Homework 6
Justify all your answers
due on Fr 11/15/24 at 11:30AM in A236WH

Exercise 1. *Let $C \subseteq \mathbb{F}_2^5$ be the linear code with generating matrix*

$$E := \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(a) *List all elements of C.*

(b) *Determine the minimum distance of C.*

(c) *Is C 1-error-correcting?*

Exercise 2. *Let $C \subseteq \mathbb{F}_2^5$ be the linear code with check matrix*

$$H := \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

(a) *List all elements of C.*

(b) *Determine the minimum distance of C.*

(c) *Is C 1-error-correcting?*

Exercise 3. *Consider the binary matrix*

$$E_0 := \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

and put $C := \mathrm{Col}(E_0)$.

Find a permutation D of C, a generating matrix E of D in standard form, and a check matrix H of D in standard form.

Exercise 4. *Let C consists of all $x = x_1 x_2 \dots x_8 \in \mathbb{F}_2^8$ such that*

$$\begin{aligned} x_1 &= x_2 + x_3 + x_4 \\ x_2 &= x_5 \\ x_3 &= x_6 \\ x_4 &= x_7 \\ x_8 &= x_5 + x_6 + x_7 \end{aligned}$$

(a) *Find a check matrix for C.*

(b) *Find a generating matrix for C.*

(c) *Determine the information rate and minimum distance of C.*

Exercise 5. (a) *Find a check matrix for a* 1*-error correcting binary linear code of length* 16 *and dimension* 11.

(b) *Find a check matrix for a Hamming code of length* 15.

Exercise 6. *Let* C *be the binary linear code with check matrix*

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

For $z = 110111011, 111010011$ *and* 101001100*: Does there exist* $a \in C$ *with* $d(a, z) \leq 1$*? If yes, find all such* a.

Exercise 7. *Let* C *be the binary linear code with check matrix*

$$H = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

(a) *Find a syndrom look-up table for* C *with respect to* H.

(b) *Let* σ *be the decision rule corresponding to the syndrome look-up table in (a). Compute*

$$\sigma(011110) \quad \textit{and} \quad \sigma(110011).$$

Exercise 8. *Which of the following binary codes are cyclic:*

(a) $C_1 = \{0000, 1100, 0011, 1111\}$.

(b) $C_2 = \{000000, 010101, 101010, 111111\}$.

(c) $C_3 = \{0000, 0111, 1011, 1101, 1110, 1000, 0100, 0010, 0001\}$.