Physics 410 -- Useful Formulas for quiz #6

I. Chemical potential: $\mu_{total} = \mu_{int} + \mu_{ext}$, where μ_{int} depends on the particle density (e.g. through the Ideal Gas relation) and μ_{ext} is a potential energy per particle added to the system).

Diffusive equilibrium between systems A and B occurs when $\mu_{total}^{A} = \mu_{total}^{B}$

Chemical potential of ideal gas in 3D: $\mu = \tau \ln\left(\frac{n}{n_Q}\right)$, where n is the concentration of particles and $n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{\frac{3}{2}}$ is the quantum concentration.

II. Grand canonical ensemble: independent variables τ, V, μ

Grand Partition function, also called the "Gibbs sum": $\mathcal{Z} = \sum_{N=0}^{\infty} \sum_{s(N)} e^{[(N\mu - \varepsilon_s)/\tau]}$,

Grand canonical distribution function (probability): $P(N,\varepsilon) = \dfrac{e^{[(N\mu-\varepsilon)/\tau]}}{\mathcal{Z}}$

The numerator of $P(N,\varepsilon)$ is called the "Gibbs factor"

Mean *total* number of particles: $\langle N \rangle = \lambda \dfrac{d(\ln \mathcal{Z})}{d\lambda}$, where $\lambda = e^{\mu/\tau}$ is the absolute activity.

III. Ideal gas: classical regime of Fermi-Dirac and Bose-Einstein distributions

Ideal gas distribution function: $f(\epsilon) = e^{\frac{\mu-\epsilon}{\tau}}$.

IV. Fermi-Dirac distribution function: $f(\epsilon) = \frac{1}{e^{\frac{\epsilon-\mu}{\tau}}+1}$

At $\tau = 0$, $f(\epsilon) = 1$ for $\epsilon < \epsilon_F$ and $f(\epsilon) = 0$ for $\epsilon > \epsilon_F$

Thermal averages via distribution function:
$\langle X \rangle = \sum_{\text{orbitals}} X f(\epsilon) = \int_0^\infty X\, D(\epsilon)\, f(\epsilon)\, d\epsilon$, where $D(\epsilon)$ is the density of states

Total number of particles: $N = \sum_{\text{orbitals}} f(\epsilon) = \int_0^\infty D(\epsilon)\, f(\epsilon)\, d\epsilon$

V. Thermodynamics

a. $W = \int p\, dV$, $Q = \int \tau\, d\sigma$, $\Delta U = Q - W$

b. Heat engine efficiency: $\eta = \frac{W}{Q_h}$; Carnot efficiency $\eta_C = \frac{\tau_h - \tau_l}{\tau_h}$

c. Coefficient of refrigerator performance: $\gamma = \frac{Q_l}{W}$,
Carnot coefficient of refrigerator performance: $\gamma_C = \frac{\tau_l}{\tau_h - \tau_l}$

d. Ideal gas in 3D: $pV = N\tau$, $U = \frac{3}{2}N\tau$, $\sigma = N(\ln n_Q/n)$

e. Photon gas in 3D: $p = A\tau^4$, $U = 3AV\tau^4$ $\sigma = 4AV\tau^3$, where $A = \frac{\pi^2}{45c^3\hbar^3}$

VI. Chemical reactions

a. Equation for chemical reaction: $\sum_i \nu_i A_i = 0$, where A_i are the particle species

b. Law of mass action: $\prod_i n_i^{\nu_i} = K(\tau)$, where n_i are the species concentrations

c. Equilibrium constant for ideal gas: $K(\tau) = \prod_i n_{Q,i}^{\nu_i}\, e^{-\frac{\nu_i F_{int,i}}{\tau}}$, where $n_{Q,i} = \left(\frac{M_i \tau}{2\pi\hbar^2}\right)^{\frac{3}{2}}$