Problem 1

Adapted from Kittel & Kroemer, Chapter 7, problem 1 [Density of orbitals in 1D and 2D.]

(a): 1 point

Find the density of states for a free electron in a one-dimensional box of length L.

(b): 1 point

Find the total ground-state energy of a gas of non-interacting electrons in 1D.

(c): 1 point

Find the density of states for a free electron in a two-dimensional square of side length L.

(d): 1 point

Find the total ground-state energy of a gas of non-interacting electrons in 2D.

Problem 2

Kittel & Kroemer, Chapter 7, problem 2 [Energy of relativistic Fermi gas.]

For electrons with an energy $\epsilon \gg mc^2$, where m is the rest mass of the electron, the energy is given by $\epsilon \simeq pc$, where p is the momentum. For electrons in a cube of volume $V = L^3$ the momentum is of the form $(\pi\hbar/L)$, multiplied by $(n_x^2 + n_y^2 + n_z^2)^{1/2}$, exactly as for the nonrelativistic limit.

(a): 1 point

Show that in this extreme relativistic limit the Fermi energy of a gas of N electrons is given by

$$\epsilon_F = \hbar\pi c(3n/\pi)^{1/3},$$

where $n = N/V$.

(b): 1 point

Show that the total energy of the ground state of the gas is

$$U_0 = \frac{3}{4}N\epsilon_F.$$

Problem 3

Kittel & Kroemer, Chapter 7, problem 3 [Pressure and entropy of degenerate Fermi gas.]

(a): 1 point

Show that a Fermi electron gas in the ground state exerts a pressure

$$p = \frac{(3\pi^2)^{2/3}}{5}\frac{\hbar^2}{m}\left(\frac{N}{V}\right)^{5/3}.$$

In a uniform decrease of the volume of a cube every orbital has its energy raised: The energy of an orbital is proportional to $1/L^2$ or to $1/V^{2/3}$.

(b): 1 point

Find an expression for the entropy of a Fermi electron gas in the region $\tau \ll \epsilon_F$. Notice that $\sigma \to 0$ as $\tau \to 0$.

Problem 4

Kittel & Kroemer, Chapter 7, problem 5 [Liquid ^{3}He as a Fermi gas]: 4 points

Read problem statement in textbook

Problem 5

Kittel & Kroemer, Chapter 7, problem 11 [Fluctuations in a Fermi gas.]: 2 points

Show for a single orbital of a fermion system that

$$\langle(\Delta N)^2\rangle = \langle N\rangle(1 - \langle N\rangle),$$

if $\langle N\rangle$ is the average number of fermions in that orbital. Notice that the fluctuation vanishes for orbitals with energies deep enough below the Fermi energy so that $\langle N\rangle = 1$. By definition, $\Delta N \equiv N - \langle N\rangle$.

Problem 6

Adapted from: Kittel & Kroemer, Chapter 7, problem 13 [Chemical potential versus concentration.]: 2 points

Sketch carefully the chemical potential versus the number of particles for a Fermi gas in volume V at temperature τ. Include both classical and quantum regimes.