Diffraction & Interference

Experiment Two

Physics 192
Michigan State University

Before Lab

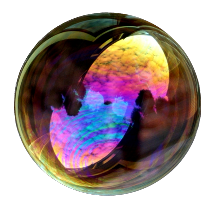
- **Due at the beginning of class (10 pt)**: answer the pre-lab theory questions
- Carefully read the entire lab guide

Experiment Overview

This is a two-week lab exploring the wavelike properties of light. You will observe the effects of diffraction and interference using a laser and various slit configurations

- Make measurements of diffraction patterns projected onto a screen
- Infer the geometry of different variations of microscopic slits by analyzing diffraction patterns
- **Produce a professionally formatted lab report**: communicate the details of your experiment to a general scientific reader, introduce each section to provide context for your work, include captions describing and interpreting plots, and show all relevant calculations

1 Wave Optics



In the previous lab, you studied reflection and refraction using geometric optics, treating light as rays which only change direction at physical boundaries. The properties of lenses, mirrors, and many other practical applications are described using the ray model. In this lab, you will observe properties of light which directly contradict the principles of geometric optics.

Many optical phenomena are a consequence of the wave properties of light. Soap bubbles consist of soap mixed with clear water, forming a thin film. The brilliant display of colors results from the interference of light waves at the boundaries between air and the bubble. When the physical dimensions of a system are near the wavelength of light, unique wave features emerge. This is the case with soap bubbles. While the wavelength of visible light is about 400–700 nm, the thickness of a typically bubble is ~ 100 nm. Despite the minuscule scale, the wave effects are easily perceived.

1.1 Interference

Interference describes how waves interact. The figure to the right shows two important measures for understanding interference. The two waves are identical, sharing the same amplitude and wavelength. The **wavelength** is the spatial period of a wave–the distance over which the wave repeats. This distance is labeled by λ, a lowercase Greek lambda. The two waves are offset by a distance labeled Δx, referred to as the **path length difference**. If $\Delta x = 0$, the two waves perfectly overlap.

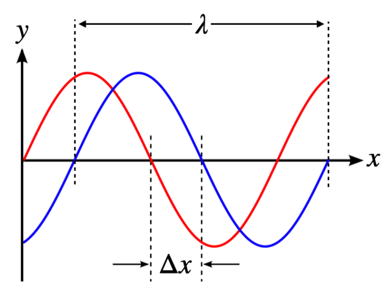


Figure 1

Waves obey the **principle of superposition**, which states that the total waveform at a given time and location is the sum total of any individual waves. Two special cases of superposition are shown in the figures below.

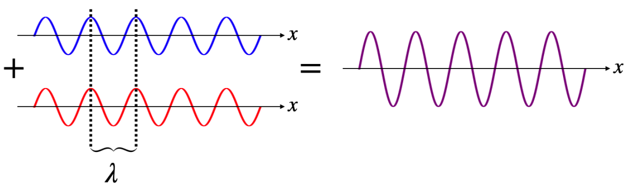


Figure 2: Constructive interference

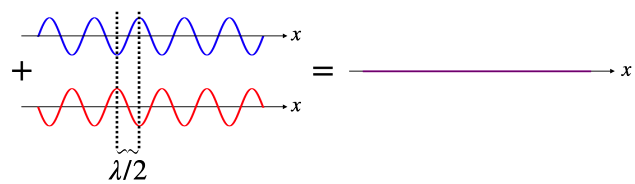


Figure 3: Destructive interference

Figure 2 shows the addition of two waves that are *in phase*–their peaks and troughs are aligned. Note that shifting one of the waves by λ leaves the situation unchanged. In fact, shifting the waves by any integer multiple of λ leaves the waves aligned. Adding the waves together results in a new wave with twice the amplitude of the individual waves. This is called **constructive interference**, with the path length criterion

$$\Delta x = m\lambda \qquad \text{(for } m = 0, \pm 1, \pm 2, ...) \tag{1}$$

where m is called the **order number**. Figure 3 shows shows the addition of two waves that are offset by a half wavelength, resulting in a phase difference of 180°. Added together in this way, the waves cancel entirely. In this case, shifting a wave by $(1 + \frac{1}{2})\lambda$ leaves the situation unchanged. This is called **destructive interference**, with the path length criterion

$$\Delta x = (m + \frac{1}{2})\lambda \qquad \text{(for } m = 0, \pm 1, \pm 2, ...). \tag{2}$$

This is the operating principle of noise canceling headphones. A small microphone sends the ambient noise to a processor that inverts the waveform while preserving the amplitude and frequency. The inverted wave is played into the ears by the speakers, combining with the external noise to provide destructive interference.

1.2 Huygens' Principle

A relationship between geometric and wave optics was first proposed by Dutch physicist Christiaan Huygens (1629—1695). Huygens formulated a model predicting how a wave will spread out as it propagates, allowing for a geometric understanding of wave motion.

Figure 4 shows a long wave moving at velocity $\vec{v}$. The entire wave can be treated as an infinite number of independent waves that spread out in all directions. If the wave is infinitely long, all red arrows cancel except for those in the direction of the velocity, leaving one long wave moving to the right. This is the essence of **Huygens' principle**, which states that every point in a propagating wave can be treated as an individual source of secondary spherical waves. By superposition, the combination of infinitely many waves form the single observed wave.

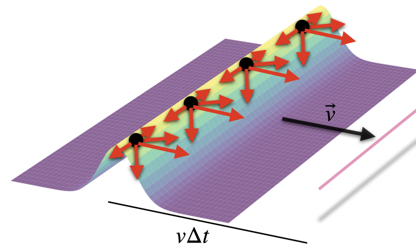


Figure 4

This construction can be used to determine the future location of a wave geometrically. Figure 5 demonstrates Huygen's principle, starting with only three point sources. The circular *wavelets* around each point have a radius corresponding to the distance the wave will travel over some time interval, $r = v\Delta t$. As point sources are added, more wavelets overlap and interfere. Drawing a line tangent to all the circles in the direction of motion gives the location of the wave after a time Δt.

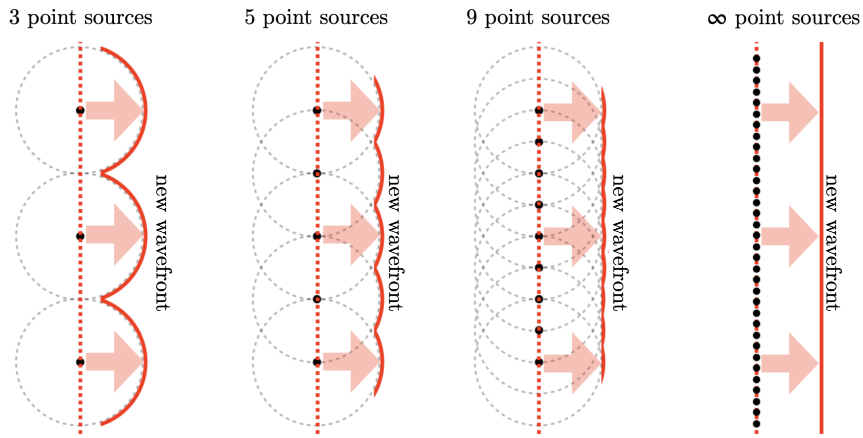


Figure 5

1.3 Diffraction

The diagram above is called a Huygens construction. These are useful for understanding how waves interact with barriers. Consider the case of a long wavefront encountering a small barrier as shown in Figure 7. Most of the incident wave is blocked by the barrier, but the portion that reaches the boundary spreads out as it passes. This effect is more pronounced in the case of a small hole in a barrier. The transmitted portion of the wave does not project in a straight line; instead, it fans outward circularly in a process called **diffraction**. The picture of the beach shows very clear diffraction effects. Waves passing through the openings interfere, leading to cancellation which is evident in the erosion pattern along the shoreline.

Figure 6

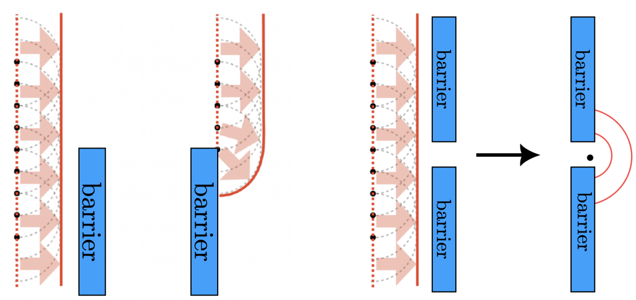


Figure 7

1.4 Double-Slit Interference

If light truly is a wave, then it should exhibit the wave properties of diffraction and interference. Through history, most scientists believed light was a particle that traveled in straight lines. In 1801, careful experiments by English scientist Thomas Young proved that light does behave as a wave[1]. Diagrams of the experiment are shown in Figure 8. For simplicity, our analysis will be restricted to **monochromatic** light, which consists of a single wavelength (color). Interference can occur if the light is **coherent**, consisting of waves that are in phase (aligned). Lasers emit light with both of these properties.

The incident light encounters a barrier with two slits no wider than the wavelength. The light diffracts through each slit much like the water waves from the previous section. The two slits act as point sources emanating waves that are in phase. The interaction of the waves can be understood using a Huygens construction. Interference can be interpreted by examining how the circular wavelets overlap.

[1]Around 100 years later, it was found that light has characteristics of *both* particles and waves. This is called wave–particle duality. With the advent of quantum mechanics, it was demonstrated that all matter exhibits this feature. Interpretation of this concept remains unresolved.

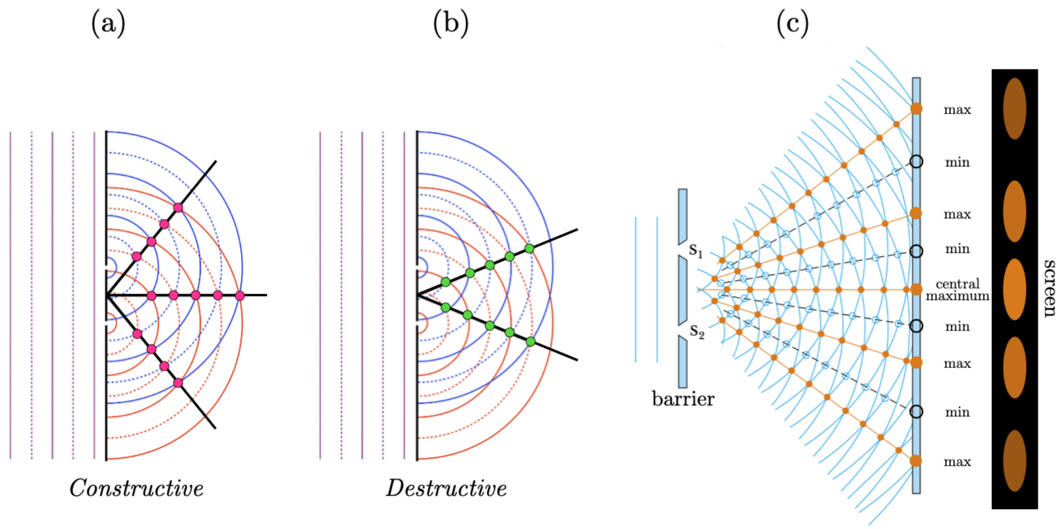


Figure 8: Waves passing through two narrow slits. The blue wavelets are from the top slit and the red wavelets are from the bottom. Diagram (a) indicates points with constructive interference. Diagram (b) shows points with destructive interference. The combined effect is shown in diagram (c).

Diagrams (a) and (b) are the same, but they are separated to illustrate different conditions. The circular solid lines are wave peaks and the dashed lines are troughs. In diagram (a), pink dots are placed on points where peaks and troughs align. Lines through these points indicate regions of *constructive* interference. Diagram (b) has green dots over points where peaks and troughs are exactly misaligned, showing regions of *destructive* interference. The combined effect is shown in diagram (c). A pattern emerges when a screen is placed behind the slits. This result is in direct contradiction with the ray model, which only predicts two bright spots projected onto the screen. The image of bright and dark spots is called an **interference pattern**.

The brightest spot is labeled as the *central maximum*, occurring directly between the two slits. A pattern of bright and darks spots called **fringes** appears symmetrically about the central maximum. The locations of the fringes can be obtained geometrically using the conditions in Equations 1 and 2. The waves from the two slits start in phase, but the wave from the bottom slit must travel a distance Δx further to reach the point P on the screen.

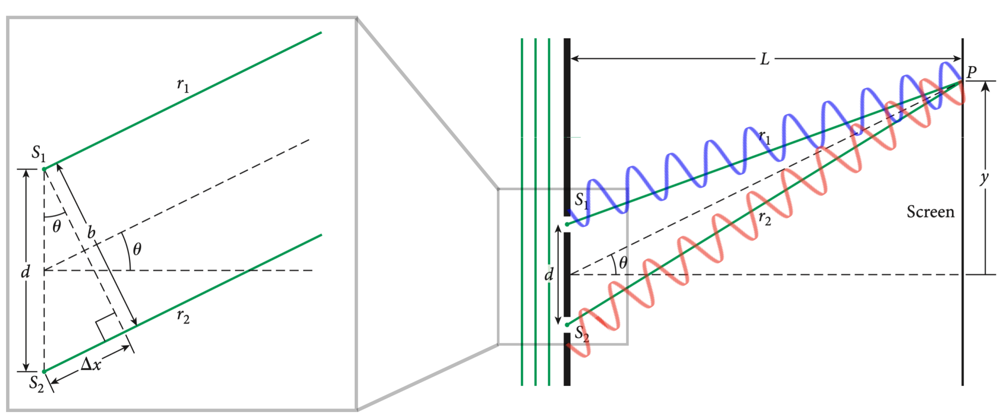


Figure 9: Two slits separated by a distance d. The red and blue waves are 180° out of phase resulting in destructive interference at P.

Figure 9 shows the geometry of double slit interference. The diagram on the right is not shown to scale.

In reality, the distance to the screen is much larger than the slit separation ($L >> d$). As a result, the to paths labeled r_1 and r_2 are effectively parallel. The gray box shows an expanded view showing how the path length difference Δx is determined:

$$\Delta x = d \sin\theta \tag{3}$$

where d is the slit separation and θ is the angle to the point in question. The result can be combined with the interference conditions in Equations 1 and 2 determine the locations of bright and dark fringes,

$$d \sin\theta_m = \begin{cases} m\lambda : & \text{constuctive (bright)} \\ (m + \frac{1}{2})\lambda : & \text{destructive (dark)} \end{cases} \qquad m = 0, \pm 1, \pm 2, \ldots \tag{4}$$

For bright fringes, the order number $m = 0$ corresponds to the central maximum. The first spots beyond the center appear at angles $\pm\theta_1$ with order number $m = \pm 1$ as shown above. The intensity distribution for very narrow slits separated by a large distance is represented by the orange curve in Figure 10. Notice that the intensity of each fringe has the same value.

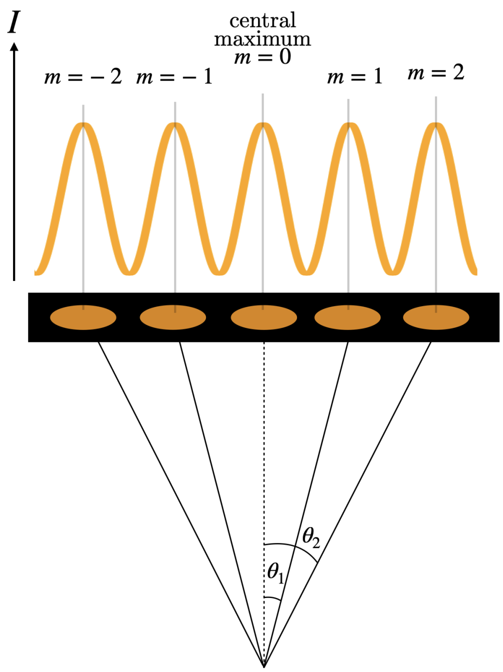


Figure 10: Double-slit interference

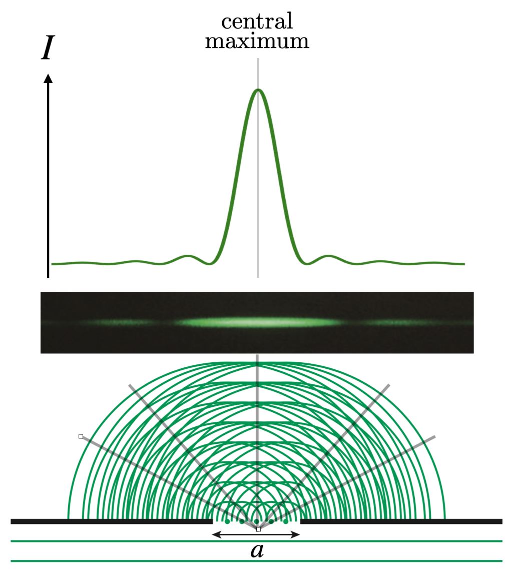


Figure 11: Single-slit diffraction

1.5 Single-Slit Diffraction

A wave diffracting through a single barrier experiences a self interaction which leads to a characteristic interference pattern as shown in Figure 11. In the previous section, we constrained the system to very narrow slits so each one could be treated as a single point source. If the width of the slit is increased to be around the size of λ, describing the wave at the slit requires the addition of more point sources. The Huygens construction in Figure 11 shows 5 point sources along the diffracting wave. The grey lines indicate regions of constructive interference. Dark fringes result from destructive interference. A **diffraction pattern** is the unique distribution of fringes resulting from self interference. Notice the difference in the character of the intensity distributions (orange and green curves). The single slit has a broad intense central maximum followed by fringes that are significantly dimmer.

Take a minute to compare Figure 11 with 8 and make sure you understand the difference. While both cases led to a patterns of bright and dark spots on a screen, double-slit interference and single-slit diffraction describe different mechanisms:

- Double-slit interference: two narrow slits with widths $a << \lambda$ act as individual point sources. The intensity maxima are equally spaced and have the same amplitude.
- Single-slit diffraction: describing a wider single slit with width $a \sim \lambda$ requires additional point sources along the width of the slit. The interfering point sources result in a broad central maximum and significant reductions in intensity beyond the central peak.

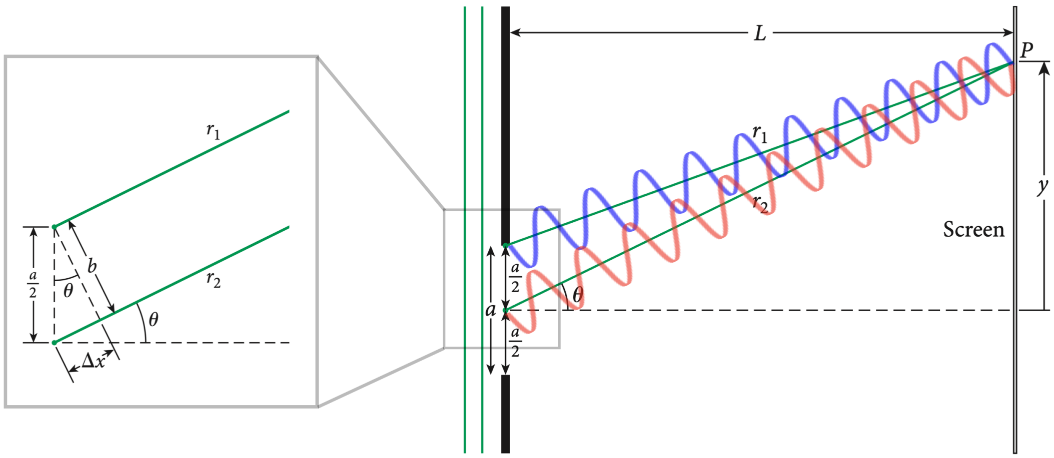


Figure 12: A single slit of width a. The red and blue waves are 180° out of phase resulting in destructive interference at P.

The geometry of single-slit diffraction is shown in Figure 12. Once again, the screen is positioned far away from the slit, $L >> a$. In this limit, the path lengths r_1 and r_2 are parallel. Basic trigonometry gives the path length difference in terms of the angle measured to the point on the screen,

$$\Delta x = \frac{a}{2} \sin\theta. \tag{5}$$

The first instance of destructive interference occurs when the path length difference is $\Delta x = \lambda/2$ (shown in Figure 3. This gives the angular location of the first dark fringe,

$$\frac{a}{\cancel{2}} \sin\theta = \frac{\lambda}{\cancel{2}} \quad \Rightarrow \quad \sin\theta = \lambda.$$

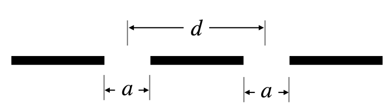
By considering the effects of additional point sources, a general expression for the locations of dark fringes can be determined:

$$a \sin\theta_n = n\lambda : \quad \text{destructive (dark) for } n = \pm1, \pm2, ... \tag{6}$$

where n is used as the order number to differentiate from double-slit interference as described in the previous section.

1.6 Double-Slit Diffraction

Consider a double-slit configuration like that shown to the right, where d is the slit separation and a is the width of each slit. The double-slit interference studied in Section 1.4 was a special case in which $a << \lambda$. In more realistic situations, the narrow slit condition is not met. When the slit width is close to the wavelength, combined effects of double-slit interference *and* diffraction will be observed.



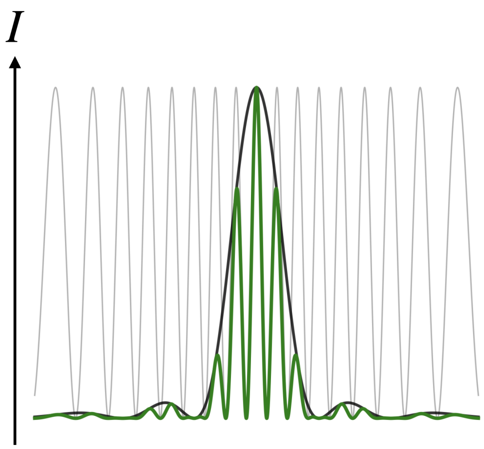


Figure 13

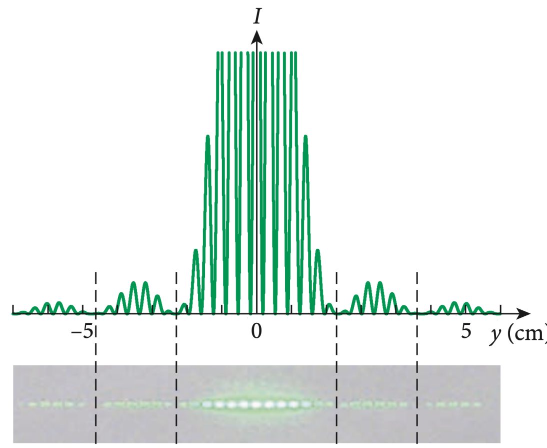


Figure 14

The resulting diffraction pattern looks like the double-slit interference pattern *enveloped* by the single-slit diffraction pattern. The dark black curve in Figure 13 is the intensity distribution for single-slit diffraction. The plot shows how diffraction effects limit the intensity of the double-slit fringes, completely eliminating those at the diffraction minima indicated by dashed lines in Figure 14.

Diffraction patterns are an incredibly powerful tool for understanding microscopic structures. Physical measurements of fringes on a screen behind a double slit can be used to determine the slit width *and* separation, two quantities which are far to small to measure by conventional methods. Diffraction techniques extend far beyond combinations of slits. X-ray crystallography employs diffraction pattern analysis to construct 3-dimensional models of molecular structures. Rosalind Franklin was an experimental chemist specializing in x-ray crystallography. In 1951, she identified key properties of DNA through interpretation of diffraction patterns, being the first to understand its coiled structure. Franklin's expertise in diffraction lead to countless discoveries related to biological molecules, viruses, and carbon materials.

1.7 Diffraction Gratings

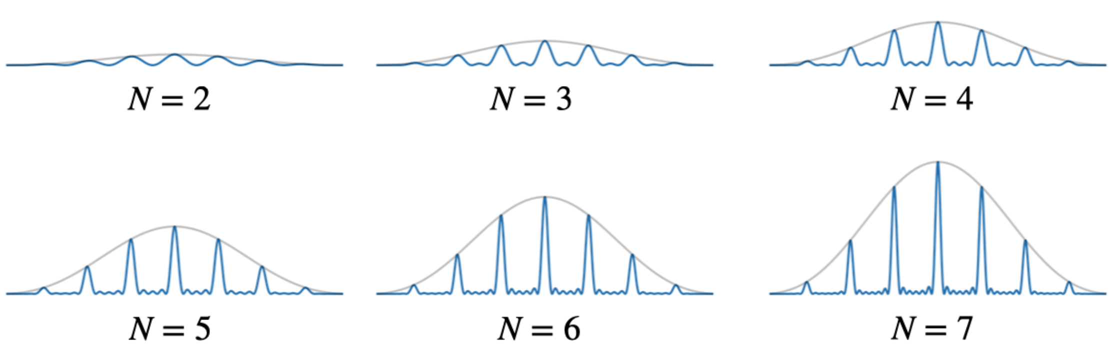


Figure 15: Intensity distributions for increasing numbers of slits

We can now consider diffraction effects for barriers with more than two slits. Figure 15 shows the intensity distributions for increasing numbers of slits N. The intensity of the bright primary fringes grows as N^2, which is why the peaks are taller with each additional slit. The entire distribution is still enveloped by the gray curve, resulting from diffraction. Adding another slit amounts to adding a new light source, creating additional interference. This results in the appearance of additional fringes between the primary maxima, which present as secondary peaks in the graphs above. As more slits are added, additional destructive interference reaches closer to the primary maxima resulting in bright, sharp peaks.

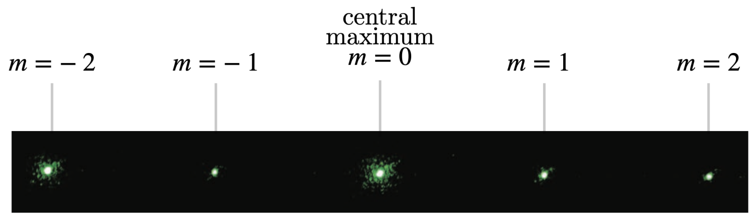


Figure 16: A diffraction pattern produced on a screen with a green laser and a grating. The high number of slits causes all secondary maxima to vanish, leaving only bright primary peaks.

A **diffraction grating** is a device that has a *very* large number of parallel slits placed very close together. A diagram similar to Figures 9 and 12 can be constructed to determine the conditions for constructive interference for a diffraction grating. The angular locations of the bright spots are determined using

$$d \sin \theta_m = m\lambda \tag{7}$$

where d is called the grating spacing. The grating spacing is like the slit separation, but with such a high number of slits (also called rulings), it is more common to use the **grating constant**,

$$G = \frac{1}{d}. \tag{8}$$

This number is a *density* of rulings, usually expressed with units of lines/mm. The sharp, widely spaced maxima allow for very precise measurements of wavelength. For this reason, diffraction gratings are an essential tool for spectroscopy.

Key Concepts

- **Interference** is the addition of two or more waves to form a composite wave. This is a property of the superposition principle.
- **Diffraction** is the spreading out of a wave encountering an obstacle, edge, or hole in a barrier. When the wavelength is around the size of the obstacle or barrier, the diffracted wave experiences interference.
- Light passing through two or more slits must travel different distances to reach the same point. The difference in path length determines if constructive or destructive interference occurs at that point. Constructive interference produces bright fringes (maxima) and destructive interference produces dark fringes (minima).
- The angular displacements for **interference maxima** are determined using

  $$d \sin \theta_m = m\lambda \tag{9}$$

  where d is the slit separation, $m = 0, \pm 1, \pm 2, \ldots$ is the order number, θ_m is the corresponding angle, and λ is the wavelength of light
- The angular locations of **diffraction minima** are determined using

  $$a \sin \theta_n = n\lambda \tag{10}$$

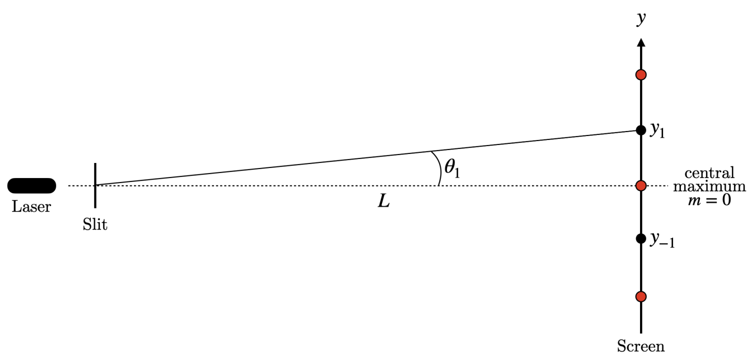
  where a is the slit width and $n = \pm 1, \pm 2, \ldots$ is the order number.
- A **diffraction grating** has a very large number of slits, producing a diffraction pattern of bright, narrow peaks at angular locations determined using

  $$d \sin \theta_m = m\lambda \tag{11}$$

  where d is grating spacing. This number is related to the grating constant by $G = 1/d$.

2 Theory Questions

1. You are given a small sheet with a tiny slit of with a in its center. In your lab, you mount a laser with known wavelength λ on a table. You place a paper screen at a distance L from the laser, and you insert the slit along the path of light as shown in the figure below. Instead of seeing a single dot on the screen, you observe a diffraction pattern with the first order ($n = 1$) diffraction minima at distances y_{-1} (bottom) and y_1 (top) from the central maximum. You only have a tool for measuring distances, not angles. Determine how to measure the width of the slit without measuring any angles.



   (a) (1 pt) Write down an equation that relates the angle θ_1 to the wavelength λ and the slit width a.

   (b) (2 pt) Use trigonometry to relate θ_1 to L and y_1. You do not need to solve for θ_1.

   (c) (2 pt) You placed the screen a large distance from the slit, so $L >> y_1$. This means that the angle θ_1 is small. For small angles, $\tan\theta \approx \sin\theta$ (see appendix). Use this fact and your answers from (a) and (b) to arrive at a relationship between y_1, L, λ, and a. Your answer should *not* depend on θ_1.

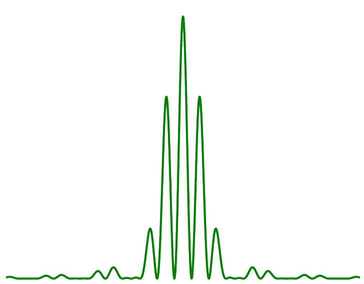


Figure 17

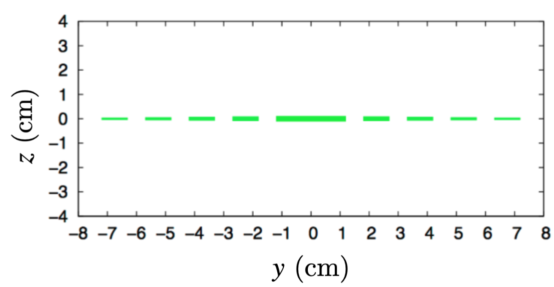


Figure 18

2. The green curve in Figure 17 is an intensity distribution for diffraction with a $\lambda = 700$ nm source.

   (a) (2 pt) Does this represent a single or double-slit configuration?

   (b) (1 pt) What is the highest order diffraction minimum visible in the graph?

3. (2 pt) Diffraction is not limited to waves passing through slits. As shown in Figure 7, diffraction occurs at the edges of any type of barrier or obstacle. The diffraction pattern resulting from a wire of thickness T is complementary to that for a thin slit, with diffraction minima determined by $T\sin\theta_n = n\lambda$. Figure 18 shows a diffraction pattern produced by a hair using a laser with $\lambda = 525$ nm at a distance $L = 1.95$ m from the screen. Determine the thickness of the hair. (*Hint*: one of the higher order diffraction minima lines up with a well defined y value).

Laser Safety

As mentioned above, lasers produce monochromatic, coherent light. In this lab, you will be working with two different lasers:

- A red laser with a wavelength of 651 nm (Pasco OS-8528A)
- A green laser with a wavelengths of 532 nm (Basic Optics OS-8458)

The lasers used in this lab have a lower power output, but the beam is still very intense. Always exercise caution when in use.

- **NEVER look directly into the beam or its reflection!**
- **Always pay attention to reflections. Do not let reflections catch your classmates.**
- **Avoid positioning your head in the plane of the laser.**

Experiment Setup

You should be familiar with most of the components used in this lab from your experience in the geometric optics experiments. You have a laser mounted on an optical rail separated by a distance L from a paper screen. Every part mounts to the rail on a post that can be rotated, raised, and lowered. You must take care to align the components whenever you make changes to your system.

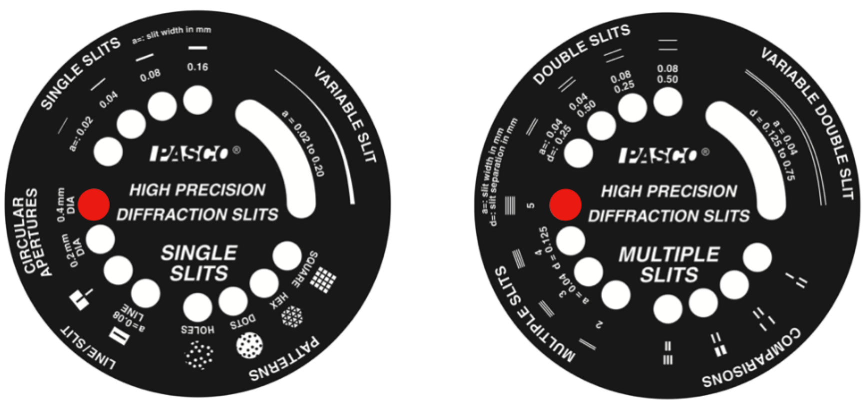


Figure 19: Single-slit (left) and double-slit (right) wheels. The red circle indicates the selected aperture.

In this experiment, you will observe diffraction patterns generated by various apertures. An **aperture** is an opening or gap in an opaque material (this is a generalization of a "slit"). At your bench, you have two Pasco aperture wheels. The wheels contain different aperture configurations which are indicated on the front face. The red dots in the figure above show the location of the opening for the beam. To get started:

- Clip a new piece of graph paper to the screen. More paper is available at the front of the room.
- Start with the green laser. When you change lasers, **do not let the metal end of the power cord touch the optical rail**.
- Place single-slit aperture wheel with the face pointed towards the laser. Align the beam with the hole indicated in red in Figure 19.
- Take your time aligning all of the components. The process is similar to the alignment in the previous lab. Try to get the reflected part of the beam to return to the center of the laser, while also centering the diffraction pattern on the screen.
- Documentation with full specifications for the aperture wheels can be found here: `https://cdn.pasco.com/product_document/Precision-Diffraction-Slits-Manual-OS-8453.pdf`

3 Single Slit Diffraction

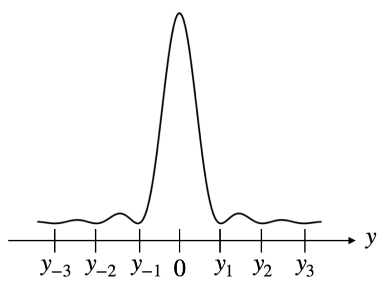
Make sure that you have all of your components aligned with the single-slit aperture wheel securely in place. Your graph paper should be flat and the screen should be tight, it should not move during your experiment. Rotate the aperture wheel to see the diffraction patterns for various configurations. Your goal is to determine the width of a slit by measuring the locations of the dark fringes.

1. Write a short introduction to this part of your experiment.

2. (1 pt) Select the variable slit segment on the aperture wheel. Describe how the slit width a effects the diffraction pattern. Does more interference occur for a narrow or large slit?

3. (1 pt) Choose a single slit that produces a well-formed diffraction pattern. Adjust the position of the screen to be as large as possible while maintaining a clear pattern. You should be able to see at least 4 minima on each side of the central peak. Measure the distance L from the aperture to the screen, and record $L \pm \delta L$ along with the manufacturer's value stated slit width, a.

4. Make sure your paper is flat and does not move and align it with your diffraction pattern. Use a pen or pencil to carefully mark the positions of the diffraction minima. Then, remove your graph paper and measure the distances y_n from the central maximum as shown below. Distances to the left of 0 should be negative (corresponding to negative values of n).



5. (1 pt) The first theory question was written to prepare you for this analysis. Write down the equation for diffraction minima. We do not have a way to measure the angles θ_n, but you *can* measure distances. Use the small angle approximation to determine an equation that relates λ, n, L, and a to the distance y_n where n is a count from the central maximum. No numbers are needed for this step. The appendix describes the origin of the small angle approximation.

6. (1 pt) Estimate your uncertainties δy_n.

7. (4 pt) You have measured distances y_n for different values of n. Use your equation from Question 5 to determine how to plot your data such that the slit width can be determined from the slope. Plot your measurements in curve.fit, including error bars and a short caption.

8. (1 pt) Determine the $a \pm \delta a$ from your fit results. Assume $\delta\lambda \approx 0$.

9. (1 pt) Compare your measured slit width with the manufacturer's stated value by computing the relative error. Comment on your results.

4 Double Slit Diffraction

Exchange the single-slit wheel for the double-slit wheel. Carefully align the system, following the same procedure as in the previous section. Fix a new piece of graph paper in place for your screen. Rotate the aperture wheel and observe the effects of different slit variations. The addition of a second slit introduces secondary interference substructure encapsulated by a diffraction envelope. Your goal is to determine the slit width and separation for one of the configurations.

1. Write a short introduction to this part of your experiment.
2. (1 pt) Select the variable separation segment on the aperture wheel. Describe how changing the separation d effects the diffraction pattern. Does more interference occur with a large or small separation?
3. (1 pt) Choose a double slit configuration that produces a well-formed diffraction pattern and appropriately adjust the position of the screen. You should see a sufficient number of diffraction and interference fringes to make a complete set of measurements. Record $L \pm \delta L$ and the manufacturer's values of both d and a.
4. With everything locked in place, mark the locations of the central maximum, the far-spaced diffraction *minima*, and the narrowly-spaced interference *maxima* on your graph paper. Once you are satisfied with your markings, remove the graph paper to measure the distances relative to the central maximum. It may help to label your measurements as
   - y_n correspond to diffraction minima (for the slit width)
   - y_m correspond to interference maxima (for the slit separation)
5. (4 pt) Follow the analytical procedure from the previous section to determine the slit width. You should produce a properly formatted plot using curve.fit. Make sure your plot has a title indicating that it corresponds to the double-slit experiment. Report your value as $a \pm \delta a$.
6. (1 pt) Write down the equation for interference maxima. Use the small angle approximation to determine an equation that relates λ, m, L, and d to the distance y_m where m is a count from the central maximum. This is just like the single-slit case, but now you are looking at *bright* fringes.
7. (4 pt) Use curve.fit to plot your measurements, and determine the slit separation from your fit. Include error bars and a good title. Report your value as $d \pm \delta d$.
8. (1 pt) Compare your measured value of a and d to the manufacturer's values by computing the relative errors. Comment on your results.

5 Diffraction Grating

In this section of the lab, you will use a diffraction grating to measure the wavelength of light emitted by the red and green lasers. Start by replacing the aperture wheel with the diffraction grating, which has a grating constant of $G = 500$ lines/mm. As discussed in the introduction, diffraction gratings form patterns with widely-spaced maxima. Place a piece of graph paper on the screen and look for the bright fringes. The grating disperses light at large angles, so you will have to reduce the distance L to the screen.

1. Write a short introduction to this part of your experiment.
2. (1 pt) Write down equation for constructive interference with a diffraction grating. You will use this relationship to determine λ for each laser. The large angular dispersion means the small angle approximation is no longer appropriate for analysis.
3. (1 pt) Once you have a clear diffraction pattern with as many maxima as possible, lock the screen in place and record the value of $L \pm \delta L$.
4. On your graph paper, mark the locations of as many bright fringes on the screen as possible. As before, the distances y_m are measured from the central maximum.
5. (1 pt) Use trigonometry to determine how to calculate the angles θ_m to the left and right of the central maximum.
6. (2 pt) Determine your best estimate of the wavelength for the first laser and determine the uncertainty using statistical methods. Recall that the uncertainty in the mean is $\delta\bar{x} = \sigma/\sqrt{N}$ where σ is the standard deviation and N is the number of measurements.

7. (2 pt) Switch to the other laser and perform the same procedure to determine its wavelength and uncertainty.

8. (2 pt) Summarize your results for both lasers in a table. Include the relative errors and fractional uncertainties for each laser.

9. (1 pt) White light is a combination of many colors (wavelengths) from the visual spectrum. If you could project a beam of white light at a diffraction grating, what would you see on a screen?

6 Hair Diameter Measurement

In this section of the lab, you will determine the thickness of a hair using diffraction techniques. At your bench, you have a rectangular frame with two clips. Use the clips to fix a hair vertically within the frame. Secure the frame on the optical rail and align the hair with the beam by rotating the frame. You should see a diffraction pattern form on the screen.

1. Try to measure the hair thickness using digital calipers. Use this measurement as motivation for this part of the lab, and write a short introduction.

2. (2 pt) Explain why you do not see a dark shadow on the screen behind the hair.

3. (6 pt) Use methods learned in previous sections to determine the thickness of the hair, including the uncertainty. You can do this using statistical methods or by plotting your data. Be sure to explain your procedure.

Appendix: Approximations

"Approximate the universe as a set of harmonic oscillators."

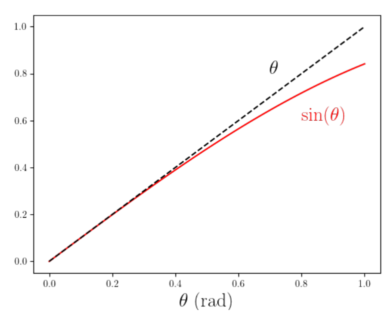
Mathematical approximations are incredibly powerful tools. In computer science, they are used to improve computation efficiency. Many physical systems can *only* be understood using approximations. Some approximations come in the form of simplifying assumptions, such as neglecting wind resistance in a free fall experiment. Others involve mathematical manipulations to simplify the form of a complicated function.

There are a number of methods used to approximate mathematical relationships. In most cases, the first choice is to use a **Taylor expansion**, which is a method for approximating a function near a specific point as a polynomial. A differentiable function $f(x)$ can be expanded about a point $x = a$ as

$$f(x) \approx f(a) + \frac{1}{1!}f'(a)(x-a) + \frac{1}{2!}f''(a)(x-a)^2 + ... + \frac{1}{k!}f^k(a)(x-a)^k \tag{12}$$

where $f'(x)$ is the first derivative, $f''(a)$ is the second derivative, and $f^k(a)$ is the k^{th} derivative evaluated at the point $x = a$. Many situations can be sufficiently approximated using only the first-order (linear) term. Using more terms improves the accuracy of the approximation.

This is the origin of the so-called "small angle approximation" used in this lab. The small angle approximations for the main trigonometric functions are



$$\sin\theta \approx \theta$$
$$\cos\theta \approx 1$$
$$\tan\theta \approx \theta$$

where the angle θ is necessarily in radians. The plot to the right shows how close $\sin\theta$ and θ are, with a significant deviation appearing beyond 0.2 radians (about 11°).