Homework1 - P493/P803

1) Which decay is allowed? List violated conservation rules.

Consider: Energy conservation, charge conservation, lepton # and flavor, baryon #, quark flavor.

a. $\gamma \rightarrow e^- + \mu^+$

Energy: $E^2 = p^2 + m^2$ → asume at rest $p = 0$

$0 \rightarrow m_e^2 + m_\mu^2$

→ not allowed based on conservation of energy ×

Charge: $0 \rightarrow -1 + 1 = 0$ ✓

lepton#: $0 \rightarrow -1 + 1 = 0$ ✓

lepton flavor: $0_e / 0_\mu \rightarrow 1_e + 1_\mu$ ×

baryon #: $0 \rightarrow 0 + 0$ ✓

quark flavor

b. $W^+ \rightarrow t + \bar{b}$

Energy: $m_W = 80\ GeV \leftrightarrow 174\ GeV + 4.3\ GeV$ ×

Charge: $+1 \rightarrow {}^2/_3 + {}^{-1}/_3 = {}^1/_3$ ×

Lepton #: $0 \rightarrow 0$ ✓ (flavor)

baryon #/ quark flavor: same generation

Not allowed based on charge and energy

c. $Z^0 \rightarrow \mu^+ + \mu^+$

Energy: $91190\ MeV \rightarrow 105\ MeV + 105\ MeV$ ✓

Charge: $0 \rightarrow 1 + 1 = 2$ ×

Lepton #: $0 \rightarrow -1 + -1 = -2$ ×

Lepton flavor: $0_\mu \rightarrow -1_\mu + -1_\mu = -2_\mu$ ×

Baryon#/ quark flavor: $0 \rightarrow 0$

Not allowed based on charge + lepton #.

1) d. $t \rightarrow W^+ + b$

Energy: 174 GeV $\rightarrow$ 80 + 4.3 GeV ✓
Charge: $2/3 \rightarrow 1 + -1/3 = 2/3$ ✓
Lepton # / flavor: $0 \rightarrow 0$
Baryon # / flavor: ✓
$\rightarrow$ This is allowed.

e. $W^- \rightarrow e^- + \nu_e$

Energy: 80 GeV $\rightarrow$ 0.511 MeV + ~0 ✓
Charge: $-1 \rightarrow -1 + 0$ ✓
lepton #: $0 \rightarrow 1 + 1 = 2$ X
lepton flavor: $0 \rightarrow 1e + 1e = 2$ X
baryon # / flavor: $0 \rightarrow 0 + 0$ ✓
Not allowed. Violates lepton # / flavor.

f. $\gamma \rightarrow \tau^- + \tau^+$

Energy: $0 \rightarrow 1.7\,GeV + 1.7\,GeV$ X
Charge: $0 \rightarrow -1 + 1 = 0$ ✓
lepton #: $0 \rightarrow -1 + 1 = 0$ ✓
lepton flavor: $0\tau \rightarrow 1\tau + -1\,\tau = 0\tau$ ✓
baryon # / flavor: $0 \rightarrow 0 + 0$ ✓
Not allowed because of energy conservation.

g. $b \rightarrow c + e^-$

Energy: 4.3 GeV $\rightarrow$ 1.2 GeV + 0.511 MeV ✓
Charge: $-1/3 \rightarrow 2/3 + -1 = -1/3$ ✓
lepton #: $0 \rightarrow 0 + 1$ X
lepton flavor: $0e \rightarrow 0e + 1e$ X
Baryon # / flavor: $\rightarrow$ crosses 2 generations
Not allowed due to violation of lepton #, flavor, baryon flavor / # (charmness, bottomness)

1) h. $\bar{b} \rightarrow Z^0 + \bar{s}$

Energy: 4.18 GeV $\rightarrow$ 91 GeV + 120 MeV ✗

Charge: $-1/3 \rightarrow 0 + -1/3$ ✓

Lepton #/ flavor: $0 \rightarrow 0$ ✓

Baryon #/ quark flavor: $-1\,b \rightarrow 0\,b$

$0\,s \rightarrow -1\,s$ ✗

Not allowed due to violation of energy conservation and also quark flavor violation (strangeness and bottomness), because it is not a charged-current weak interaction. ($Z^0 \rightarrow$ NC weak interaction).

2) Determine which processes are allowed (and if not, why not). Draw Feynman diagrams for those that are allowed.

a. $\Omega^- \rightarrow \bar{\Xi}^- + \pi^-$

$sss \rightarrow d\,ss + d\bar{u}$

Energy: 1672 MeV $\rightarrow$ 1321 MeV + 139 MeV = 1460 MeV ✓

Charge: $-1 \rightarrow -1 + -1 = -2$ ✗

Baryon #: $-1 \rightarrow -1 + 0 = -1$ ✓ $\rightarrow$ Note: There is no rule that meson # be conserved.

Lepton #: $0 \rightarrow 0 + 0$

Not allowed due to charge conservation.

2) b. $\Sigma^+ \rightarrow \pi^+ + \pi^0$

$uus \rightarrow u\bar{d} + \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$

Energy: 1189 MeV → 139 + 134 MeV ✓

Charge: 1 → 1 + 0 ✓

Baryon #: 1 → 0 + 0 ✗

Lepton #/flavor: 0 → 0 + 0 ✓

Not allowed because of baryon # conservation.

c. $\pi^0 \rightarrow \mu^+ + e^- + \bar{\nu}_e$

$\frac{u\bar{u} - d\bar{d}}{\sqrt{2}} \rightarrow \mu^+ + e^- + \bar{\nu}_e$

Energy: 134 MeV → 105 MeV + 0.51 MeV + 0 = 105.51 MeV ✓

Charge: 0 → 1 + −1 + 0 = 0 ✓

Baryon #: 0 → 0 ✓

Lepton #: 0 → −1 + 1 + −1 = −1 ✗

Lepton flavor: 0 e/μ → −1μ + 1e + −1e = −1μ ✗

Not allowed due to lepton # and flavor violation.

d. $\bar{D} \rightarrow K^+ + \pi^- + \pi^-$

$c\bar{d} \rightarrow u\bar{s} + d\bar{u} + d\bar{u}$

Energy: 1896 MeV → 493 MeV + 139 MeV + 139 MeV = 771 MeV ✓

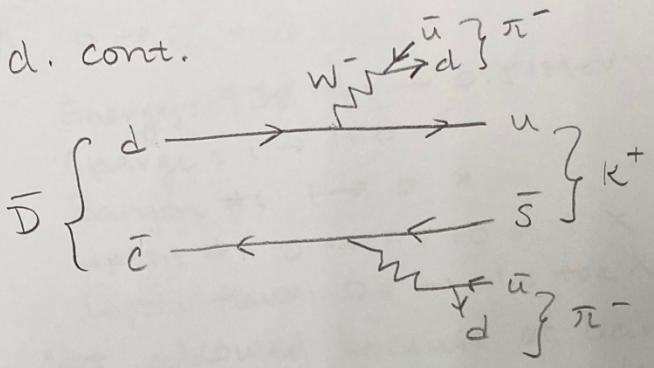
Charge: −1 → 1 + −1 + −1 = −1 ✓

Baryon #: 0 → 0 + 0 + 0 ✓

Lepton #/flavor: 0 → 0 ✓

Allowed. Is it a weak decay? Yes, because quark flavor is not conserved.

2) d. cont.



e. $p + \bar{p} \rightarrow \pi^- + \pi^+$

$uud + \bar{u}\bar{u}\bar{d} \rightarrow d\bar{u} + u\bar{d}$

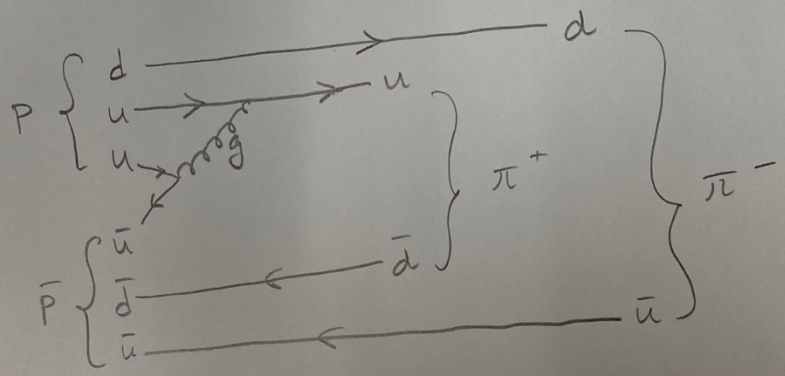
Energy: 938 MeV + 938 MeV → 139 MeV + 139 MeV ✓

Charge: $1 + -1 \rightarrow -1 + 1 \Rightarrow 0 \rightarrow 0$ ✓

Baryon #: $-1 + 1 \rightarrow 0 + 0 \Rightarrow 0 \rightarrow 0$ ✓

Lepton # / flavor: $0 \rightarrow 0$ ✓

Allowed. Could be strong because quark flavor is conserved.



2) f. $p \rightarrow e^+ + \gamma$

Energy: 938 MeV $\rightarrow$ 0.511MeV + 0 ✓

Charge: 1 $\rightarrow$ 1 + 0 ✓

Baryon #: 1 $\rightarrow$ 0 ✗

Lepton #: 0 $\rightarrow$ -1 + 0 = -1 ✗

Lepton flavor: 0e $\rightarrow$ -1e + 0e ✗

Not allowed because of baryon #, lepton # and lepton flavor are violated.

3)

Counting rate:

$$W = \sigma \cdot \rho \cdot \frac{N_A}{M_A} \cdot t \cdot I$$

Integrate over detector:

$$W = \int \frac{dW}{d\Omega} \cdot d\Omega$$

$$= \int \rho \frac{N_A}{M_A} I t \frac{d\sigma}{d\Omega} d\Omega$$

Where $\rho = 19.7\ g/cm^3$

N_A = Avogadro's # =

$M_A = 197$

$I = 10^5\ \alpha/s$

$t = 0.1\ cm$

And

$$\frac{d\sigma}{d\Omega} = \left(\frac{z Z e^2}{16 \pi \epsilon_0} \frac{1}{E_{kin}}\right)^2 \text{Cosec}^4 \frac{\theta}{2}$$

where $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \rightarrow \frac{e^2}{4\pi\epsilon_0} = \alpha \hbar c$, $\alpha = \frac{1}{137}$

$$= \left(\frac{z Z \alpha \hbar c}{4} \frac{1}{E_{kin}}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

$z_\alpha = 2$, $Z_{gold} = 79$, $\alpha = \frac{1}{137}$, $\hbar c = 197\ MeV \cdot fm$

$E_{kin} = 10\ MeV$, $\theta = \pi/4$

$$= \left(\frac{2 \cdot 79 \cdot 197}{4 \cdot 137 \cdot 10}\right)^2 \frac{1}{\sin^4(\pi/8)}$$

$$= 1505\ fm^2 = 1.51 \times 10^{-23}\ cm^2$$

$$d\Omega = \frac{S}{r^2} = \frac{1\,cm \times 1\,cm}{(100\,cm)^2} = 10^{-4}$$

$$\rightarrow \quad W = 1.51 \times 10^{-23}\,cm^2 \times 19.7\,g/cm^3 \times 6.023 \times 10^{23} \times \frac{1}{197}\,\frac{1}{g}$$

$$\times\ 10^5\,\alpha/s \quad \times\ 0.1\,cm \times 10^{-4}$$

$$= 1.51 \times 19.7 \times 6.023 \times \frac{1}{197} \times 10 \times 0.1 \cdot \frac{\cancel{cm^3}}{\cancel{cm^3}}\,\frac{\cancel{g}}{\cancel{g}}\,\frac{\alpha}{s}$$

$$\boxed{W = 0.91\ Hz}$$

4) a. Explain why $\Lambda^0 \to p + \pi^-$ is allowed but not $\Lambda^0 \to \pi^+ + \pi^-$.

$\Lambda^0 \to \pi^+ + \pi^-$ does not conserve baryon #.

In this case: $uds \to u\bar{d} + d\bar{u}$

→ not allowed → two mesons, no baryons.

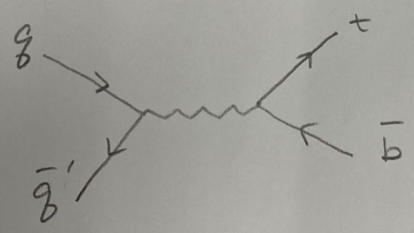
$\Lambda^0 \to p + \pi^-$ does conserve baryon # and is allowed

$uds \to uud + d\bar{u}$: $1 \to 1 + 0$ ✓

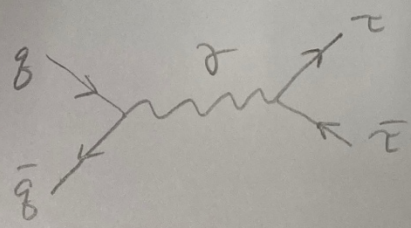
b. The following processes occur:

1. $W^+ \to t + \bar{b}$

→ if the W is virtual,



2. $\gamma \to \tau^+ + \tau^-$ → if the γ is virtual



4) b. cont.

3. $\bar{b} \rightarrow Z^0 + \bar{s}$

→ Allowed if Z is virtual and decays
→ penguin-style diagram.

Example diagram: $B^+ \rightarrow K^+ + \pi^0$