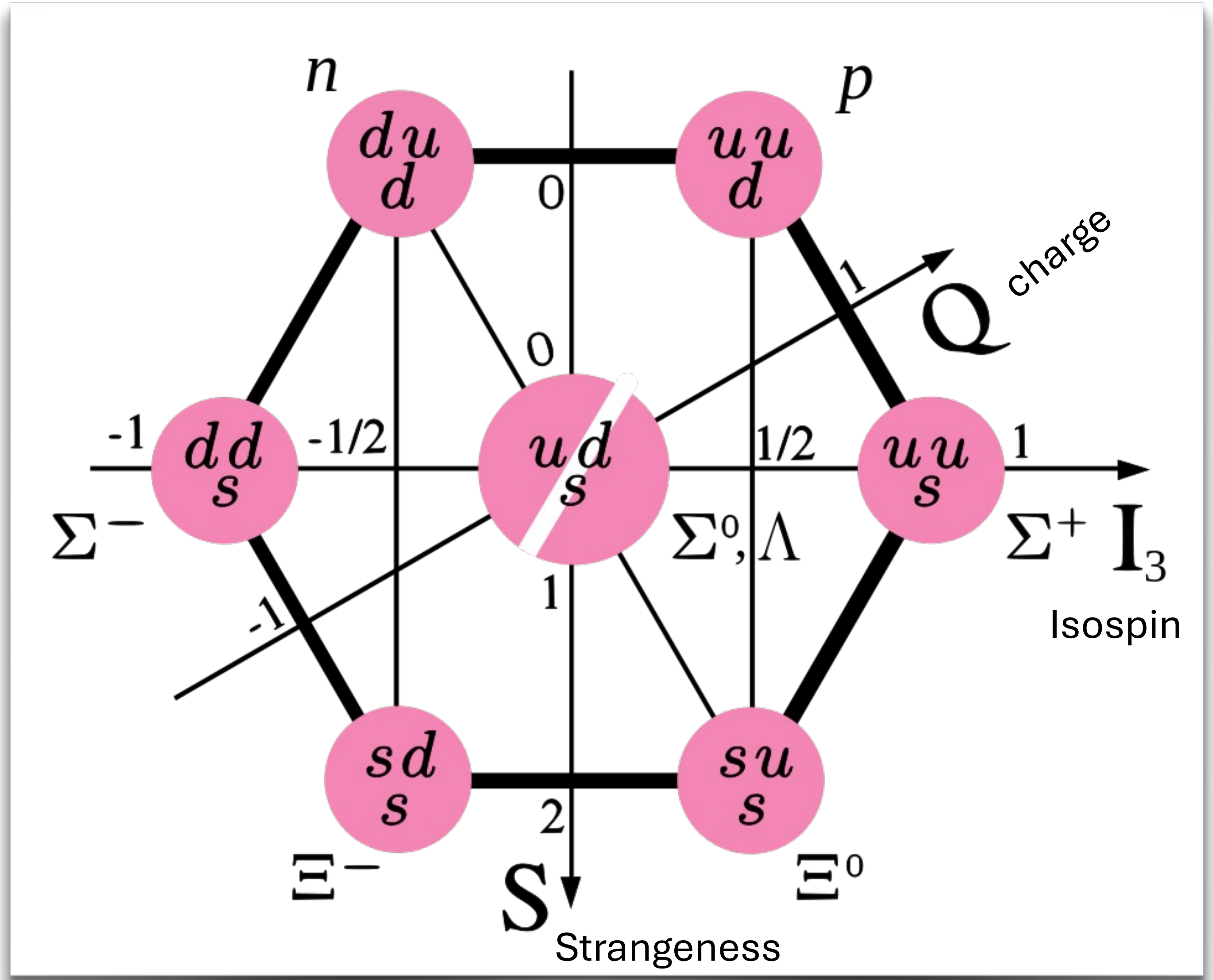
$$\pi^+ = (u\bar{d}) = |1, +1>$$
$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) = |1, 0>$$
$$\pi^- = (\bar{u}d) = |1, -1>$$

- What about the other add-mixture $u\overline{u} + d\overline{d}$
- This is the eta meson, η^0, |0,0>
  - (However, the η^0, also contains strange quarks: it's a superposition of $u\overline{u} + d\overline{d}$ and $u\overline{u} + d\overline{d}$ states in addition to $s\overline{s}$)

Isospin: Σ baryons

- Σ baryons contain a strange quark, in addition to u and/or d
- Σ^+ = uus, isospin |1,1>
- Σ^- = dds, isospin |1,-1>
- Σ^0 = uds, isospin |1,0>
  - The corresponding isospin singlet is Λ^0 = uds, isospin |0,0>
    - Since quark content of Λ^0 is the same, but lower in mass, Σ^0 considered an excited state of Λ^0
    - The main decay of Σ^0 is to $\Lambda^0+\gamma$

Isospin



n
p
d u
d
u u
d
0
0
1
Q
charge
-1
d d
s
-1/2
u d
s
1/2
u u
s
1
Σ⁻
Σ⁰, Λ
Σ⁺
I₃
Isospin
-1
1
s d
s
s u
s
2
Ξ⁻
S
Ξ⁰
Strangeness

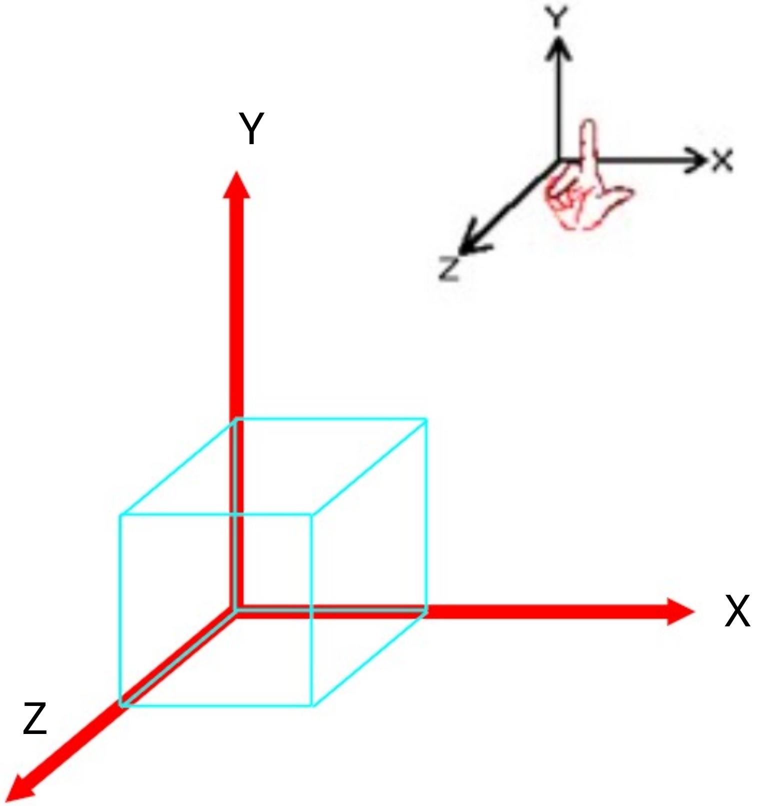
Discrete Symmetries

We have several important symmetries that are discrete

- Parity
- Charge conjugation
- Time inversion
- Combined operations
- Conserved quantities
- Violation of C, P and CP
- CPT?

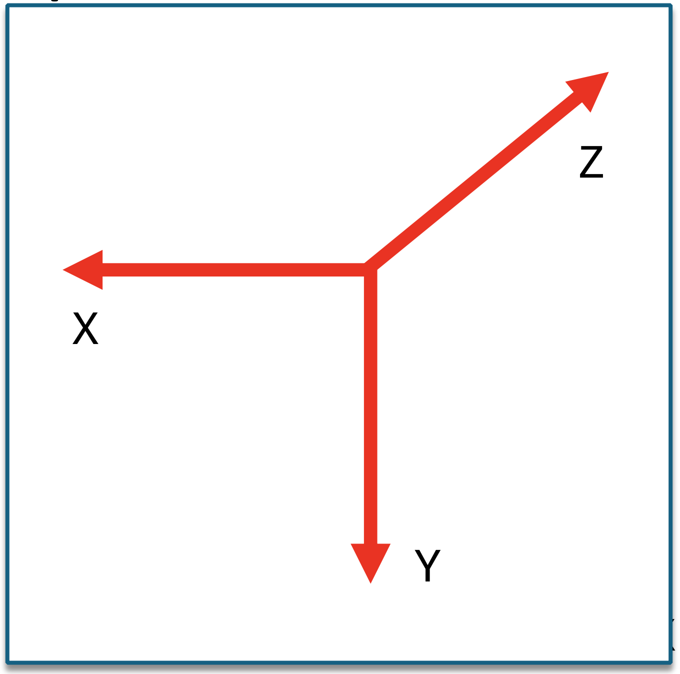
Parity transformation

- XYZ coordinate system



- right-handed

- Parity transformation flips all axes, $\vec{r} \to -\vec{r}$

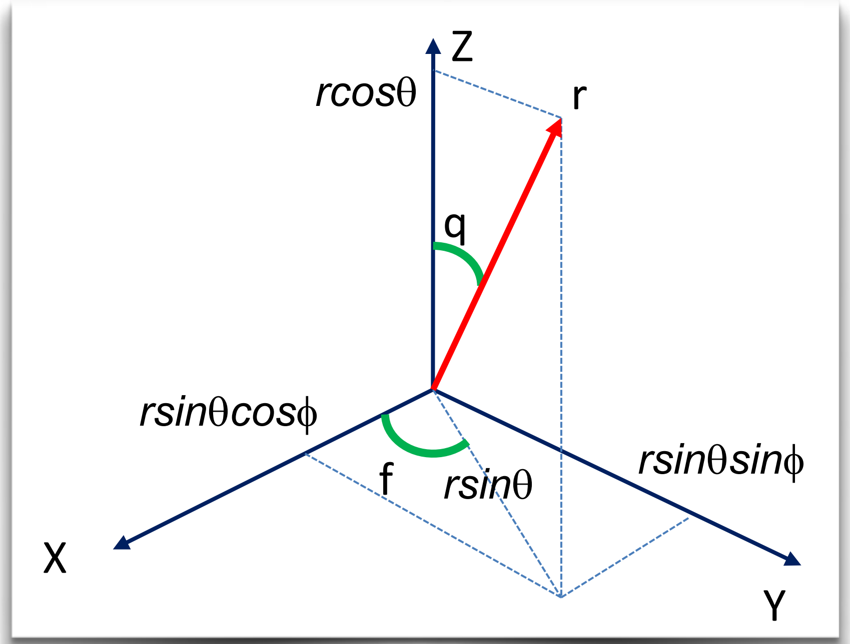


- left-handed

Spatial Reflection: Parity

Spherical polar co-ordinates:
$(x,y,z) \rightarrow (r,\theta,\phi)$

$$X = r \sin\theta \cos\phi$$
$$Y = r \sin\theta \sin\phi$$
$$Z = r \cos\theta$$



$\hat{P}$: Parity Operator P : Parity

In these coordinates, ***r*** → - ***r*** implies:

$$\begin{cases} r \rightarrow r \\ \theta \rightarrow \pi - \theta \\ \phi \rightarrow \pi + \phi \end{cases}$$

It can be shown for the spherical harmonics function that

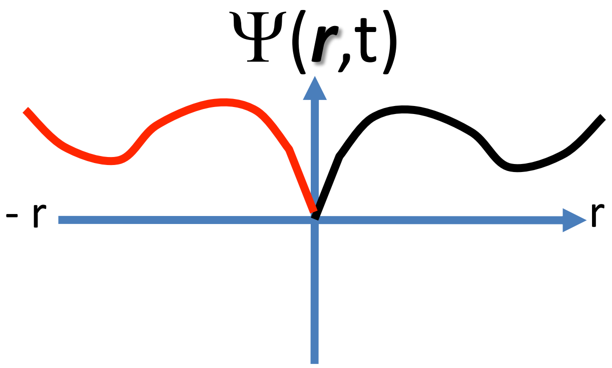
$$\hat{P}\, Y^l m(\theta,\phi) = (-1)^l\, Y^l m(\theta,\phi)$$

Intrinsic Parity (P)

The behavior of a state under a coordinate transformation $\boldsymbol{r} \rightarrow -\boldsymbol{r}$

Point particles are not extended objects BUT
Particles have intrinsic parity and it's conserved!

Particles are in definite eigenstates of the parity operator:



$$\hat{P}\,\Psi(\boldsymbol{r},t) = P\,\Psi(-\boldsymbol{r},t)$$

$$\begin{cases} \hat{P}^2\,\Psi(\boldsymbol{r},t) = \Psi(\boldsymbol{r},t) \\ \hat{P}^2\,\Psi(\boldsymbol{r},t) = P\hat{P}\,\Psi(-\boldsymbol{r},t) = P^2\,\Psi(\boldsymbol{r},t) \end{cases}$$

$$P^2 = 1, \quad P = \pm 1$$

Particles can have parity of +1 or - 1

Intrinsic Parity (P)

We assign intrinsic parity values to each particle & bound state

- It's arbitrary, but we assign fermions as P=+1 and anti-fermions as P=-1
- The parity of bound states is the multiplicative product of the constituents

$$\pi^0, \pi^{\pm}\ (q\bar{q}):\ P = (+1)(-1) = -1$$

$$p, n\ (qqq):\ P = (+1)^3 = +1$$

Parity: Looking it up

Particles and bounds states can be in complicated parity eigenstates

- Eigenstates of intrinsic parity
- Eigenstates of angular momentum parity

$$\hat{P}\psi(a) = P_a(-1)^{\ell}\psi(a)$$

$$N(938): \quad I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$$

Neutron

I = Isospin
J = Total angular momentum
P = Parity

$$N(1675): I(J^P) = \frac{1}{2}(\frac{5}{2}^-)$$

Pentaquark

(5 quarks)

Vector vs Pseudovector

- Vector - spin 1, for example Rho, Omega, J/Psi mesons or the W, Z bosons
  - Vectors have parity -1, P(v) = -v
  - Vectors in daily life are velocity, momentum, distance, electric field
- Pseudovector - spin 1, for example a_1 meson
  - Or axial vector
  - Pseudovectors have parity +1, P(a) = a
  - Pseudovectors in daily life are angular velocity, angular momentum, magnetic field - anything that is a cross-product

Parity Transformations

Scalar	***P(s) = s***
Pseudoscalar	***P(p) = -p***
Vector	***P(v) = -v***
Pseudovector (or axial vector)	***P(a) = a***