

Homework 4

- 1) 50 mCi source, emits α w/ 5 MeV
- assume α lose all energy in the detector
 - energy to create e+ion pair = 30eV
 - collection efficiency for charge: 89%

1. How many α/s ? $1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$, $1 \text{ Bq} = 1 \text{ decay/s}$

$$\rightarrow 50 \times 10^{-3} \text{ Ci} \cdot 3.7 \times 10^{10} \frac{\text{Bq}}{\text{Ci}} = 185 \times 10^7 \text{ Bq or } \alpha/\text{s}$$

2. How many e-ion pairs per α ?
 $5 \text{ MeV } \alpha/s$ \rightarrow assume all converted to ioniz.
 $\rightarrow 30 \text{ eV to ionize}$

$$\rightarrow 5 \text{ MeV}/30 \text{ eV} = \# \text{ electrons from ionization}$$

$$5 \times 10^6 \text{ eV}/30 \text{ eV} = 1.67 \times 10^5 \text{ electrons}$$

3. Charge of electrons:
 for each electron $q = 1.602 \times 10^{-19} \text{ C}$

$$\rightarrow q_{\text{tot}} = 1.67 \times 10^5 \text{ electrons} \times 1.602 \times 10^{-19} \text{ C/elect.}$$

$$= 2.7 \times 10^{-14} \text{ C}$$

4. Current, including efficiency:

$$I = 185 \times 10^7 \alpha/\text{s} \times 2.7 \times 10^{-14} \text{ C} \times 0.89$$

$$= 444 \times 10^7 \times 10^{-14} = \boxed{4.44 \times 10^{-5} \text{ A}}$$

1) b. 5 MeV particle stopped in ionization chamber
Voltage undergoes step change of 2mV.

What is the capacitance?

Assume electronics/chamber time constant
is long compared to charge deposition
by δs .

$$C = \Theta / V$$

$$\text{where } \Theta = 2.7 \times 10^{-14} C \times 0.89 \rightarrow \begin{matrix} \text{take detection} \\ \text{efficiency into} \\ \text{account} \end{matrix}$$

$$= 2.4 \times 10^{-14} C$$

$$C = \frac{2.4 \times 10^{-14} C}{2 \times 10^{-3} V} = \boxed{1.2 \times 10^{-11} F = C}$$

2) a. $f = 30 \text{ MHz}$, accelerates to V_i . What is λ ?
 $\lambda = V \Delta t$, $\Delta t = 1/2f \rightarrow \text{half cycle}$

$$\boxed{\lambda = V_i / 2f}$$

b. Assume particles are O^{16} ions, acc. to 80 MeV
 $= p$

$$V = \beta c, \text{ where } \beta = p/E$$

Assume $E = m \rightarrow p = 80 \text{ MeV}$ much smaller than
mass of ^{16}O

$$\rightarrow E = m = 16u = 16 \times 930 \text{ MeV} = 14,880 \text{ MeV}$$

$$\rightarrow V = \frac{p c}{E} = \frac{80 \text{ MeV} \times 3 \times 10^8 \text{ m/s}}{14,880 \text{ MeV}} = \boxed{1.6 \times 10^6 \text{ m/s} = V}$$

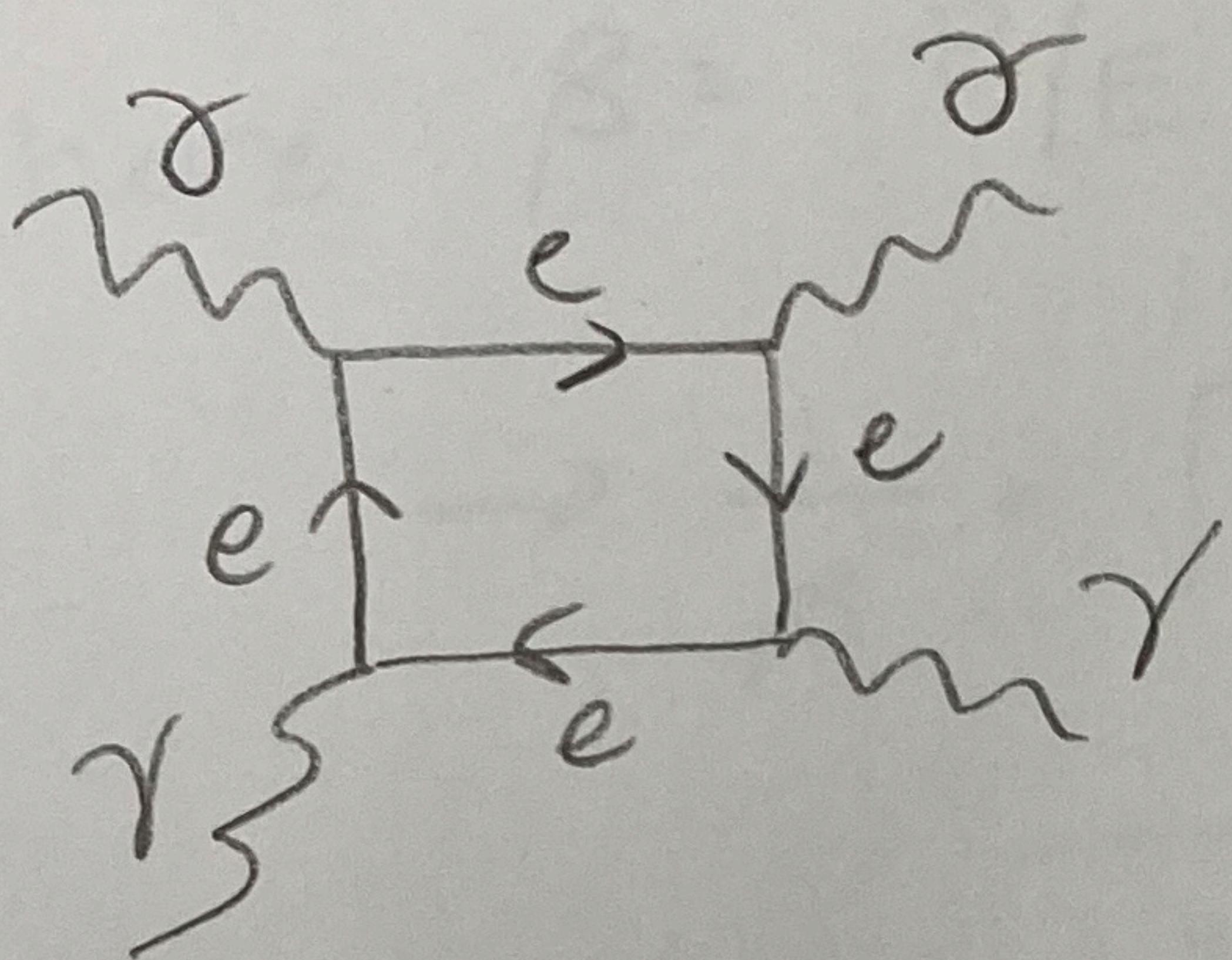
2) b. Cont:

$$l = \frac{v_i}{2f} = \frac{1.6 \times 10^6 \text{ m/s}}{2 \cdot 30 \times 10^6 \text{ /s}} = 0.0267 \text{ m} = 2.7 \text{ cm}$$

3) a. $\gamma + \gamma \rightarrow \gamma + \gamma$

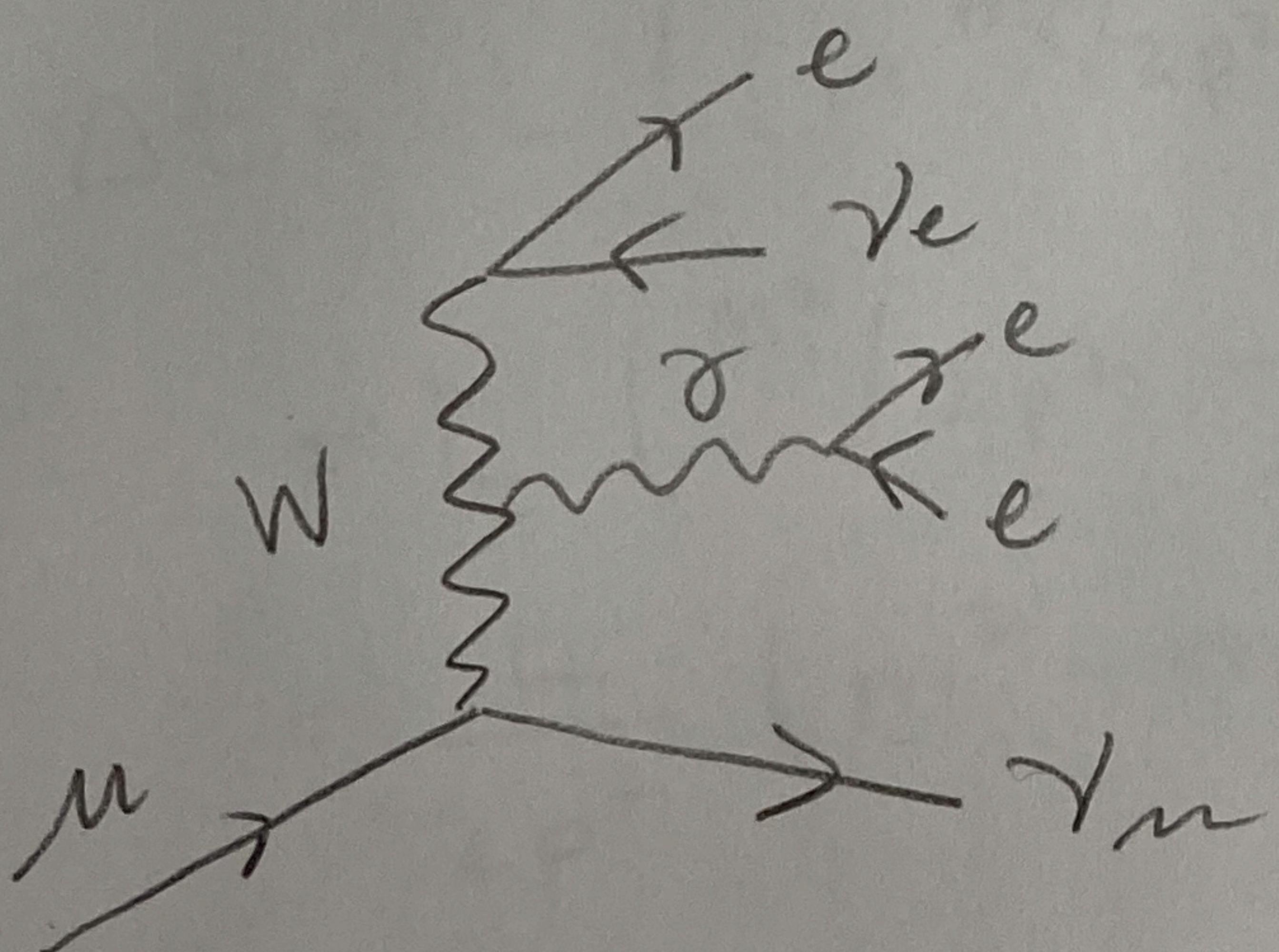
Base vertex:

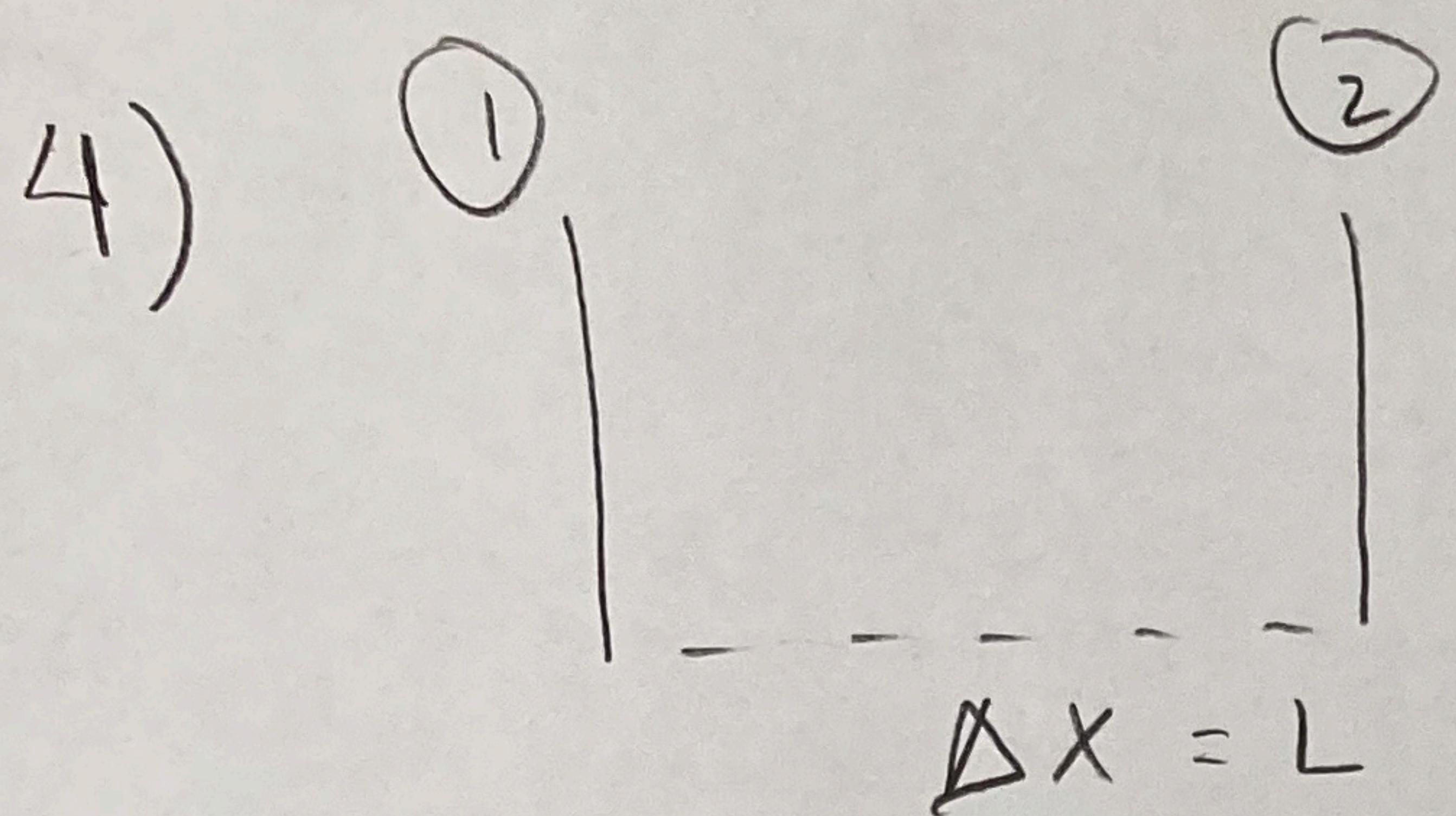
\rightarrow need 2 to get 2 γ in/out



b. $\mu^- \rightarrow e^- + e^- + e^+ + \gamma_\mu + \bar{\nu}_e$

Neutrinos \rightarrow must be weak + charged, since not
 $\nu_{\text{in}} + \nu_{\text{out}}$ of interaction w/ flavor conservation





$$\Delta t = \frac{L}{v_1} - \frac{L}{v_2} = L \left(\frac{1}{\beta_1} - \frac{1}{\beta_2} \right)$$

where $\beta = P/E = \frac{P}{\sqrt{p^2 + m^2}}$

$$\rightarrow \frac{1}{\beta} = \sqrt{1 + \frac{m^2}{p^2}}$$

Approximate: $\sqrt{1 + \alpha} \approx 1 + \alpha/2$

$$\rightarrow \frac{1}{\beta} \approx 1 + \frac{m^2}{2p^2}$$

$$\begin{aligned} \text{So } \Delta t &= L \left(\left(1 + \frac{m_1^2}{2p^2} \right) - \left(1 + \frac{m_2^2}{2p^2} \right) \right) \\ &= L \left(\frac{m_1^2}{2p^2} - \frac{m_2^2}{2p^2} \right) \\ &= \frac{L}{2p^2} (m_1^2 - m_2^2) \end{aligned}$$

b. $\Delta t = 200 \text{ ps}, p = 3 \text{ GeV/c}$

From q:

$$L = \frac{2p^2 \Delta t}{c(m_1^2 - m_2^2)} = \frac{2 \times 3000 \times c \times 200 \times 10^{-12}}{493.7^2 - 139.6^2} = \boxed{4.82 \text{ m}}$$