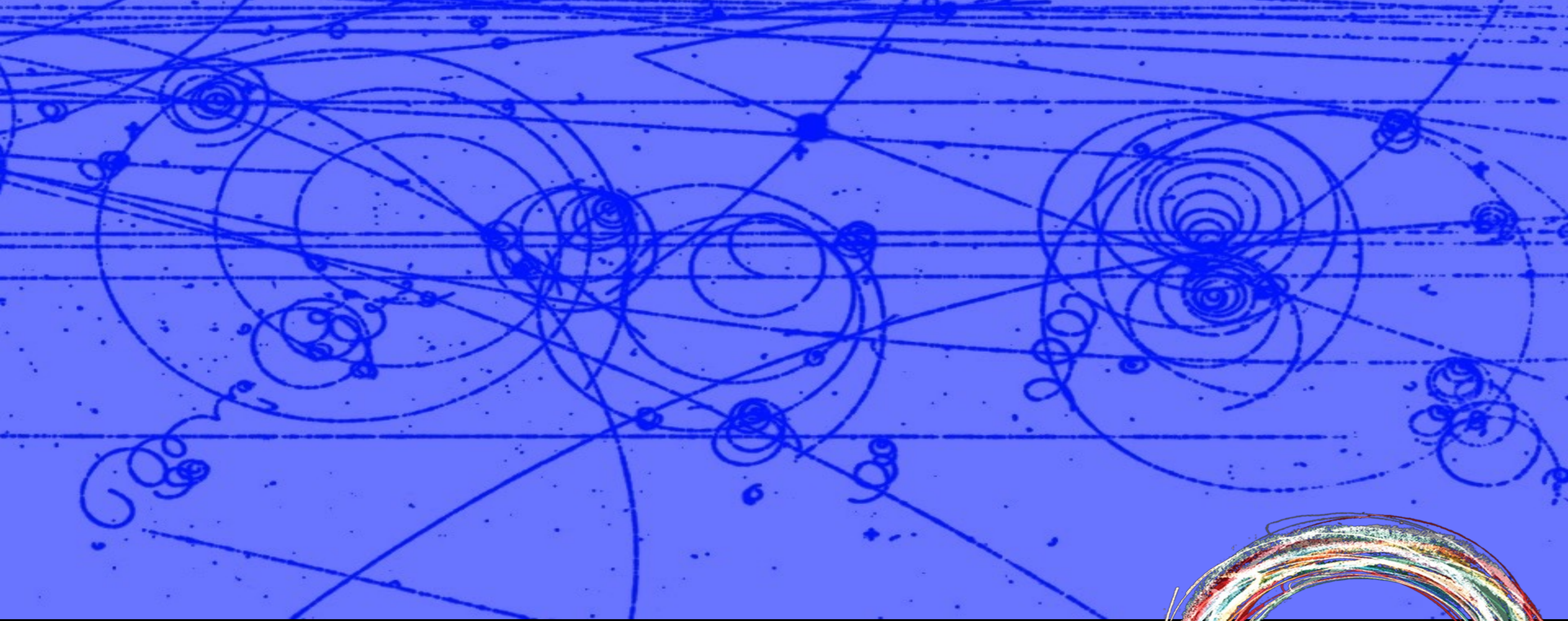
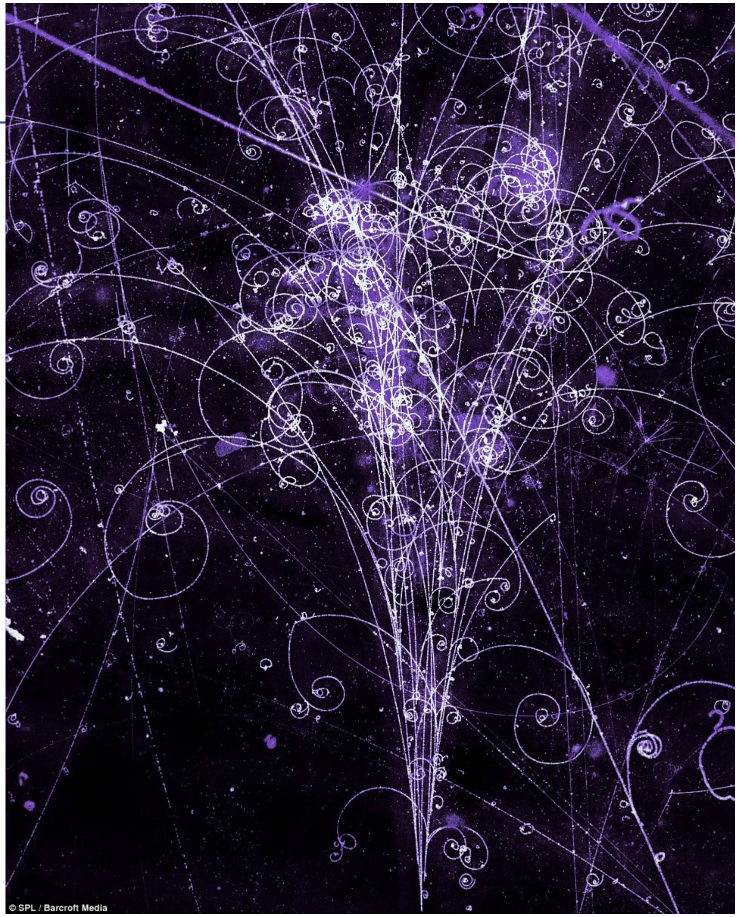


LECTURE 6
BOUND STATES

PHY 493 / 803

Announcements



Quiz:
- Pick up Friday's quiz after class
-Next quiz: Wednesday this week

Homework:
- Second homework due today at 3pm.
- Submit on gradescope.
- Third HW will be posted soon

Paper:
- Topic due Monday Feb. 17th. Please reply to this google form before then:
  https://forms.gle/MmCk8NtrMm7RdfLC7
- Outline due on gradescope: Feb 28 at 3pm

Midterm: Friday, Feb 21
Will cover material through "Bound States"
Equation sheet: 1 letter-sized (8 ½ by 11 inches) page front and back, handwritten

Last Time

Important discrete symmetries in particle physics:

1. Parity (mirror world)
2. Charge conjugation
3. Time reversal

$$K^0_L \to \pi^+ + e^- + \underline{\nu}_e$$

$$K^0_L \to \pi^- + e^+ + \nu_e$$

The EM and strong forces conserve parity, but the weak does not!

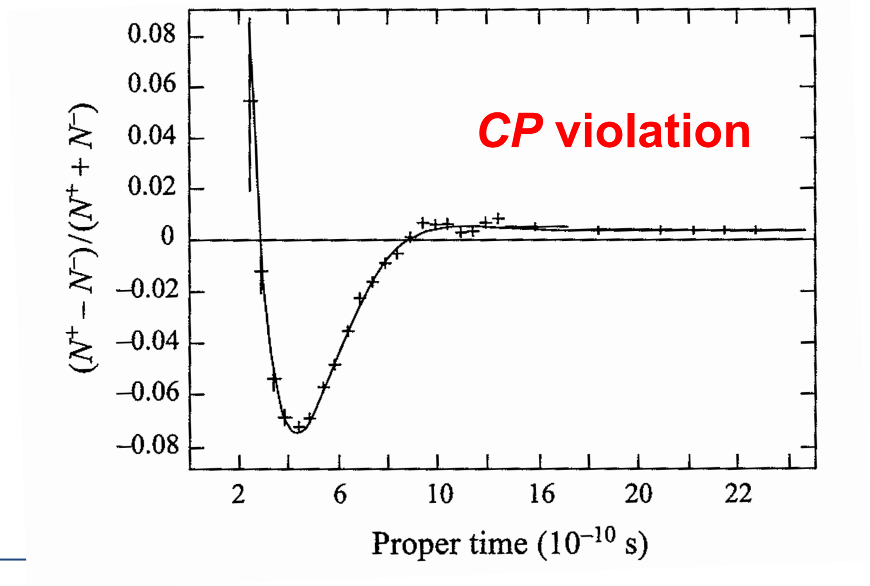
$$CP[\pi^+ + e^- + \underline{\nu}_e] = \pi^- + e^+ + \nu_e$$

What about the combination of C and P together?

-Works for muon decay

- Kaons exhibit CP violation

e+ vs e- Asymmetry



How can we rely on these symmetries to make a coherent theory (SM) given these observations?

What about CPT together?

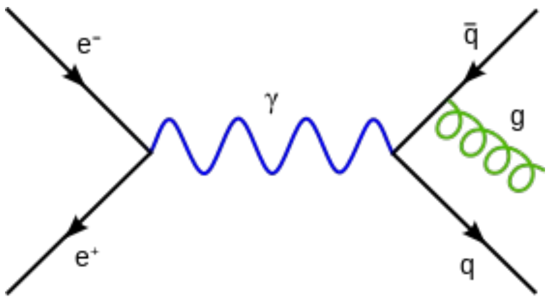
CP violation implies T-Symmetry violation!

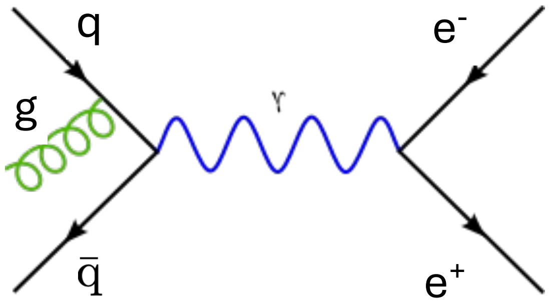
I cannot invert time axes and get back to where I was.

But for a closed system, we strongly believe that the combined CPT operation results in a valid symmetry.

Violation of CPT also violates Lorentz symmetry, a requirement for relativistic quantum field theories.

$$\begin{aligned}\hat{C}\hat{P}\hat{T}\psi(\vec{p},t,\sigma,q) &= \hat{C}\hat{T}\psi(-\vec{p},t,\sigma,q)\\ &= \hat{T}\psi(-\vec{p},t,\sigma,-q)\\ &= \psi(-\vec{p},-t,-\sigma,-q)\\ &= \psi(\vec{p},t,-\sigma,-q)\end{aligned}$$



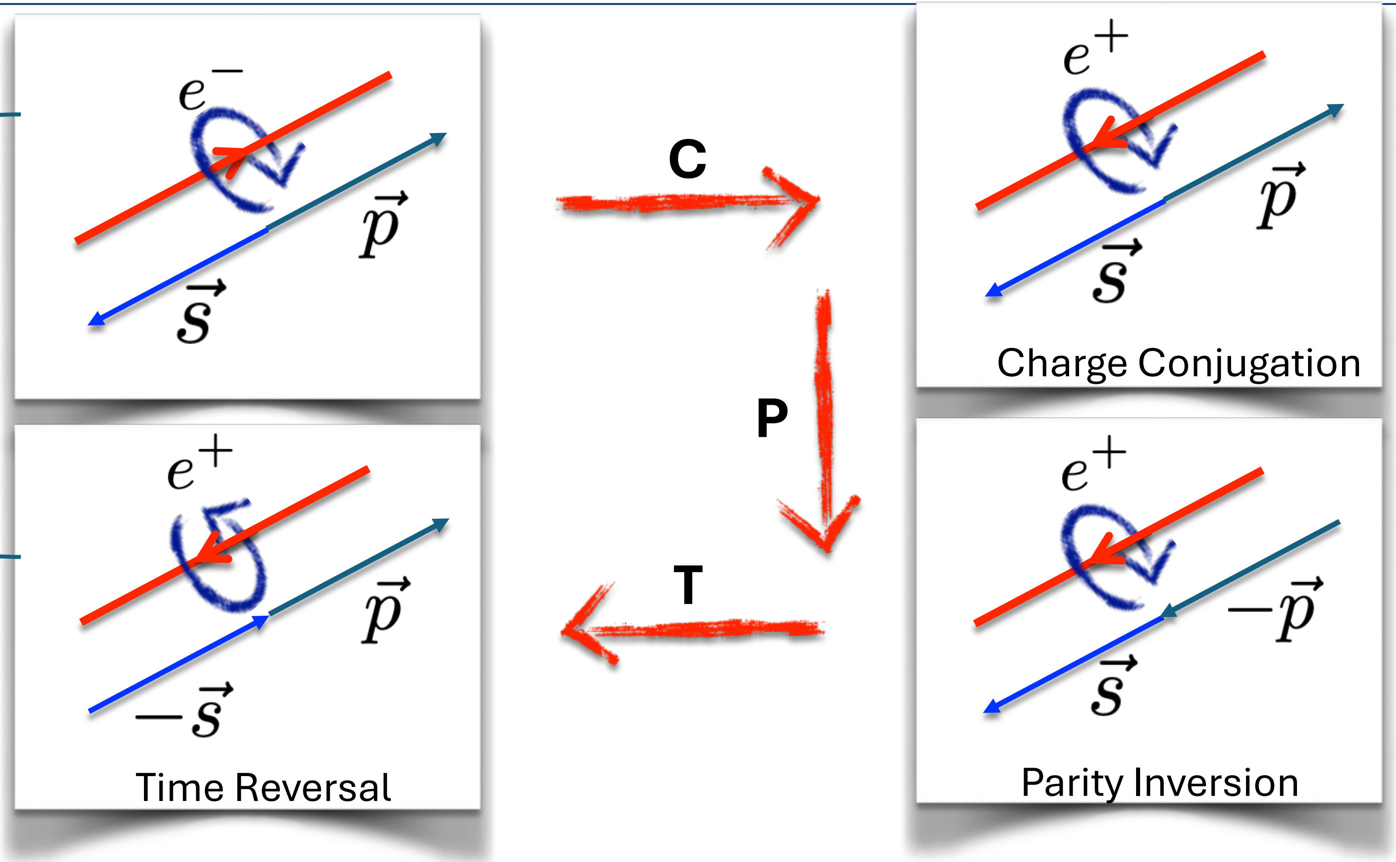


CPT implies that the cross section we calculate for the top diagram is the same as for the bottom diagram.

-> So far this is what we measure

CPT: Operations on an Electron

CPT says these states should be the same



Flip e- to e+

Momentum changes direction

Spin and momentum change directions

Nature seems to tell us that our time axis only goes forward

T-symmetry violation

Nature seems to be able to prefer matter over antimatter

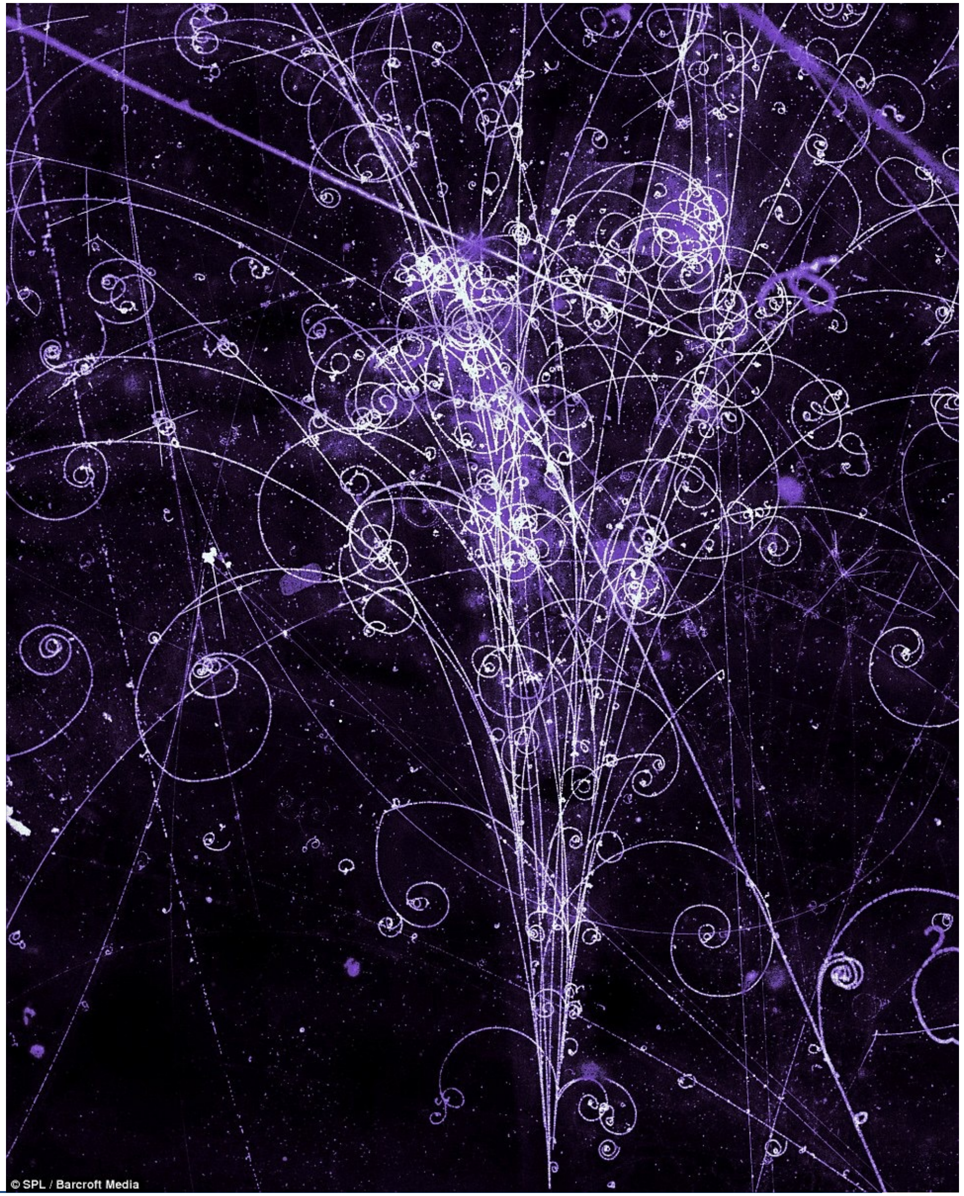
CP-symmetry violation

CPT seems to help us understand the apparent fabric of our universe

CPT Symmetries

Quantity	Electromagnetism	Strong Force	Weak Force
Parity (P)	Yes	Yes	No
Charge Conjugation Parity (C)	Yes	Yes	No
CP	Yes	Yes	No
Time Reversal (T)	Yes	Yes	No (?)
CPT	Probably	Probably	Probably

Recap / Up Next



This time:

Symmetries
- Group Theory
- Operators

Conservation Laws
- Physical Symmetries

Next time:

Bound States
- Hydrogen
- 'Onium
- Mesons/Baryons

The Schrodinger Equation

Foundation of non-relativistic quantum theory

Schrodinger Equation:

$$\underbrace{(-\frac{\hbar^2}{2m}\nabla^2 + V)}\psi = i\hbar\frac{\partial}{\partial t}\psi$$

Hamiltonian – description of total energy:

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V$$

Time-independent solution to the Schrodinger Equation:

Assume (or know!) that the potential V does not depend on time

This wave function

$$\psi(\vec{r},t) = \psi(\vec{r})e^{-iEt/\hbar}$$

Solves time-independent Schrodinger equation:

$$(-\frac{\hbar^2}{2m}\nabla^2 + V)\psi = E\psi$$

Eigenvalue form:

$$H\psi = E\psi$$

The Schrodinger Equation

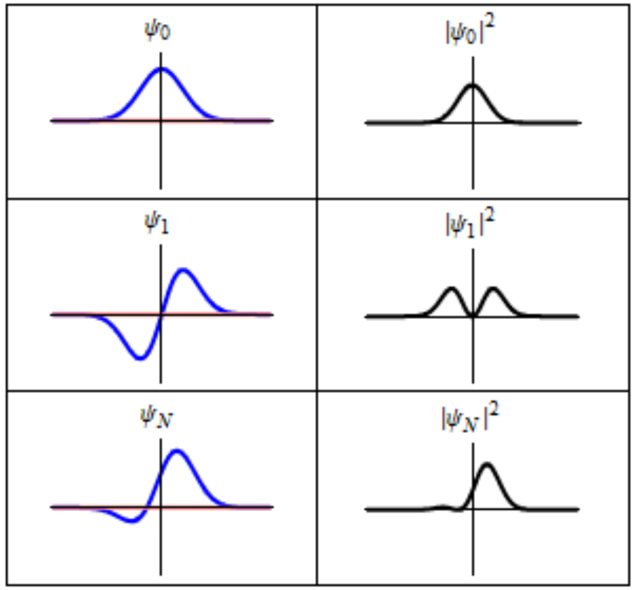
Foundation of non-relativistic quantum theory

Schrodinger's equation is based on the Hamiltonian, a description of the total energy of a system.

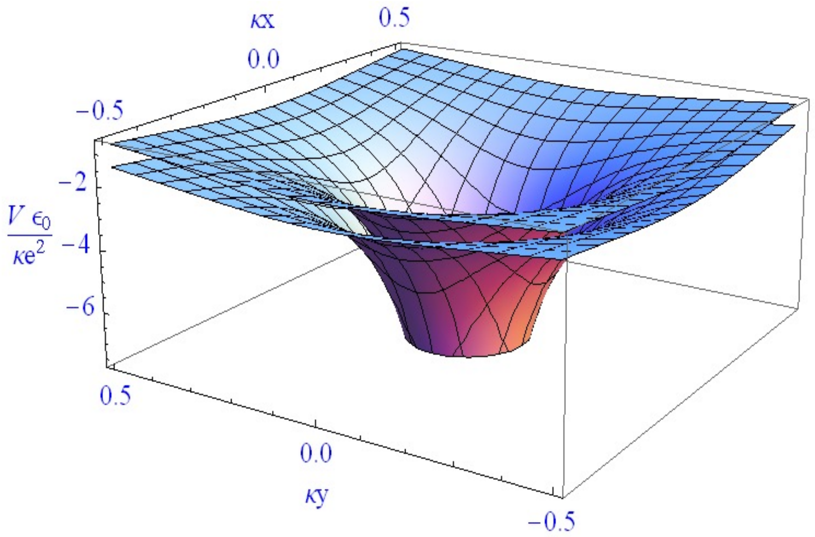
$$\psi(\vec{r},t) = \psi(\vec{r})e^{-iEt/\hbar}$$

$$H\psi = E\psi$$

Wave function Amplitude



The Central Potential



The Coulomb potential is a classic example

We can separate wave function into radial and angular states

$$\psi(r,\theta,\phi) = \frac{u(r)}{r} Y_l^{m_l}(\theta,\phi)$$

l,m = orbital angular momentum quantum numbers

Only the radial wave function depends on the potential!

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V(r) + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u = Eu$$

The angular wave functions are the same for all spherically symmetric potentials.

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{-i\varphi}\cdot\sin\theta$$

$$Y_1^{0}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\cos\theta$$

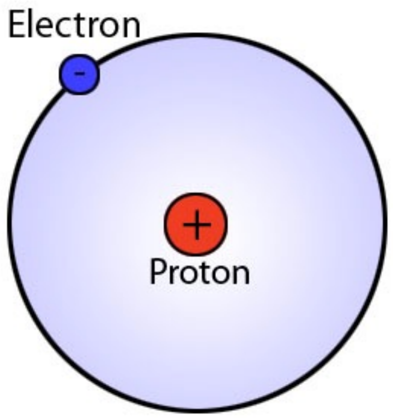
$$Y_1^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{i\varphi}\cdot\sin\theta$$

Can be looked up in PDG

Hydrogen

Electrons in the Hydrogen atom are bound in a central potential

Special case in which $M_e \ll M_p$



E&M potential:

Coulomb Potential $$V(r) = -\frac{e^2}{r}$$

Bound state energy levels

$$E_n = -\frac{me^4}{2\hbar^2 n^2} = -\alpha^2 m^2 \left(\frac{1}{2n^2}\right) = -13.6 \text{ eV}/n^2$$

Hydrogen Radial Solutions

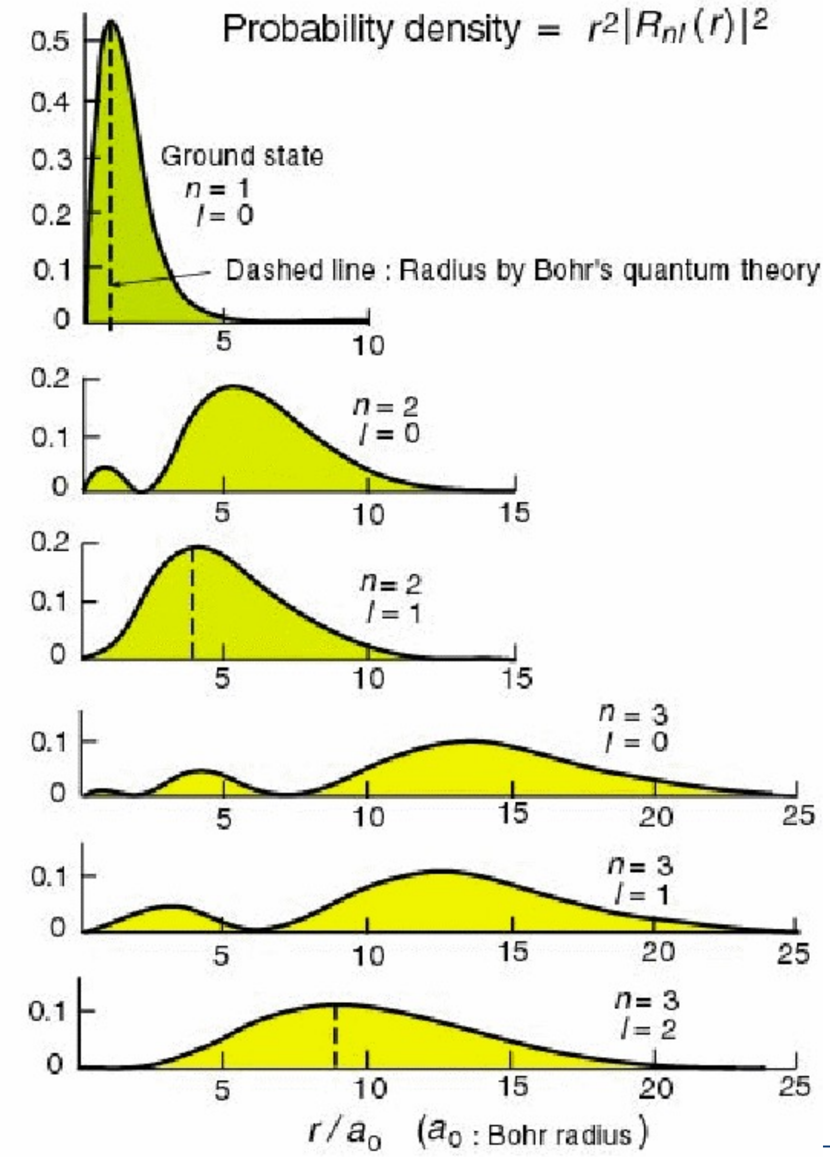
Low energy levels

Stable state

High energy levels

More options for electron position

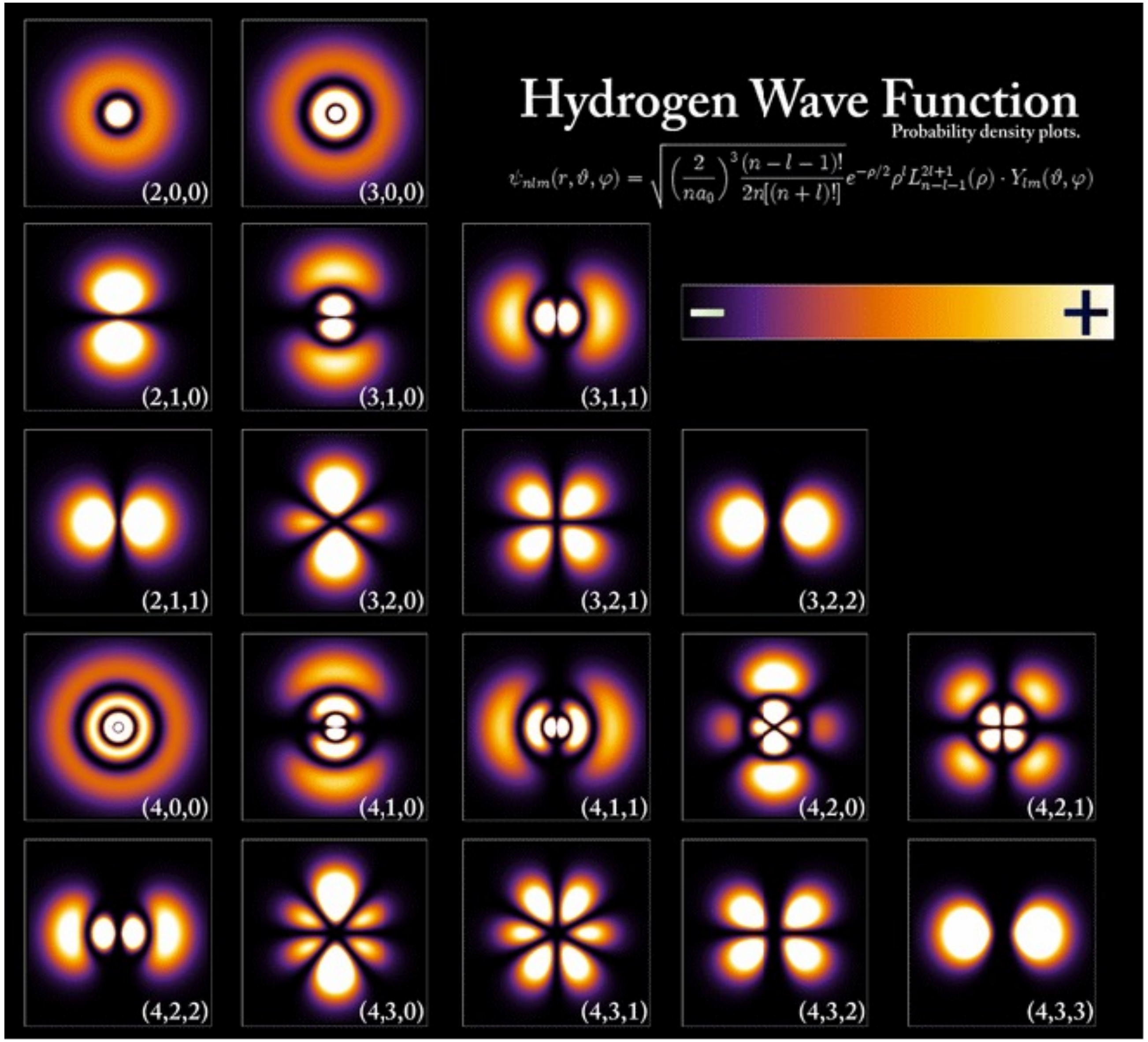
Probability



Radial distance (r)

Hydrogen Wave Functions

High probability = white
Low probability = black



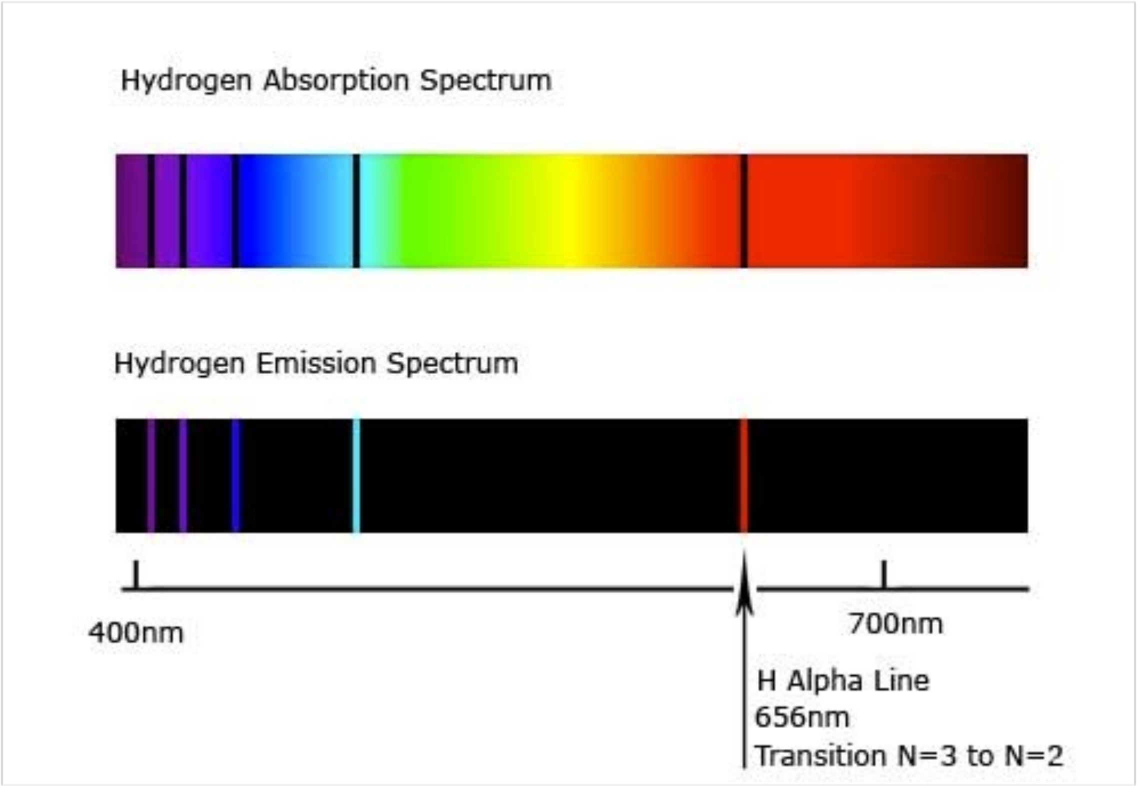
"probability cloud"
of electrons

Hydrogen Spectrum: Observation

Shine light through hydrogen and look at what is absorbed:

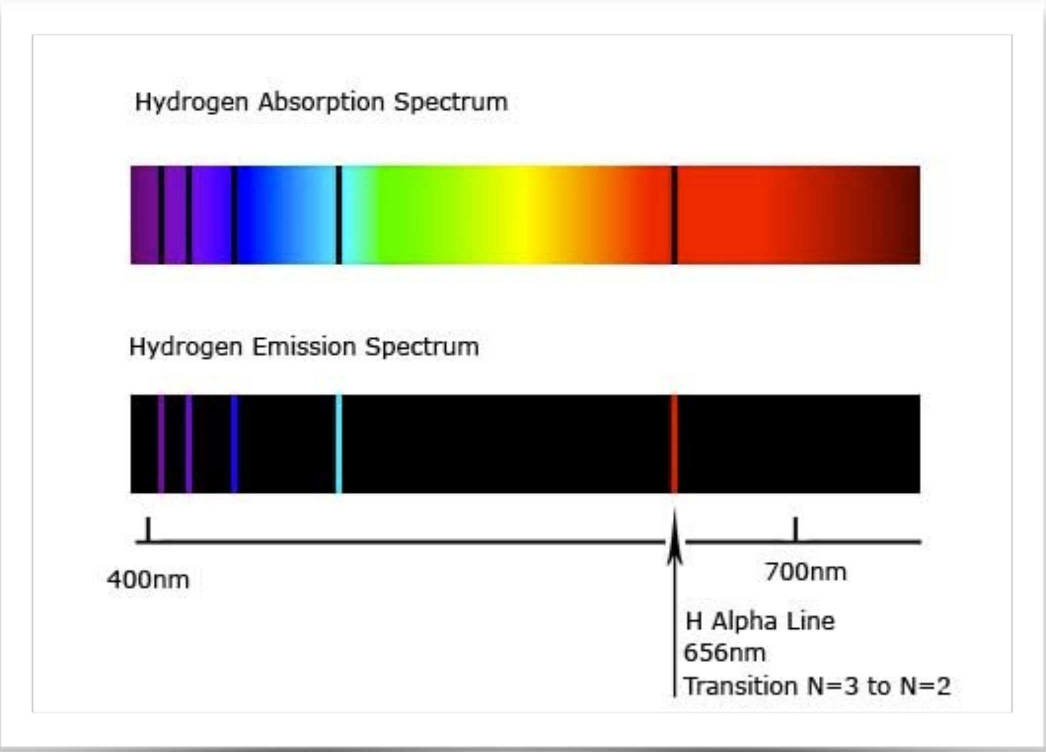
Excite hydrogen and look at emitted light:

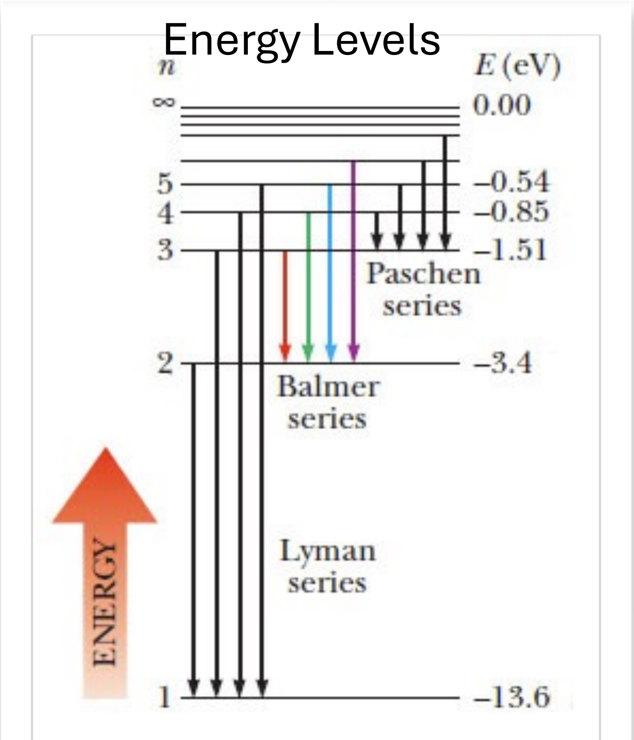
Will at some time deexcite. Detect deexcitation photons.



$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad R = \frac{me^4}{4\pi} = 1.09737 \times 10^5/\mathrm{cm}$$

Hydrogen Spectrum: Observation



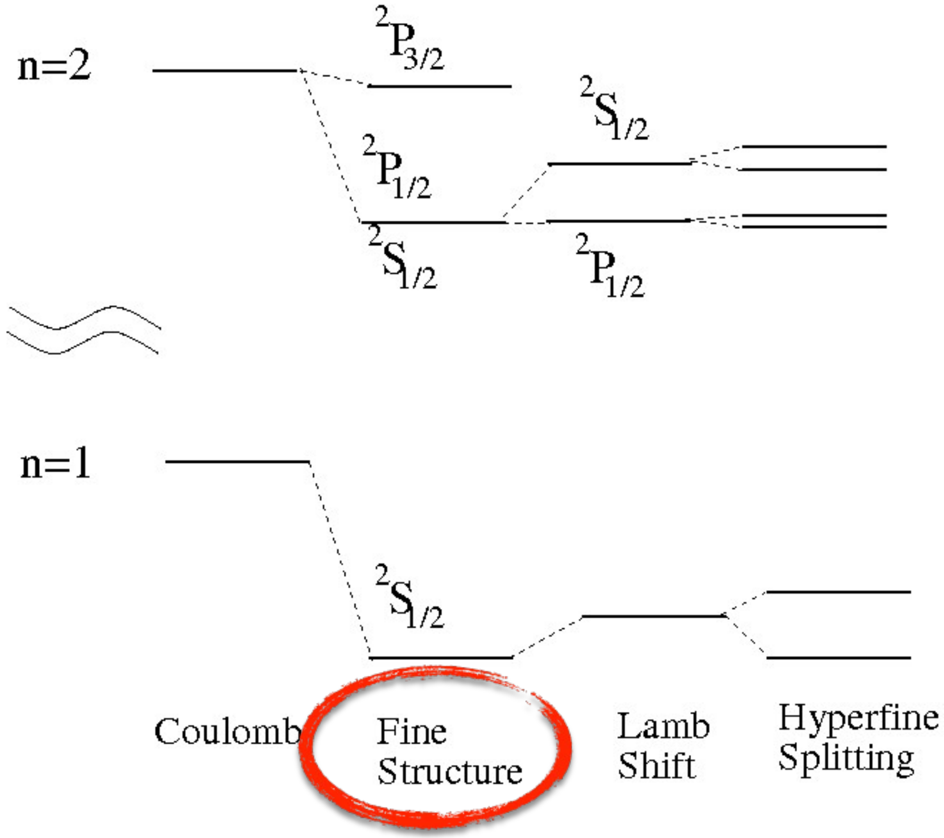


An energy level diagram for hydrogen.

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad R = \frac{me^4}{4\pi} = 1.09737 \times 10^5/\text{cm}$$

Hydrogen Spectrum*

Energy levels:



*Magnitude of corrections not drawn to scale (or even relative scale)!

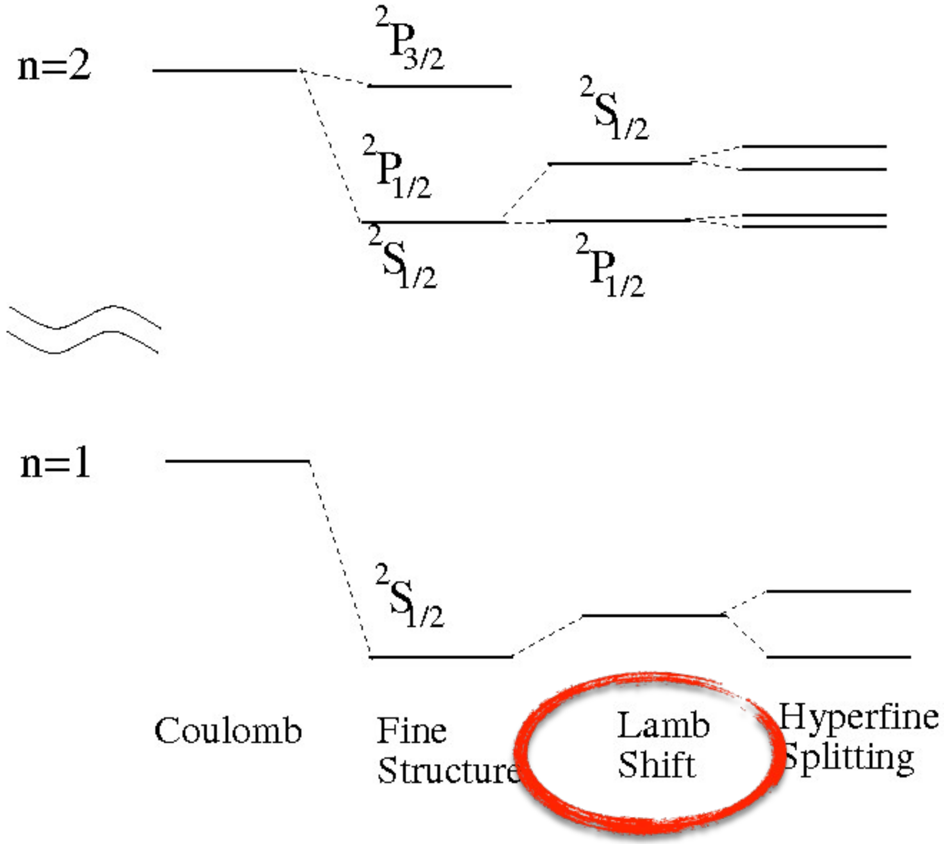
Fine structure:

interaction between spin and magnetic field B

caused by orbital angular momentum

$$\Delta E = -\mu_e \cdot \mathbf{B} = \frac{e}{mc}\mathbf{S} \cdot \mathbf{B}$$

Hydrogen Spectrum*

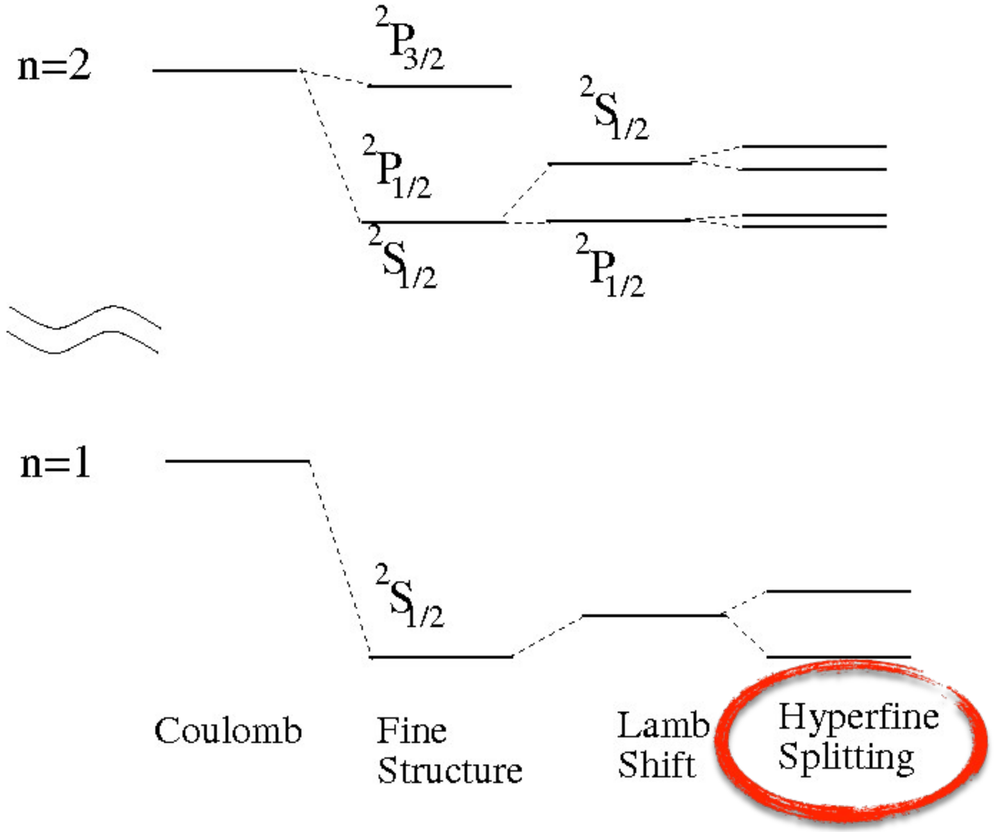


*Magnitude of corrections not drawn to scale (or even relative scale)!

Lamb shift: Quantization of electromagnetic field

E&M is exchange of quantized photons; only certain transitions allowed

Hydrogen Spectrum*

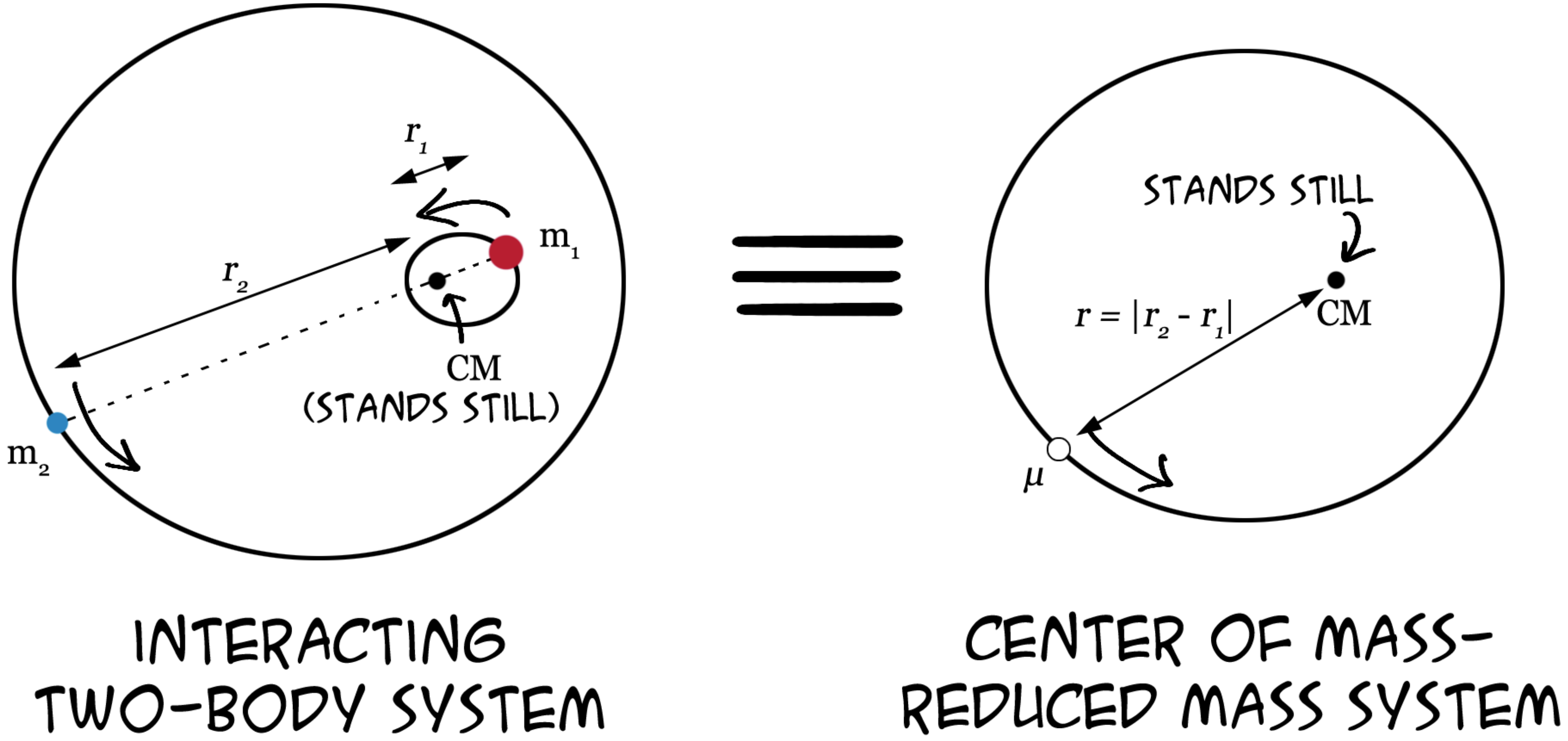


*Magnitude of corrections not drawn to scale (or even relative scale)!

Hyperfine splitting due to proton spin-orbit and spin-spin interactions.

Correction terms add variations, even just to the ground state of hydrogen

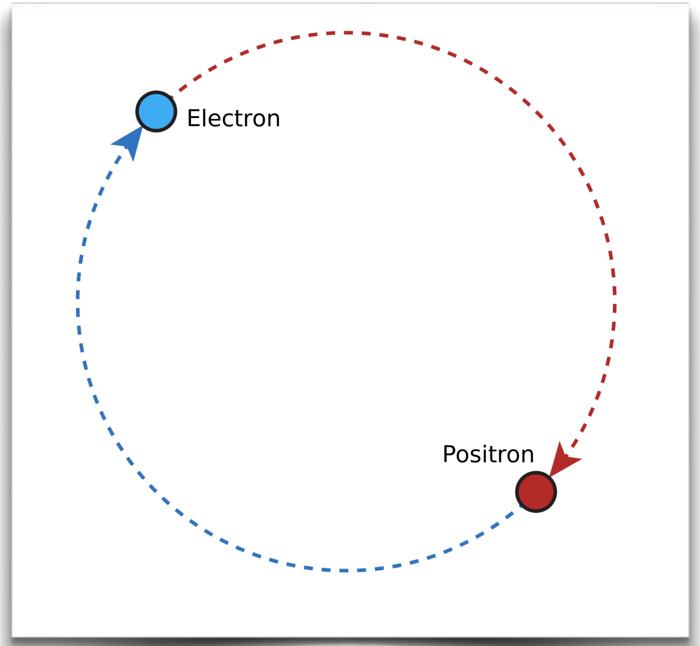
Definition: Reduced Mass

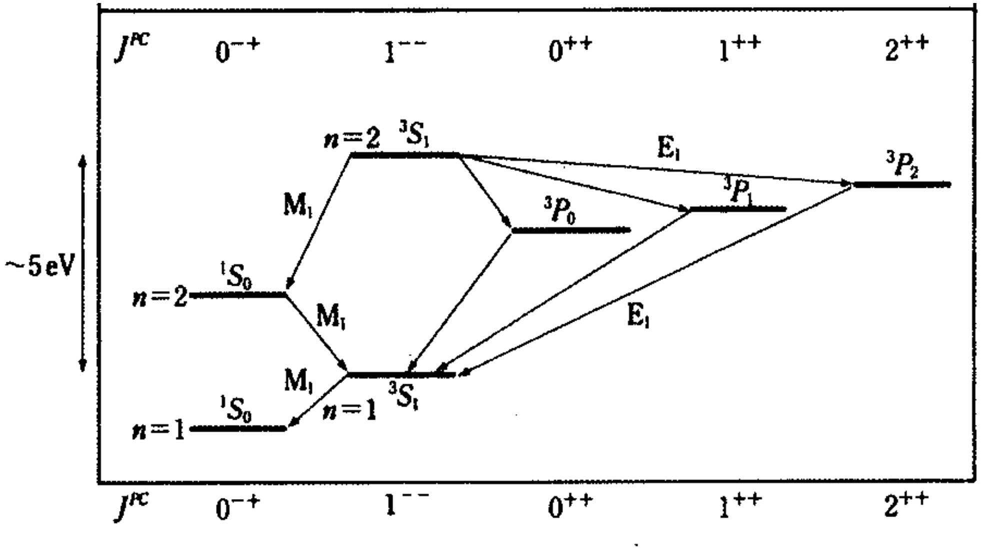


For an atom, the reduced mass μ is close to the electron mass because the proton is so heavy.
Not necessarily true for other bound states.

Positronium

electron-positron bound state





Energy States:

$$E_n^{\text{pos}} = \frac{1}{2} E_n = -\alpha^2 m \left(\frac{1}{4n^2}\right)$$

Reduced mass:

$$m_{\text{red}} = \frac{m_1 m_2}{m_1 + m_2}$$

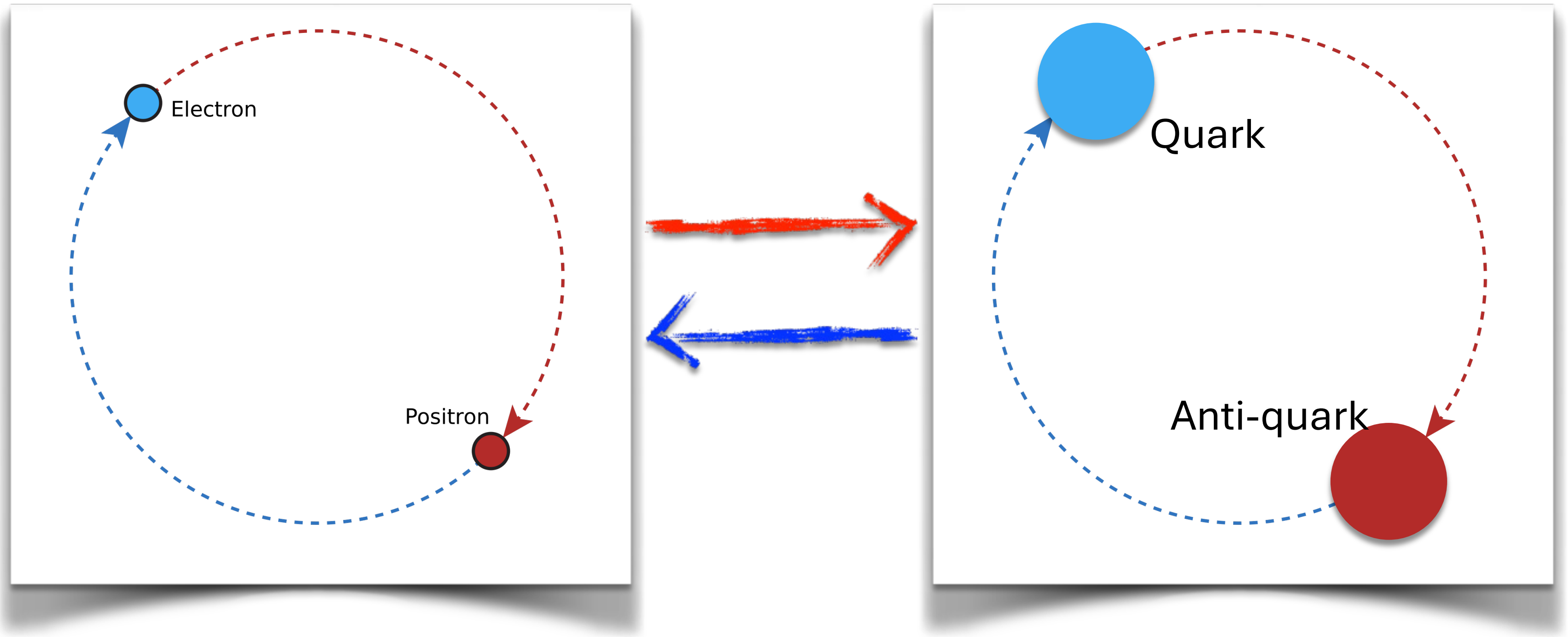
Lifetime:

$$\tau = \frac{2\hbar}{\alpha^5 m c^2} = 1.25 \times 10^{-10}\ \text{sec}$$

Quarkonium

Bound states of same-flavor quark and anti-quark pairs also exist.

We will use the convenience of the Hydrogen and positronium models to try to learn something about meson structure.



For quarks need to add strong force, and can never study the quarks in isolation.

Static potential between q and $\bar{q}$

Heavy mesons, which consist of a heavy quark and an anti-quark, can be treated without relativity to a first approximation, since the quarks are heavy and they move "slowly".

Mesons : $q + \bar{q}$

Static potential (QCD) → energies of hadrons

same approach

Hydrogen atom : $p + e^-$

Static potential (Coulomb potential) → energy levels of atoms

Flavor Independence

Flavor Independence :

The strong force between two quarks at a fixed distance apart is independent of which quark flavors *u, d, s, c, b, t* are involved. (after taking into account quark mass differences)

u $\Leftrightarrow$ s equals d $\Leftrightarrow$ s

but differs from $\bar{u}$ $\Leftrightarrow$ s (not just flavor, also particle->anti-particle)

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but differs from $\bar{u} \Leftrightarrow$ s (not just flavor, also particle->anti-particle)

A consequence of flavor independence is hadrons in families have approximately the same masses (charge multiplets).

Example :

π^+	139.57 (MeV/c^2)	p	938.27 (MeV/c^2)
π^0	134.97 (MeV/c^2)	n	939.57 (MeV/c^2)
π^-	139.57 (MeV/c^2)		

Different quark content, similar masses

Small correction: electromagnetic force

J/Ψ meson

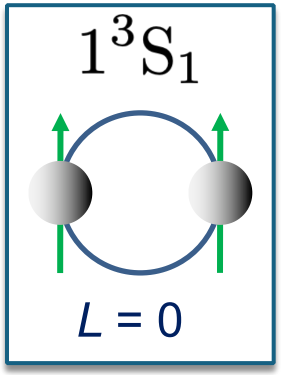
Discovery of the charm quark (1974)

J/Ψ meson ($c\bar{c}$) : Richter at SLAC $e^{-}+e^{+}$ collision
Ting at BNL $p+p$ collision

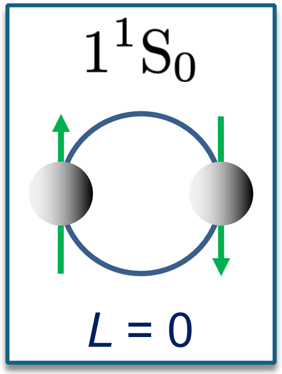
Charm quarks were theoretically predicted, but not in a meson form with a long lifetime

$J/\Psi(3097)$ is one of states with the name charmonium
(by analogy with positronium $e^{+}e^{-}$), and has the quantum state 1^3S_1

$$n^{(2s+1)}l_j$$



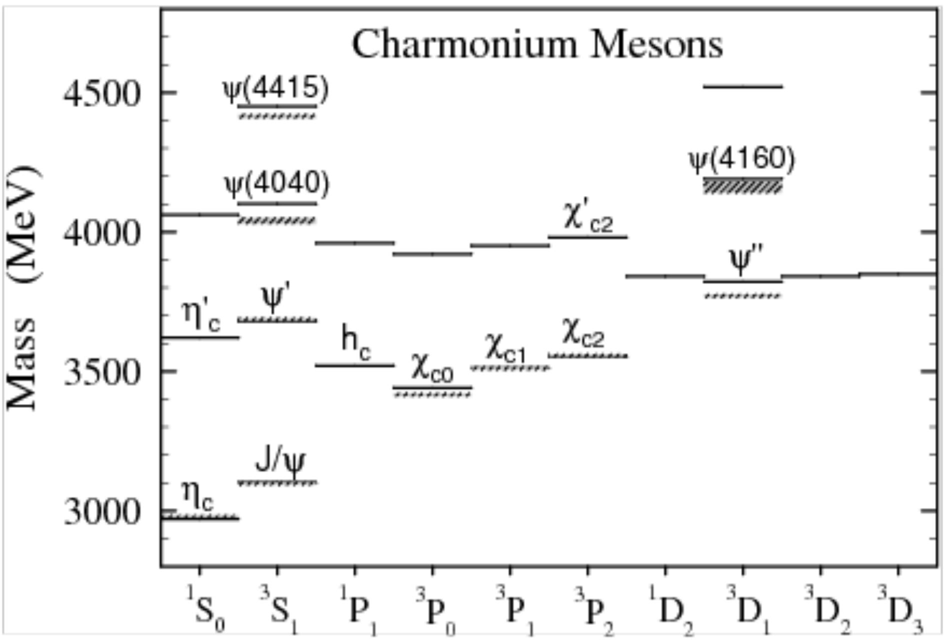
Also, we have the η_c



Just like hydrogen, many different states, depending on energy level and spin alignment

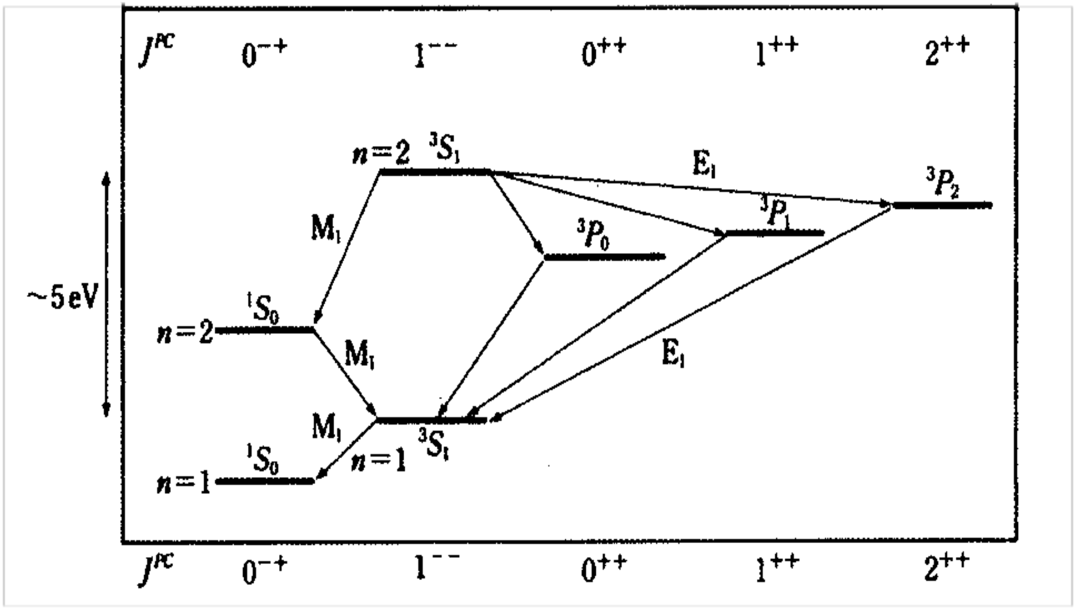
J/Ψ meson

Charmonium



States

Positronium

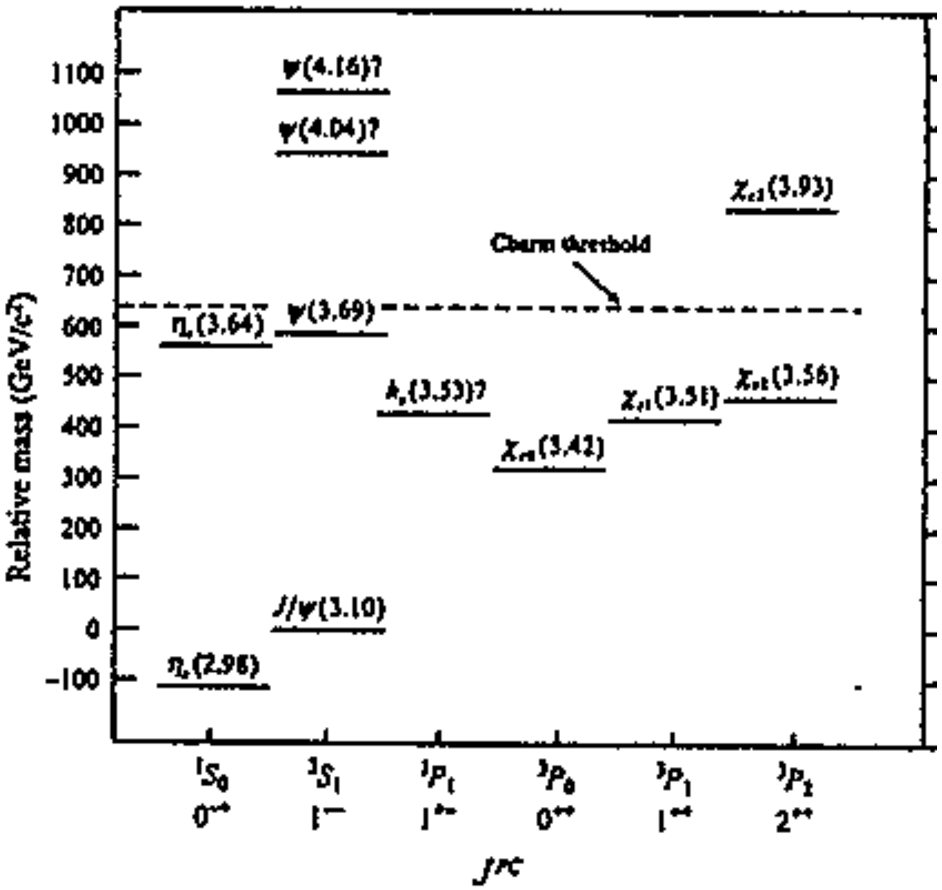


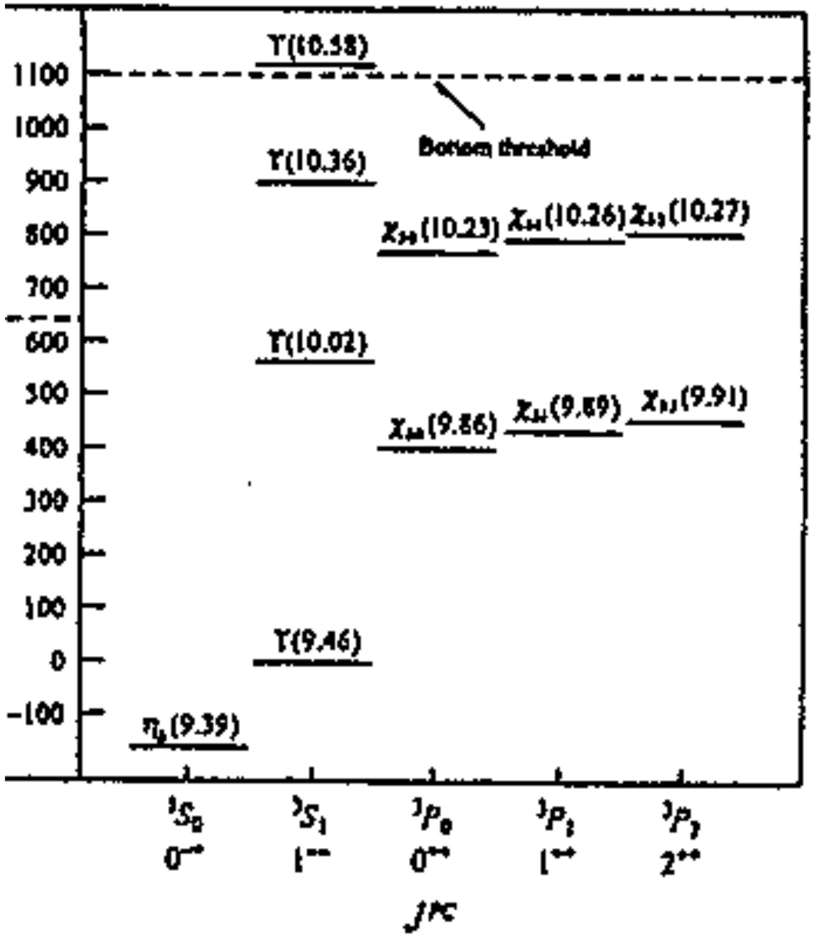
Same quarks involved, differences from dynamics lead to different masses

For mesons, the mass has to do with the binding energy as much as the mass of the constituent quarks

Flavor Independence: “Charmonium" and "Bottomonium"

Examples of charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) with various $^{2S+1}L_j$



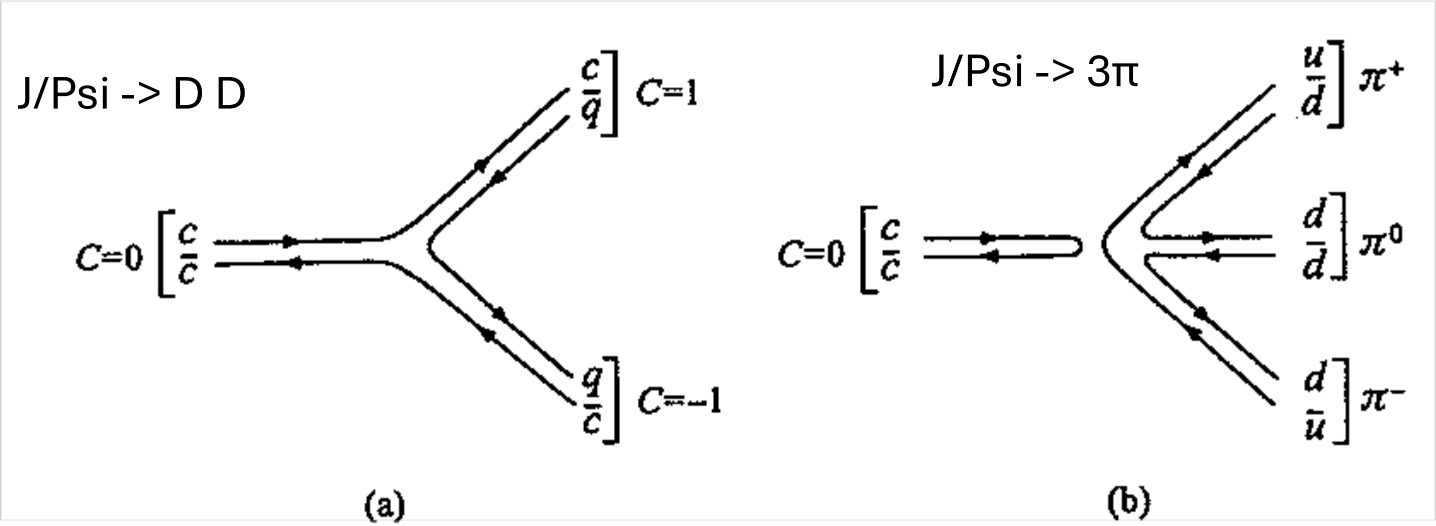


Similarity in energy levels indicates **flavor independence** of the strong force

Decay of J/Ψ

J/Ψ (3097) has an anomalously long lifetime for a meson due to the nature of its decays

Possible decay diagrams



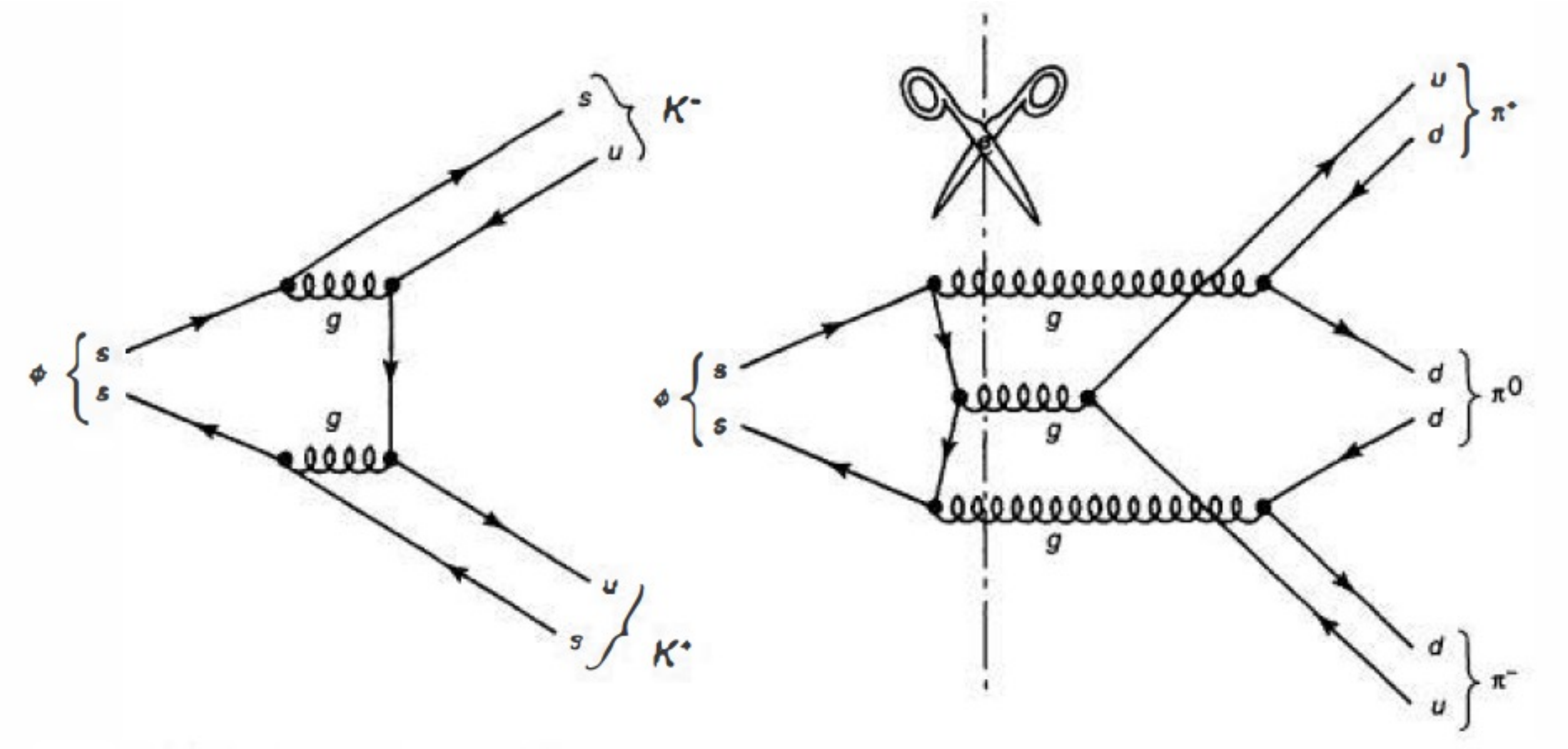
Forbidden by energy conservation
mass difference
$M_{J/\psi} < 2M_D$ (charm threshold)
$M_D = 1869$
the mass of the lightest meson with *c*

Possible, but highly suppressed as explained by the OZI rule

initial and final quark lines are disconnected

The OZI Rule (Susumu Okubo, George Zweig and Jugoro Iizuka)

Final states that can be separated purely by gluon lines are highly suppressed.



The reason is that when quarks are close together, the gluons have relatively low energy (recall asymptotic freedom). Therefore, they are unlikely to have enough energy to generate these reactions.

One-gluon exchange potential

The level structure of charmonium ($c\bar{c}$) and bottomium ($b\bar{b}$) is similar to that seen in the positronium

There should be a major contribution from a single-particle exchange with a "Coulomb-like" form.
(one gluon exchange)

$$V_{qcd}(r) = -\frac{4}{3}\frac{\alpha_s \hbar c}{r} \longleftrightarrow V_{EM}(r) = -\frac{\alpha \hbar c}{r}$$

α_s is proportional to the strong interaction analogue of the fine structure constant α in QED

One-gluon exchange potential

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There should be a major contribution from a single-particle exchange with a "Coulomb-like" form.
(one gluon exchange)

$$V_{qcd}(r) = -\frac{4}{3}\frac{\alpha_s \hbar c}{r}$$

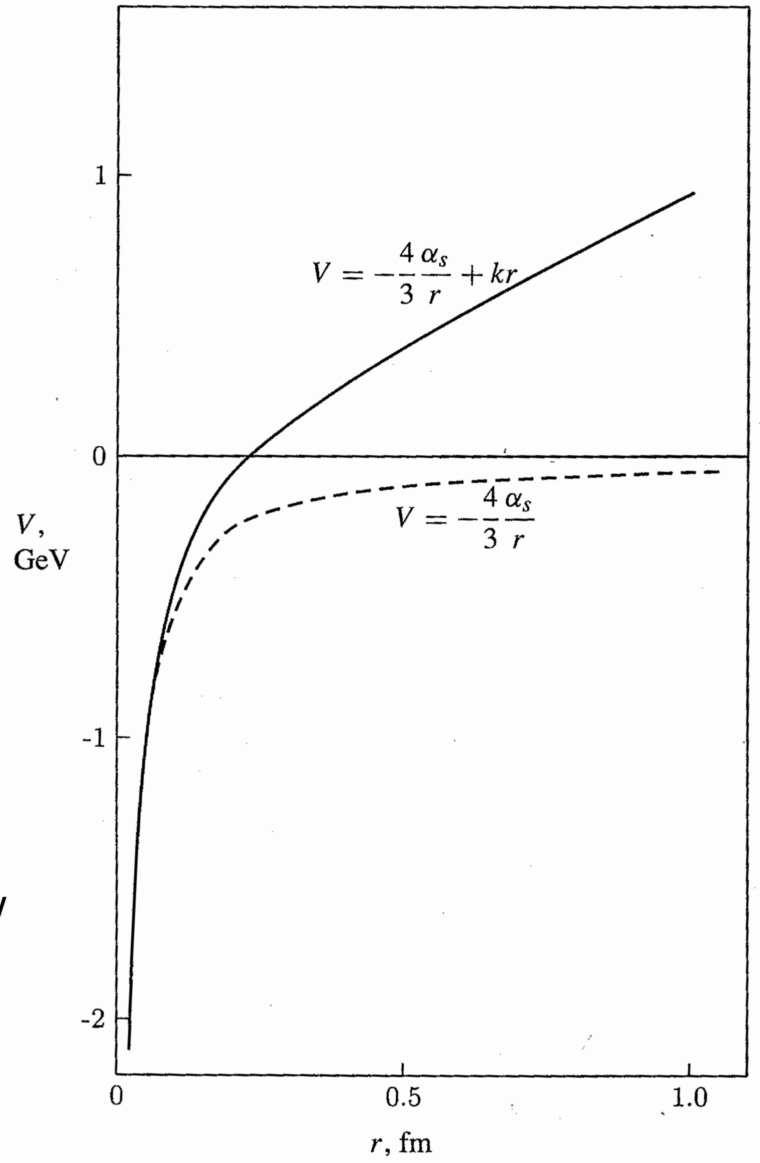
$$V_{EM}(r) = -\frac{\alpha \hbar c}{r}$$

To account for the quark confinement, we need to add a confining potential

$$V_{qcd}(r) = -\frac{\alpha_s \hbar c}{r} + F_o r$$

Mass of the $c\bar{c}$ and $b\bar{b}$ systems can be explained
with the same values of F_o (flavor independence)

The QCD Quasi-Potential



Differ at large distances

Strong force

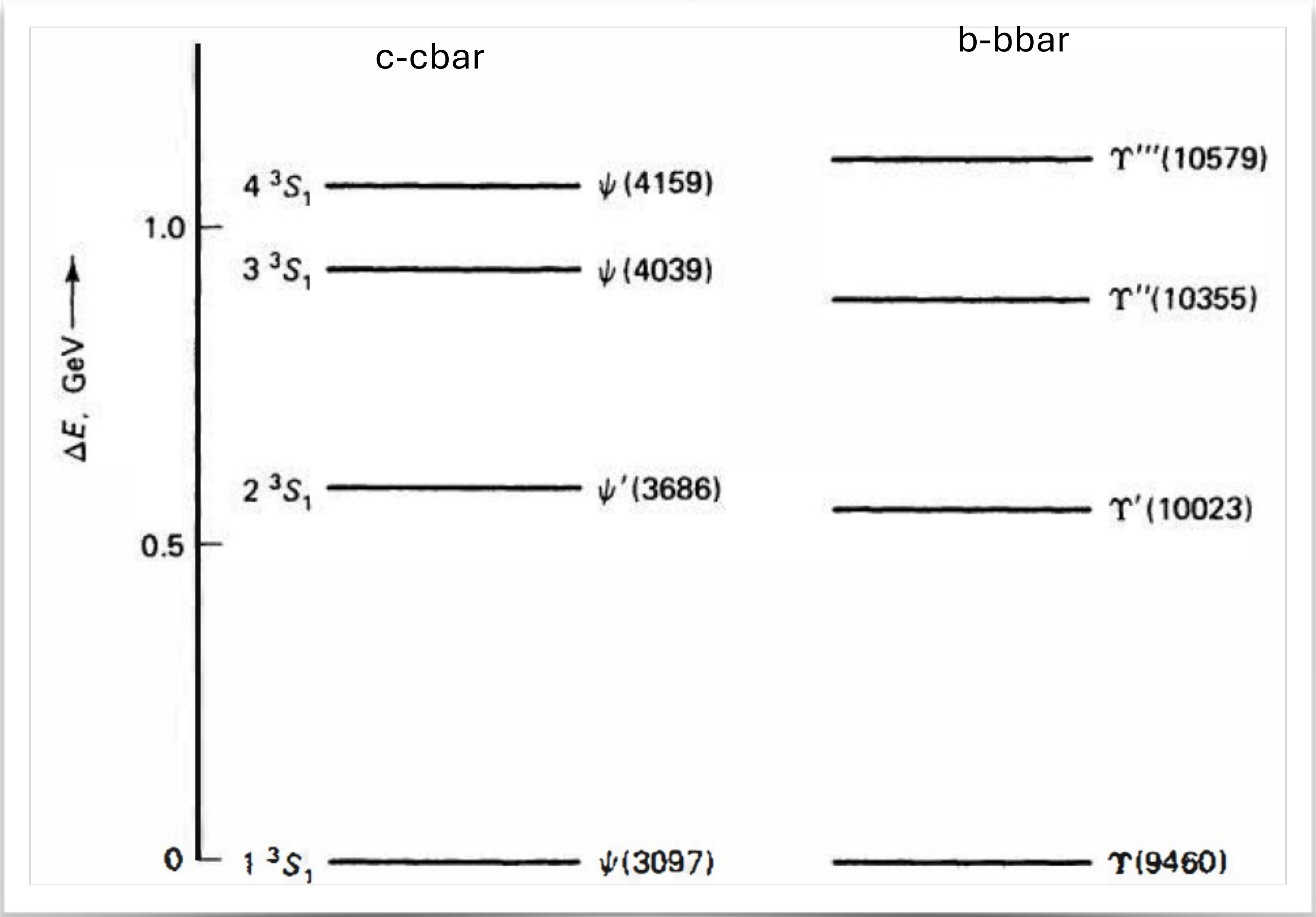
Does not go to 0 at large distances
(never see quarks in isolation)

E&M

Goes to 0 at large distances

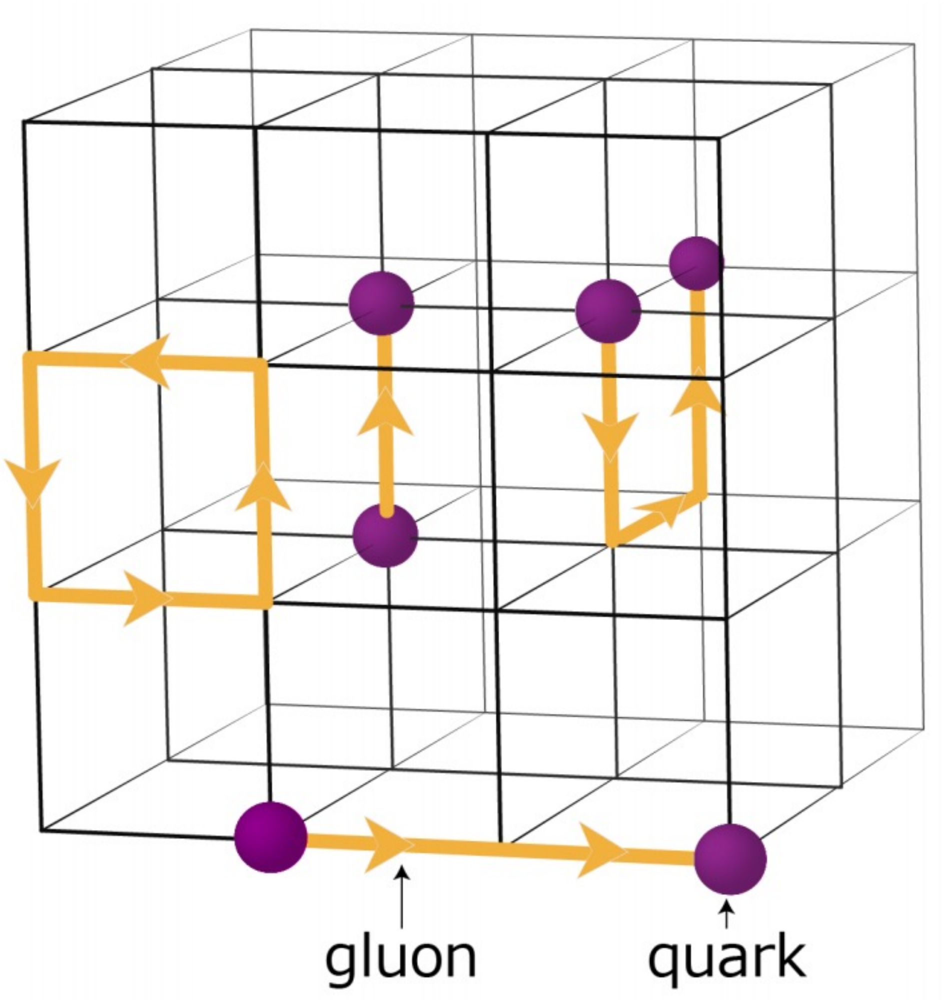
Same at low distances

The QCD Quasi-Potential

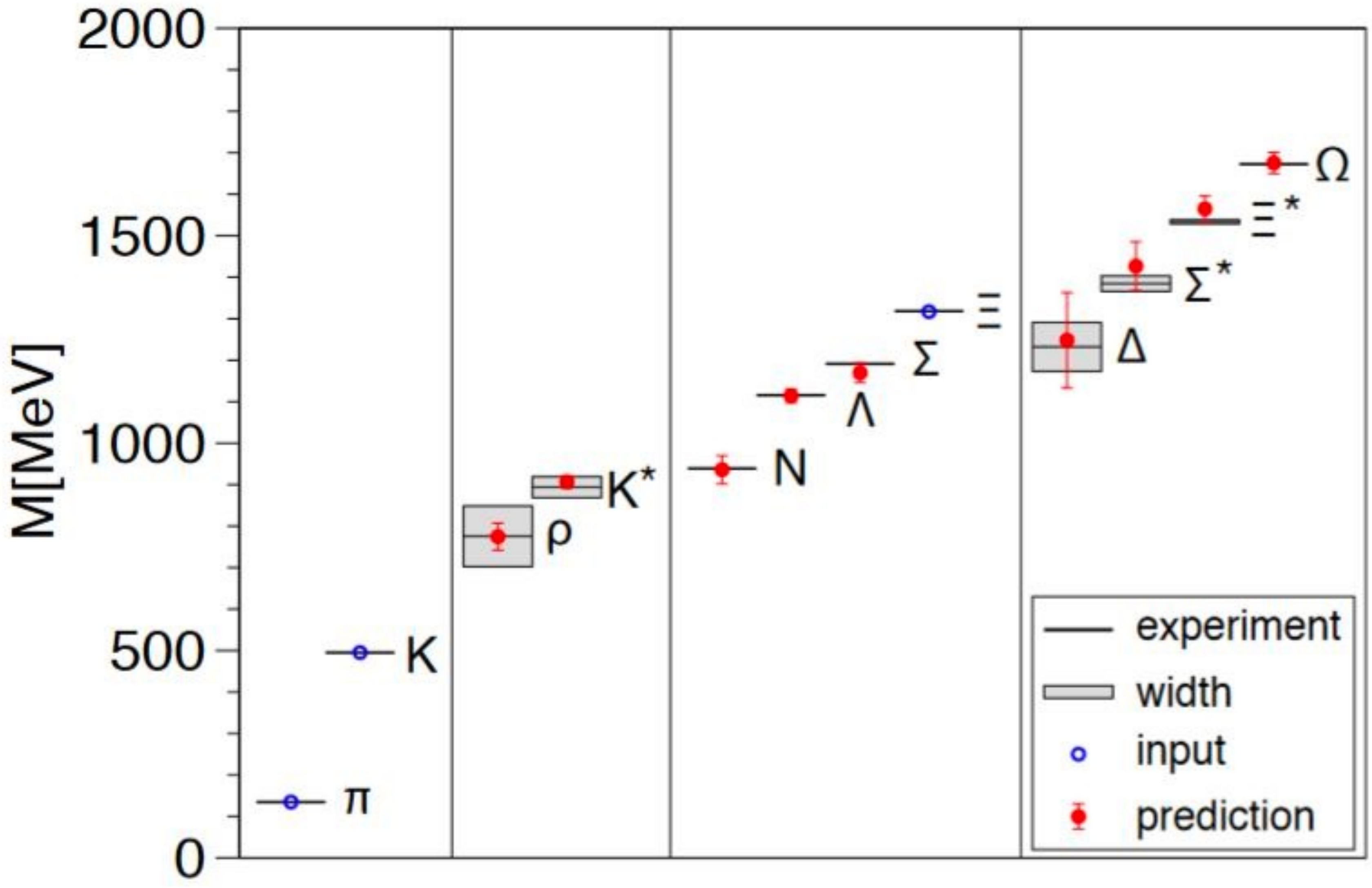


More accurate predictions from Lattice QCD

- Lattice QCD addresses QCD calculations at hadronic energy scales, $\lesssim 1$ GeV
  - Otherwise not calculable
- Implement full QCD on discrete space-time points
- Precise calculations require large, parallel computing facilities
- Inputs are quark masses and strong coupling
- Calculate parameters related to hadrons and their interactions
  - Hadron mass spectrum
  - Decay constants, dipole moments, parton distribution functions, etc



Lattice QCD computation of light hadron masses



Predict spectrum, and energy levels within spectrum