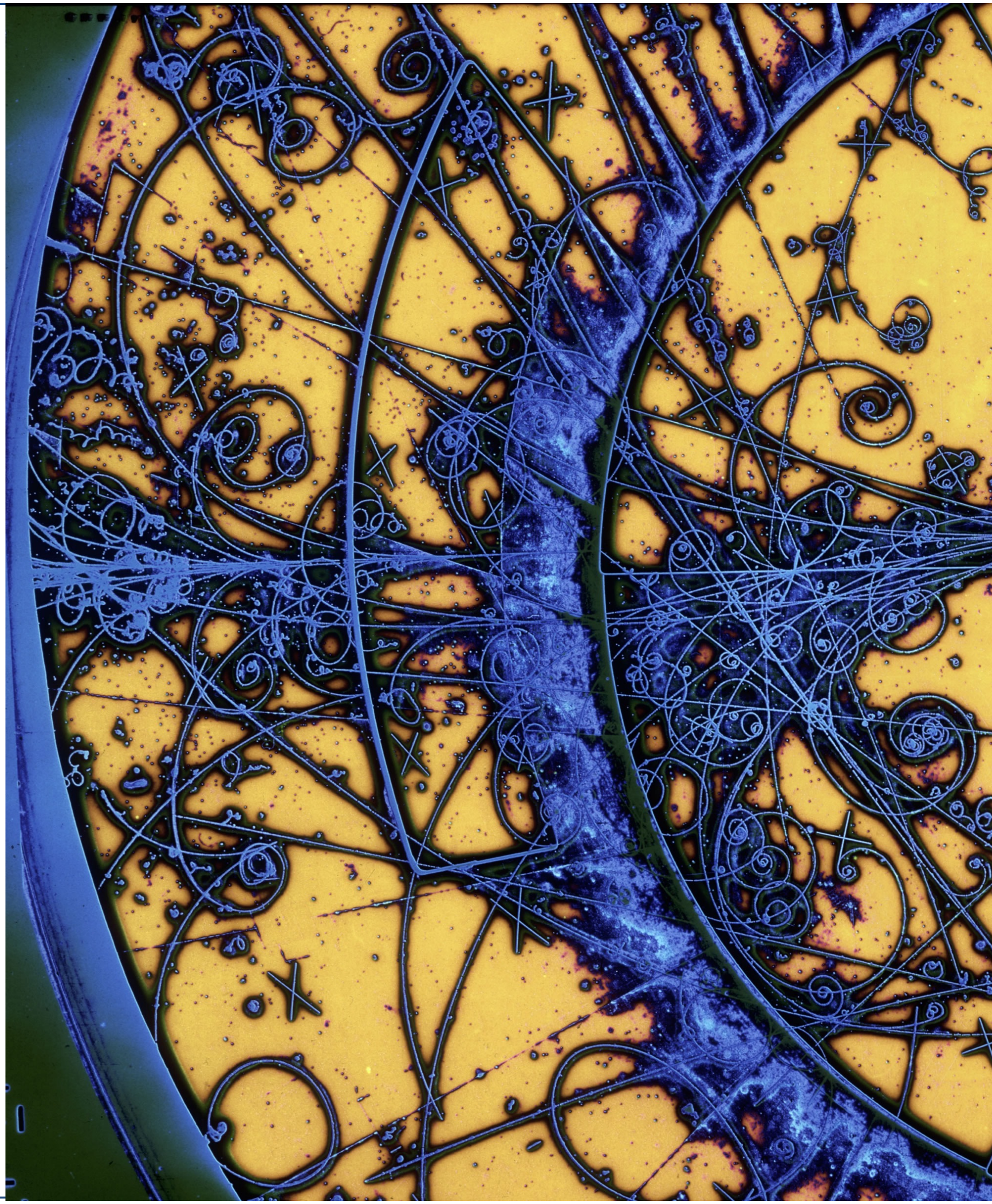
Announcements

1. First homework assignment will be posted by end of the day today.

2. Next Monday is MLK day; there will be no class. We will next meet on Wednesday, January 22

3. In class quizzes (open note/book) will start next Friday

4. Reminder: reading for this week is Chapter 1 of Griffiths



Grading Scale

4.0: >90%
3.5: >85%
3.0: >80%
2.5: >75%
2.0: >70%
1.5: >65%
1.0: >60%
0.0: <60%

Some Historical Context

It has taken us some time to discover all of these particles!

A few important milestones:

- Discovery of the electron by J.J. Thompson (1897)
- Discovery of the nucleus by Rutherford (1911, also Geiger & Marsden)
- Discovery of the proton by Rutherford (1917)
- Discovery of the electron (anti-)neutrino (1956)
- Discovery of the quark (1968)
- Discovery of the W and Z bosons (1983)
- Discovery of the tau neutrino (2001)
- Discovery of the Higgs boson (2012)

A little about these today

Radiation

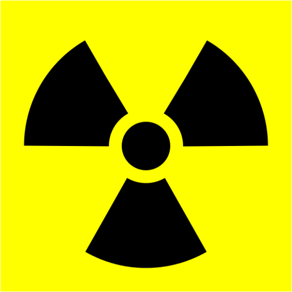
Early on, we only really knew that certain materials emitted what we now call “Radiation”.

No one really knew what this consisted of and it was all referred to as “rays”.

- Discovered first by Henri Becquerel in 1896 in Uranium
- Subsequently Marie & Pierre Curie in Thorium.

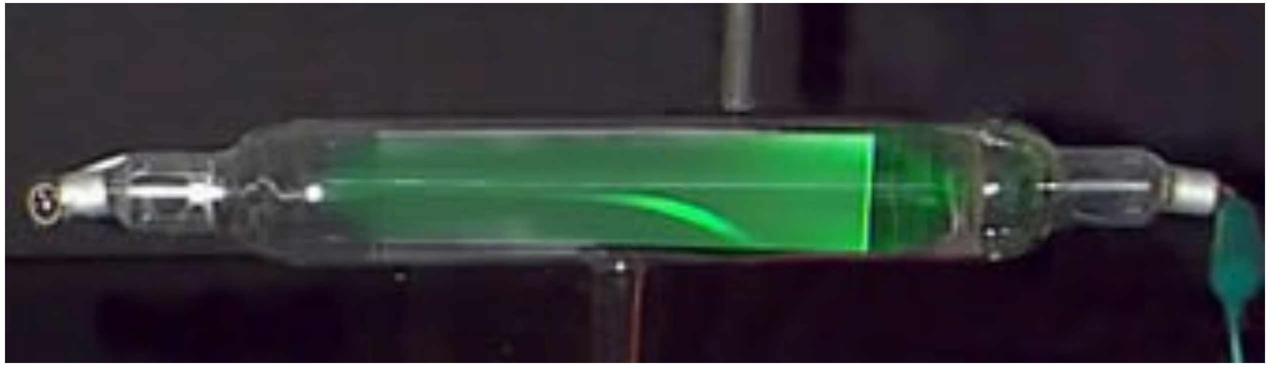
You can guess the order of discovery via the Greek alphabet:

- <u>Alpha rays (1899):</u> Rutherford. You know them as alpha particles or Helium-4 ions.
- <u>Beta rays (1899/1900):</u> Rutherford. You know them as electrons and positrons.
- <u>Gamma rays (1900-1903):</u> Villard & Rutherford. Didn’t realize at the time they were photons.



Cathode Ray Puzzle

Mystery of why these tubes glow:

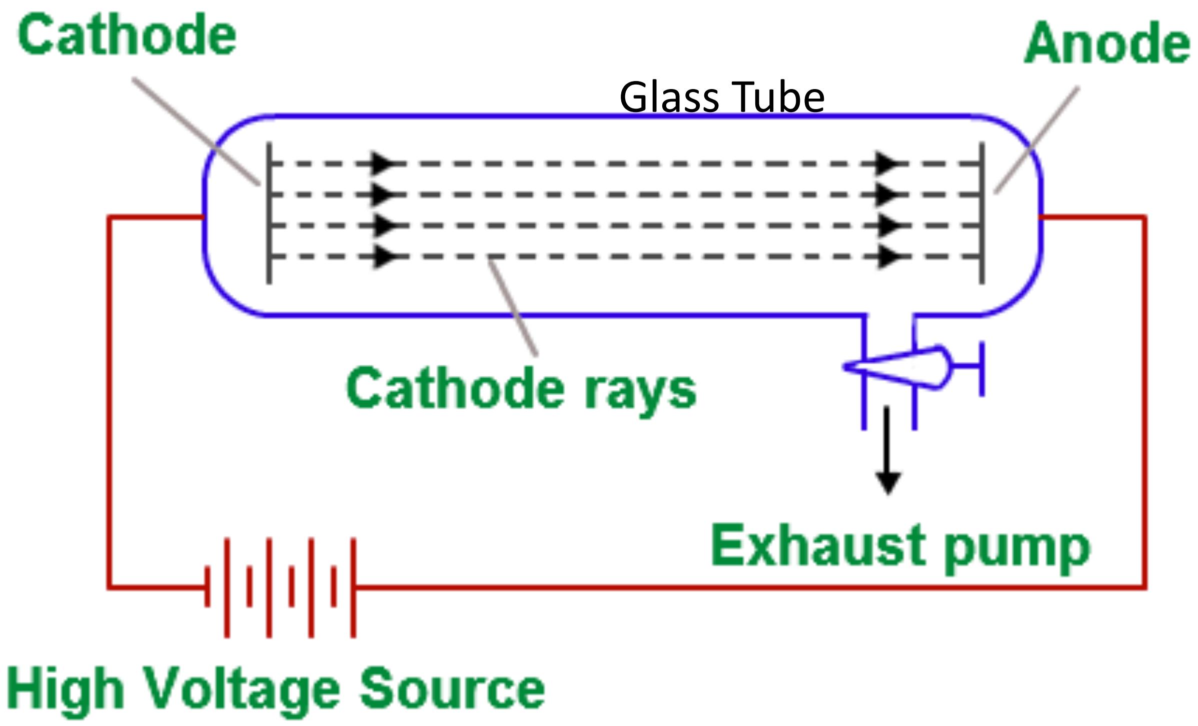


Waves travelling in ether?
Ordinary atoms?

"The most diverse opinions are held as to these rays... It would seem at first sight that it ought not to be difficult to discriminate between views so different, yet experience shows that this is not the case..." -- J.J. Thomson, *"Cathode Rays"* (1897).

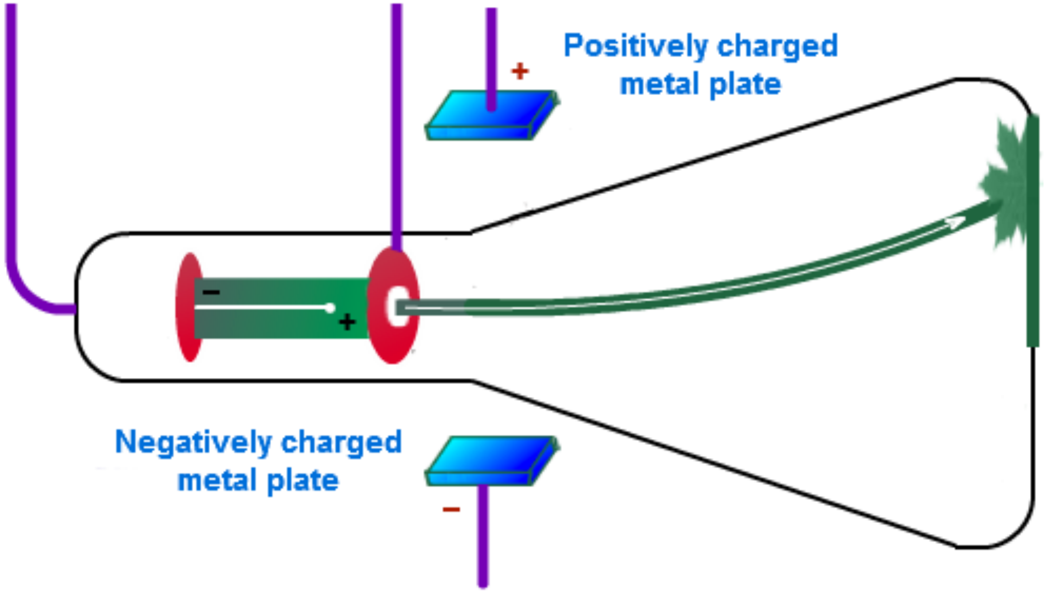
https://history.aip.org/exhibits/electron/jjhome.htm

What is a cathode ray tube?



Glass tube with wires on either end. Apply voltage. Evacuate tube, or add some gas (for florescence).
HV causes ”cathode rays” to be ejected from the cathode and accelerated in electric field towards anode.

Discovery of the Electron

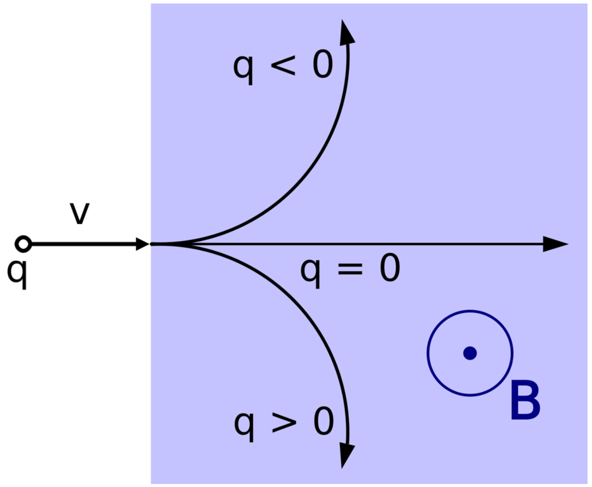


Early experiments showed:

Cathode rays bend in magnetic fields

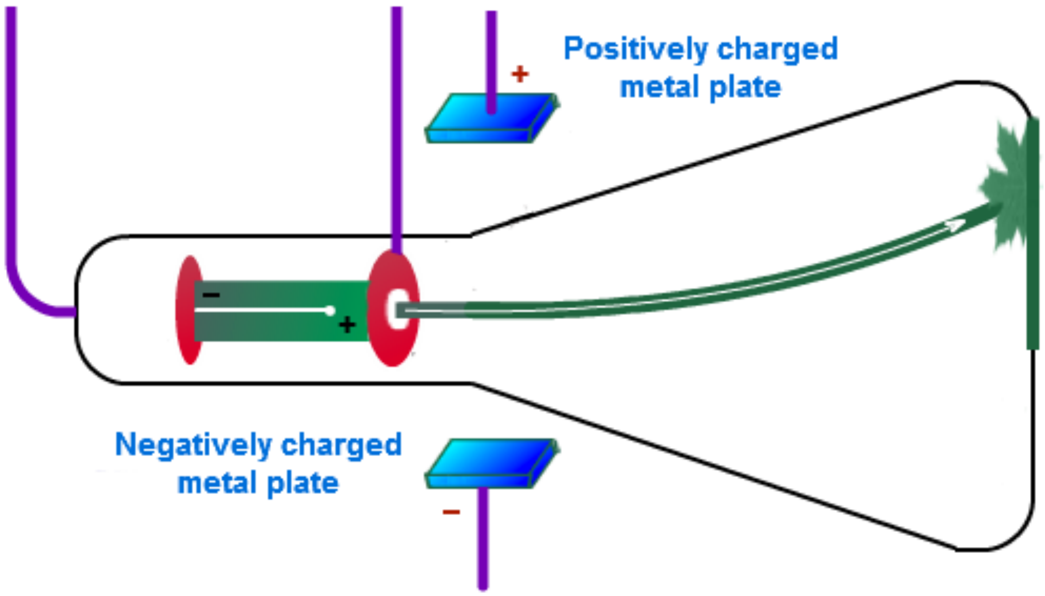
Lorentz Force

$$\vec{F} = q\vec{v} \times \vec{B}$$



Magnetic field out of the board

Discovery of the Electron



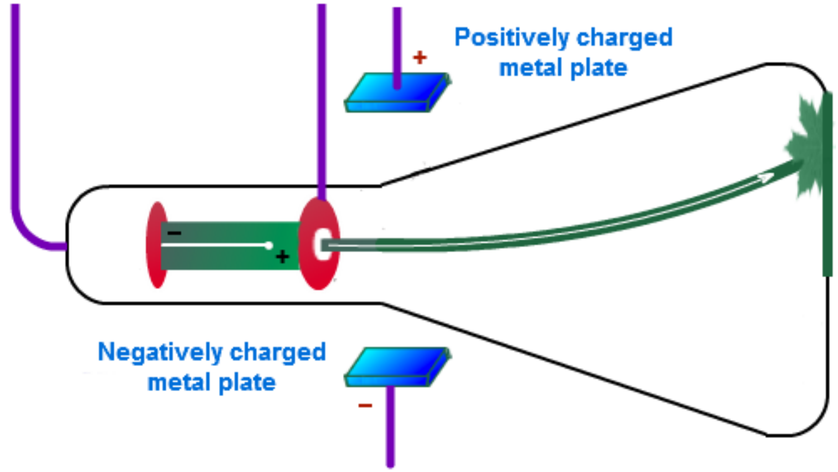
Cathode rays also bend in electric fields:

-This means they are electrically charged
not possible until the tube was fully
evacuated (gas acting as conductor)

Discovery of the Electron

With these tools have learned that:

1. Cathode rays bend in magnetic fields
2. Cathode rays bend in electric fields



<u>An experiment:</u> Place E and B fields at right angles (bend "cathode rays" in opposite directions), adjust cathode/anode voltage until there is no deflection. Bend "cathode rays" in opposite directions.

- Measures charge-to-mass ratio of the Cathode rays

$$\vec{F_B} = q\,\vec{v} \times \vec{B}$$

$$\vec{F_E} = q\vec{E}$$

Discovery of the Electron

$$\vec{F_B} = q\,\vec{v} \times \vec{B}$$

$$\vec{F_E} = q\vec{E}$$

1) Forces at right angles, so if no deflection $F_B = F_E$:

$$\longrightarrow \qquad \vec{v} = \vec{E}/\vec{B}$$

2) Lorentz force only (turn off E field): particles bend in magnetic field

$$F = mv^2/r = qvB$$

$$q/m = \frac{v}{rB} = \frac{E}{rB^2}$$

q/m = fundamental quantity for these “cathode rays”. Does not depend on experimental setup.

Discovery of the Electron

J.J. Thomson

- Cathode rays are particles with a specific q/m
- q/m is ~2000 times larger than the Hydrogen ion, the lightest known element
- Thus Cathode rays are particles, not ions!

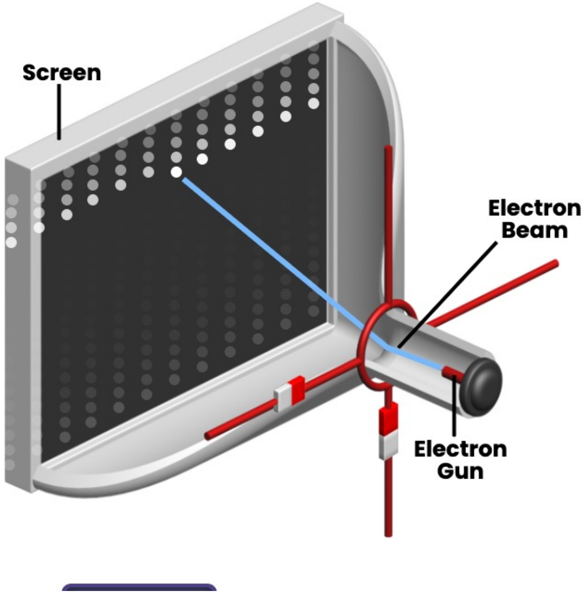
Becquerel

- Applied Thomson's q/m method to Beta rays
- They have the same q/m ratio as Cathode rays
- They are the same!

Discovery of the Electron

This seems (and is!) very fundamental, but the methodologies are not that far in our past.

Cathode Ray Tube



https://nationalmaglab.org/magnet-academy/watch-play/interactive-tutorials/cathode-ray-tube-television/

Most TVs/ computer monitors up until mid-2000s

Change path of particle by changing field
Change path very quickly and draw image (~50Hz)

Discovery of the Nucleus: Rutherford Experiment

Measurement of α scattering through materials

Top figure (Fig D): Schematic view from above, showing radioactive alpha source (R), foil scattering target (F) and microscope "detector" with a fluorescent screen (M): see flash of light in the microscope

Bottom figure (Fig E): Same thing from the side.

Move the microscope to look at the number of alpha scatters at different angles

Fig. (D) Scattering of α rays by an atom

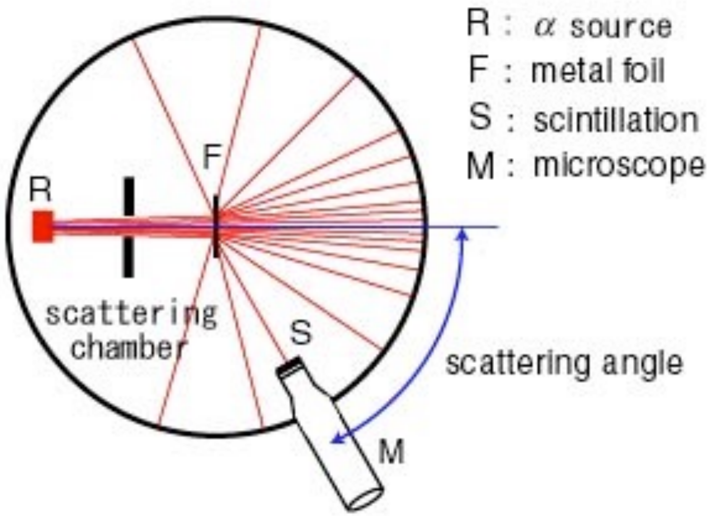
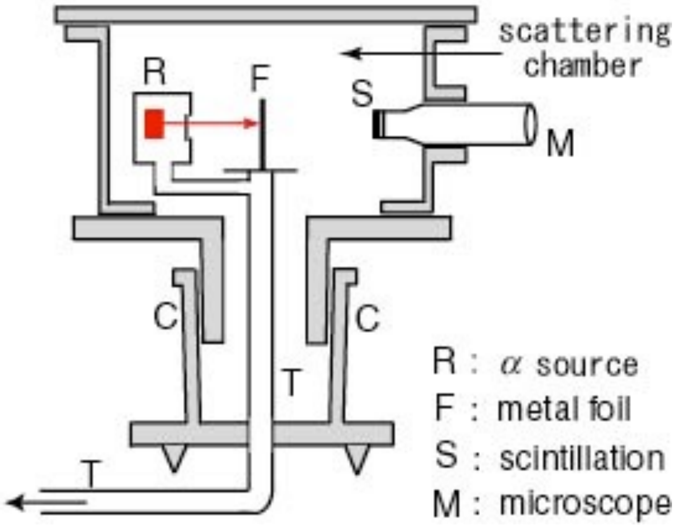
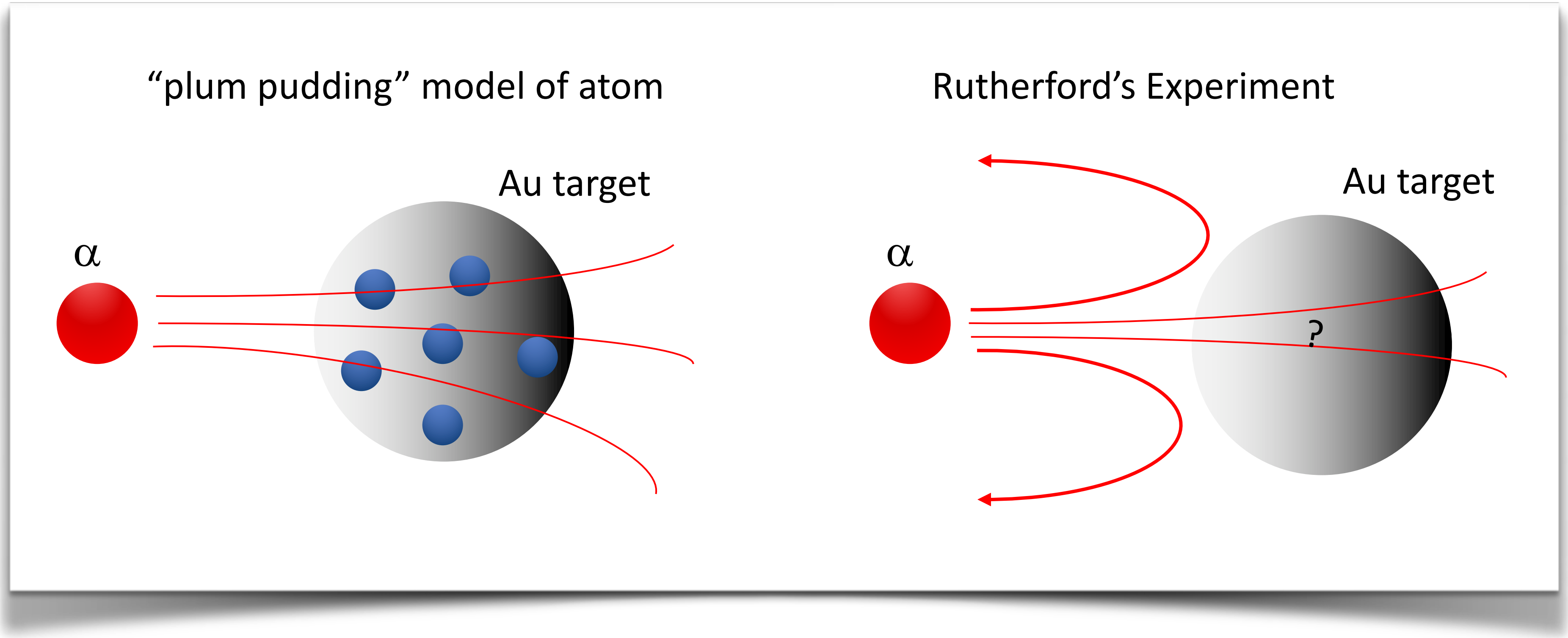


Fig. (E) Setting of the experiment



Rutherford Scattering

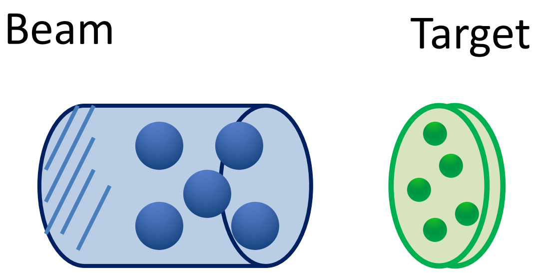


"...it was quite the most incredible event that has ever happened to me in my life.
It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you..." Rutherford, 1936.

Reaction Rate

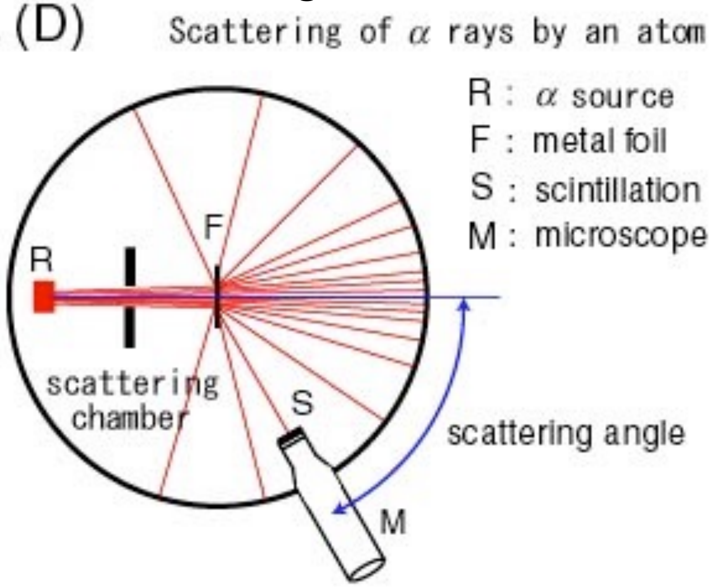
Reaction rate (W) is proportional to:

1. Number of particles in the beam
2. Number of targets
3. Probability of a collision between beam and target



Beam = R
Target = F

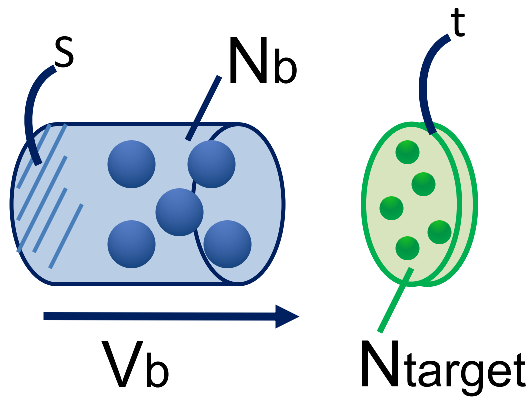
Fig. (D)



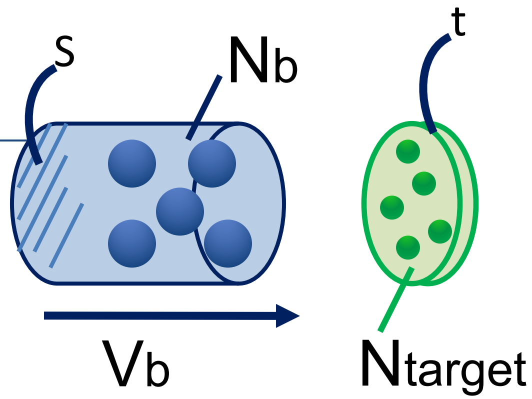
Reaction Rate

Reaction rate (W) is proportional to:

1. Number of particles in the beam
   - Beam flux J = beam rate / unit area
   - $J = N_b * V_b$
     - N_b =number density of beam particles
     - V_b = beam velocity
   - Beam intensity I = *J* * S (S: beam area)
2. Number of targets: N_t
3. Probability of a collision between beam and target
   - Cross section (σ)



Cross Section



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   - Beam intensity I = *J* S (S: beam area)
2. Number of targets:
3. Probability of a collision between beam and target: Cross section (σ)

$$
\begin{aligned}
W &= \sigma \cdot N_{\text{target}} \cdot J \\
&= \sigma \cdot N_{\text{target}} \cdot I/S \\
&= \sigma \cdot (n_t \cdot V) \cdot I/S \\
&= \sigma \cdot (n_t \cdot t) \cdot I \\
&= \sigma \cdot \rho \cdot (N_A/M_A) \cdot t \cdot I
\end{aligned}
$$

n_t: number of target particles/unit volume, V: Volume

t: thickness of target

ρ: target density, N_A: Avogadro’s constant, M_A: mass

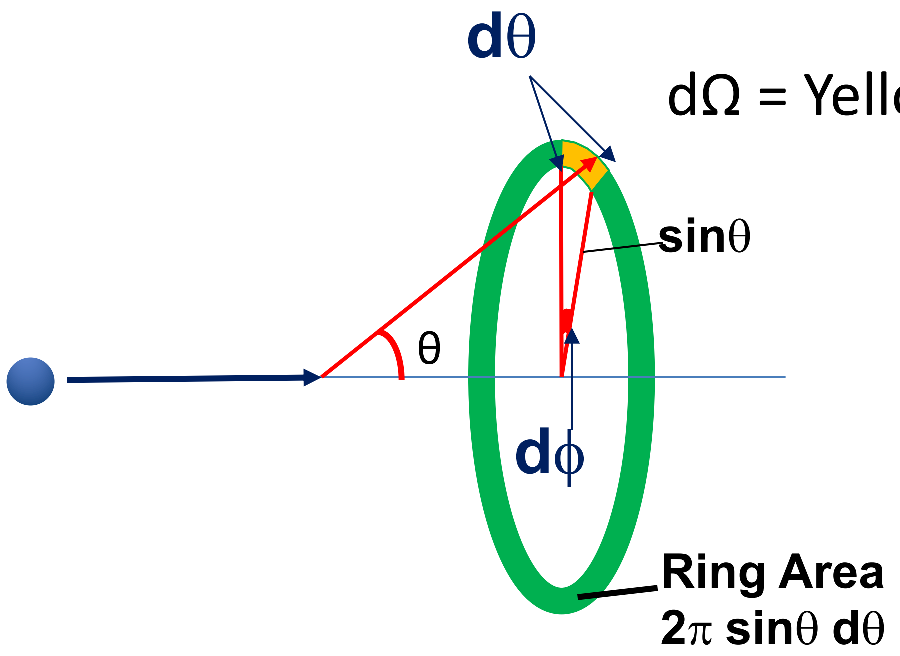
Differential cross section

Reaction rate:

$$W = \sigma \cdot N_{\text{target}} \cdot J$$

Differential Cross Section:

$$\mathrm{d}W = J \cdot N_{\text{target}} \cdot \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \cdot \mathrm{d}\Omega$$



Trigonometry for the area of an arc:

$$\mathrm{d}\Omega = \mathrm{d}\theta \cdot \sin\theta \cdot \mathrm{d}\phi$$

Integrate in phi because expect it to be symmetric going around the ring:

$$\sigma = \int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\mathrm{d}\Omega$$

$$= \int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi} \sin\theta \cdot \mathrm{d}\theta \cdot \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$

To get total cross section integrate theta over π
Otherwise for differential cross section only care about small slice in dθ

Rutherford Scattering

An incoming charged particle (α particle) scatters of a fixed point, the nucleus (i.e. no recoil, gold atom is fixed)

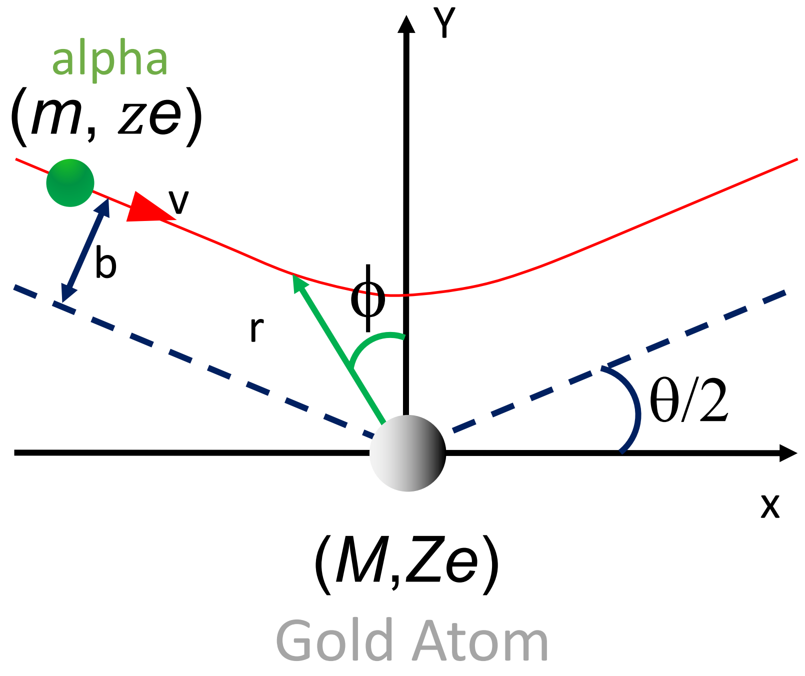
Treat like a classical elastic collision with repulsive Coulomb force

Impact parameter b: how close the alpha comes to the gold atom

Impact parameter tells us about angle θ, use to compute differential cross-section

The incoming particle moves with initial velocity v

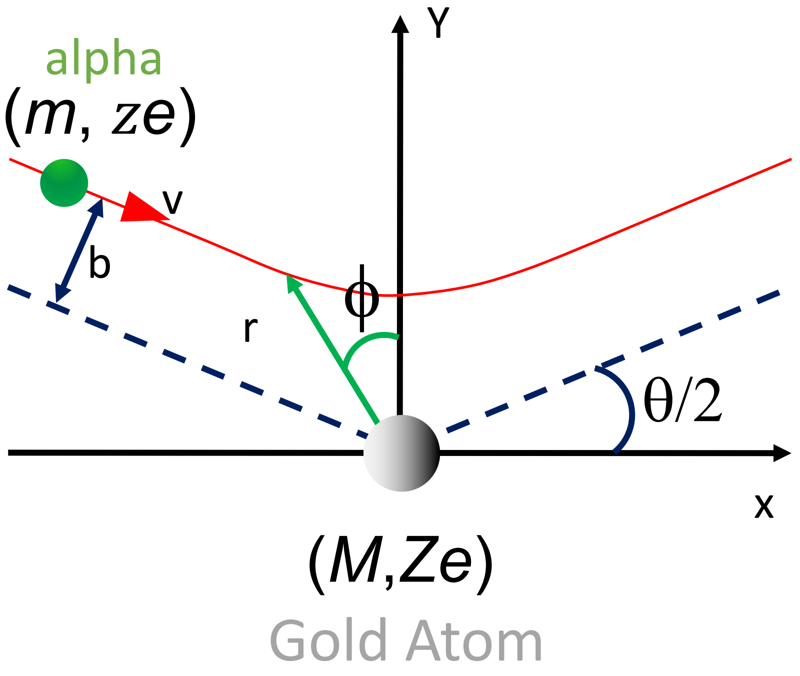
Angular momentum is conserved (m**v**×**r**) – elastic collision



Rutherford Scattering

Momentum :

$$\Delta p = 2mv\sin\frac{\theta}{2}$$



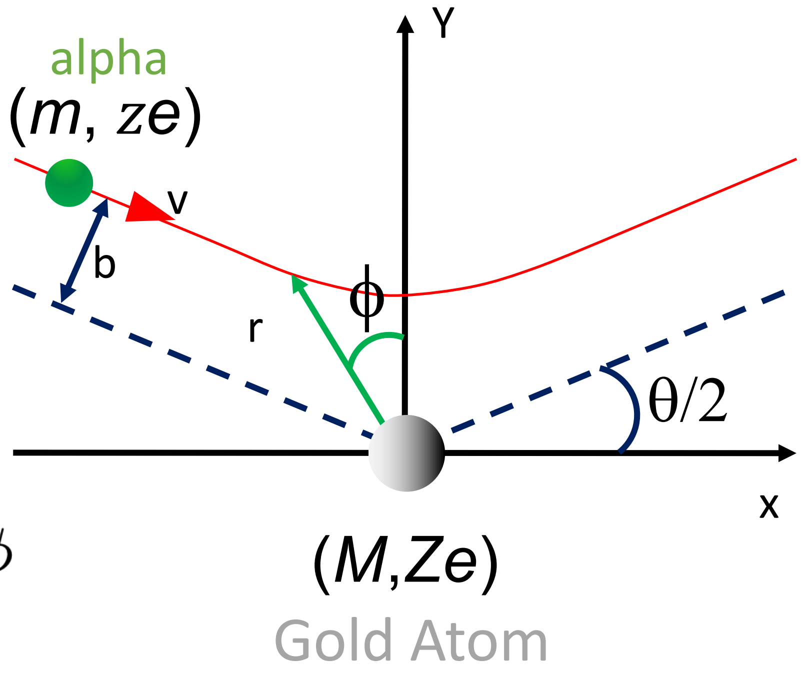
And also, using momentum and force relationship:

$$\Delta p = \int_{-\infty}^{\infty} dt\, F\,(t)$$

$$\Delta p = \int_{-\infty}^{+\infty} \frac{zZe^2}{4\pi\epsilon_0 r^2}\cos\phi\,\mathrm{dt}$$

Rutherford Scattering

$$\Delta p = 2mv\sin\frac{\theta}{2} \;\; = \int_{-\infty}^{+\infty} \frac{zZe^2}{4\pi\epsilon_0 r^2}\cos\phi \,\mathrm{dt}$$



Use conservation of angular momentum to convert to dφ:

$$\Delta p = \int_{-\infty}^{+\infty} \frac{zZe^2}{4\pi\epsilon_0 r^2}\cos\phi \,\mathrm{dt} = \frac{zZe^2}{4\pi\epsilon_0}\left(\frac{1}{bv}\right)\int_{-(\pi-\theta)/2}^{(\pi-\theta)/2}\cos\phi \,\mathrm{d}\phi$$

$$= \frac{zZe^2}{2\pi\epsilon_0}\left(\frac{1}{bv}\right)\cos\frac{\theta}{2}$$

Relationship for impact parameter as a function of scattering angle:

$$b = \frac{zZe^2}{8\pi\epsilon_0}\frac{1}{E_{\mathrm{kin}}}\cot\frac{\theta}{2} \qquad E_{\mathrm{kin}} = \frac{1}{2}mv^2$$

Impact Parameter "b"

$$\begin{cases} \Delta p = 2mv \ \sin\dfrac{\theta}{2} \\ \Delta p = \dfrac{zZe^2}{2\pi\epsilon_0}\left(\dfrac{1}{bv}\right)\cos\dfrac{\theta}{2} \end{cases}$$

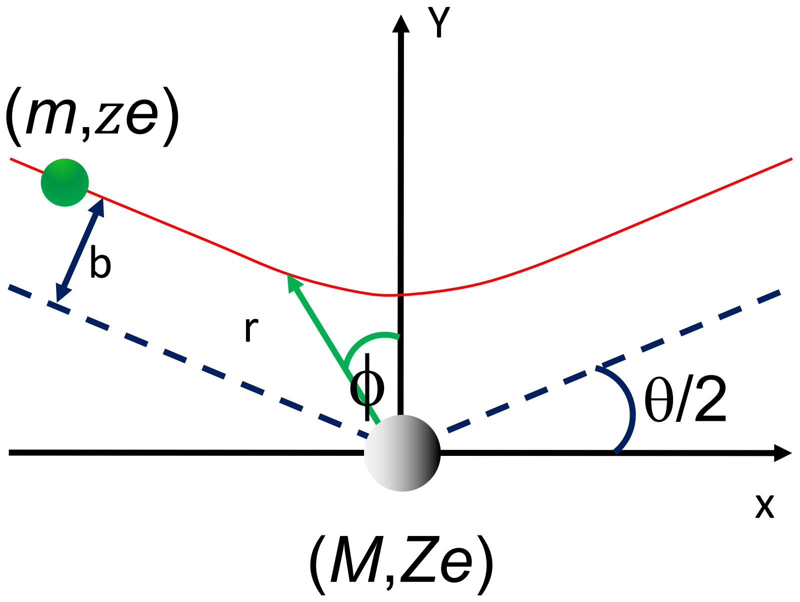
Relation between impact parameter and angle:

$$b = \frac{zZe^2}{8\pi\epsilon_0}\frac{1}{E_{\text{kin}}}\cot\frac{\theta}{2} \qquad E_{\text{kin}} = \frac{1}{2}mv^2$$

Closer to nucleus is deflected more:

Small *b*, large θ

Large *b*, small θ



"Cross sectional" area of a Gold nucleus

$b = 7\,\text{fm}$ ->measured area of closest approach

$r = 7\,\text{fm} = 7\times10^{-15}$

$A = \pi r^2 = 154\ \text{fm}^2 = 1.54\times10^{-28}\ \text{m}^2$

$= 1.54\ \text{barns}$

$1\ \text{barn} = 1\times10^{-28}\ \text{m}^2 = (10\ \text{fm})^2$

Rutherford Scattering

Currently in terms of b, but measure is differential cross section. Rewrite in terms of something we can measure in angle: differential cross section $\dfrac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$

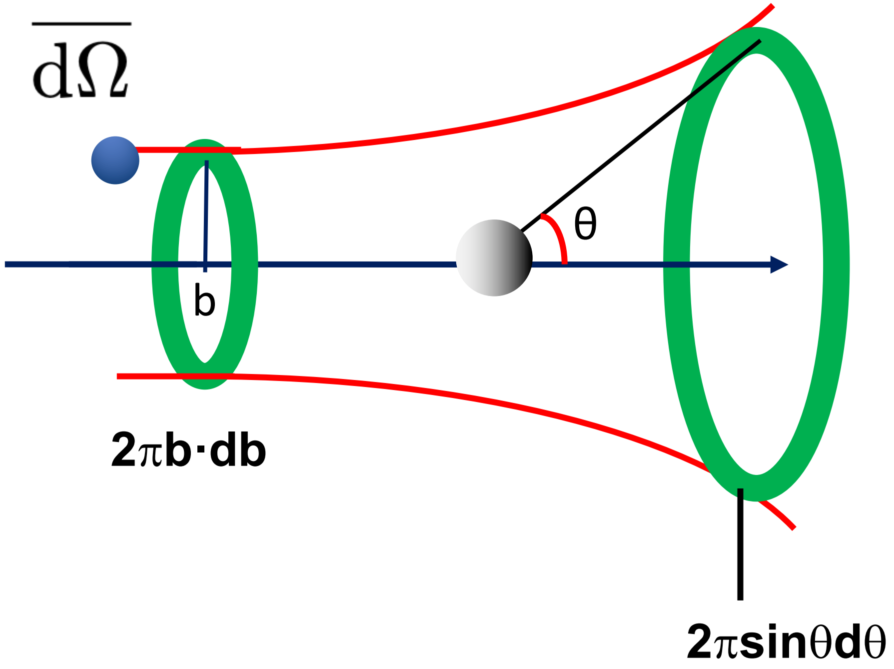
$$J\ 2\pi b\ |\mathrm{d}b|$$

$$= J\ \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\ 2\pi \sin\theta\,\mathrm{d}\theta$$

***J* (flux)**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{b}{\sin\theta}\left|\frac{\mathrm{d}b}{\mathrm{d}\theta}\right|$$

$$b = \frac{zZe^2}{8\pi\epsilon_0}\frac{1}{E_{\mathrm{kin}}}\cot\frac{\theta}{2}$$



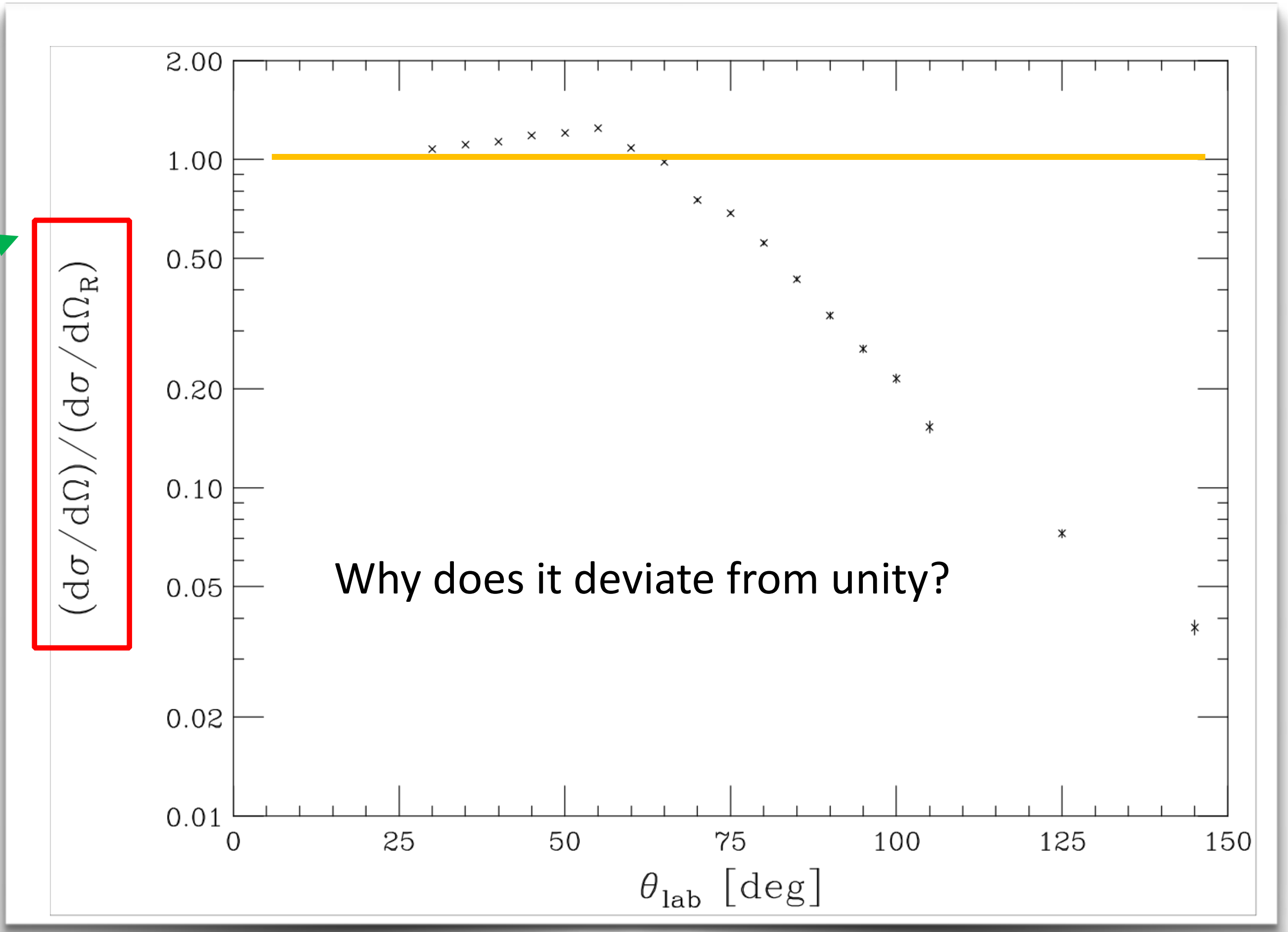
$$\left|\frac{\mathrm{d}b}{\mathrm{d}\theta}\right| = \frac{zZe^2}{16\pi\epsilon_0}\frac{1}{E_{\mathrm{kin}}}\operatorname{cosec}^2\frac{\theta}{2}$$

$$\left|\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right| = \left(\frac{zZe^2}{16\pi\epsilon_0}\frac{1}{E_{\mathrm{kin}}}\right)^2 \operatorname{cosec}^4\frac{\theta}{2}$$

Rutherford scattering data

Rutherford Scattering - Data for 27MeV α + ^{197}Au target

Ratio between measured and calculated (Rutherford) cross sections.



Large angle = small impact parameter

Recap / Up Next

This time:

The particle Zoo

- Quarks, leptons, bosons

Basic concepts

- Units, decays, anti-particles

Some history, 2 important cases

1. Discovery of the electron
2. Discovery of the nucleus

Next time:

Particle dynamics & interactions

Exchange forces

Conservation laws