Problem 1

Kittel & Kroemer, Chapter 6, problem 1 [Derivative of Fermi-Dirac function.]: 2 points

Show that $-\partial f/\partial\epsilon$ evaluated at the Fermi level $\epsilon = \mu$ has the value $(4\tau)^{-1}$. Thus the lower the temperature, the steeper the slope of the Fermi-Dirac function.

Problem 2

Kittel & Kroemer, Chapter 6, problem 2 [Symmetry of filled and vacant orbitals.]: 2 points

Let $\epsilon = \mu + \delta$, so that $f(\epsilon)$ appears as $f(\mu + \delta)$. Show that

$$f(\mu + \delta) = 1 - f(\mu - \delta).$$

Thus the probability that an orbital δ above the Fermi level is occupied is equal to the probability an orbital δ below the Fermi level is vacant. A vacant orbital is sometimes known as a **hole**.

Problem 3

Kittel & Kroemer, Chapter 6, problem 4 [Energy of gas of extreme relativistic particles.]: 3 points

Extreme relativistic particles have momenta p such that $pc \gg Mc^2$, where M is the rest mass of the particle. The de Broglie relation $\lambda = h/p$ for the quantum wavelength continues to apply. Show that the mean energy per particle of an extreme relativistic ideal gas is 3τ if $\epsilon \sim pc$, in contrast to $\frac{3}{2}\tau$ for the nonrelativistic problem.

Problem 4

Kittel & Kroemer, Chapter 6, problem 9 [Gas of atoms with internal degree of freedom.]

Consider an ideal monatomic gas, but one for which the atom has two internal energy states, one an energy Δ above the other. There are N atoms in volume V at temperature τ. Find the...

(a): 1 point

...chemical potential.

(b): 1 point

...free energy.

(c): 1 point

...entropy.

(d): 1 point

...pressure.

Problem 4 continued on next page. . .

(e): 1 point

...heat capacity at constant pressure.

Problem 5

Kittel & Kroemer, Chapter 6, problem 12 [Gas of atoms with internal degree of freedom.]

(a): 1 point

Find the chemical potential of an ideal monatomic gas in two dimensions, with N atoms confined to a square of area $A = L^2$. The spin is zero. The temperature is τ,

(b): 1 point

Find an expression for the energy U of the gas.

(c): 1 point

Find an expression for the entropy σ.

Problem 6

Kittel & Kroemer, Chapter 6, problem 14 [Ideal gas calculations.]

Consider one mole of an ideal monatomic gas at 300 K and 1 atm. First, let the gas expand isothermally and reversibly to twice the initial volume; second, let this be followed by an isentropic expansion from twice to four times the initial volume.

(a): 1 point

How much heat (in joules) is added to the gas in each of these two processes?

(b): 1 point

What is the temperature at the end of the second process?

(c): 1 point

Suppose the first process is replaced by an irreversible expansion into a vacuum, to a total volume twice the initial volume. What is the increase of entropy in the irreversible expansion, in joules per kelvin?