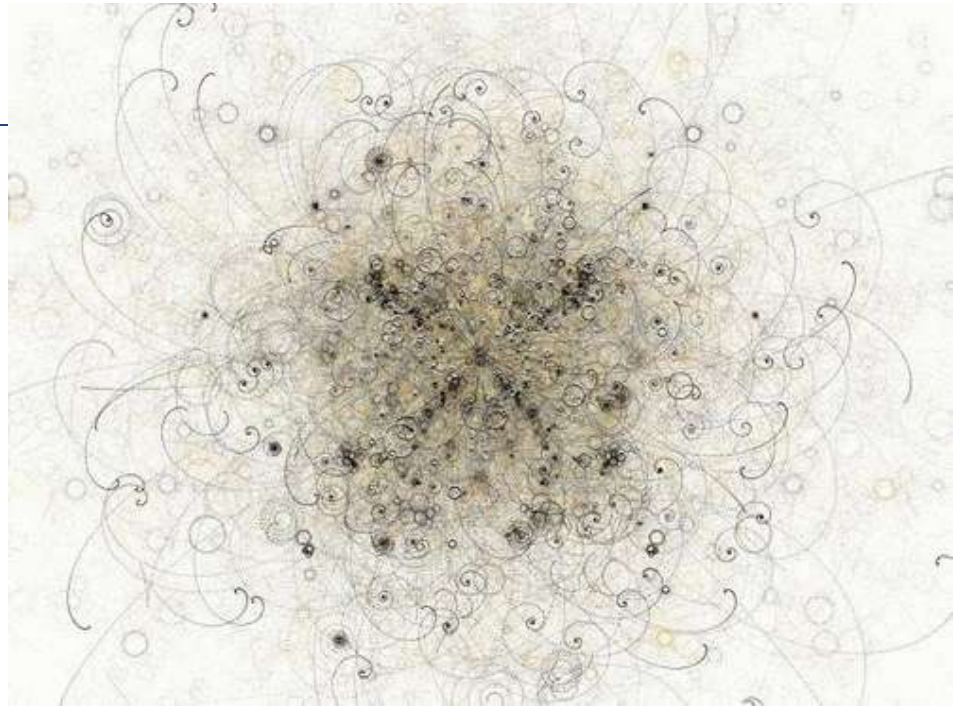
Announcements



Quiz:
- Assorted quizzes from earlier weeks. Pick up after class.
- Next quiz on Friday

Homework:
Fourth HW posted. Due date **March 24 at 3pm** on gradescope

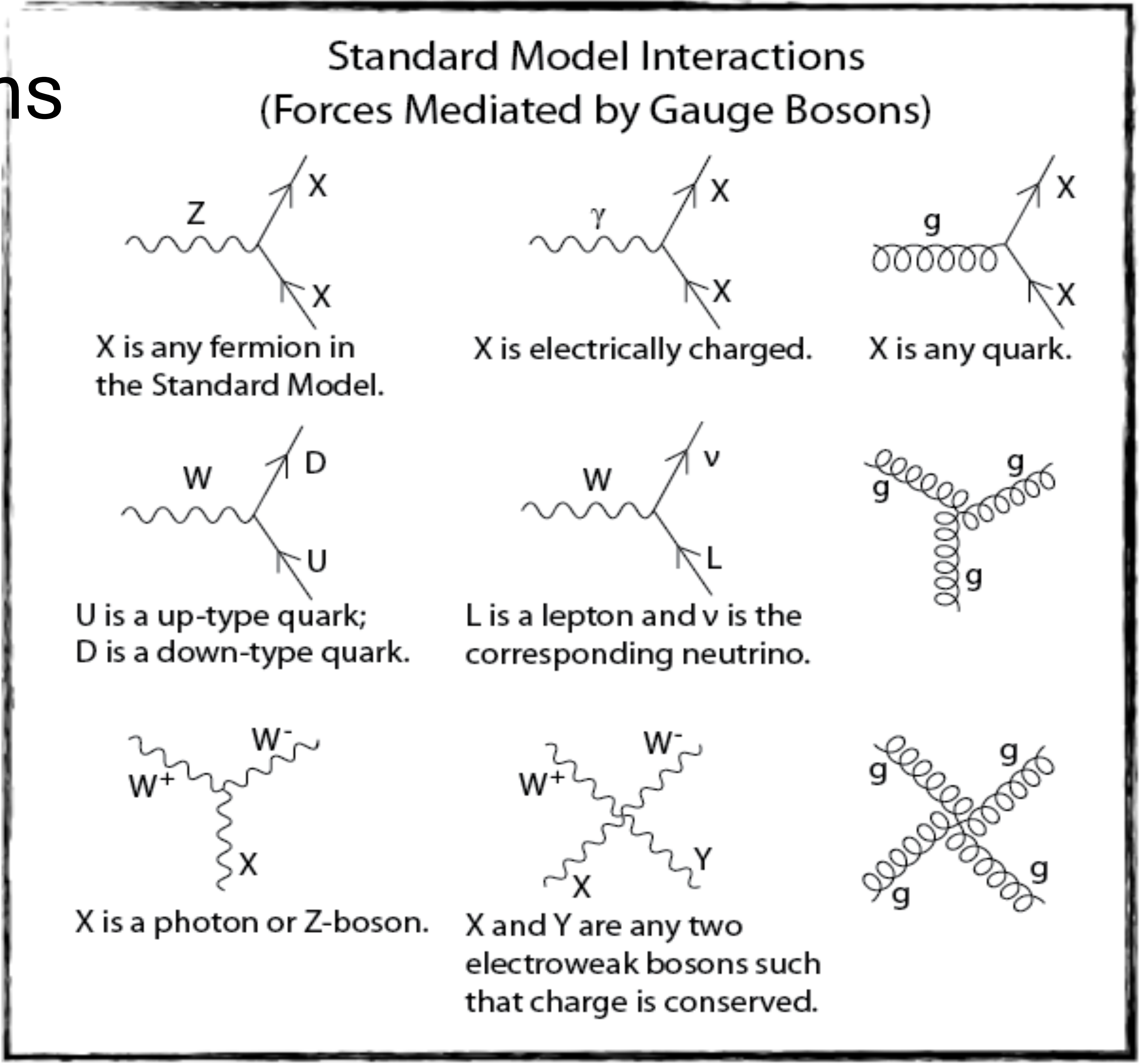
Paper:
-Outlines returned on gradescope – please take a look; reach out if you have questions
-Draft deadlines:
    - **<u>Optional</u>**, 3/28 in class: bring a paper copy to me by this date if you want feedback
    - For credit, 4/11 at 3pm, on gradescope

Midterm:
-Pick up graded midterms after class
-Will be curved to add 10 points to the score at the top of the page
- Please note: your grade is the sum of your best four questions on the exam, the fifth question is not extra credit

Feynman Diagrams

- Basic vertices

(everything except for the Higgs)



Bethe Bloch Equation

$$-\frac{dE}{dx} = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right] \times \rho$$

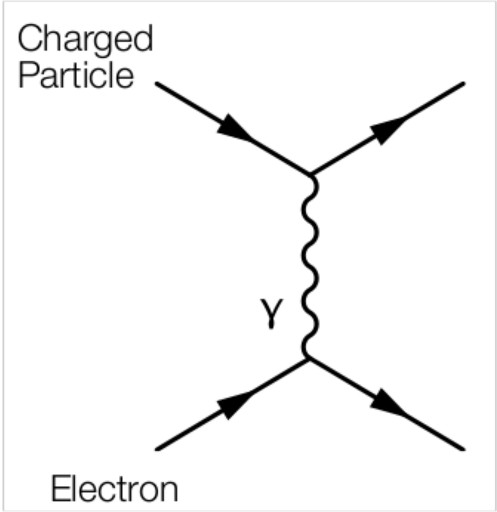
$K = N_A e^2/\epsilon_0$ = 0.307 MeV cm^2 / g

$$T_{\max} = 2m_e c^2 \beta^2 \gamma^2 / (1 + 2\gamma m_e/M + (m_e/M)^2)$$

z : charge of incident particle
M : mass of incident particle

Z : charge number of medium
A : atomic mass of medium

I : mean excitation energy of medium
δ : density correction [dielectric screening for highly relativistic particles]

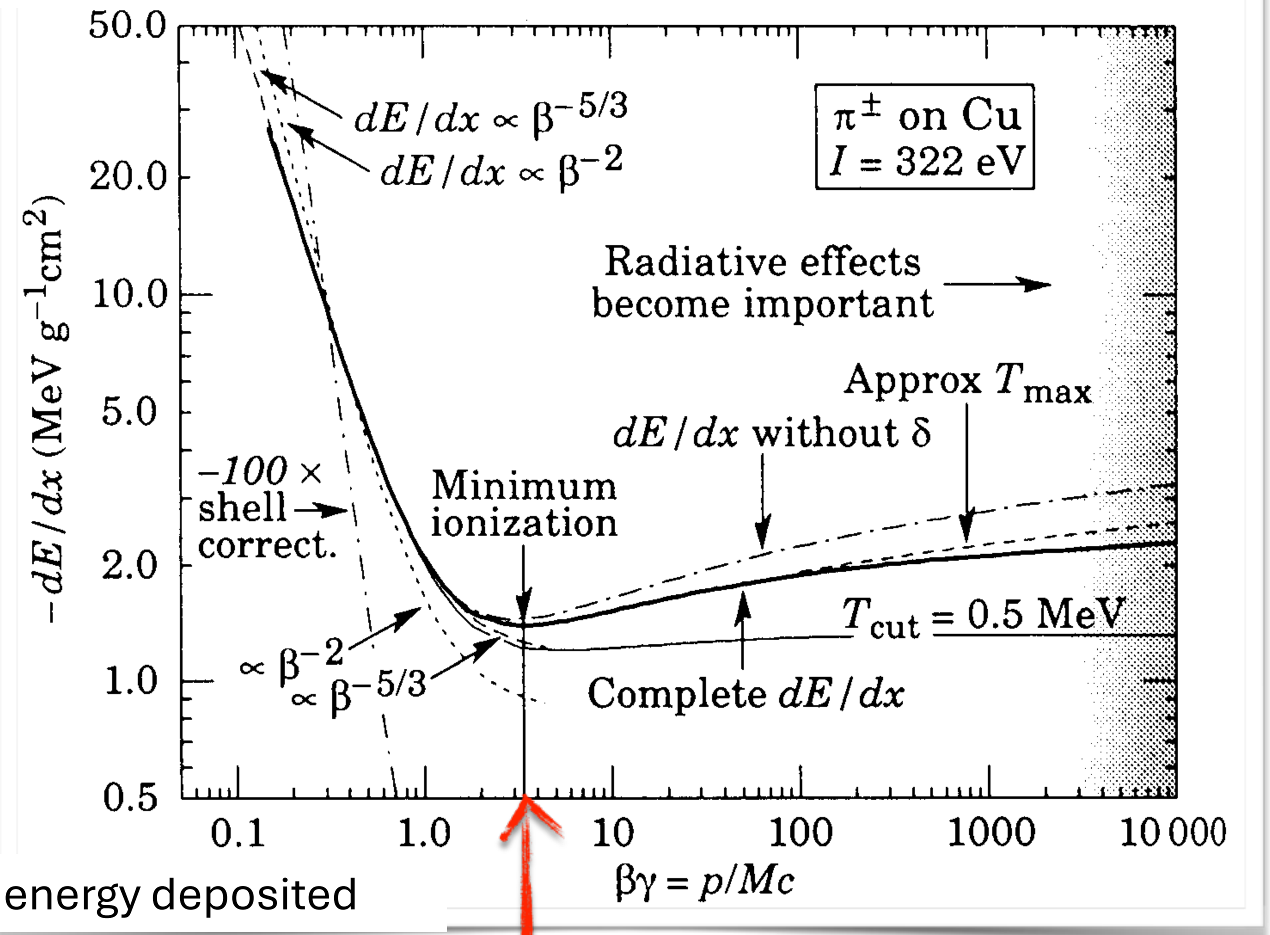


Valid for 0.5<βγ<500
M > m_μ

(Not for electrons)

Energy loss of pions in Cu

Lots of energy deposited



Small energy deposited

Minimum ionizing particles (MIPs): $\beta\gamma$=3-4

dE/dx falls ~$\beta^{-5/3}$; non-relativistic regime

dE/dx rises ~ $\ln(\beta\gamma)^2$; relativistic regime

Saturation at large ($\beta\gamma$) due to polarization of the medium.

Units: MeV cm^2 / g
MIP loses ~ 13 MeV/cm for copper

minimum ionization = minimum energy deposited: $\left(-\frac{dE}{dX}\right)_{min} \approx 3.5\frac{Z}{A}$ **MeV/(g cm^{-2}) ≈ 13 MeV/cm here**

Understanding Bethe-Bloch

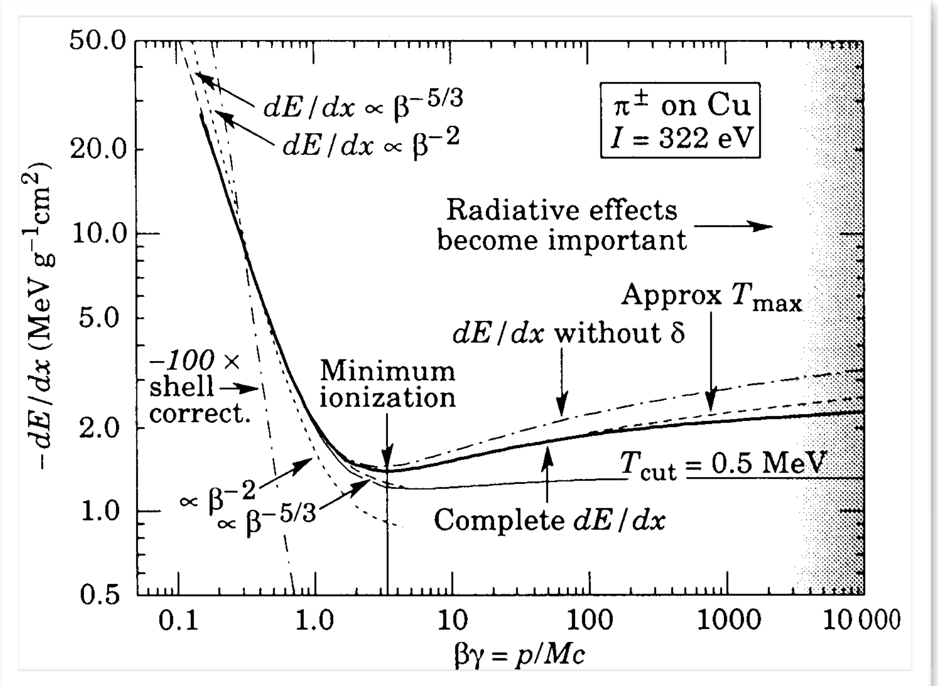
1/β² dependence:

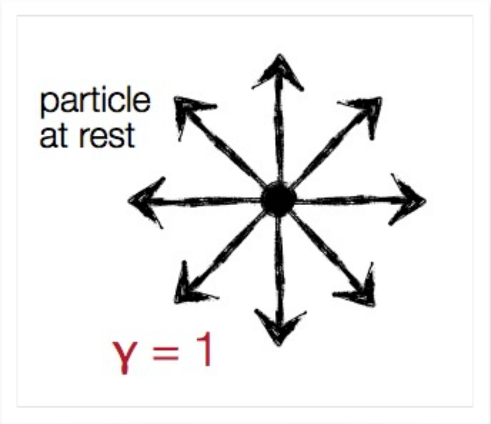
Recall: $$\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dx}{v}$$

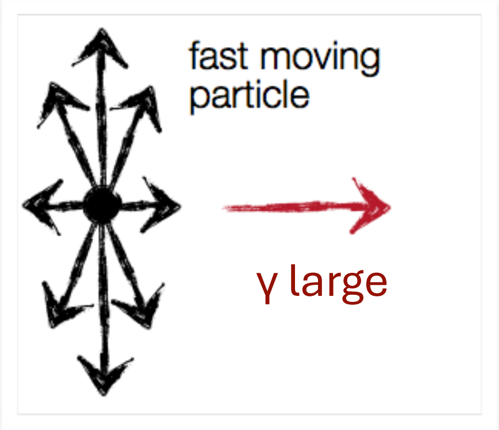
ie, slower particles feel electric force of atomic electron for a longer time.

Relativistic rise for βγ > 4:

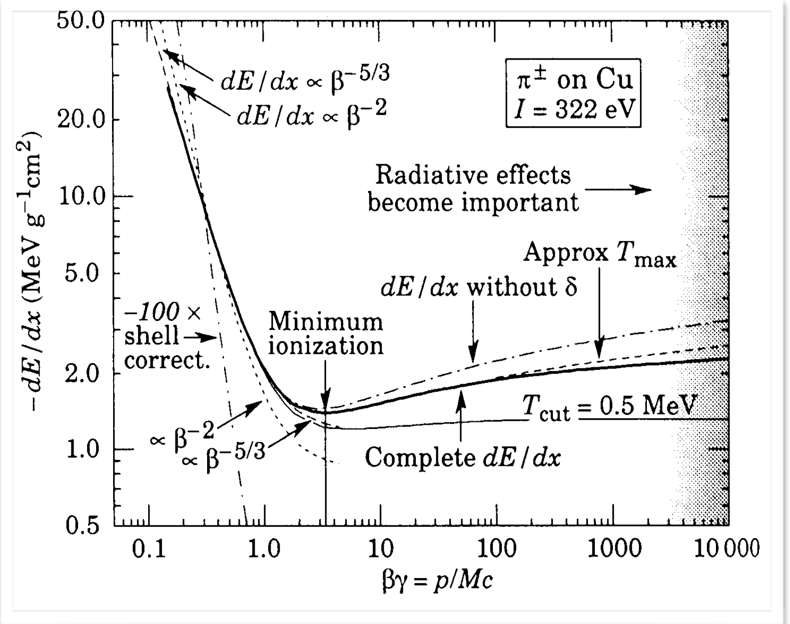
High energy particle: transverse electric field increases due to Lorentz transform: $E_\perp \rightarrow \gamma E_\perp$. Thus the interaction "strength" increases.







Understanding Bethe-Bloch



$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Density Correction:

- Polarization effect, which is density dependent.
- Arises from the shielding of the electrical field far from the particle path; effectively cuts off the long-range contribution.
- More relevant at high γ: increased transverse range = larger b_{max}.

Energy loss of charged particles

Dependence on:

— Mass (A)
— Charge (Z) of the target nucleus

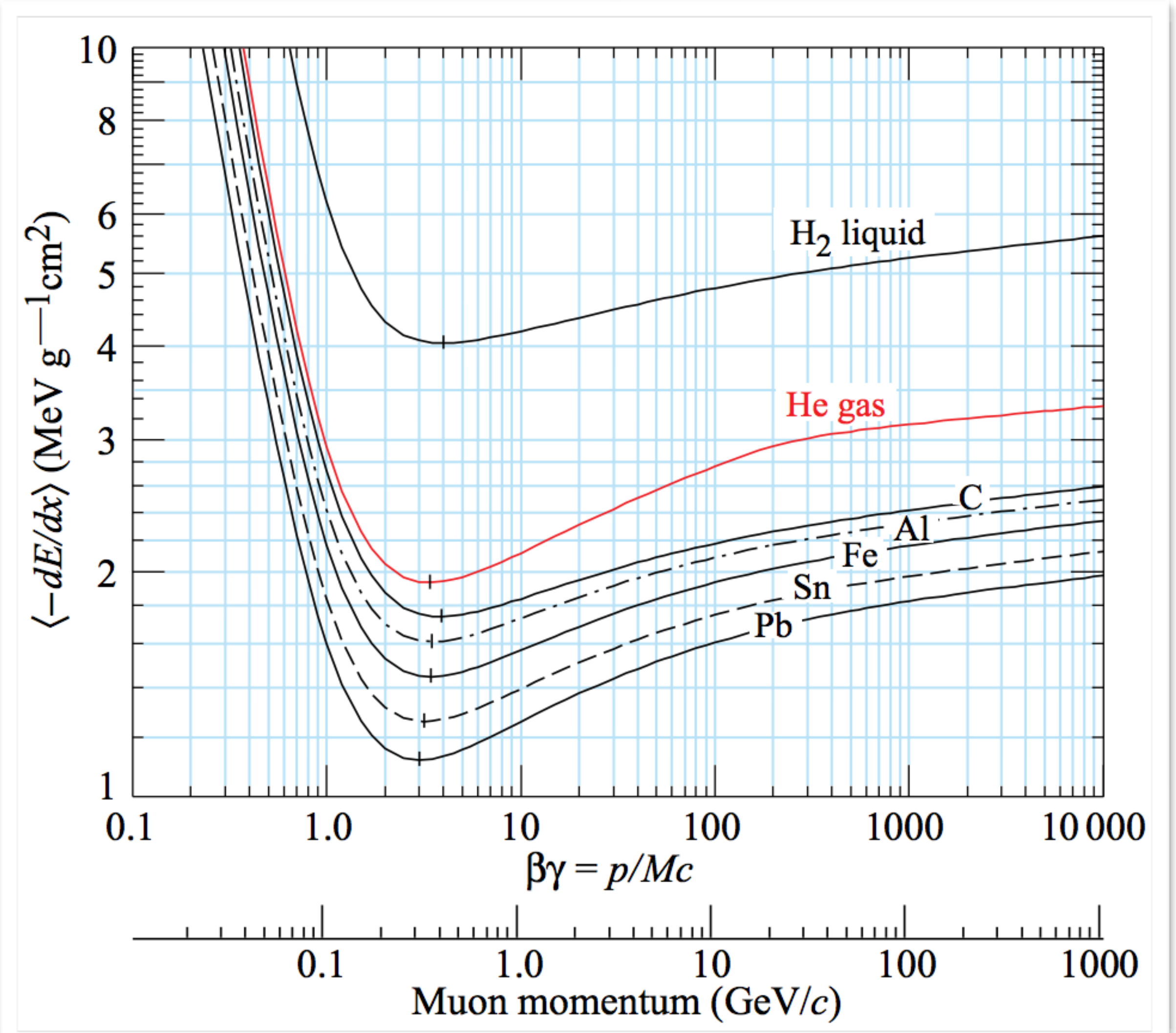
Minimum ionization

Average 1-2 MeV cm^2/g

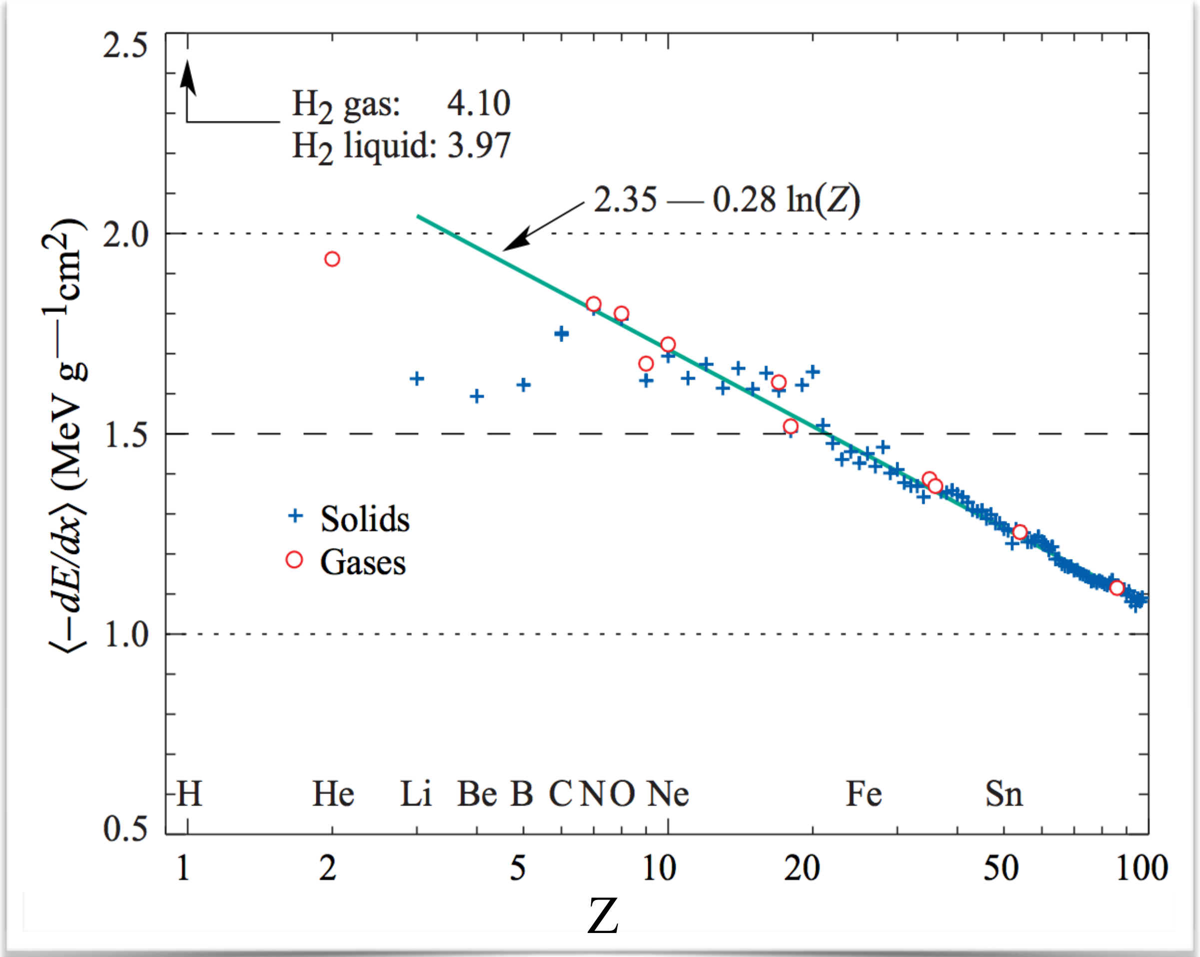
H_2: 4 MeV cm^2/g

Same Bethe-Bloch form, but different energy loss

More energy lost in lighter mediums: fewer electrons, generally stable and harder to knock out through ionization



Stopping Power at Minimum Ionization

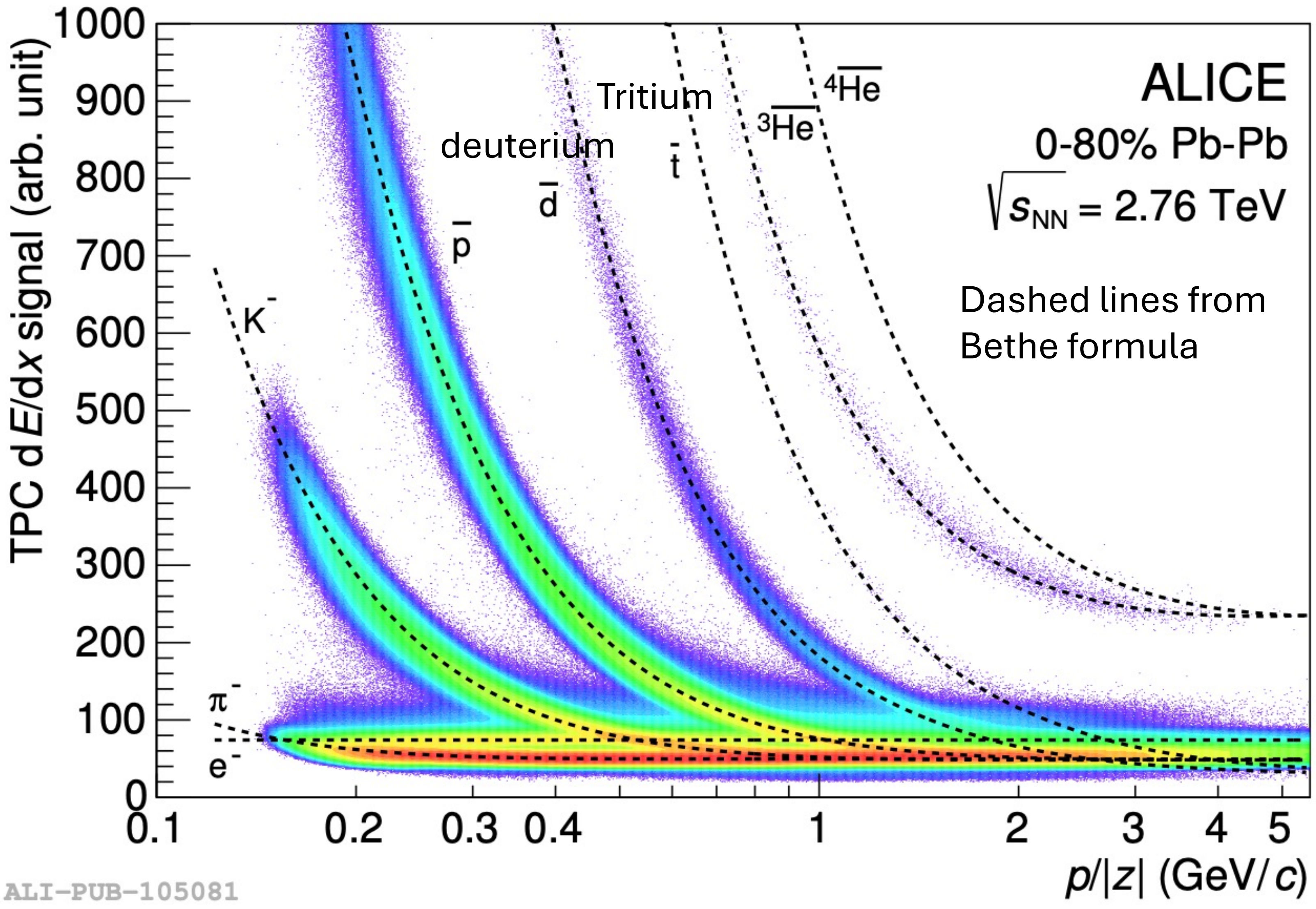


Stopping power at minimum ionization for the chemical elements.

The straight line is fitted for Z > 6. A simple functional dependence on Z is not to be expected, since <−dE/dx> also depends on other variables.

Note: this is just for ionization!

dE/dx and Particle Identification - ALICE TPC



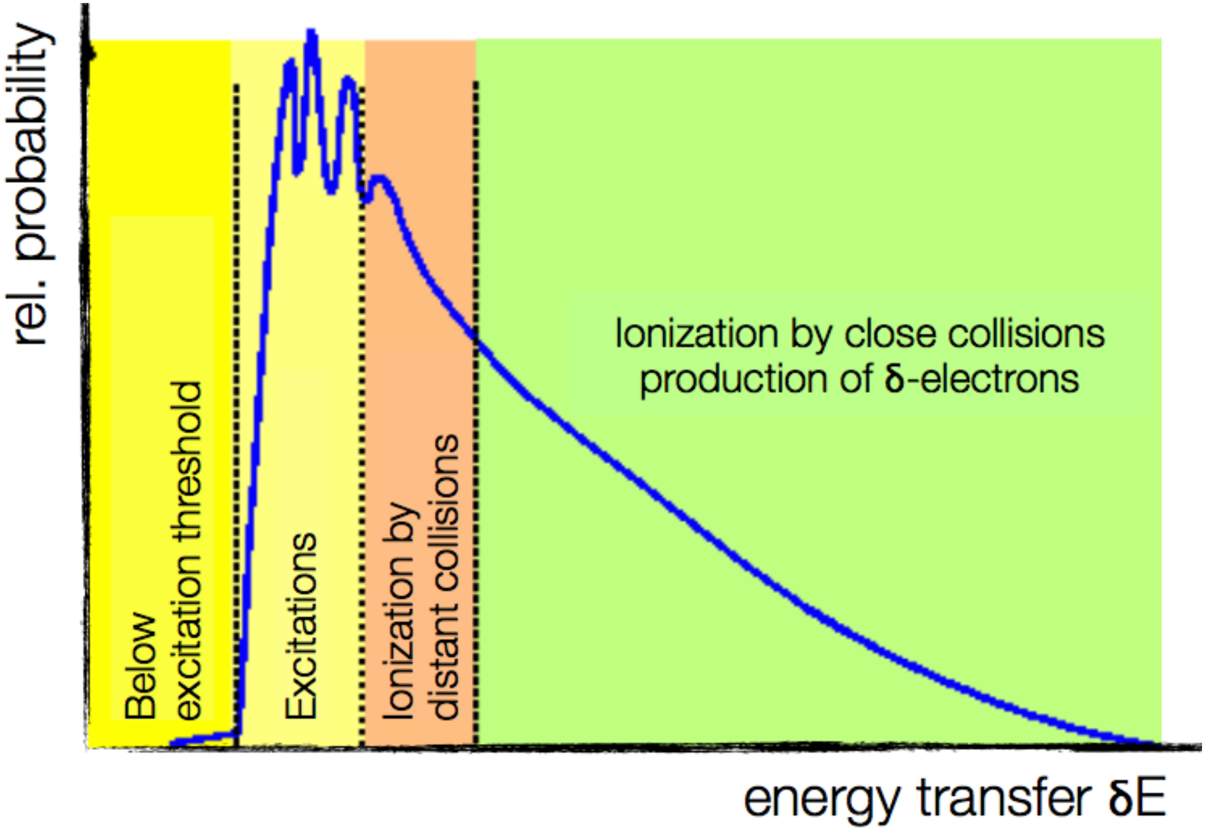
dE/dx Fluctuations

Bethe-Bloch only describes the mean energy loss

- For N collisions, each one giving a random δE:

$$\Delta E = \sum_{n=1}^{N} \delta E_n$$

Probability of a given energy transfer



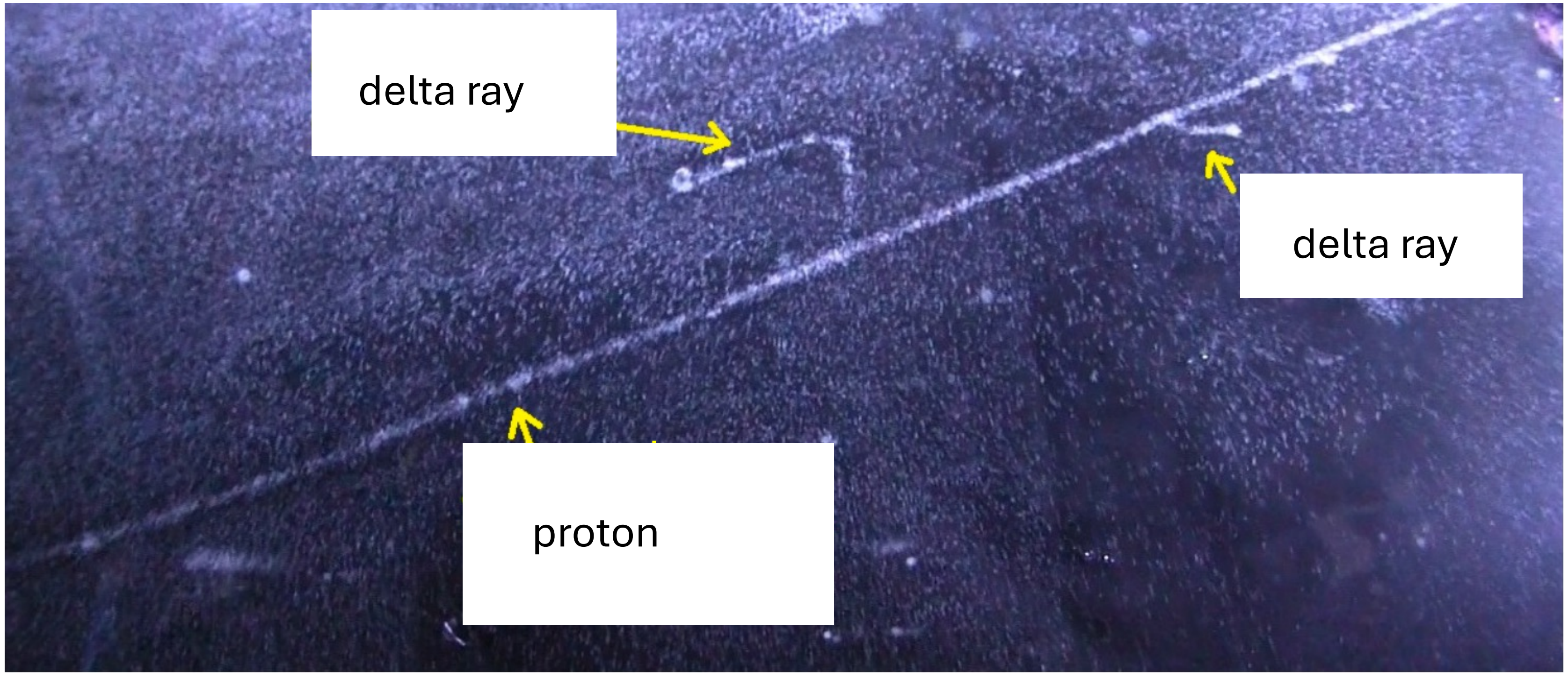
Ionization loss δE is distributed statistically.

Referred to as energy loss “straggling”

- At low δE: Excitation of atoms in the material
- At intermediate δE: Ionization through collisions with low energy transfer to one electron (large impact parameter)
- At high δE: δ-electrons that have enough energy to make secondary interactions

δ-electrons

- At small impact parameters, particles traveling through material can transfer a lot of energy to the atomic electrons
- This produces δ electrons
- Example: proton traveling through a cloud chamber at CERN



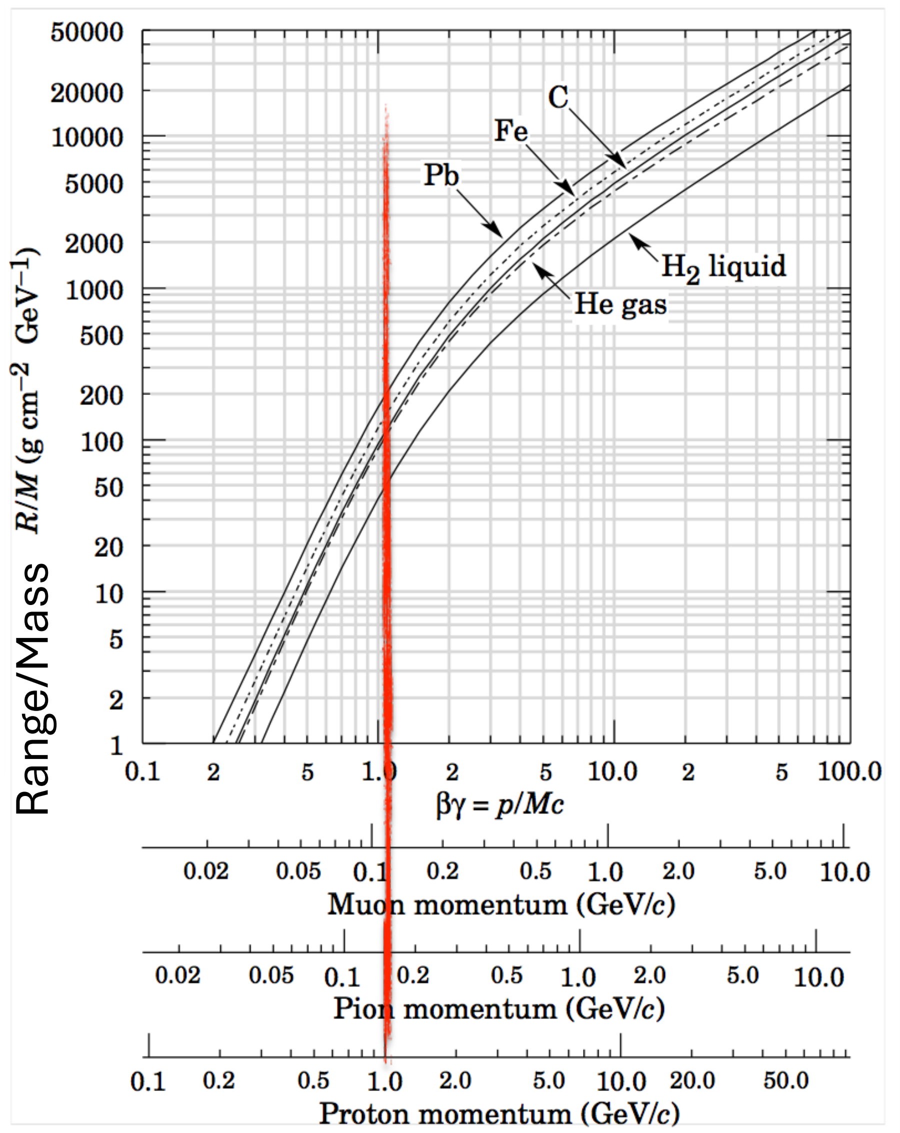
Mean Particle Range

Integrate over energy losses: from E down to 0

$$R = \int_{E_0}^{0} \frac{dE}{dE/dx} = \int_{\beta_0}^{0} \frac{dx}{dE}\frac{dE}{d\beta}\,d\beta = \int_{0}^{\beta_0} \frac{-dx}{dE}\frac{dE}{d\beta}\,d\beta = \frac{M}{z^2 e^2 n_e} F(\beta_0)$$

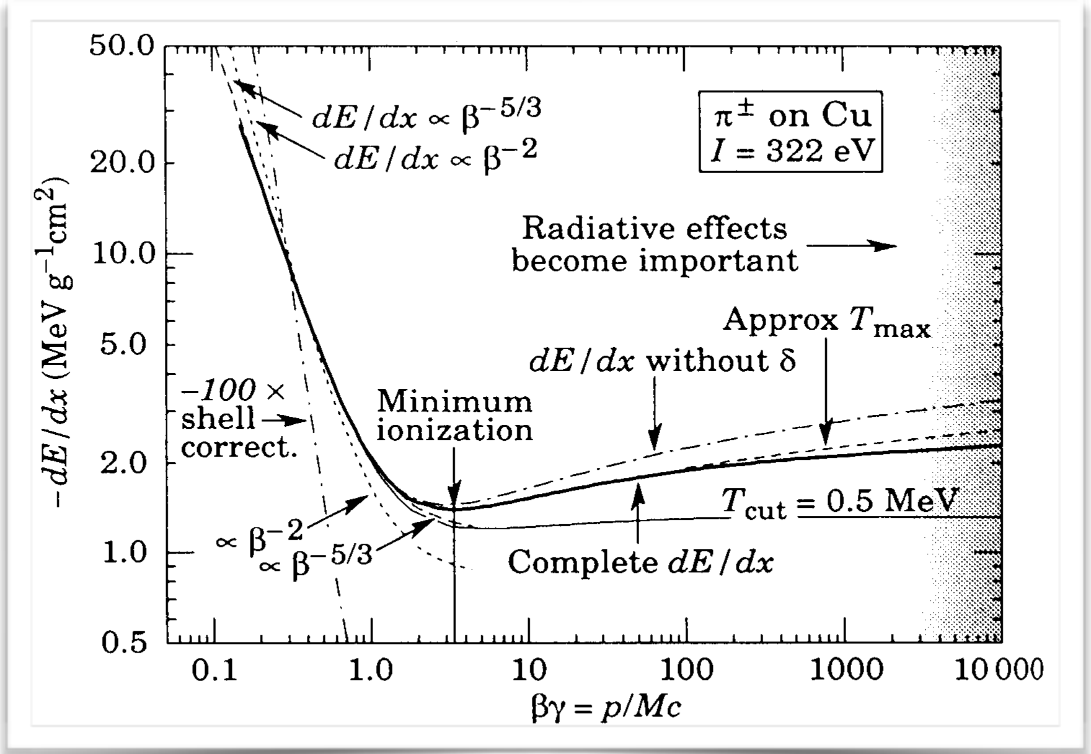
<u>Example:</u>
Proton with p = 1 GeV
Target: lead with ρ = 11.34 g/cm^3

R/M = 200 g cm^{-2} GeV^{-1}
→ R = 200/11.34/1 cm ~ 20 cm



Mean Particle Range

Think of this as an active diagram:



Moving Slower

Moving Faster

Minimum ionization ~3.5 for pions in copper

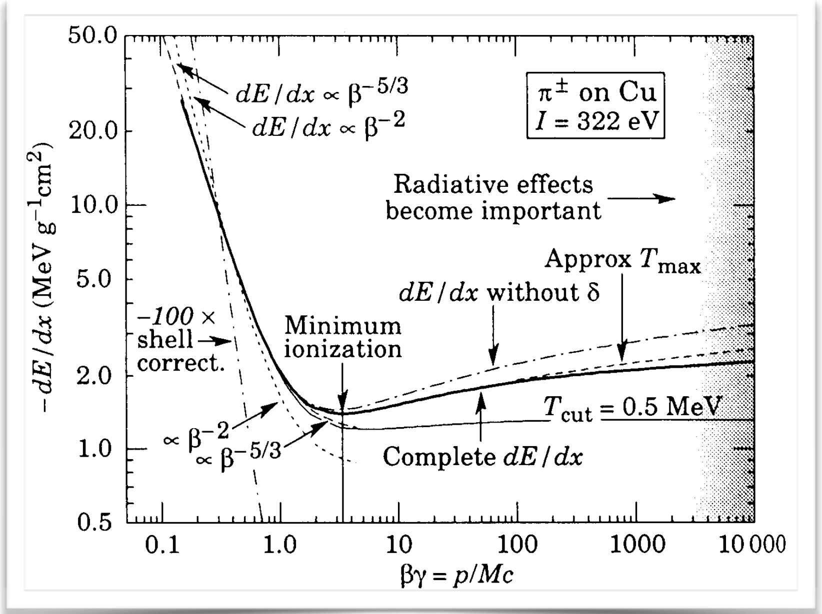
$\beta\gamma > 3.5$:

$$\left\langle \frac{dE}{dx} \right\rangle \approx \left.\frac{dE}{dx}\right|_{\min}$$

$\beta\gamma < 3.5$:

$$\left\langle \frac{dE}{dx} \right\rangle \gg \left.\frac{dE}{dx}\right|_{\min}$$

Mean Particle Range



$\beta\gamma > 3.5$:

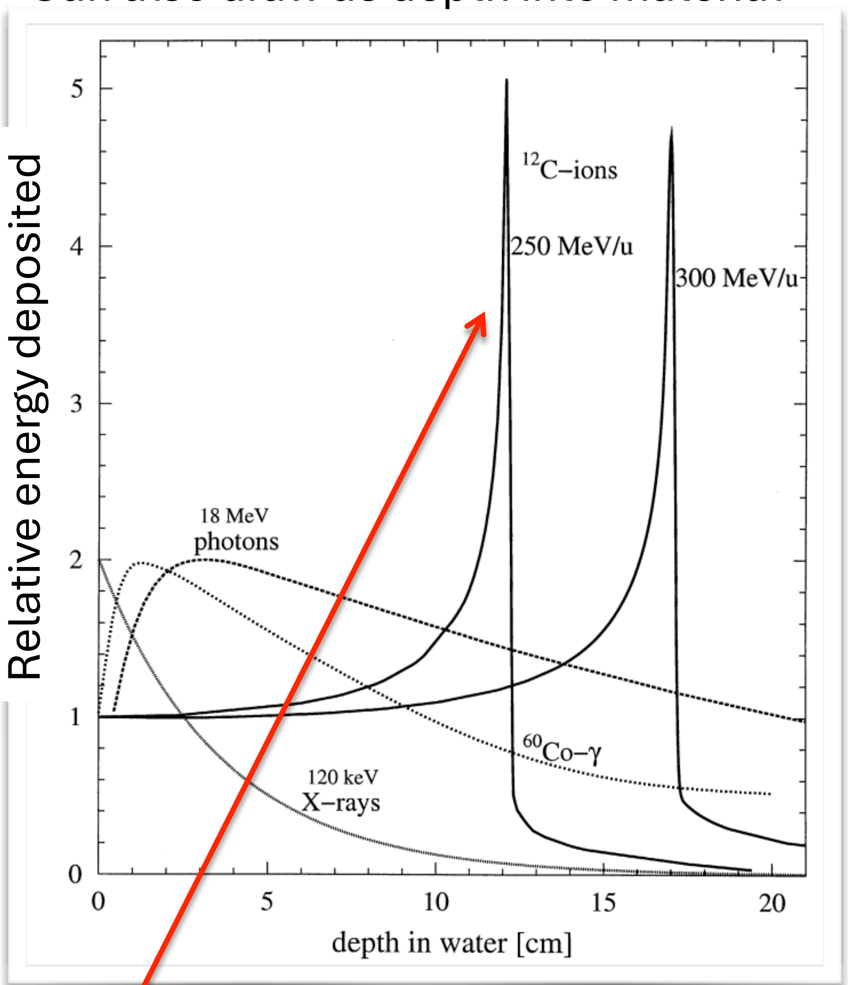
$$\left\langle \frac{dE}{dx} \right\rangle \approx \left.\frac{dE}{dx}\right|_{\min}$$

$\beta\gamma < 3.5$:

$$\left\langle \frac{dE}{dx} \right\rangle \gg \left.\frac{dE}{dx}\right|_{\min}$$

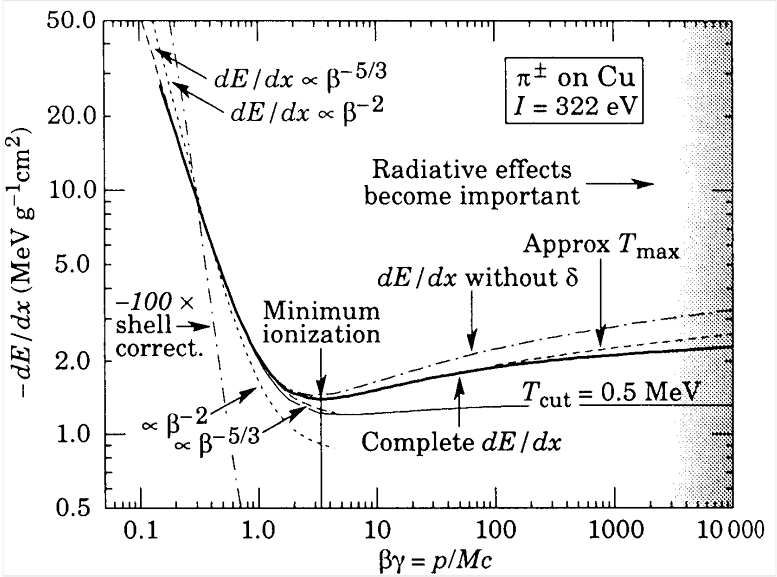
Minimum Ionization
~3.5

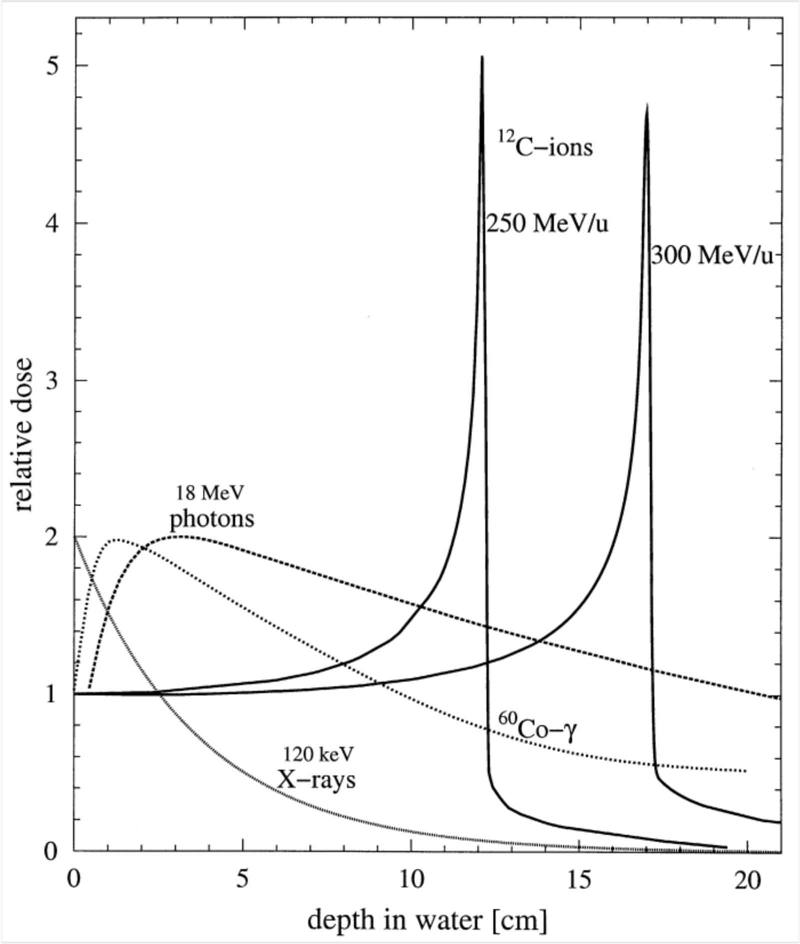
Can also draw as depth into material



Bragg Peak: sharp fall after all energy deposited

Mean Particle Range





Applications:

1) Tumor therapy

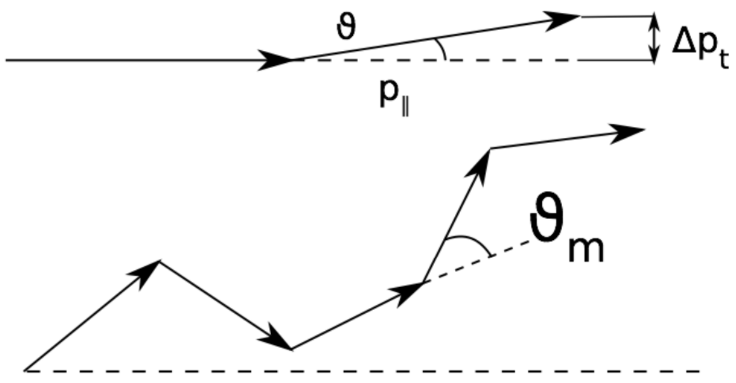
Possibility to precisely deposit dose at well defined depth by E_{beam} variation

2) Particle Calorimetry

We can sample dE/dx as the particle traverses a medium, ultimately stopping it and thus measuring total energy.

Multiple scattering

- Particles can be deflected by elastic scattering off the nucleus
  - Can happen many times - stochastic process
  - Molière theory
- Multiple scattering limits tracking resolution pointing



- Average deflection of particle with charge ze in one plane in material:

$$\theta_{RMS} = \frac{13.6\mathrm{MeV}}{\beta p} z \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \ln(x/X_0)\right]$$

p = particle momentum
Z = material particle is traveling in
x = distance traveled

where X_0 is the radiation length

-> average length will travel into material before interacting

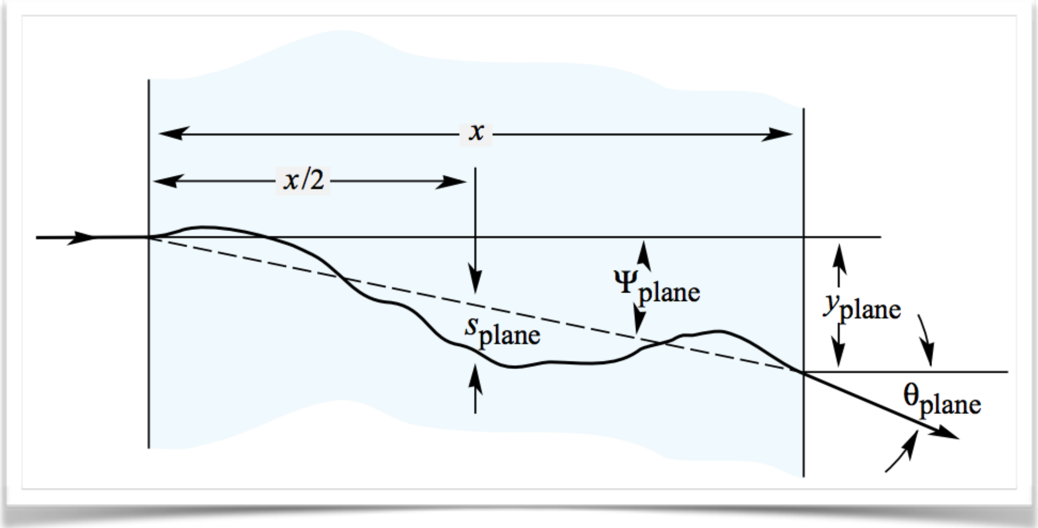
Multiple Scattering

Gaussian Approximation:

When deriving energy loss via ionization
we considered transverse momentum transfer ...

$$\Delta p_t = \frac{2Zze^2}{bv}$$

$$\theta \approx \frac{\Delta p_t}{p_{||}} \approx \frac{\Delta p_t}{p} = \frac{2Zze^2}{b}\frac{1}{pv}$$



After k collisions:

$$\langle \theta_k^2 \rangle = \sum_{m=1}^{k} \theta_m^2 = k\langle \theta^2 \rangle$$

$$\sqrt{\langle \theta^2 \rangle} = \frac{13.6 \text{ MeV}}{\beta c p}\, z\, \sqrt{x/X_0}\left[1 + 0.038 \ln(x/X_0)\right]$$

By averaging over many collisions and
integrating over b ...

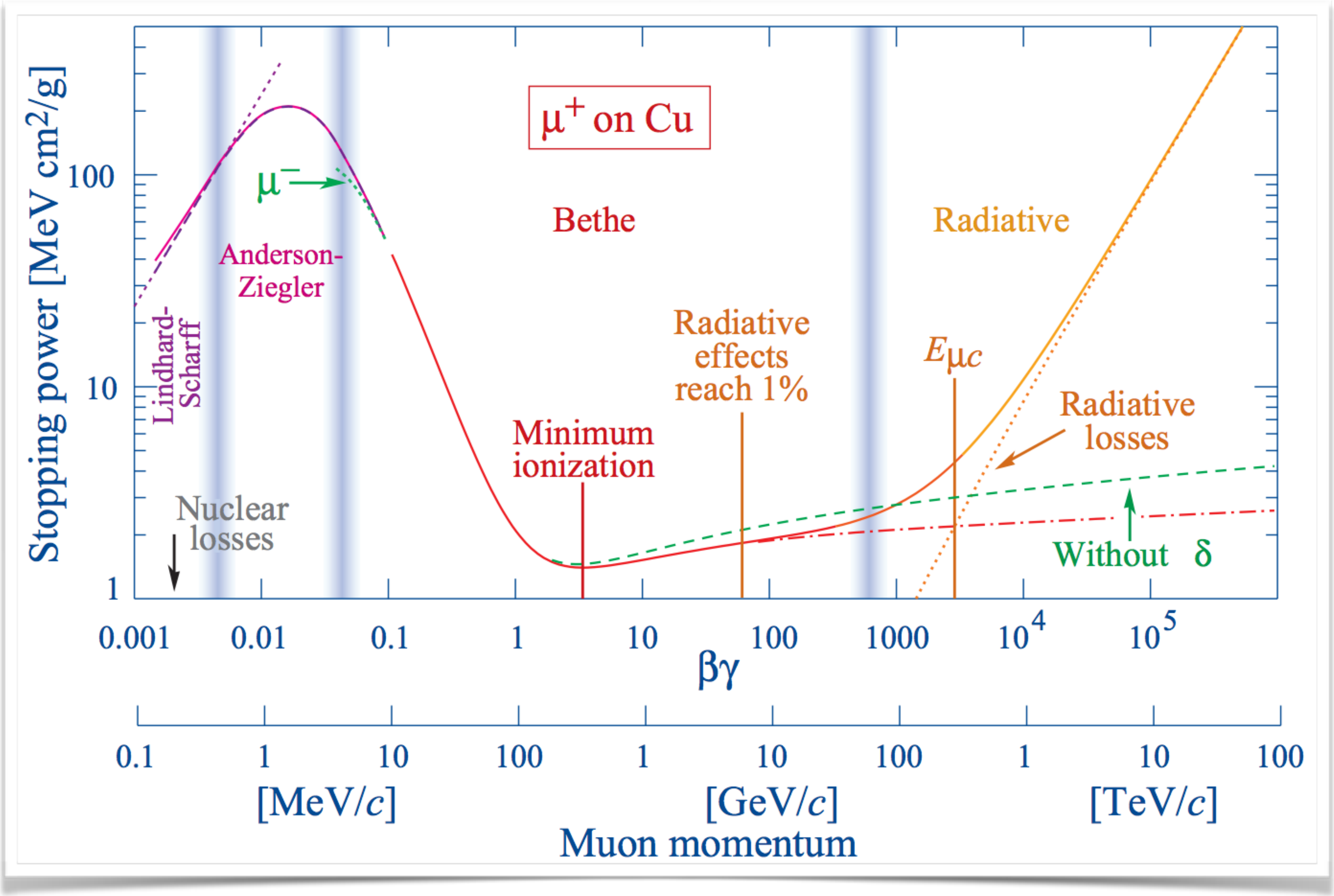
Radiation length

- How far will a particle travel into a material
- Radiation length characterizes material
  - Definition based on high-energy electrons and photons losing energy through Bremsstrahlung
  - Energy loss is exponential, $-\frac{dE}{dx} = \frac{E}{X_0}$

E = Energy of incoming particle
X0 = radiation length

- Radiation length typically given in g/cm^2
  - Multiply by density to get X_0 in cm
- See PDG tables at https://pdg.lbl.gov/2020/AtomicNuclearProperties/index.html
  - In lead: 0.56 cm, in water: 36 cm

Energy Loss for Muons in Cu



Energy Loss by Electrons

Electrons and positrons lose energy via ionization just as other charged particles.

Bethe-Bloch formula needs modification

- Incident and target electron have same mass
- Scattering of identical, indistinguishable particles

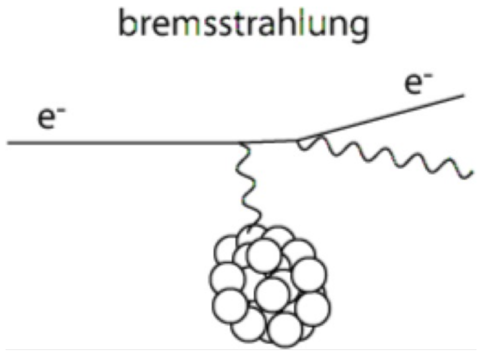
$$-\left\langle \frac{dE}{dx} \right\rangle_{\text{el.}} = K \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{m_e \beta^2 c^2 \gamma^2 T}{2I^2} + F(\gamma) \right]$$

However, their smaller masses mean they lose significant energy due to radiation as well:

- Bremsstrahlung
- Elastic scattering
- Pair production and electromagnetic showers

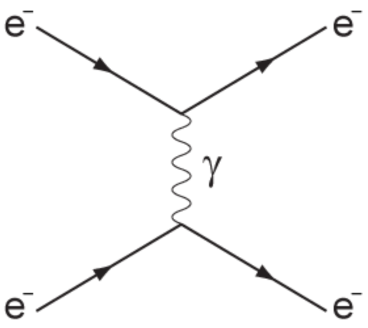
Bremsstrahlung, Elastic scattering

- **Bremsstrahlung:** Radiation of photons as particle is deflected near nucleus
  - Travels close to atomic nucleus
  - Feels coulomb potential, can be deflected and emit photon
  - Like synchrotron radiation

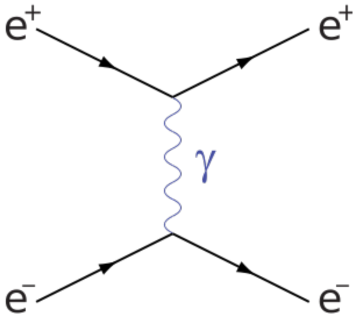


- **Møller (e^- - e^-) and Bhabha (e^+ - e^-) scattering**
  - Elastic scattering of electrons and electrons (or positrons)

Møller



Bhabha



(Cross section is slightly different for these due to the different electric charges)

Bremsstrahlung

Bremsstrahlung arises if particles are accelerated in Coulomb field of nucleus

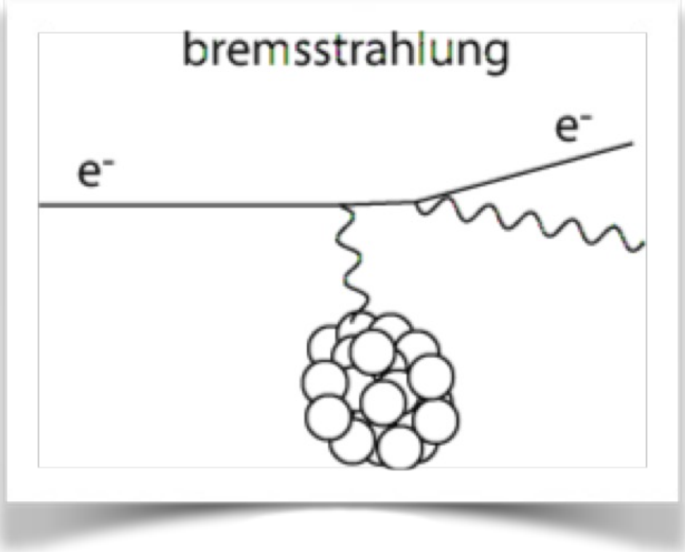
$$\frac{dE}{dx} = 4\alpha N_A \frac{z^2 Z^2}{A} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \ \ln \frac{183}{Z^{\frac{1}{3}}} \propto \frac{E}{m^2}$$

i.e. energy loss proportional to $1/m^2$

Consider electrons:

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \ \ln \frac{183}{Z^{\frac{1}{3}}}$$

$$\frac{dE}{dx} = \frac{E}{X_0} \qquad X_0 = \frac{A}{4\alpha N_A \ Z^2 r_e^2 \ \ln \frac{183}{Z^{\frac{1}{3}}}}$$



Radiation Length

Radiation length (X_0) :

The average thickness
of material that reduces the
mean energy of an electron (positron)
by a factor of *e:* $E(x=X_0) = E_0/e$

$$X_0 = \frac{A}{4\alpha N_A \, Z^2 r_e^2 \, \ln \frac{183}{Z^{\frac{1}{3}}}}$$

$$E = E_0 e^{-x/X_0}$$

Consider electrons:

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \, \ln \frac{183}{Z^{\frac{1}{3}}}$$

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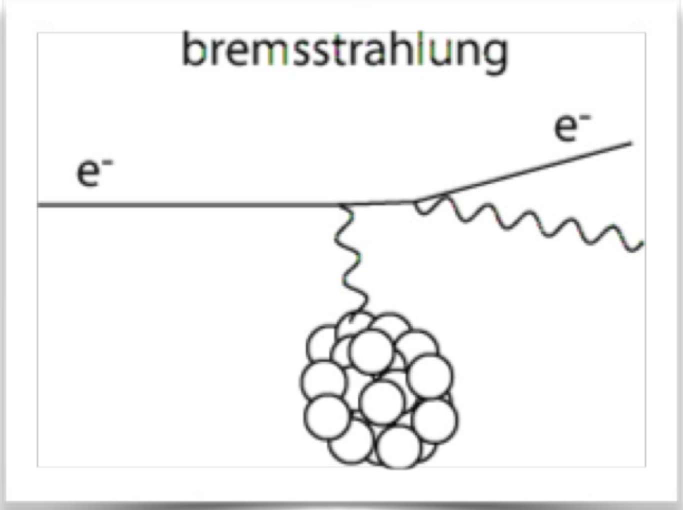
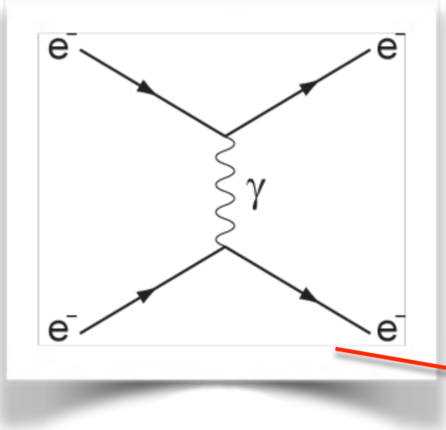
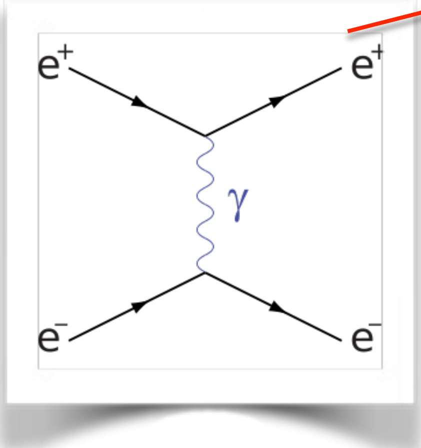


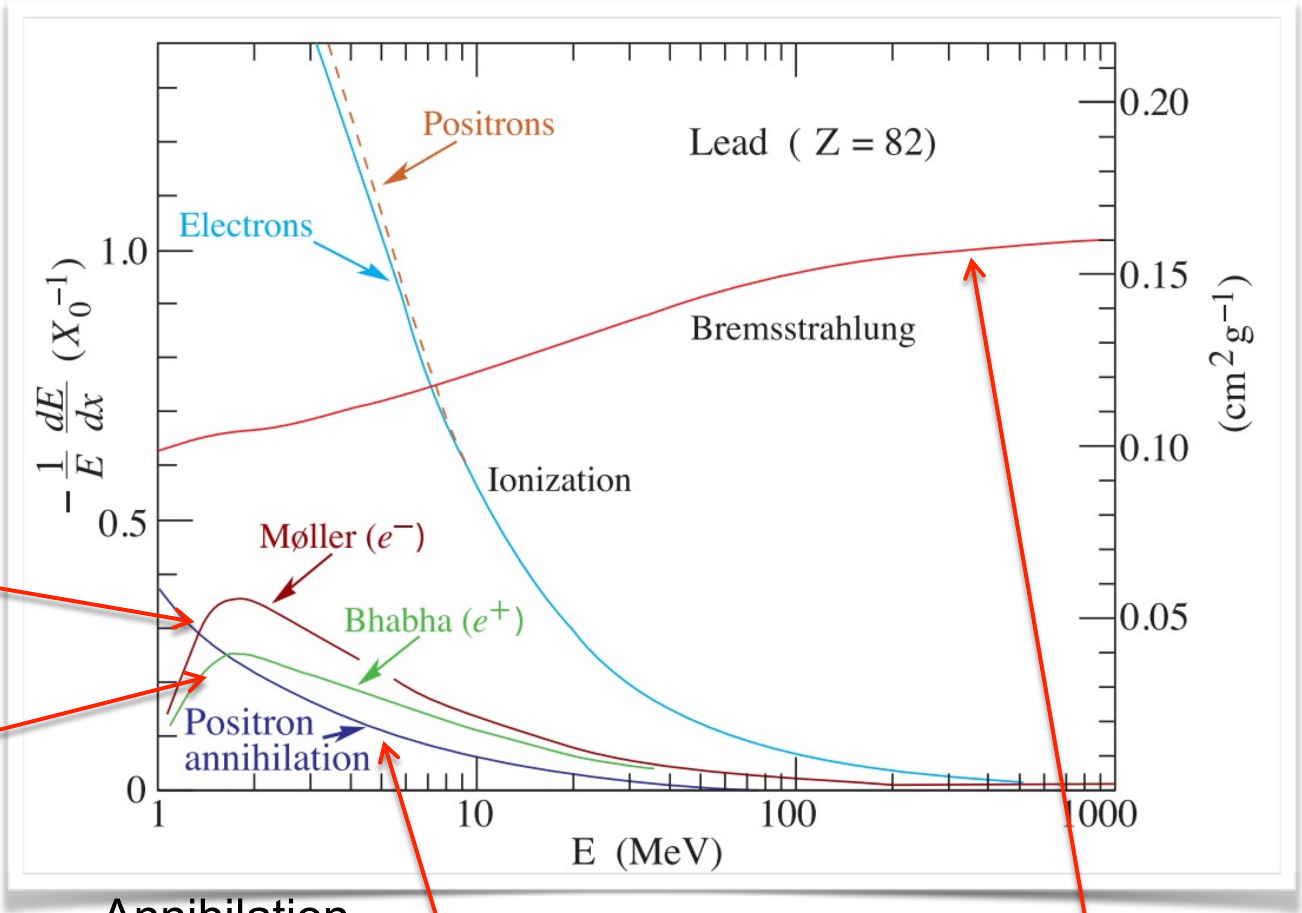
Radiation Energy Loss by Electrons

Møller



Bhabha





Annihilation