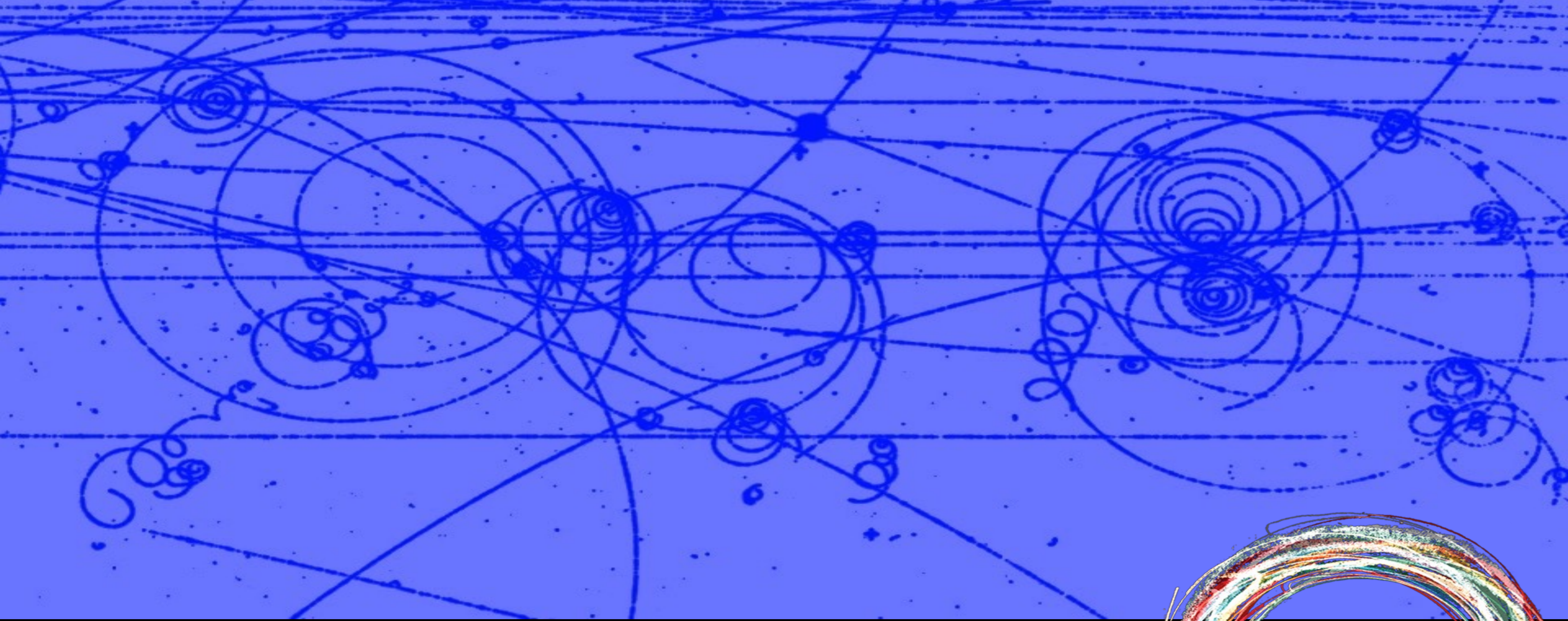
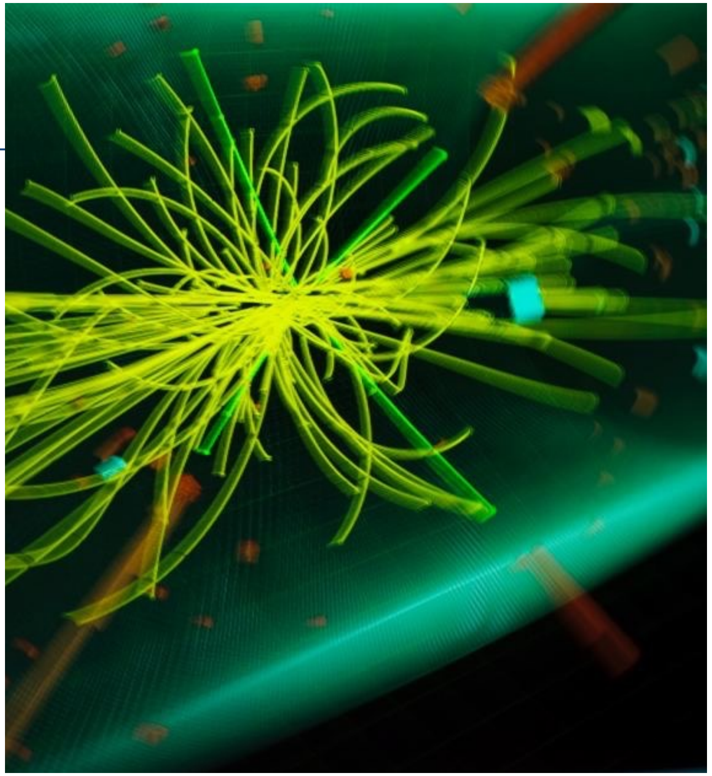


LECTURE 5
SYMMETRIES

PHY 493 / 803

Announcements



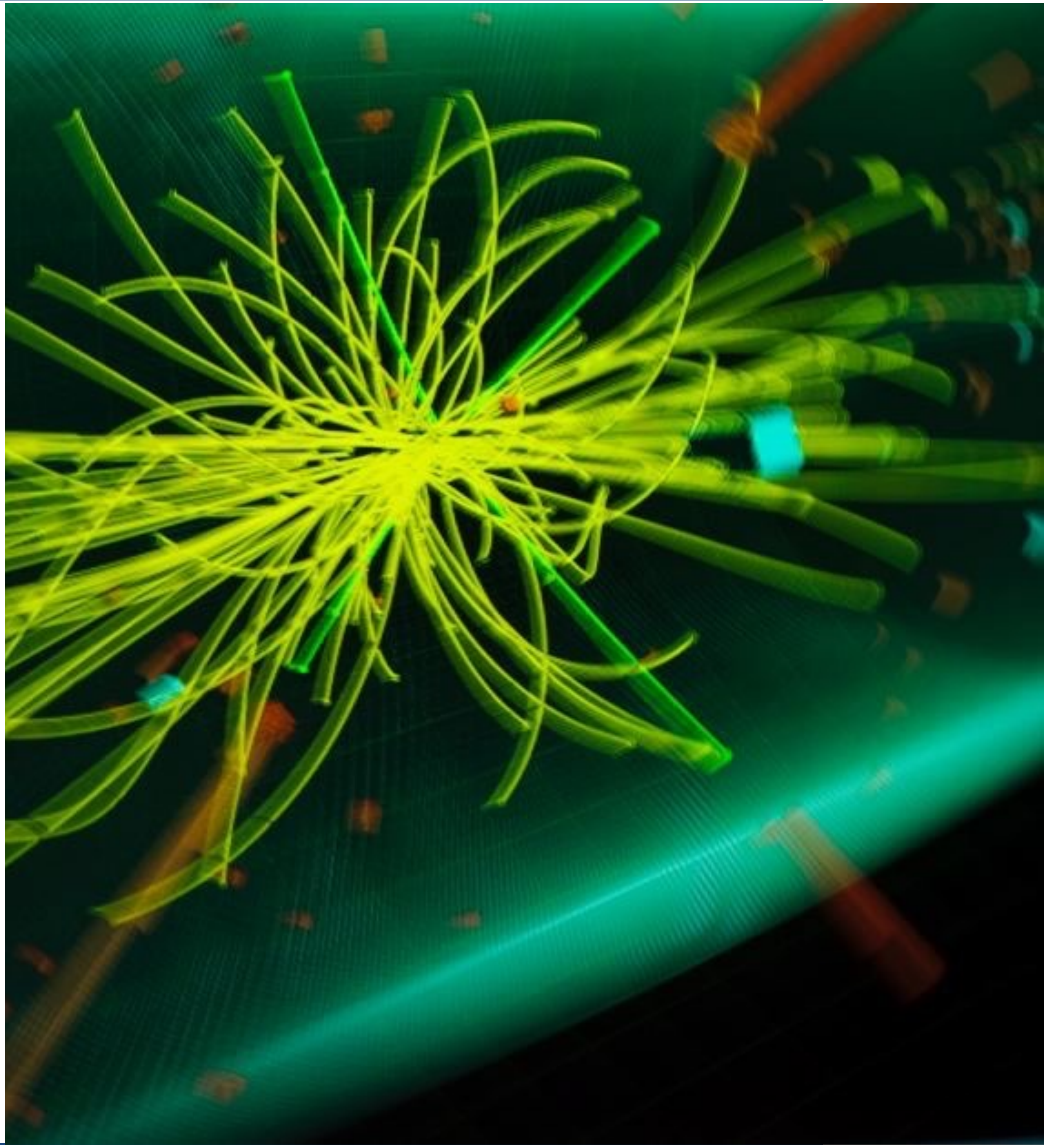
Quiz:
- Pickup Friday's quiz after class
- Next quiz on Friday.

Homework:
- Second homework due Feb 10 at 3pm. Submit on gradescope.

Paper: Topic due **Monday, Feb. 17th at 3pm**
Please reply to this google form before then:
https://forms.gle/MmCk8NtrMm7RdfLC7

Next week: no quiz on Friday, Feb 14th; will be on Wednesday instead

Recap / Up Next



Last time:

Special Relativity

4-Vector Notation

Relativistic Collisions

This time:

Symmetries

- Group Theory
- Operators

Conservation Laws

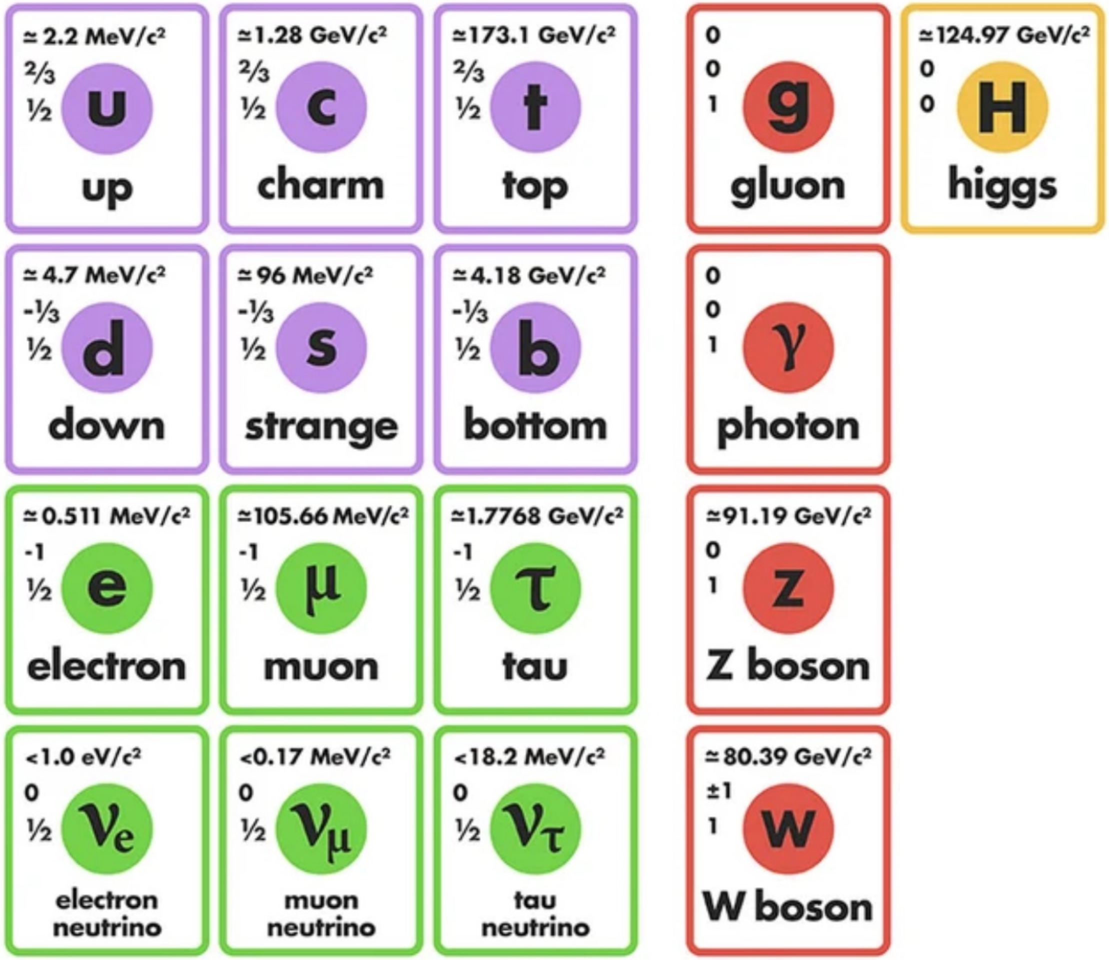
- Physical Symmetries

Symmetries and Data

Group → ↓Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo
		*	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
		**	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

It is common that experimental observations precede a full understanding of the patterns and symmetries in our data.
The periodic table is fundamentally explained by arrangements of nucleons, protons and neutrons.

Symmetries and Data



It is common that experimental observations precede a full understanding of the patterns and symmetries in our data.

Symmetry: before 1961

Willis Lamb, 1955 Nobel Prize acceptance speech:

When the Nobel Prizes were first awarded in 1901, physicists knew something of just two objects which are now called "elementary particles": the electron and the proton. A deluge of other "elementary" particles appeared after 1930; neutron, neutrino, μ meson (sic), π meson, heavier mesons, and various hyperons. I have heard it said that "the finder of a new elementary particle used to be rewarded by a Nobel Prize, but such a discovery now ought to be punished by a $10,000 fine".

(more like $100,000 fine in 2025)

82 NATURE January 15, 1949 Vol. 163

OBSERVATIONS WITH ELECTRON-SENSITIVE PLATES EXPOSED TO COSMIC RADIATION*

By Miss R. BROWN, U. CAMERINI, P. H. FOWLER, H. MUIRHEAD and Prof. C. F. POWELL
H. H. Wills Physical Laboratory, University of Bristol
and D. M. RITSON
Clarendon Laboratory, Oxford

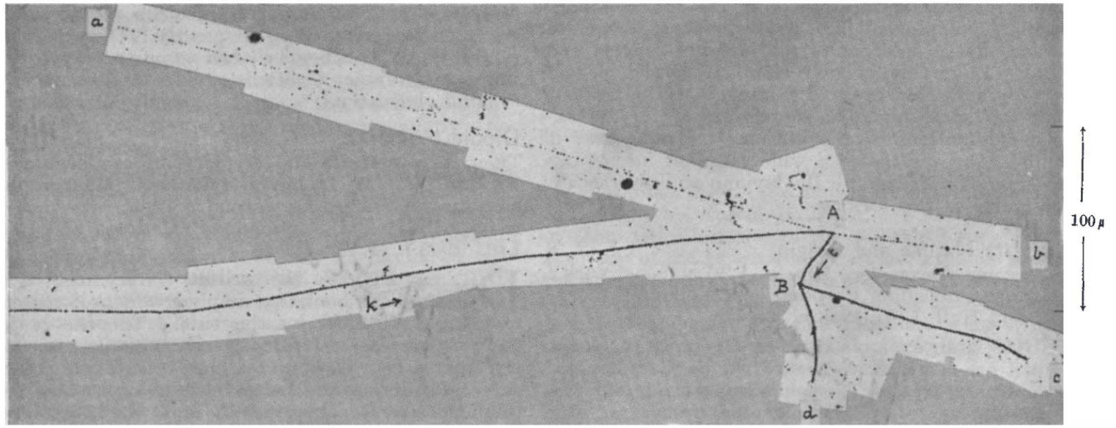
PART 2. FURTHER EVIDENCE FOR THE EXISTENCE OF UNSTABLE CHARGED PARTICLES, OF MASS ~ 1,000 m_e, AND OBSERVATIONS ON THEIR MODE OF DECAY

ONE of the first events found in the examination of electron-sensitive plates exposed at the Jungfraujoch is represented in the mosaic of photomicrographs shown in Fig. 8. There are two centres, *A* and *B*, from which the tracks of charged particles diverge, and these are joined by a common track, *t*. Because of the short duration of the exposure, and the small number of disintegrations occurring in the plate, the chance that the observation corresponds to a fortuitous juxtaposition of the tracks of unrelated events is very small—of the order 1 in 10^7. It is therefore reasonable to exclude it as a serious possibility. Further observations in support of this assumption are presented in a later paragraph.

that it carried the elementary electronic charge; and that it had reached, or was near, the end of its range at the point *A*. We therefore assume that the particle *k* initiated the train of events represented by the tracks radiating from *A* and *B*. It follows that the particle producing track *t* originated in star *A*, and produced the disintegration *B*. In order to analyse the event, we first attempted to determine the mass of the particle *k*.

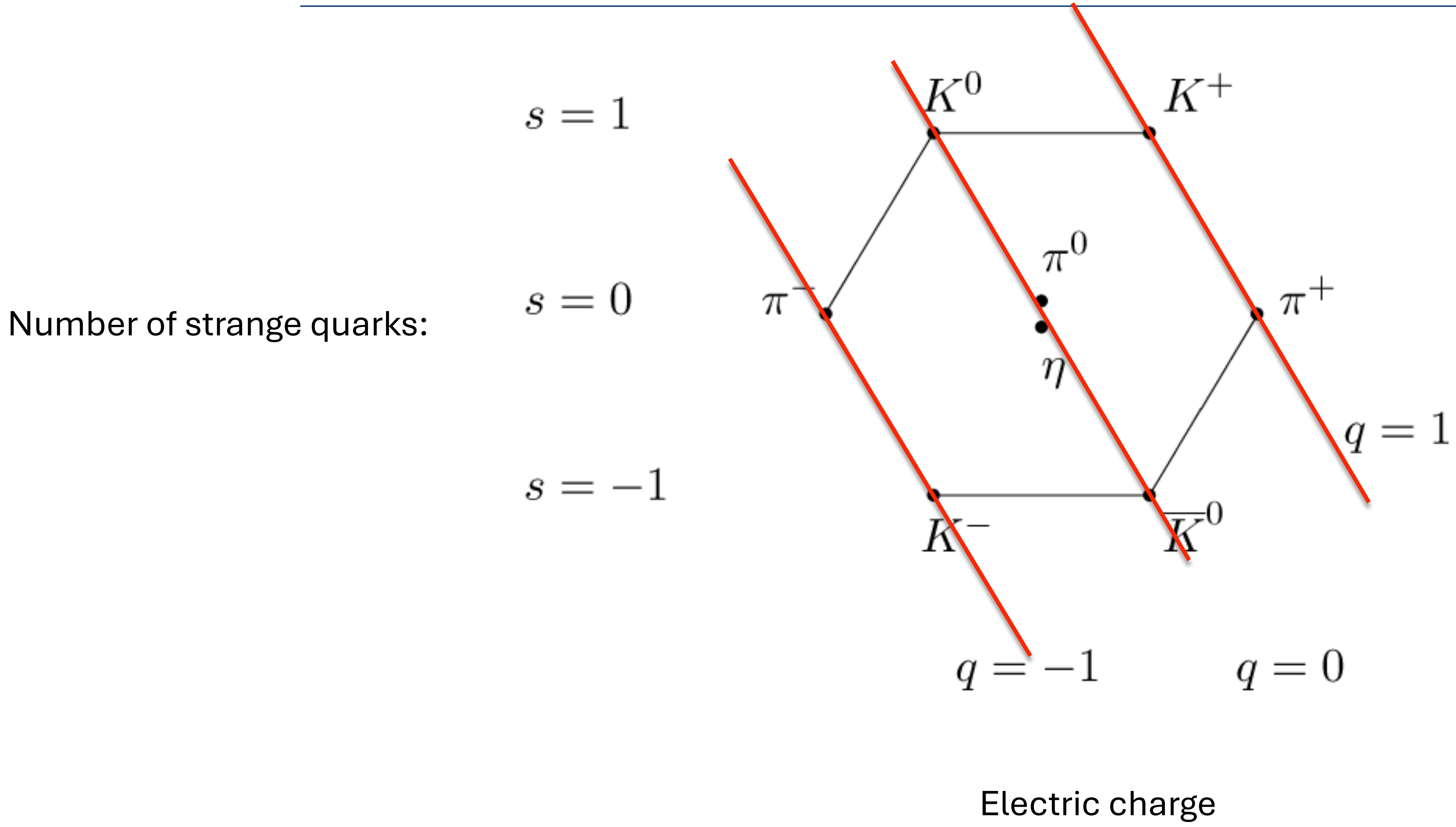
Mass Determinations by Grain-Counts

About a year ago, experiments were made in this Laboratory to determine the ratio, m_π/m_μ, of the masses of π- and μ-mesons, by the method of grain-counting[5], and by studying the small-angle scattering of the particles in their passage through the emulsion[4]. The values obtained by the two methods were $m_\pi/m_\mu = 1{\cdot}65 \pm 0{\cdot}11$, and $m_\pi/m_\mu = 1{\cdot}35 \pm 0{\cdot}10$*, respectively. Recent experiments at



Observer: Mrs. W. J. van der Merwe
Fig. 8

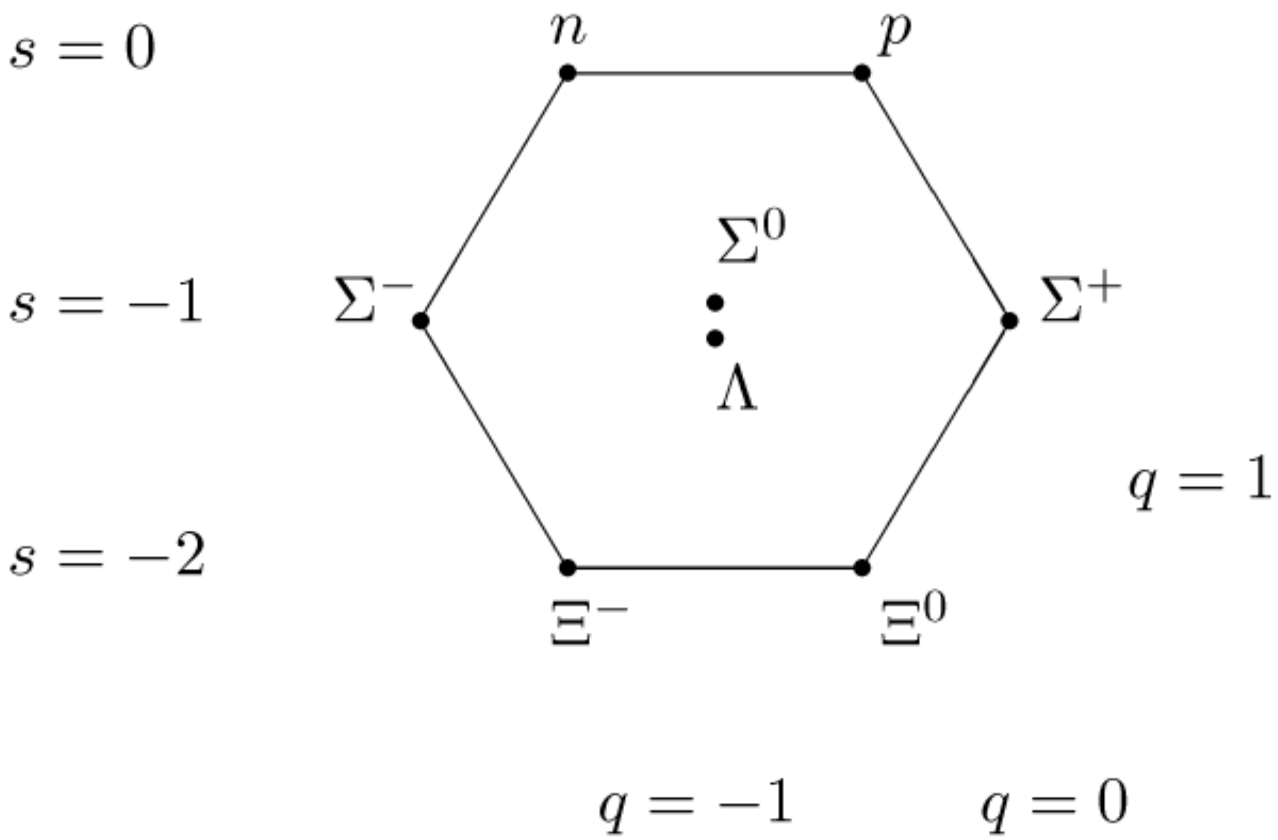
Symmetries and Data: Mesons



Discoveries of more and more mesons reveals a structure.

Symmetries and Data: Baryons

Number of strange quarks:

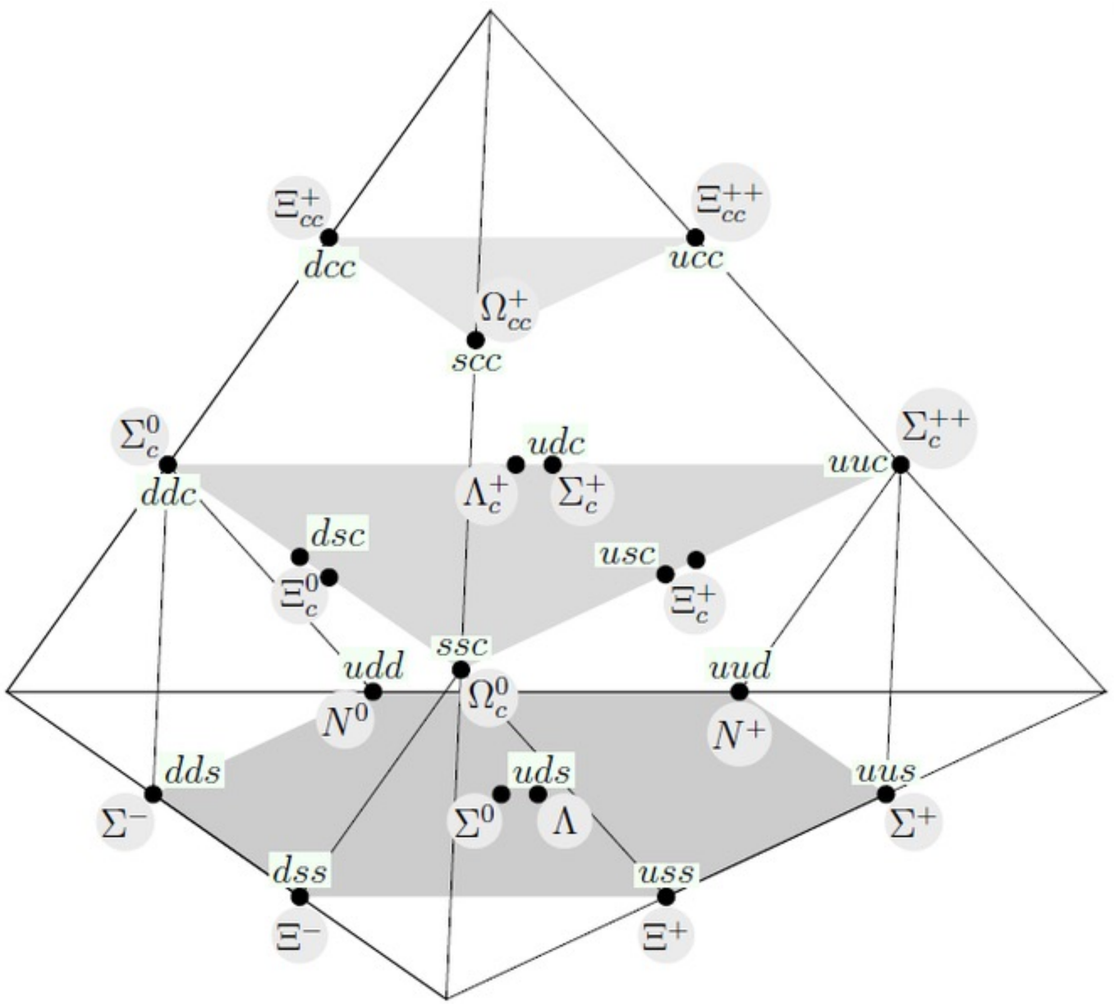


Electric charge

Discoveries of more and more baryons reveal a structure and symmetries

Symmetries and Data

Can be expanded beyond just strange-charge axes:



It is common that experimental observations precede a full understanding of the patterns and symmetries in our data.

Symmetry

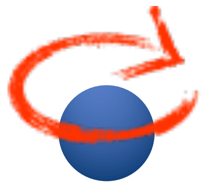
Symmetries leave a system invariant:

Symmetry

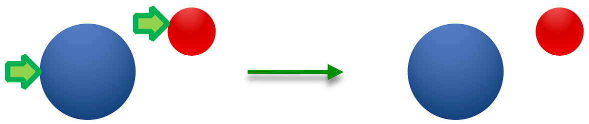
Symmetries leave a system invariant:

Continuous symmetries:

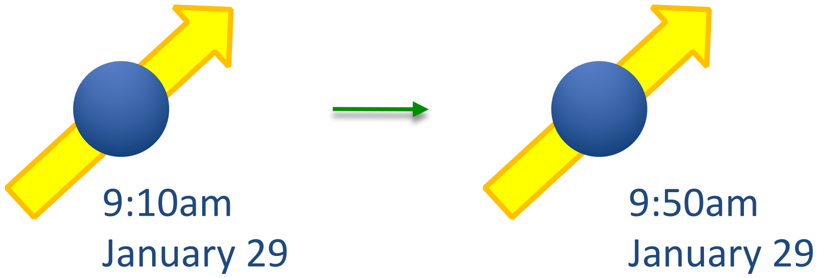
Rotational symmetry



Transitional symmetry



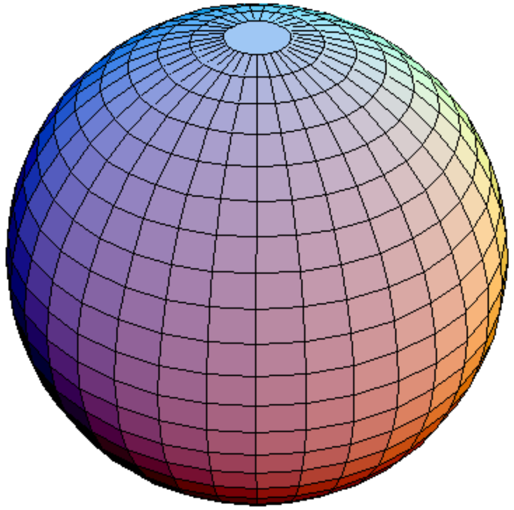
Time translation symmetry



Important discrete symmetries in Nuclear and Particle Physics

- Parity (P) : Reflection
- Charge conjugation (C)
- Time (time reversal) (T)

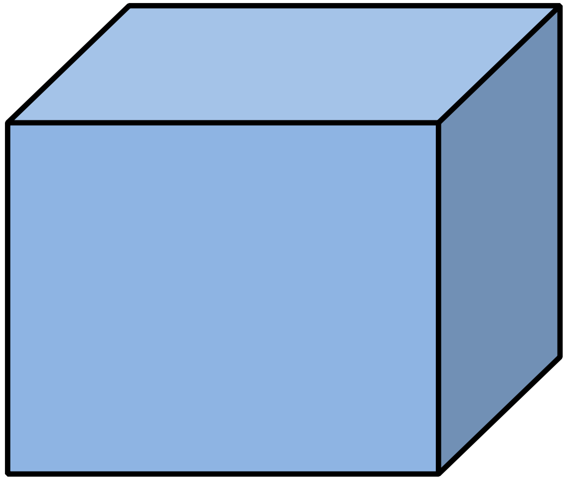
Continuous vs Discrete Symmetries



Continuous Symmetry:

The operation describes a continuous change in the geometry of the system. Also, arbitrary-sized operations result in a valid symmetry.

Consider rotations of a sphere: any rotation, big or small, returns a symmetrical result. Each rotation can be built with infinitesimally small rotations.



Discrete Symmetry:

The operation describes a discontinuous change in the geometry of the system. Only rotations of fixed size can result in a valid symmetry.

Consider rotations of a cube: only rotations of 90º in any direction result in a symmetrical outcome.

Noether's Theorem (1918)

Emmy Noether
(1882-1935)

Every symmetry of nature has a conservation law
OR
Every conservation law reflects a symmetry

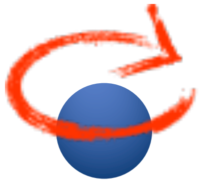
Noether's Theorem (1918)

Emmy Noether
(1882-1935)

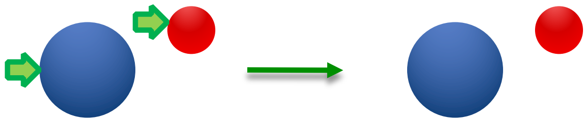
Every symmetry of nature has a conservation law
OR
Every conservation law reflects a symmetry

(Applies only to continuous symmetries!)

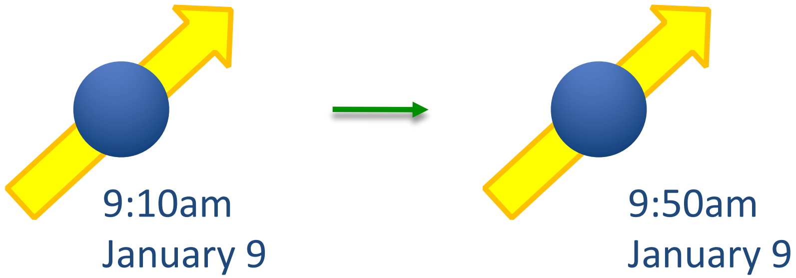
Rotational symmetry:
Angular momentum conservation



Transitional symmetry:
Momentum conservation



Time translation symmetry:
Energy conservation



Groups and Symmetries

Requirements of a symmetry are the same as the requirements of a group

Group: a set, together with an operation that combines any two elements to form a third element of the set

Closed by multiplication: any element can be constructed by multiplying other elements

Inverses: an inverse is defined that undoes the original operation

Groups and Symmetries

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Group: a set, together with an operation that combines any two elements to form a third element of the set

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Properties of groups:

- <u>Closure:</u> products of operators R_i are also valid operators
- <u>Identity:</u> Element $\mathbb{I}$ exists such that $\mathbb{I}\, R_i = R_i\, \mathbb{I}$
- <u>Inverse:</u> $R_i R_i^{-1} = R_i^{-1} R_i = \mathbb{I}$
- <u>Associative:</u> $(R_i R_j) R_k = R_i (R_j R_k)$

Example : Integers with addition

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- <u>Associative:</u> $(R_i\, R_j)\, R_k = R_i\, (R_j\, R_k)$

Example group: integers, with addition

$$\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

Closure: Any integer a + b = c

Identity: 0 is the identity, such that any integer a + 0 = a

Inverse: For every integer a, there is an integer b, such that a + b = 0, i.e 1 + (-1) = 0.

Associative: (a + b) + c = a + (b + c)

Other Groups You May Encounter

- Closure: products of operators R_i are also valid operators
- Identity: Element $\mathbb{I}$ exists such that $\mathbb{I}\, R_i = R_i\, \mathbb{I}$
- Inverse: $R_i R_i^{-1} = R_i^{-1} R_i = \mathbb{I}$
- Associative: $(R_i R_j) R_k = R_i (R_j R_k)$

Group	nxn matrices in group
U(n)	Unitary ($U^{T*}U = \mathbb{I}$)
SU(n)	Unitary and Det = 1
O(n)	Orthogonal ($O^T O = \mathbb{I}$)
SO(n)	Orthogonal and Det = 1

$\mathbb{I}$ = identity matrix
Unitary: $U^{-1} = U^{T*}$
S = special (determinant = 1)

Orthogonal = real unitary matrix

Example: SO(3)

Rotations about the origin of 3 dimensional Euclidean space (R3) are described by the 3D rotation group: SO(3)

- Represented by 3x3 matrices (Det = 1, each unique matrix is orthogonal to all other elements of the group)

Rotation about y axis

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

Rotation about z axis

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

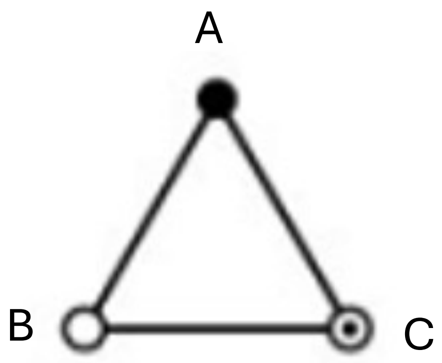
Orthogonal Group Operations: $\mathbf{U^T U} = \mathbb{I}$

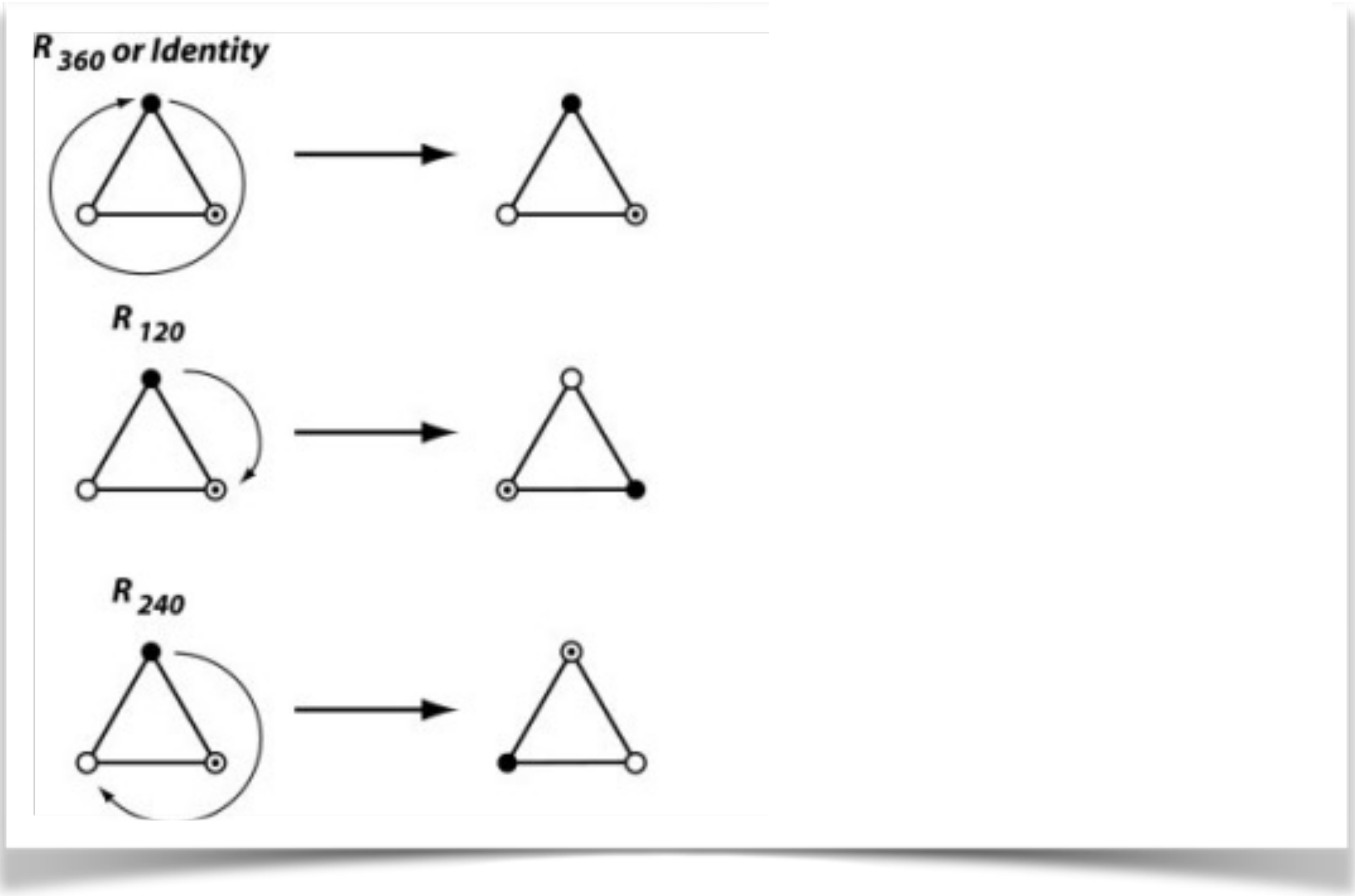
$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos^2\theta + \sin^2\theta & 0 & 0 \\ 0 & \cos^2\theta + \sin^2\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Operation on a vector

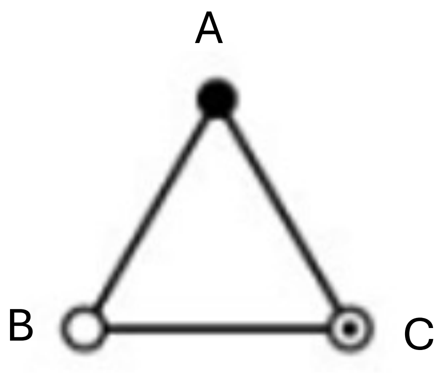
$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x\cos\theta - y\sin\theta \\ x\cos\theta + y\sin\theta \\ z \end{pmatrix}$$

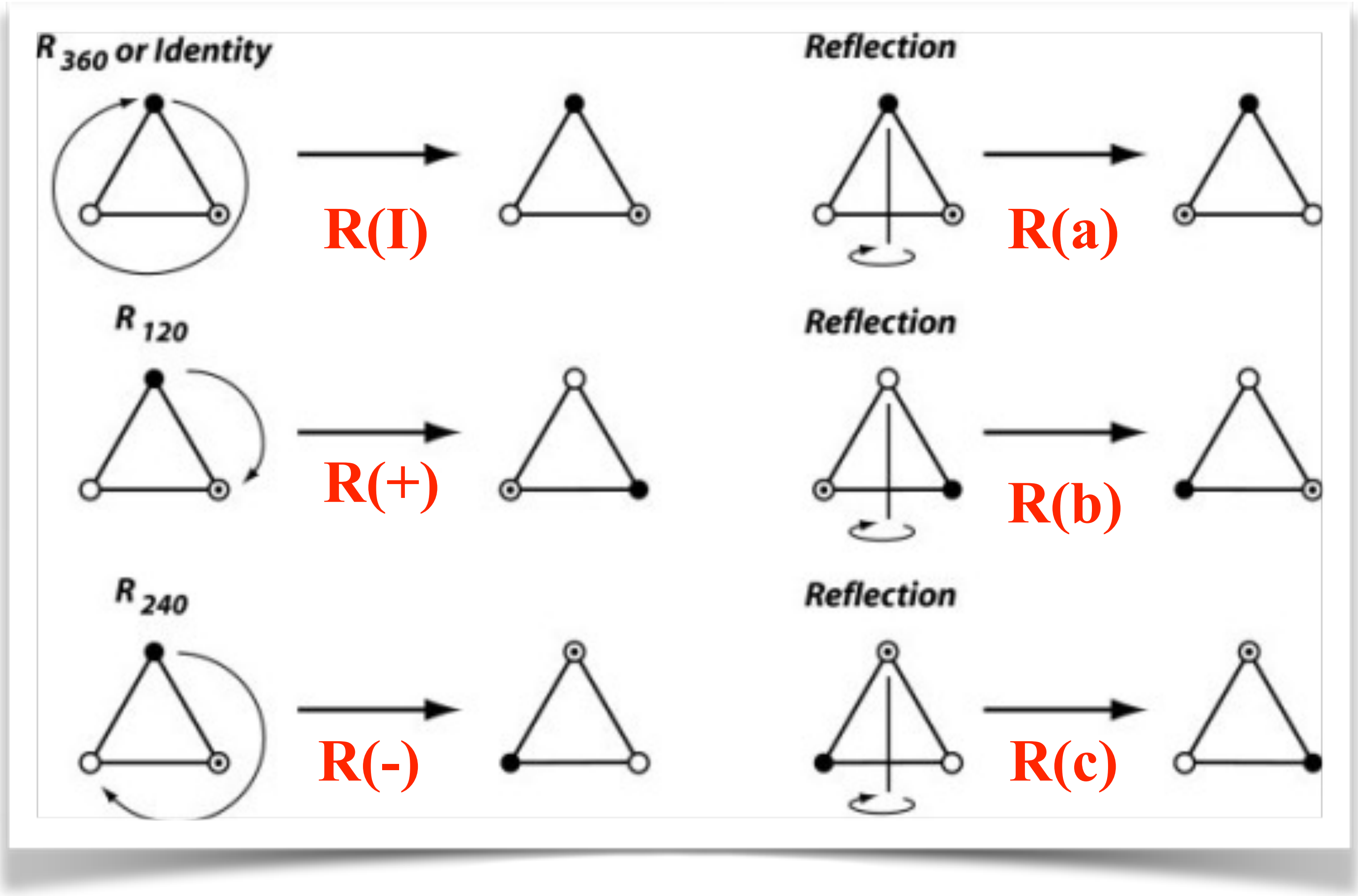
Triangle Symmetry Group





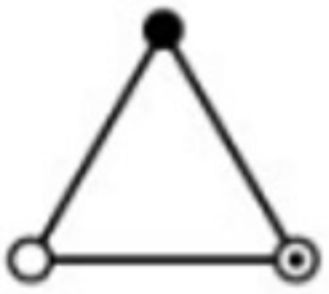
Triangle Symmetry Group





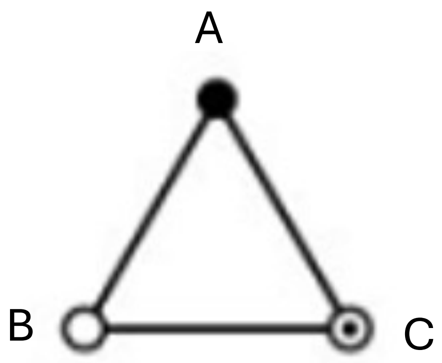
R(+) = + 120, R(-) = - 120

Multiplication Table



	Rotations		Reflections			
	I	R_+	R_-	R_a	R_b	R_c
I	I	R_+	R_-	R_a	R_b	R_c
R_+	R_+	R_-	I	R_b	R_c	R_a
R_-	R_-	I	R_+	R_c	R_a	R_b
R_a	R_a	R_c	R_b	I	R_-	R_+
R_b	R_b	R_a	R_c	R_+	I	R_-
R_c	R_c	R_b	R_a	R_-	R_+	I

Multiplication Table



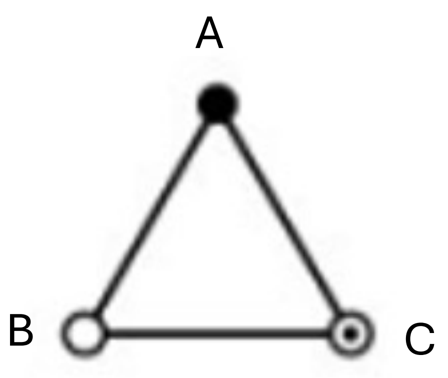
Rotations Reflections

	I	R_+	R_-	R_a	R_b	R_c
I	I	R_+	R_-	R_a	R_b	R_c
R_+	R_+	R_-	I	R_b	R_c	R_a
R_-	R_-	I	R_+	R_c	R_a	R_b
R_a	R_a	R_c	R_b	I	R_-	R_+
R_b	R_b	R_a	R_c	R_+	I	R_-
R_c	R_c	R_b	R_a	R_-	R_+	I

$$R_- R_+ = \mathbb{I}$$

0º - 120º +120º = 0º

Multiplication Table



Rotations | Reflections

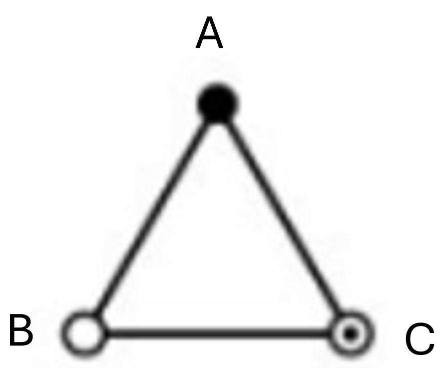
	I	R_+	R_-	R_a	R_b	R_c
I	I	R_+	R_-	R_a	R_b	R_c
R_+	R_+	R_-	I	R_b	R_c	R_a
R_-	R_-	I	R_+	R_c	R_a	R_b
R_a	R_a	R_c	R_b	I	R_-	R_+
R_b	R_b	R_a	R_c	R_+	I	R_-
R_c	R_c	R_b	R_a	R_-	R_+	I

$$R_- R_- = R_+$$

0° - 120° -120° = -240° = +120°

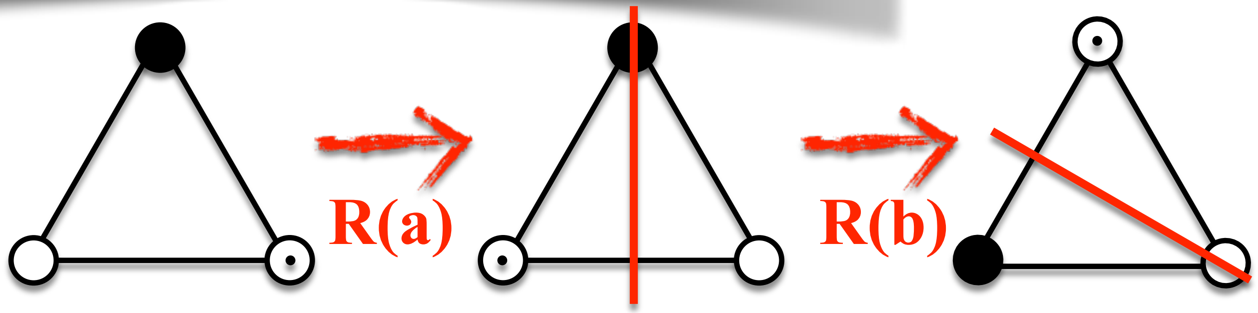
Multiplication Table

Rotations Reflections



	I	R_+	R_-	R_a	R_b	R_c
I	I	R_+	R_-	R_a	R_b	R_c
R_+	R_+	R_-	I	R_b	R_c	R_a
R_-	R_-	I	R_+	R_c	R_a	R_b
R_a	R_a	R_c	R_b	I	R_-	R_+
R_b	R_b	R_a	R_c	R_+	I	R_-
R_c	R_c	R_b	R_a	R_-	R_+	I

$$R_a R_b = R_-$$



Multiplication Table

Rotations Reflections

	I	R_+	R_-	R_a	R_b	R_c
I	I	R_+	R_-	R_a	R_b	R_c
R_+	R_+	R_-	I	R_b	R_c	R_a
R_-	R_-	I	R_+	R_c	R_a	R_b
R_a	R_a	R_c	R_b	I	R_-	R_+
R_b	R_b	R_a	R_c	R_+	I	R_-
R_c	R_c	R_b	R_a	R_-	R_+	I

Groups for which all operators commute [$R_1R_2 = R_2R_1$] are called Abelian groups. Otherwise they are referred to as non-Abelian groups.

Multiplication Table

	I	R_+	R_-	R_a	R_b	R_c
I	I	R_+	R_-	R_a	R_b	R_c
R_+	R_+	R_-	I	R_b	R_c	R_a
R_-	R_-	I	R_+	R_c	R_a	R_b
R_a	R_a	R_c	R_b	I	R_-	R_+
R_b	R_b	R_a	R_c	R_+	I	R_-
R_c	R_c	R_b	R_a	R_-	R_+	I

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Internal Symmetries

In addition to the symmetries of physical systems, there are also symmetries related to internal properties of particles.

Invariance	***Conserved Quantities***
U(1) Gauge Transformation	***electric charge, lepton number, hyper charge***
U(2) [U(1)xSU(2)]	***electroweak charge***
SU(3)	***quark flavor (approximate), baryon number***
SU(3) Gauge Transformation	***quark color***

A few definitions

— A ***gauge theory*** is a type of field theory in which the Lagrangian is invariant under a continuous group of local transformations.

  — The term ***gauge*** refers to redundant degrees of freedom in the Lagrangian.

  — The transformations between possible gauges are called ***gauge transformations***.

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— The transformations between possible gauges are called ***gauge transformations***.

— Gauge transformations form a ***Lie group***, which is referred to as the ***symmetry group*** or the ***gauge group*** of the theory.

— A ***Lie group*** is a group that is also a ***differentiable manifold***.

— A differentiable manifold is a description of a topological space that resembles Euclidean space at each point & is smooth enough to allow calculus.

— Each Lie group has a Lie algebra of ***group generators***.

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— A differentiable manifold is a description of a topological space that resembles Euclidean space at each point & is smooth enough to allow calculus.

— Each Lie group has a Lie algebra of ***group generators***.

— For each group generator there is a vector field, called the ***gauge field***.

— Gauge fields are included in the Lagrangian to ensure invariance under the local group transformations, aka ***gauge invariance***.

EM Gauge Transformation Example

Consider an electric potential, V, and a (magnetic) vector potential A:

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

$$\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

EM Gauge Transformation Example

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$$\mathbf{B} = \nabla \times \mathbf{A}$$

Adding a constant to the potential does not change the fields, and thus does not change the forces.

(Just redefining the axes)

$$V \rightarrow V + C$$

This represents a **global gauge transformation**. It is just like a rigid rotation of a geometric coordinate system.

EM Gauge Transformation Example

Consider an electric potential, V, and a (magnetic) vector potential A:

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

But what if we made a local gauge transformation using a function (f) that depends on position and time:

$$V \to V - \frac{\partial f}{\partial t}$$

$$\mathbf{A} \to \mathbf{A} + \nabla f$$

The fields also remain unchanged. This as a **local gauge transformation**. Maxwell’s equations thus have a local gauge symmetry.