Problem 1

Kittel & Kroemer, Chapter 9, problem 1 [Thermal expansion near absolute zero.]

(a): 1 point

Prove the three Maxwell relations

$$(\partial V/\partial \tau)_p = -(\partial \sigma/\partial p)_\tau,$$
$$(\partial V/\partial N)_p = +(\partial \mu/\partial p)_N$$
$$(\partial \mu/\partial \tau)_N = -(\partial \sigma/\partial N)_\tau.$$

(b): 1 point

Show with the help of the first Maxwell relation above and the third law of thermodynamics that the volume coefficient of thermal expansion

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial \tau}\right)_p$$

approaches zero as $\tau \to 0$.

Problem 2

Kittel & Kroemer, Chapter 9, problem 2 [Thermal ionization of hydrogen.]

Consider the formation of atomic hydrogen in the reaction e + H $\leftrightarrow$ H, where e is an electron, as the adsorption of an electron on a proton H^+.

(a): 1 point

Show that the equilibrium concentrations of the reactants satisfy the relation

$$[\mathrm{e}][\mathrm{H}^+]/[\mathrm{H}] \simeq n_Q e^{-I/\tau},$$

where I is the energy required to ionize atomic hydrogen, and $n_Q \equiv (m\tau/2\pi\hbar^2)^{3/2}$ refers to the electron. Neglect the spins of the particles; this assumption does not affect the final result. The result is known as the Saha equation. If all the electrons and protons arise from the ionization of hydrogen atoms, then the concentration of protons is equal to that of the electrons, and the electron concentration is given by

$$[\mathrm{e}] = [\mathrm{H}]^{1/2} n_Q^{1/2} e^{-I/2\tau}.$$

A similar problem arises in semiconductor physics in connection with the thermal ionization of impurity atoms that are donors of electrons.

(b): 1 point

Let [H(exc)] denote the equilibrium concentration of H atoms in the first excited electronic state, which is $\frac{3}{4}I$ above the ground state. Compare [H(exc)]with [e] for conditions at the surface of the Sun, with

$[\mathrm{H}] \sim 10^{23}\mathrm{cm}^{-3}$ and $T \sim 5000$ K.

Problem 3

Kittel & Kroemer, Chapter 9, problem 3 [Ionization of donor impurities in semiconductors.]: 2 points

A pentavalent impurity (called a donor) introduced in place of a tetravalent silicon atom in crystalline silicon acts like a hydrogen atom in free space, but with e^2/ϵ playing the role of e^2 and an effective mass m^* playing the role of the electron mass m in the description of the ionization energy and radius of the ground state of the impurity atom, and also for the free electron. For silicon the dielectric constant $\epsilon = 11.7$ and, approximately, $m^* = 0.3m$. If there are 10^{17} donors per cm^3, estimate the concentration of conduction electrons at 100 K.

Problem 4

Kittel & Kroemer, Chapter 9, problem 4 [Biopolymer growth.]

Consider the chemical equilibrium of a solution of linear polymers made up of identical units. The basic reaction step is monomer + Nmer = $(N+1)$mer. Let K_N denote the equilibrium constant for this reaction.

(a): 1 point

Show from the law of mass action that the concentrations $[\cdots]$ satisfy

$$[N+1] = [1]^{N+1}/K_1K_2K_3\cdots K_N$$

(b): 1 point

Show from the theory of reactions that for ideal gas conditions (an ideal solution):

$$K_N = \frac{n_Q(N)n_Q(1)}{n_Q(N+1)}e^{(F_{N+1}-F_N-F_1)/\tau}.$$

Here

$$n_Q(N) = (2\pi\hbar^2/M_N\tau)^{-3/2},$$

where M_N is the mass of the Nmer molecule, and F_N is the free energy of one Nmer molecule.

(c): 1 point

Assume $N \gg 1$, so that $n_Q(N) \simeq n_Q(N+1)$. Find the concentration ratio $[N+1]/[N]$ at room temperature if there is zero free energy change in the basic reaction step: that is, if $\Delta F = F_{N+1} - F_N - F_1 = 0$. Assume $[1] = 10^{20}\ \mathrm{cm}^{-3}$, as for amino acid molecules in a bacterial cell. The molecular weight of the monomer is 200

Problem 4 continued on next page. . .

(d): 1 point

Show that for the reaction to go in the direction of long molecules we need $\Delta F < -0.4$ eV, approximately. This condition is not satisfied in Nature, but an ingenious pathway is followed that simulates the condition.

Problem 5

Kittel & Kroemer, Chapter 9, problem 5 [Particle-antiparticle equilibrium.]

(a): 1 point

Find a quantitative expression for the thermal equilibrium concentration $n = n^+ = n^-$ in the particle-antiparticle reaction $A^+ + A^- = 0$. The reactants may be electrons and positrons: protons and antiprotons; or electrons and holes in a semiconductor. Let the mass of either particle be M; neglect the spins of the particles. The minimum energy release when A^+ combines with A^- is Δ. Take the zero of the energy scale as the energy with no particles present.

(b): 1 point

Estimate n in cm^{-3} for an electron (or a hole) in a semiconductor $T = 300$ K with a Δ such that $\Delta/\tau = 20$. The hole is viewed as the antiparticle to the electron. Assume that the electron concentration is equal to the hole concentration; assume also that the particles are in the classical regime.

(c): 1 point

Correct the result of (a) to let each particle have a spin 1/2. Particles that have antiparticles are usually fermions with spins of 1/2.