PHY 481 - Fall 2024
Homework 09

Due Saturday, November 9, 2024

Preface

Homework 09 helps you further investigate the multipole expansion.

1 The Beauty of the Multipole expansion

The multipole expansion is a very powerful approximation that arises in a number of different kinds of field theories. The beauty of it is that it can provide a simple approximate form for the field in question far from the sources that produce the field. Often, this is helpful when solving problems where you only care about the dominant contributions because the others only provide small corrections to the behavior.

In this problem, you will explore the the multipole expansion for the charge configuration shown below.

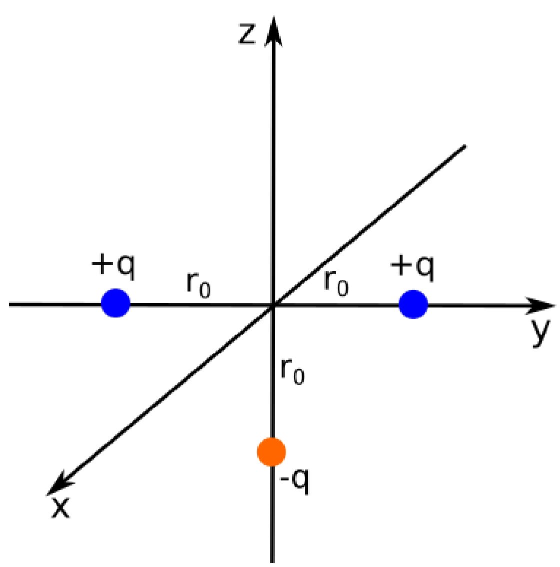

Figure 1: Three Charges

1. For the three charges shown above, determine the approximate potential at a distance far from the origin of coordinates. Keep only the two lowest non-vanishing orders of the expansion. *Notice that each is a distance r_0 from the origin.*

2. Explain how you know the two terms you find are the lowest non-vanishing terms for the potential.

3. Using your answer to part 1, find the approximate electric field produced by this system of charges far from the origin. Express your answer in spherical coordinates.

2 A Sphere of Charge

Consider a sphere of radius R with the volume charge density inside the sphere given by:

$$\rho(r, \theta) = \mu r \sin(\frac{3\theta}{2})$$

where μ is a known constant and θ is the usual polar angle in spherical coordinates.

1. Calculate the total charge, Q, on the sphere.

2. Calculate the dipole moment, $\vec{\mathbf{p}}$, of the sphere.

3. Use your results from Parts 1 and 2 to find $V(r, \theta)$ when you are far from the sphere ($r >> R$)

3 Python: Dipole Animation

The purpose of this problem is study the Electric Field due to a dipole at different distances from the dipole as the separation between the two charges changes. As a reminder, the electric field due to a point charge is:

$$\vec{E} = kq \left[\frac{\vec{r}_{\text{observation}} - \vec{r}_{\text{source}}}{|\vec{r}_{\text{observation}} - \vec{r}_{\text{source}}|^3} \right] \tag{1}$$

This exercise will also provide you with an opportunity to learn how to make an animated gif. The generation of the animation make take some time (a few minutes) depending on your computer so please be patient. The output is an animated gif file named "temp.gif" saved under the same folder as your Python file - this file can be opened in a web browser for viewing. If your computer is fast enough, then you can see the animation in real time within the notebook. Otherwise, viewing the animated gif file in a web browser or opening the saved gif file may be the better option. The template Jupiter notebook can be downloaded from D2L.

Problem to Solve

Calculate the Electric Fields: Put in the correct formulas for the x and y components of the electric field due to the positive charge (Epx and Epy) and the negative charge (Emx and Emy) as well as for the total electric field (Ex and Ey). In the template file, all are set to X and Y as placeholders.

Upload on D2L both the Jupyter notebook file and the animated gif file.