Problem Set #4 - Michael

Question 1

Suppose we observe a person making the following pattern of choices:

$$\$71 \text{ Now} \prec \$75 \text{ in 1 Week}$$
$$\$72 \text{ Now} \prec \$75 \text{ in 1 Week}$$
$$\$73 \text{ Now} \succ \$75 \text{ in 1 Week}$$
$$\$74 \text{ Now} \succ \$75 \text{ in 1 Week}$$

Under the "usual assumptions" (see Topic 3c), what can we infer about the person's D(7 days)?

What can we infer about the person's average yearly discount rate applied to a 7-day delay?

If we go through the descisions:

$$D(0) = 1 D(7) > \frac{v(72)}{v(75)}$$
$$D(7) < \frac{v(73)}{v(75)}$$

So $D(7)$ is between $.96$ and $.9733$.

This yields $\rho = 52/7(-\ln(.97)) = 22.6\%$

Question 2

Suppose we observe that a person has the following indifference points:

$$\$140 \text{ Now} \sim \$150 \text{ in 1 Year}$$
$$\$147 \text{ in 1 Year} \sim \$150 \text{ in 2 Years}$$
$$\$149 \text{ in 2 Years} \sim \$150 \text{ in 3 Years}$$

(a) Under the "usual assumptions" (see Topic 3c), what can we infer about the person's D(1 Year), D(2 Years), D(3 Years)? Discuss whether, under the "usual assumptions", this data consistent with exponential discounting, hyperbolic discounting, and present bias.

"Usual assumptions:" $v(x) = x$ and no expectation utility. With exponential discounting we have

$$D(1)/D(0) = \frac{140}{150} = .933$$
$$D(2)/D(1) = \frac{147}{150} = .98$$
$$D(3)/D(2) = \frac{149}{150} = .993$$

Exponential discounting assumes each of these are equivalent, but they vary quite greatly. Lets consider the other two. Pure hyperbolic discounting implies $D(x) = \frac{1}{1+kx}$, with $\frac{D(x)}{D(x-1)} = \frac{1+kx-k}{1+kx} = 1 - \frac{k}{1+kx}$

$$\frac{k}{1+k} = 1 - .933 \implies k = 0.0718$$
$$\frac{k}{1+2k} = 1 - .98 \implies k = 0.02$$
$$\frac{k}{1+3k} = 1 - .993 \implies k = 0.007$$

No luck here either. The 5% change in discounting from .93 to .98 is best explained by present bias, that money now is much more valuble than future money. Even so, there's still unexpected change between $D(2)$ and $D(3)$ for both models.

(b) Instead of assuming that "utility" is linear in the amount, let's assume that the person evaluates gains according to a value function $v(x) = x^{0.5}$. Under this alternative, what can we infer about the person's $D(1\text{ Year})$, $D(2\text{ Years})$, $D(3\text{ Years})$? Discuss whether, under this alternative, this data consistent with exponential discounting, hyperbolic discounting, and present bias.

Now $v(140, 147, 149, 150) = 11.83, 12.12, 12.21, 12.25$

Adjusting the previous ratios in exponential discounting we have

$$11.83/12.25 = .966$$
$$12.12/12.25 = .989$$
$$12.21/12.25 = .997$$

While more closely packed, there's still a general lack of consistency and the issues present in part (a) are stil present here.

For hyperbolic this yields k values of

$$\frac{k}{1+k} = 1 - .966 \implies k = 0.004$$
$$\frac{k}{1+2k} = 1 - .989 \implies k = 0.011$$
$$\frac{k}{1+3k} = 1 - .997 \implies k = 0.003$$

Okay still not quite consistent, but it doesn't vary by an order of magnitude. I still believe present bias + exponential discounting is most consistent, but it still misses something

(c) If a person actually values money according to a concave value function, but we incorrectly assume that her utility is linear in the amount, will we over-estimate or under-estimate how patient the person is?

We will under-estimate, since we predict a greater discount rate for linear in part a compared to our prediction for a concave function in part b. This makes sense considering that we'll think they're putting too much weight into lower, instant values when really their value function states that further increases in wealth aren't as highly weighted

Question 3:

This question asks you to reconsider the "doing-it-once" environment that we studied in class (Example 1 and Example 2 from Topic 3d), except that we now consider costs and rewards that do not rise monotonically. For both parts, suppose that you value rewards and costs linearly, and that you have β, δ preferences with $\beta = 1/2$ and $\delta = 1$.

(a) Suppose there is an onerous task that you must complete on one of the next 6 days. If you complete the task on day t, the cost is c_t, where $(c_1, c_2, c_3, c_4, c_5, c_6) = (10, 24, 18, 38, 34, 70)$. There is no reward.

(i) When is the best time to complete the task (given long-run preferences)?

(ii) When do naifs complete the task? When do sophisticates complete the task?

(iii) Discuss how the outcomes here compare to the outcomes in Example 1.

Lets compute! If we were a souless and ideal corporate entity with vast wealth, we'd do it on the first day since that is when costs are lowest. However, we discount the future costs according to $U(t) = \beta_{t \neq 1} \delta^{t-1} v(c_t)$ which becomes

$$U(t) = \begin{cases} c_t & t = 1 \\ \beta c_t & t > 1 \end{cases}$$

where t is the action date. Remember we are in costs, so we want to minimize U

$$U(1) = 10,\ U(2) = 12,$$
$$U(3) = 9,\ U(4) = 19,$$
$$U(5) = 17,\ U(6) = 35,$$

(i) clearly we want to do it on c_3, the lowest cost

(ii) A naif won't succeed in completion in c_3, since by then $U_3(3) = 18$ and $U_3(5) = 17$, so they'll wait. By U_5 they'll have $U_5(5) = 34,\ U_5(6) = 35$, and complete the task then. A sophisticate will realize this behavior and avoid ending up there. Considering that if they wait till period 3, they'll re-evaluate and go for period 5, which has a higher cost than period 1, $U(1) = 10$ vs $U(5) = 17$, they'll do the task immediately to avoid ending up in period 5.

(iii) This outcome mirrors the sophisticate in Example 1, with a correction to reach the best possible outcome according to the period 1 value. The naif obtains a less ideal state, but not the worst possible as in Example 1. This is because the discounted final state is still not as bad as the immediate state

(b) Suppose there is a pleasurable task that you get to complete on one of the next 6 days. If you complete the task on day t, the reward is v_t, where $(v_1, v_2, v_3, v_4, v_5, v_6) = (6, 10, 8, 18, 34, 30)$. There is no cost.

(i) When is the best time to complete the task (given long-run preferences)?

(ii) When do naifs complete the task? When do sophisticates complete the task?

(iii) Discuss how the outcomes here compare to the outcomes in Example 2.

A pleasure-seeking algorithm with $\beta = \delta = 1$ will wait until day 5 to maximize pleasure, but we are fun-hungry monkeys. Fungry.

$$U(1) = 6,\ U(2) = 5,$$
$$U(3) = 4,\ U(4) = 9,$$
$$U(5) = 17,\ U(6) = 15,$$

(i) Despite our fungriness, we still end up valuing day 5 the most.

(ii) Naifs will hit a roadblock on day 4: $U_4(4) = 18$, so they won't make it to the final day. Sophisticates cannot control their future selfs, and fall into the same trap ($U(4)$ is still the least bad according to original prefrences)

(iii) In example 2, the naif ended up in a better position than the sophisticate, but here the sophisticate ends up in the same place since they value period 4 at all previous periods

Question 4

This question also asks you to reconsider the "doing-it-once" environment, except that we now consider an activity that generates BOTH rewards and costs. Throughout, suppose that you value rewards and costs linearly, and that you have β, δ preferences with $\beta = 0.7$ and $\delta = 1$.

Suppose there is an activity that you will complete on one of the next 5 days. As a function of when you do the activity, your reward will be v_t and your cost will be c_t, where $(c_1, c_2, c_3, c_4, c_5) = (45, 45, 45, 62, 87)$ and $(v_1, v_2, v_3, v_4, v_5) = (35, 45, 60, 60, 60)$. However, there are two possible cases for **when** you receive these payoffs:

Immediate costs: you incur the cost when you do it, and receive the reward in the future.

Immediate rewards: you receive the reward when you do it, and incur the cost in the future.

For instance, if you do the activity in period 4, your cost is $c_4 = 62$ and your reward is $v_4 = 60$. For immediate costs, the cost $c_4 = 62$ is incurred in period 4 while the reward $v_4 = 60$ is received sometime later. For immediate rewards, the reward $v_4 = 60$ is received in period 4 while the cost $c_4 = 62$ is incurred sometime later. Since we are assuming $\delta = 1$, exactly when later is irrelevant.

(a) When is the best time to complete the task (given long-run preferences)? How does your answer depend on the timing of rewards and costs?

We are dealing with two sets of outcomes - immediate costs and immediate rewards.

Immediate costs: β is applied to reward upon action period. As such, all rewards are multiplied by β

$Uic(1:5) = [-20.5, 0.0, 10.5, -1.4, -18.9]$. Best reward: period 3

Immediate rewards: β is applied to cost upon action period. As such, all costs are multiplied by β

$Uir(1:5) = [3.5, 0.0, 10.5, -1.4, -18.9]$. Best reward: period 3

Calculations come from code at the bottom of the page

(b) Consider the case of immediate costs. When do naifs complete the task, and when do sophisticates complete the task?

Lets check out the naifs. By period 3, period 4 looks nicer $U_3(3:5) = [-3, -1.4, -18.9]$. By period 4, period 5 looks nicer. They ends up in their second-to-worst situation, the final period.

The all-wise sophisticate wishes to obtain period 3. Fortunately, their period 3 self is still a sophisticate and devalues period 5 below period 4, so they'll end up acting in period 3 and successfully hit the ideal completion period

(c) Consider the case of immediate rewards. When do naifs complete the task, and when do sophisticates complete the task?

Naif: By period 2, $U_2(2) = 13.5$ is the highest value, and they complete the task

Soph: Sophs value period 1 over period 2, so they complete the task in period 1

In [ ]:
```
import numpy as np

period = np.array([1,2,3,4,5])
c = np.array([45,45,45,62,87])
v = np.array([35,45,60,60,60])

#data = np.array([period,c,v])

beta = 0.7
```

In [ ]:
```
Uic = [] # Generates results for part a
Current_period = 1

for p in period:
    i = p-1

    if p == Current_period:
        Uic.append(beta*v[i]-c[i])
    if p >> Current_period:
        Uic.append(beta*(v[i]-c[i]))

Uir = []
for p in period:
    i = p-1

    if p == Current_period:
        Uir.append(v[i]-beta*c[i])
    if p >> Current_period:
        Uir.append(beta*(v[i]-c[i]))
```

In [ ]:
```
print(Uic,Uir)
```

```
[-20.5, 0.0, 10.5, -1.4, -18.9] [3.5000000000000036, 0.0, 10.5, -1.4, -18.9]
```

In [ ]:
```
Uic_futures = [] # Generates results for part b
Uir_futures = []


for now in period:
    Uic = []
    Current_period = now

    for p in period:
        i = p-1

        if p == Current_period:
            Uic.append(beta*v[i]-c[i])
```

```
        if p > Current_period:
            Uic.append(beta*(v[i]-c[i]))
    Uic_futures.append(Uic)

    Uir = []
    for p in period:
        i = p-1

        if p == Current_period:
            Uir.append(v[i]-beta*c[i])
        if p > Current_period:
            Uir.append(beta*(v[i]-c[i]))
    Uir_futures.append(Uir)
```

In [ ]:
```
print(Uic_futures)
print(Uir_futures)
```

```
[[-20.5, 0.0, 10.5, -1.4, -18.9], [-13.500000000000004, 10.5, -1.4, -18.9], [-3.0, -1.4, -18.9],
[-20.0, -18.9], [-45.0]]
[[3.5000000000000036, 0.0, 10.5, -1.4, -18.9], [13.500000000000004, 10.5, -1.4, -18.9], [28.50000
0000000004, -1.4, -18.9], [16.6, -18.9], [-0.8999999999999986]]
```

Question 5:

This question asks you to explore further the relationship between present bias and health-club usage. Suppose there are 30 days in a month, and that on each of these days you consider going to the health club.

Each visit to the health club generates a future benefit of 35. *Each* visit to the health club also carries an immediate cost (because exercise requires effort). However, because your motivation varies from day to day, this immediate cost will vary from day to day. Specifically, assume that for m days each month you will have a low cost of 10, for n days each month you will have a medium cost of 20, and for $(30-m-n)$ days each month you will have a high cost of 30.

The health club offers two contracts:

A: No monthly fee, but you pay \$10 per visit.

B: Monthly fee of \$$X$, but then you pay nothing per visit.

You must choose your contract in advance (prior to the first day of the month).

Finally, suppose you treat any money spent as a future cost (linear in the amount of money spent). For instance, under Contract A, if you visit the club on a low-cost day, then you incur an immediate cost of 10 (effort), you incur a future cost of 10 (price paid), and you receive a future benefit of 35 (health benefits).

(a) Suppose you are a standard exponential discounter with $\delta = 1$.

(i) If you choose Contract A, on which days (low-cost, medium-cost, high-cost) will you visit the gym? From a prior perspective, what would be your total utility for the month?

(ii) If you choose Contract B, on which days (low-cost, medium-cost, high-cost) will you visit the gym? From a prior perspective, what would be your total utility for the month as a function of X?

(iii) For what values of X would you choose Contract B?

(iv) If you chose Contract B, what would end up being your average price per visit as a function of X? Can we say how this compares to \$10?

I accidentally went ahead and solved for all β, at least it looks cool

(i) Money spent is always $\beta 10$. In order to visit the gym, there must be positive future gain in health $\beta 35$ that overcomes effort cost $c \in [10, 20, 30]$, so you go to the gym when

$$\beta 35 - \beta 10 - c > 0$$
$$\beta 25 > c$$

For $\beta > 0.8$ They will go for both n and m days, yielding a prior-oriented utility $\beta 25 - 10$ on n days and $\beta 25 - 20$ on m days. For $\beta > 0.4$ this is only scored on n days, and any less leads to zero utility. This yields the utility function

$$U = \begin{cases} (\beta 25 - 10)n + (\beta 25 - 20)m & \beta > 0.8 \\ (\beta 25 - 10)n & 0.8 \geq \beta > 0.4 \end{cases}$$

assuming $\beta \leq 1$.

(ii) When to visit the gym: Now $\beta 10$ is out of the picture when considering days to go to the gym. I'll go to the gym when

$$\beta 35 - c > 0$$
$$\beta 35 > c$$

This yields the utility

$$U = \begin{cases} (\beta 35 - 10)n + (\beta 35 - 20)m + (\beta 35 - 30)(30 - n - m) & \beta > .857 \\ (\beta 35 - 10)n + (\beta 35 - 20)m & .857 > \beta > .571 \\ (\beta 35 - 10)n & .517 > \beta > .286 \end{cases}$$

However we need to account for our subscription fee

$$U + \beta X = \begin{cases} (\beta 35 - 10)n + (\beta 35 - 20)m + (\beta 35 - 30)(30 - n - m) & \beta > .857 \\ (\beta 35 - 10)n + (\beta 35 - 20)m & .857 > \beta > .571 \\ (\beta 35 - 10)n & .517 > \beta > .286 \end{cases}$$

(iii) The choice of subscription fee depends on both the number of n and m days as well as β. Lets consider each case separately for when $U = 0$ to see the break even point. Each case breaks down into

$$\beta > .857 \implies X = \frac{(\beta 35 - 10)n + (\beta 35 - 20)m + (\beta 35 - 30)(30 - n - m)}{\beta}$$

$$.857 > \beta > .571 \implies X = \frac{(\beta 35 - 10)n + (\beta 35 - 20)m}{\beta}$$

$$.517 > \beta > .286 \implies X = \frac{(\beta 35 - 10)n}{\beta}$$

However we're up against the utility of contract A, so

$$0 = U_B - U_A$$

is the break even point, where U_B has X baked in Considering the cases, this gets very large very fast, so I'll leave it here

(iv) If β is above $.857$, simply divide X by 30. If $X/30 < 10$, choose contract B. If it is above 10, choose contract A. For lower values, divde by $n + m$ or n.

(b) Repeat part (a), except suppose you have sophisticated present bias with $\beta = .7$ and $\delta = 1$.

(B) $\beta = 0.7$. Let's plug into contract A

$$U_A = 7.5n$$

Simple enough. When does this outdo contract B?

$$\begin{gathered} U_B = 14.5n + 4.5m - 0.7X \\ U_A = U_B \rightarrow \text{(indifference point)} \\ 7.5n = 14.5n + 4.5m - 0.7X \\ X = 10n + 6.4m \end{gathered}$$

As such, the sophisticate will choose contract B if it costs less than $10n + 6.4m$. If we have an even spread of n, m and $30 - n - m$ this is $\$164$

(c) Repeat part **(a)**, except suppose you have naive present bias with $\beta = .7$ and $\delta = 1$. Note: For parts (i)-(iii), you should be deriving what naifs predict for their own future behavior as they think about this problem from a prior perspective—which is when they decide between Contract A and Contract B.

(C) Naifs have an issue with appreciating their future descision making. From the outset, practically all values are multiplies by β, since they are all future values. For part (i) everything is multiplied by β, which has the same effect as multiplying nothing by β when it comes to guessing descision making. The present-day costs are also downscaled.

Now, when conisdering exercise on extra-hard days $c = 30$, we have $U(c = 30) = \beta(35 - 10 - 30) < 0$, but on medium days $\beta(35 - 10 - 20) > 0$ and we obtain the new utility function

$$U_A = 0.7(15n + 5m)$$

However when the naif rolls around to those m days they'll chick out since $0.7 * 25 = 17.5 < 20$, so portion of prior utility they obtain is

$$U_{A'} = 0.7 * 15n$$

For part (ii) a similar process yields action on the extra-hard $c = 30$ days according to the prior utility, but it isn't actually realized. We believe our utilty function to be

$$U_B + 0.7X = 0.7(25n + 15m + 5(30 - n - m))$$

But the actual utility is

$$U_{B'} + 0.7X = 0.7 * 25n + 0.7 * 15m$$

For part (iii) we're initially willing to pay according to our prior utility, so

$$\begin{aligned} 0.7X &= 0.7(25n + 15m + 5(30 - n - m)) - 0.7(15n + 5m) \\ X &= \frac{25n + 15m + 5(30 - n - m) - 15n - 5m}{0.7} \\ &= (150 + 5n + 5m)/0.7 = 214 + 7.14n + 7.14m \end{aligned}$$

Any price less than $214 + 7.14n + 7.14m$ leads to choosing option B. Funnily enough, if they have 30 straight hard days, they'll pay $214 to go zero times

(iv) The naif believes they will go every day, and will pay an average $\frac{214+7.14n+7.14m}{30}$ per day, which has a range between $7.13 and$14.27. In theory this works out, but really they only go $n + m$ days, and the real average cost will increase. This becomes

$$\bar{X} = \frac{214 + 7.14n + 7.14m}{n + m}$$

Which has a range between $14.27 and \infty$

Even with every day being an easy day, they'll still pay more than the $10 they'd pay with the per-visit rate.