PHY493/803 Spring 2025, Intro to Elementary Particle Physics
Homework 3 - Due Date 24th of February

Please clearly state any assumptions, show all your work, number the equations, and indicate logical connections between lines.

1. (5+10+5 pts) Consider the production of a pair of pions in e^+ e^- collisions. This is an electromagnetic process that proceeds via a virtual photon.

   (a) What are the possible final states and what is the isospin of these pions?

   (b) The virtual photon can have isospin 0 or 1 because EM interactions do not preserve isospin. From the Clebsch-Gordan tables, find the decay amplitudes for both photon isospin states, |0,0> and |1,0>. Write down the particles corresponding to the isospin states for each coefficient.

   (c) Only one of the two decays (|0,0> and |1,0>) is actually allowed. Determine which one and explain why this is the case.

2. (5+5+5 pts) It would be great if we could accelerate and store muons to produce powerful neutrino beams for a muon collider. Suppose you start out with a million muons.

   (a) If the muons are at rest, how many would still be around 2.2×10^{-5} seconds later?

   (b) If the muons have an energy of 1 GeV and a mass of 100 MeV, how many would still be around 2.2×10^{-5} seconds later?

   (c) If the muons have an energy of 10 GeV and a mass of 100 MeV, how many would still be around 2.2×10^{-5} seconds later?

3. (5+5+5+5+5 pts)

Consider the elastic scattering reaction A + B → A + B in the lab frame (B initially at rest) and assume that the initial energy E_1 of the incoming A particle satisfies $E_1 \ll m_B$ so that the recoil of the target can be neglected.

a) Use the Golden Rule for scattering to show that the differential cross section is given by:

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{(8\pi m_B)^2}$$

b) Write down the lowest order diagram(s) for this scattering process in ABC theory.

c) Calculate the scattering amplitude using the Feynman rules for ABC theory (express your result using the Mandelstam variables s, t and/or u as relevant).

d) Combine the results from (a) and (c) to obtain the differential cross section (in the limit $E_1 \ll m_B$ and assuming that m_A and m_C are tiny compared to m_B).

e) Show that the total cross-section is

$$\sigma = \frac{g^4}{4\pi m_B^6}$$

under the conditions stated in part (d).

4. (10+10 pts) **{Required for PHY803 students only. +20 pts extra credit for PHY493 students.}**

a) Repeat problem 4(d) assuming the reaction A + B → A + B occurs in the center-of-momentum frame, and take $m_A = m_B = m$ and $m_C = 0$. You should report your answer in terms of the incident particle energy ($E_A = E_B = E$) and the scattering angle for particle A (θ).

Useful relation: $1 + \cos\theta = 2\cos^2(\theta/2)$

b) Without actually performing the integral, explain what the total cross section would be for this process. Does it converge or is it divergent? Compare to the solution in problem 4e) and explain any differences.