Homework 2

1) Particle described by $P_\mu = (200, 30, 100, 150)$ GeV

$\rightarrow$ lab frame

a. What is particle mass?

$$m^2 = E^2 - p^2$$
$$= 200^2 - 30^2 - 100^2 - 150^2$$
$$\rightarrow \boxed{m = 80 \text{ GeV}}$$

$\rightarrow$ W Boson

b. What is β / γ for this particle?

$$E = \gamma m \rightarrow \gamma = \frac{E}{m} = \frac{200}{80} = \boxed{2.5 = \gamma}$$

$$P = \gamma \beta m \rightarrow \beta = \frac{P}{\gamma m} = \frac{P}{E} = \frac{\sqrt{30^2 + 100^2 + 150^2}}{200} = \boxed{0.91 = \beta}$$

c. 4-vectors in particle rest frame:

$$\boxed{P_W = \begin{pmatrix} 80 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{GeV}}$$

Conservation of momentum:

$$P_W = P_\nu + P_e$$
$$\rightarrow P_\nu = -P_e$$

Energy conservation:

$$E_W = E_\nu + E_e$$
$$80 = \sqrt{m_\nu^2 + p_\nu^2} + \sqrt{m_e^2 + p_e^2} \rightarrow m_e, m_\nu = 0 \rightarrow \text{small enough to be negligible}$$
$$= \sqrt{p_\nu^2} + \sqrt{p_e^2}$$
$$= 2 p_e \rightarrow p_e = 40 \text{ GeV}$$

1) c. Cont. So, writing as 4 vectors

$$\boxed{P_e = \begin{pmatrix} 40 \\ 0 \\ 0 \\ -40 \end{pmatrix} \text{GeV} \qquad P_\nu = \begin{pmatrix} 40 \\ 0 \\ 0 \\ 40 \end{pmatrix} \text{GeV}}$$

d. Boost the electron back to moving frame.

$\gamma = 2.5$, $\beta = 0.91$ from pt. b.

Maximum when motion aligns with W.
Boost electron into lab frame using direction of W motion as z' axis:

$$p = \gamma p' + \gamma \beta E'$$
$$= (2.5)(40) + (2.5)(0.91)(40)$$
$$= 191 \text{ GeV}$$

Minimum when motion of electron is in the opposite direction of the W

$$p = -\gamma p' + \gamma \beta E'$$
$$= -2.5(40) + 0.91(2.5)(40)$$
$$= -9 \text{ GeV}$$

$\rightarrow$ magnitude: $p = 9 \text{ GeV}$

2) a. $A + B \rightarrow C_1 + C_2 \ldots + C_n$

Initial state 4-vector:

$$P_i = \begin{pmatrix} E_A + m_B \\ \vec{P}_A \end{pmatrix}$$

Threshold energy → final state particles have no 3-momentum.

Final state 4-vector:

$$P_f = \begin{pmatrix} \sum_i m_{ci} \\ \vec{0} \end{pmatrix}$$

Use the invariant to compare across these frames:

$$P_i^2 = P_f^2$$

Where

$$P_i^2 = (E_A + m_B)^2 - |\vec{P}_A|^2$$
$$= E_A^2 + 2E_A m_B + m_B^2 - |\vec{P}_A|^2$$
$$= \underbrace{E_A^2 - |\vec{P}_A|^2}_{= m_A^2} + 2E_A m_B + m_B^2$$
$$= m_A^2 + 2E_A m_B + m_B^2$$

$$P_f^2 = \left(\sum_i m_{ci}\right)^2 - 0^2 = \left(\sum_i m_{ci}\right)^2 = M^2$$

2) a. Cont.

$$m_A^2 + 2E_A m_B + m_B^2 = M^2$$

$$\rightarrow E_A = \frac{M^2 - m_A^2 - m_B^2}{2m_B}$$

$\rightarrow$ see next pg. for 2b.

3) $\eta \rightarrow \pi^0 + \pi^0 + \pi^0$

and

$\eta \rightarrow \pi^+ + \pi^- + \pi^0$

What is the parity of the η?

1. Parity is conserved in EM decays

$\eta \rightarrow \pi^0 + \pi^0 + \pi^0$

Parity of decay: $\eta \rightarrow (-1)(-1)(-1) = -1$

$\eta \rightarrow \pi^+ + \pi^- + \pi^0 : (-1)(-1)(-) = -1$

Why not $\eta \rightarrow \pi^0 + \pi^0$ or $\eta \rightarrow \pi^+ + \pi^-$?

Parity not conserved : $(-1)(-1) = 1 \rightarrow$ but η must have -1 parity

2) b. Use answer from pt. a to calculate threshold energies for the following processes.

i. $\pi^- + p \rightarrow K^0 + \Sigma^0$

$m_\pi = 139.6$ MeV $\quad m_{K_0} = 497.7$ MeV

$m_p = 938.3$ MeV $\quad m_{\Sigma_0} = 1192.6$ MeV

$\rightarrow E_{thresh} = 1043$ MeV

ii. $p + p \rightarrow p + p + \pi^0$

$m_p = 938.3$ MeV $\quad m_{\pi_0} = 135$ MeV

$E_{thresh} = 1218$ MeV

iii. $\pi^- + p \rightarrow p + p + n$

$m_\pi = 139.6$ MeV $\quad m_n = 939.6$ MeV

$m_p = 938.3$ MeV

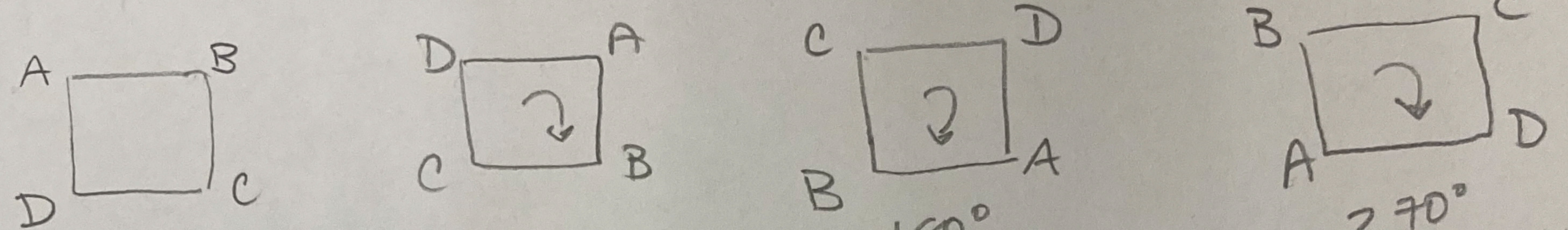
$E_{thresh} = 3747$ MeV

4) Symmetry group of the square:

a. Elements:

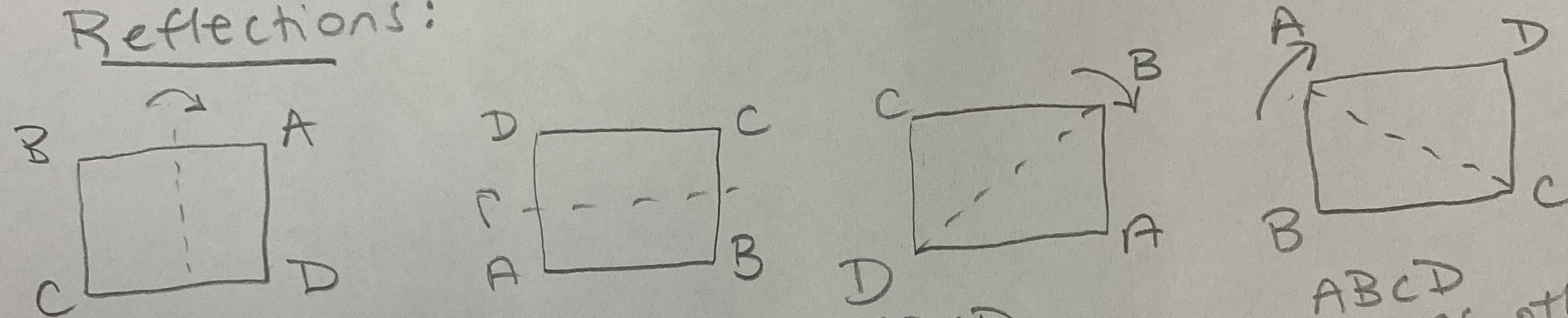
Identity: Rotations:

ABCD DABC CDAB BCDA



90° clockwise 180° clockwise 270° clockwise

Reflections:



ABCD across vertical axis

ABCD across horizontal axis

ABCD across diagonal

ABCD across other diagonal

b. Is this Abelian?

Abelian requires $R_1 R_2 = R_2 R_1$

Example operators that do not commute:

Rotation + axis inversion:

$$\begin{pmatrix}\text{90° clockwise}\\ \text{rotation}\end{pmatrix} + \begin{pmatrix}\text{Horizontal}\\ \text{reflection}\end{pmatrix} \neq \begin{pmatrix}\text{Horizontal}\\ \text{reflection}\end{pmatrix} + \begin{pmatrix}\text{90° clockwise}\\ \text{rotation}\end{pmatrix}$$

$$CBAD \neq ADCB$$