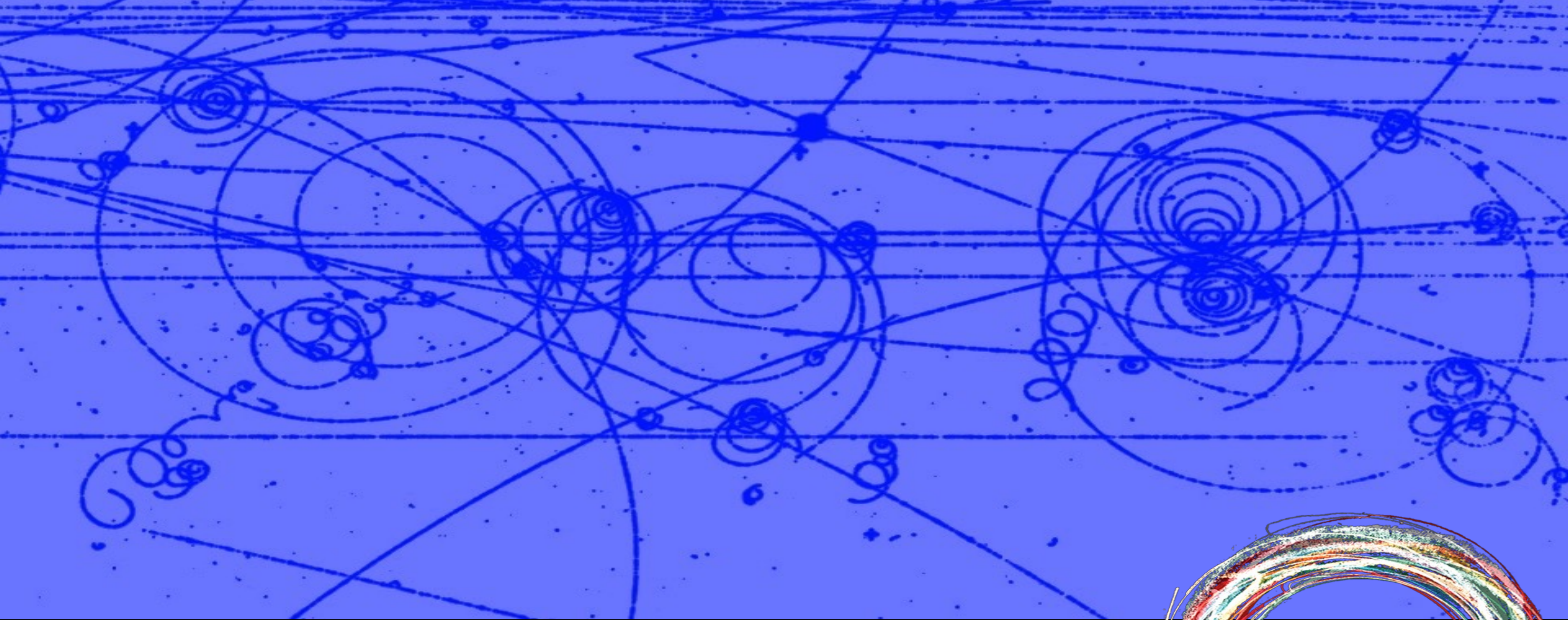
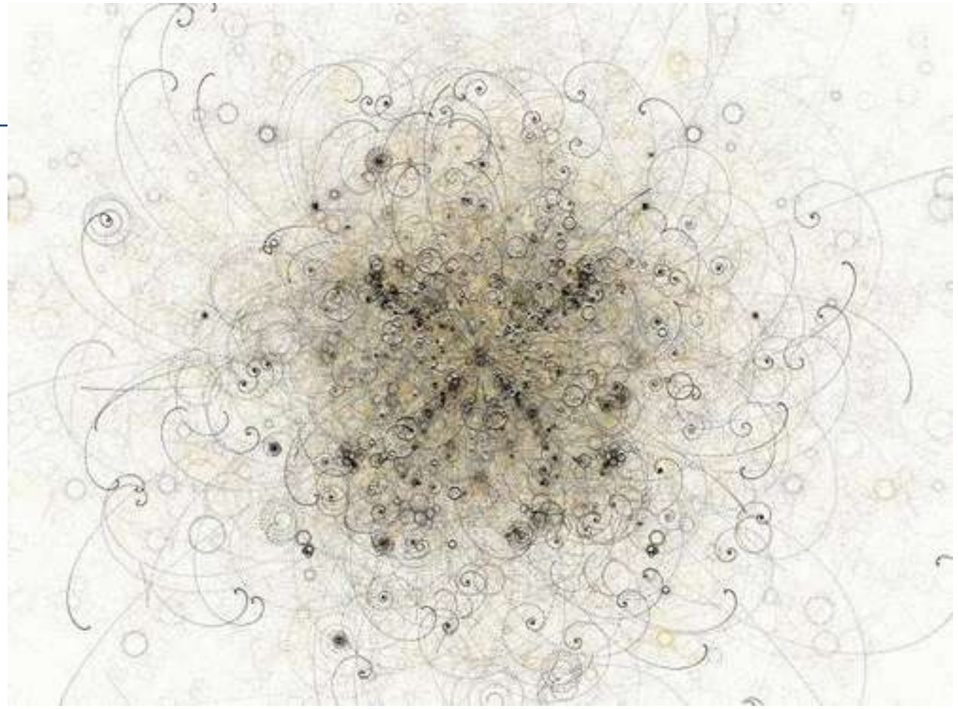


LECTURE 9
PARTICLE INTERACTIONS IN MATTER

PHY 493/803

Announcements



Quiz:
- Pick up quiz after class
- Next quiz on Friday

Homework:
Fourth HW will be posted later today

Paper: Outlines returned with some comments on gradescope later today.

Outline Comments

- Overall, really nice job!

- Abstracts: typically ~ paragraph length, can include some details related to the main conclusions of the paper.

- Include a reference section! References should also be included throughout the text

Recap / Up Next

Last time:

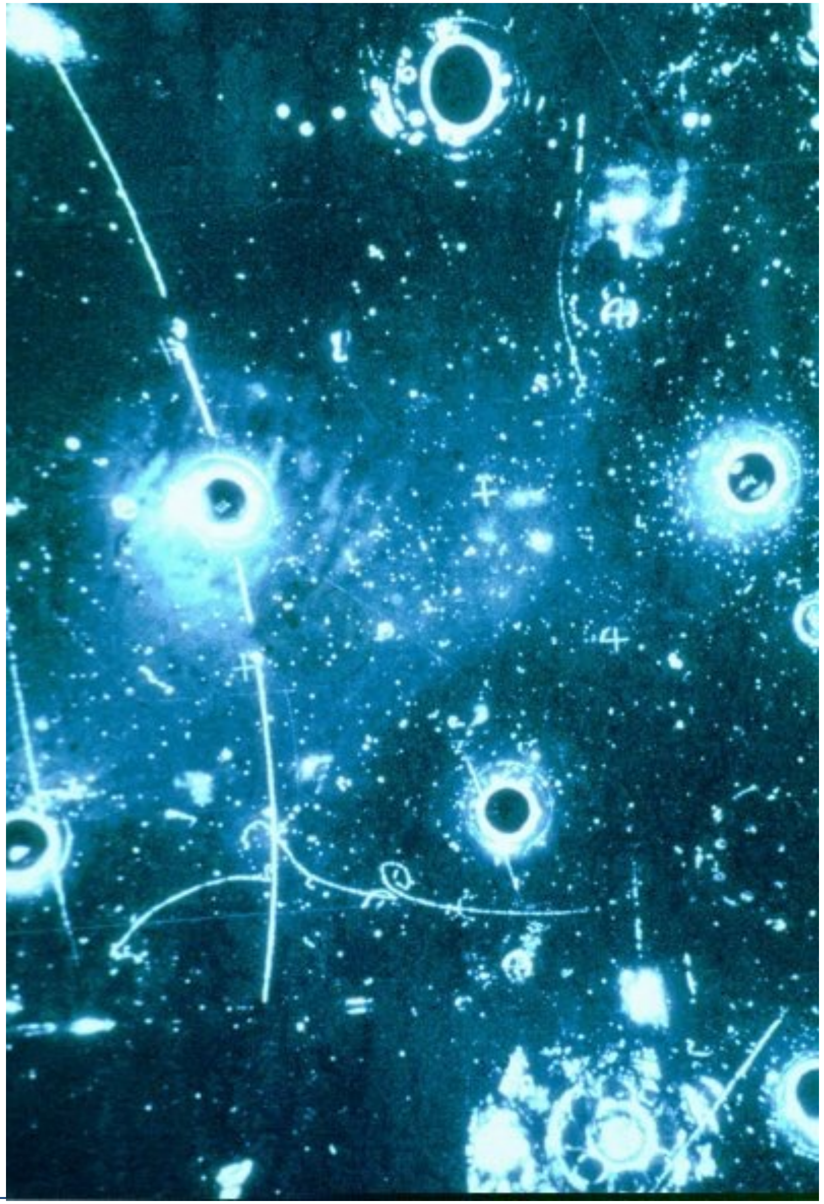
Particle Accelerators

- Cyclotrons
- Synchrotrons
- The Large Hadron Collider

This time:

Particle Interactions with Matter

- Ionization
- Radiation
- High/low energy losses



- Material for this week, last week and next week is not covered in the textbook
- Instead, read the corresponding PDG reviews
- This week: PDG review of particle interactions with matter, https://pdg.lbl.gov/2021/reviews/rpp2021-rev-passage-particles-matter.pdf
  - This is chapter 34 in the PDG

Particle Detection Principles

We need to detect particles to:

- Understand their properties
- Study particles that don't exist at everyday energies
- Take advantage of their properties in applications

In order to detect a particle, it must:

- Interact with the material of the detector
- Transfer energy in some identifiable manner

To do so, we must exploit particle properties themselves:

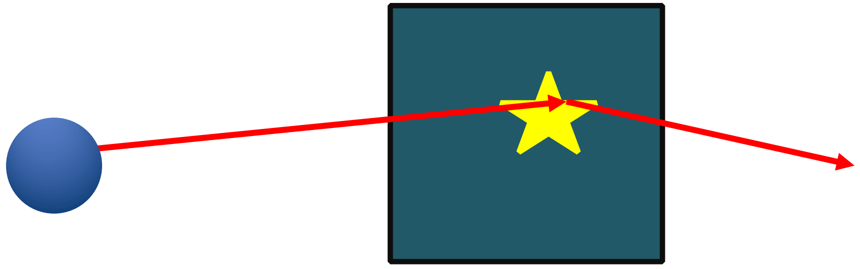
- Elementary particles have quantized (& known) electric charge
- Only quarks and gluons interact via the strong nuclear force
- Neutrinos interact very rarely

Particle Interactions with Matter

In order to be detected, a particle must have an interaction with the material of a detector.

Particles can interact with:

- **atoms/molecules**
- **atomic electrons**
- **nucleus**



Short-range interaction

- strong nuclear interaction
- weak interaction

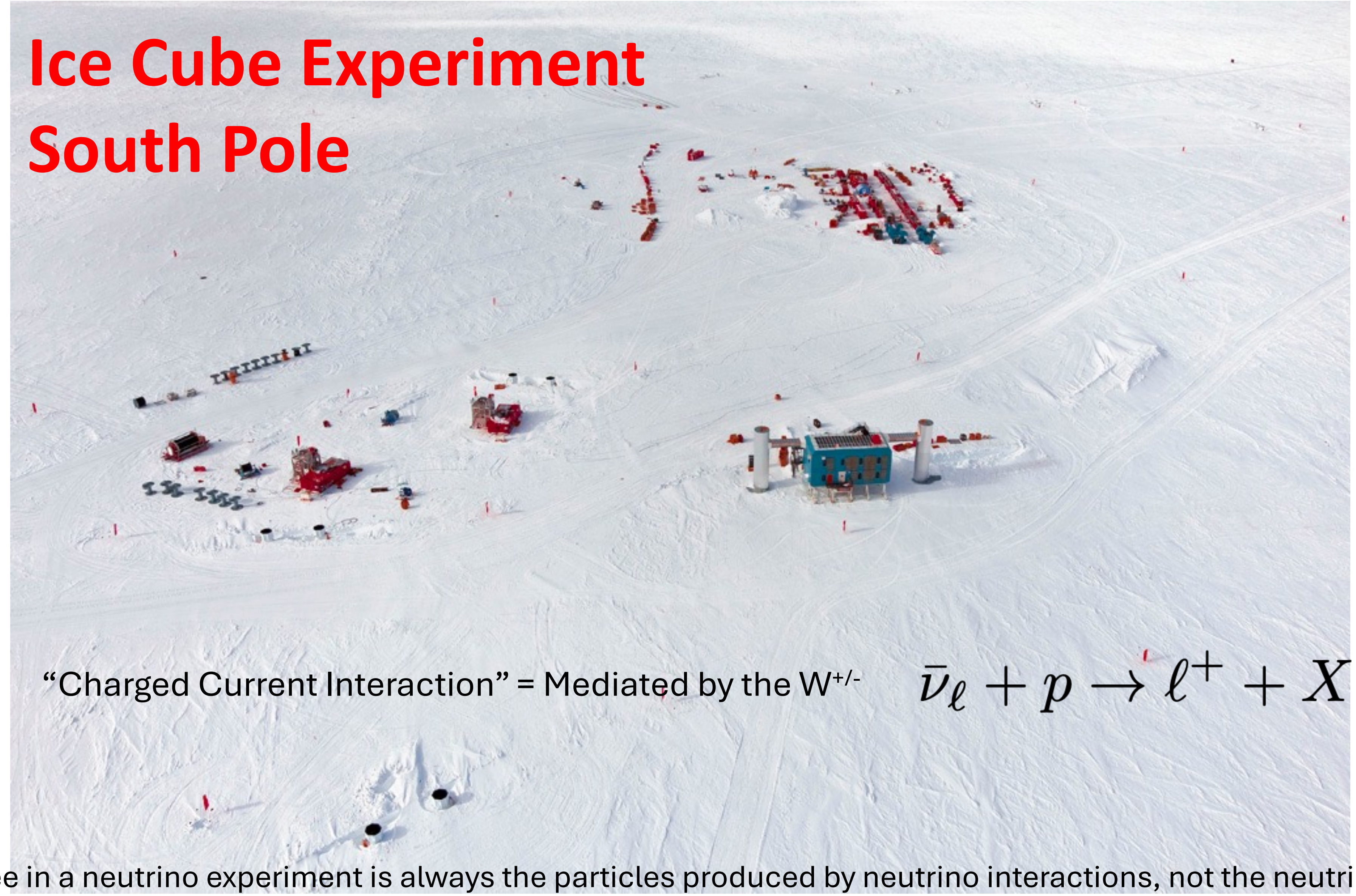
Long-range interaction

- electromagnetic interaction
  - ionization energy loss
  - radiation energy loss
  - pair production

Specific Examples:

Charged Particles	Ionization, Bremsstrahlung, Cherenkov…
Hadrons	Nuclear interactions
Photons	Photo/Compton effect, pair production
Neutrinos	Weak interactions

A Motivating Example



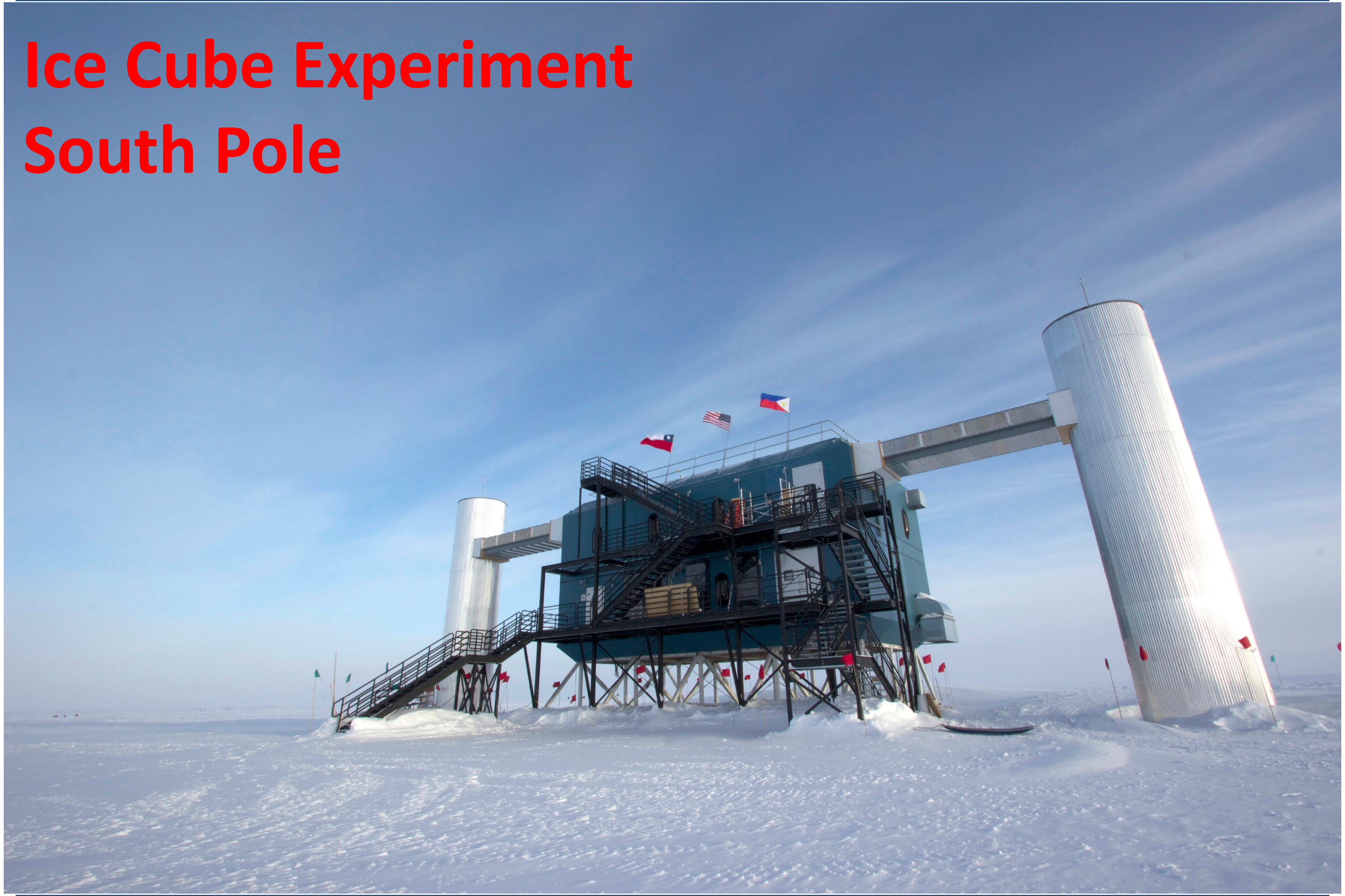
Ice Cube Experiment

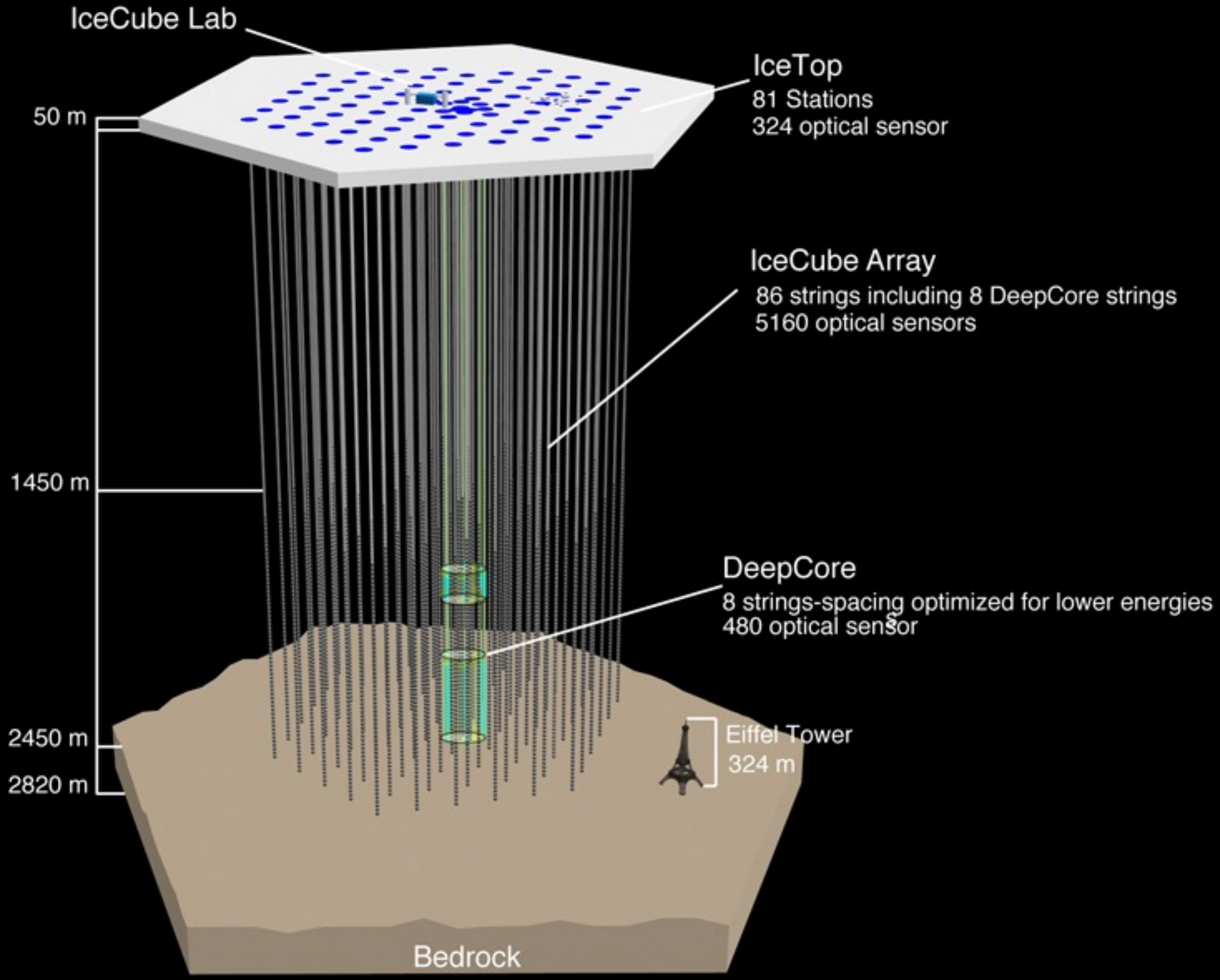
South Pole

"Charged Current Interaction" = Mediated by the $W^{+/-}$ $\quad \bar{\nu}_\ell + p \to \ell^+ + X$

What you see in a neutrino experiment is always the particles produced by neutrino interactions, not the neutrinos themselves

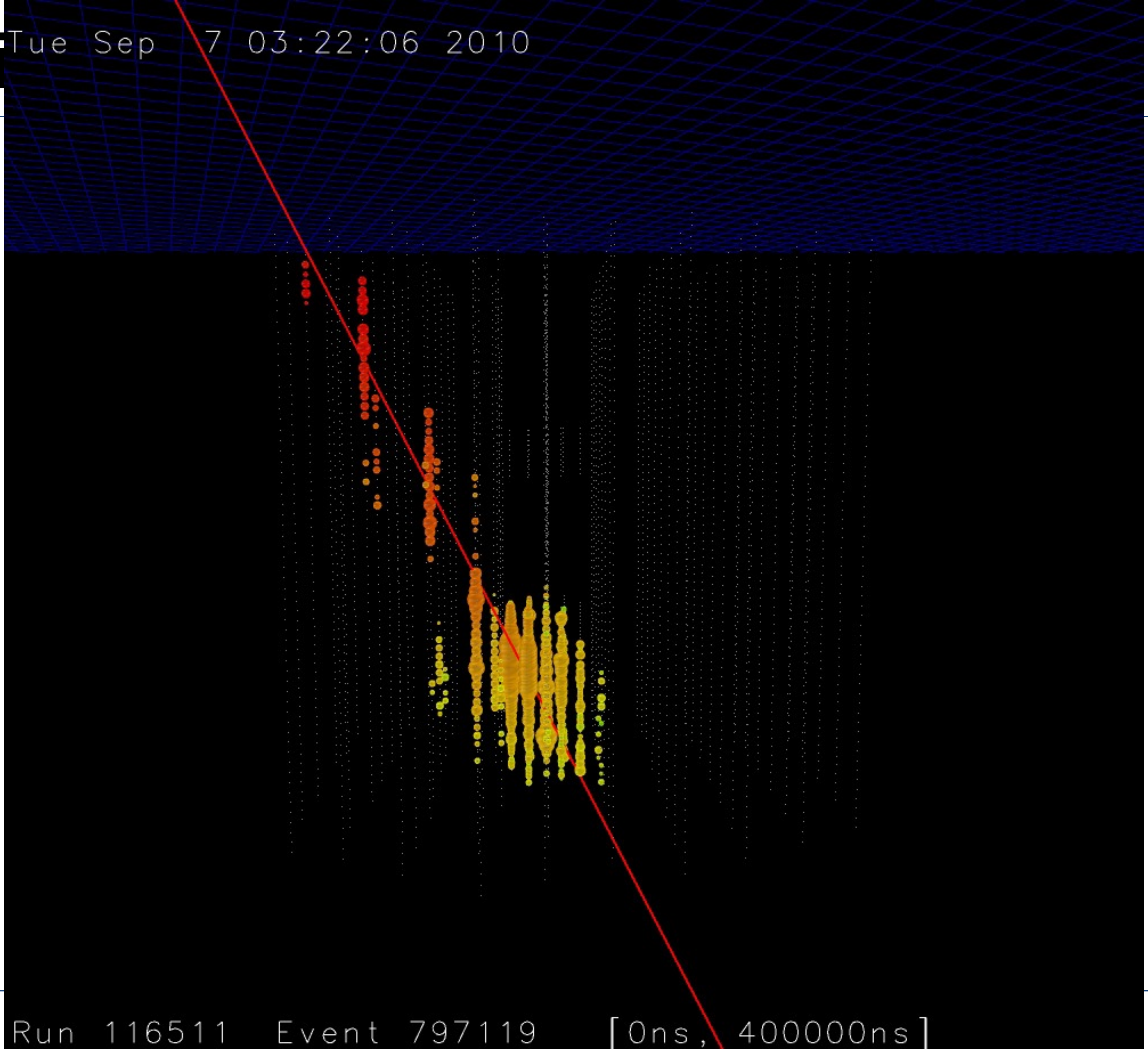
A Motivating Example

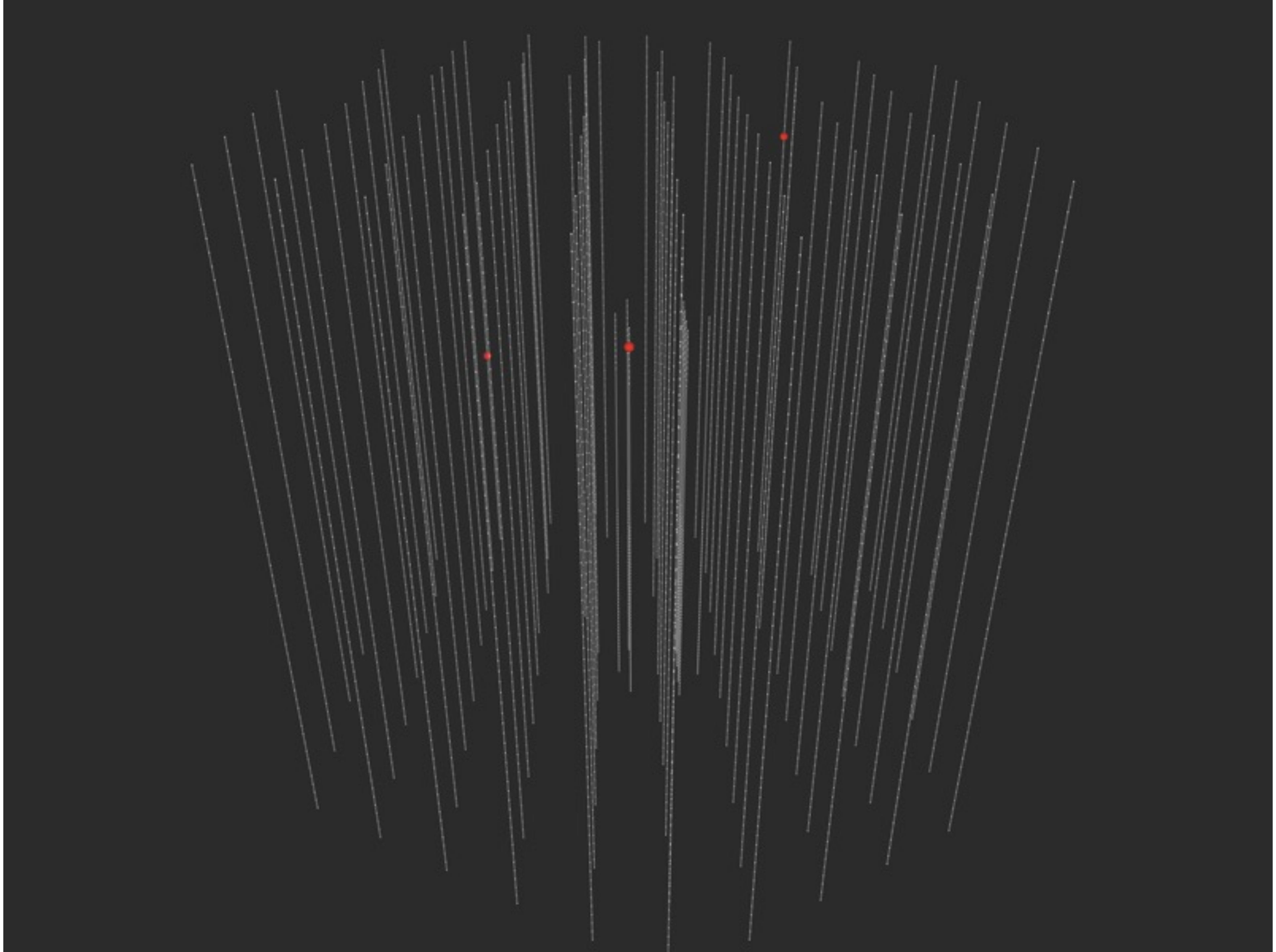



IceCube Lab
IceTop
81 Stations
324 optical sensor
50 m
IceCube Array
86 strings including 8 DeepCore strings
5160 optical sensors
1450 m
DeepCore
8 strings-spacing optimized for lower energies
480 optical sensor
2450 m
2820 m
Eiffel Tower
324 m
Bedrock

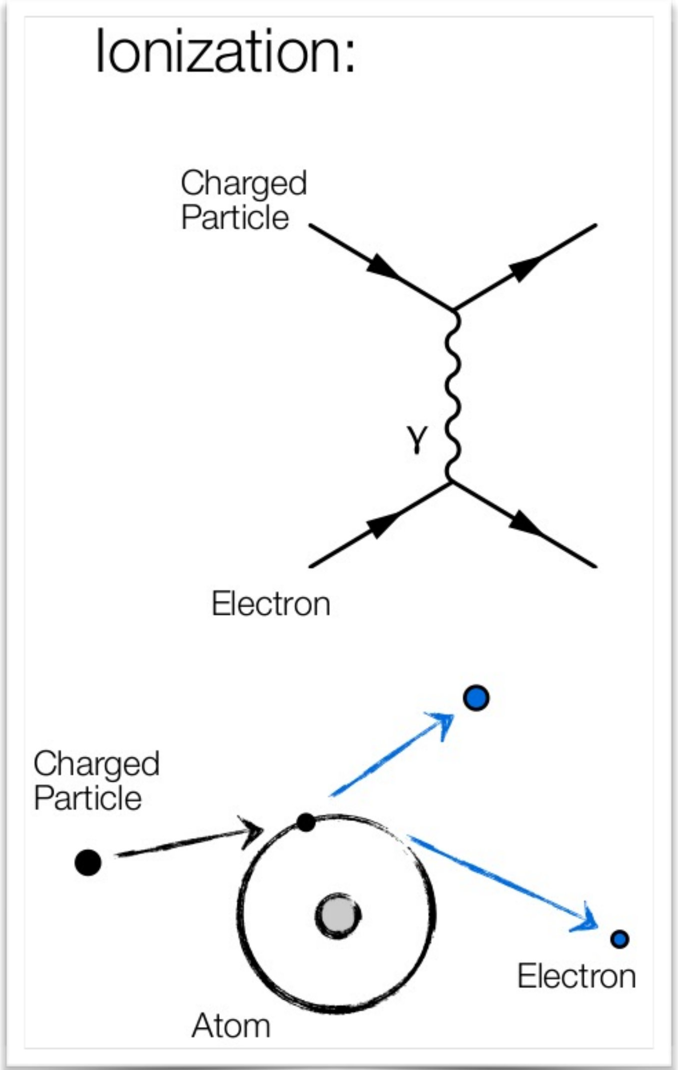
A Motivating Example

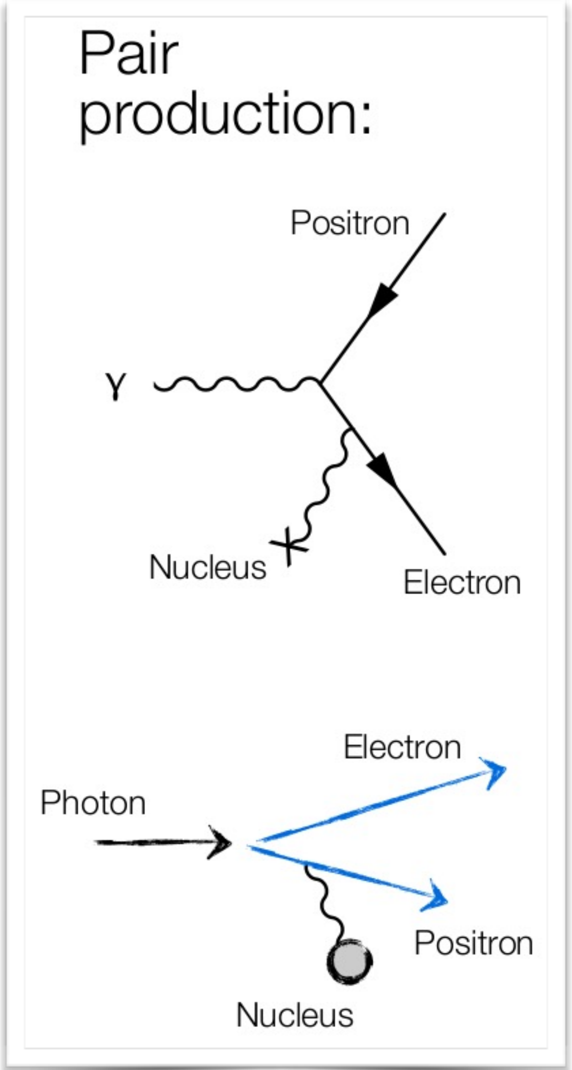
Ti

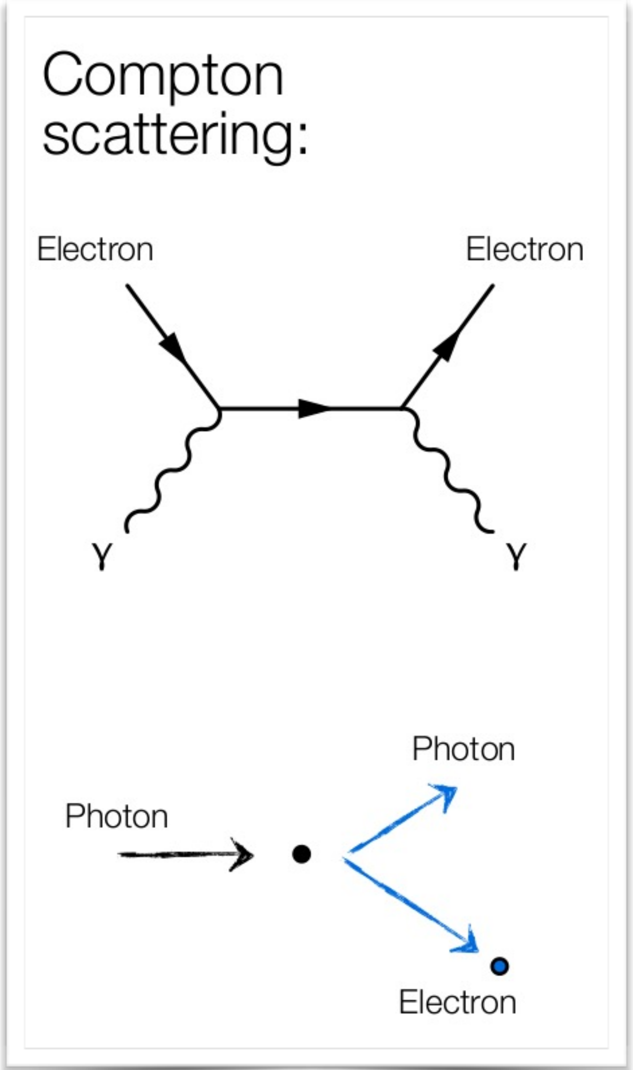




Particle Interaction Examples







Short-range Interactions with Nuclei

The probability of a particle interaction occurring as the particle traverses a small thickness *dx* of material is given by

$$dP = n\ \sigma_{\text{tot}}\ dx$$

n : number of nuclei per unit volume or N/V,

σ_{tot} : total cross section of hadron-nucleus scattering

Collision length (*Lc*) : the mean distance travelled before an interaction occurs

$$L_c = 1/n\sigma_{\text{tot}}$$

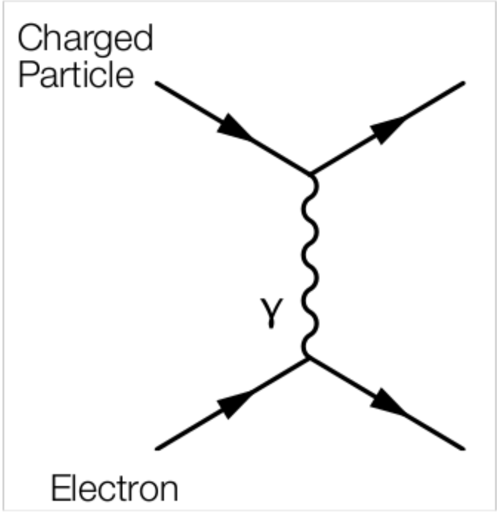
Absorption length (*La*) : the probability of an inelastic collision
(an incident particle is absorbed or changed)

$$L_a = 1/n\sigma_{\text{inel}}$$

Energy Loss by Ionization — dE/dx

For now, assume that: $Mc^2 \gg m_e c^2$

i.e. energy loss is for heavy, charged particles
[*dE/dx for electrons more difficult to parametrize*]



Ionization interactions are dominated by elastic collisions with electrons:

Bethe-Bloch Formula

Energy loss per distance:

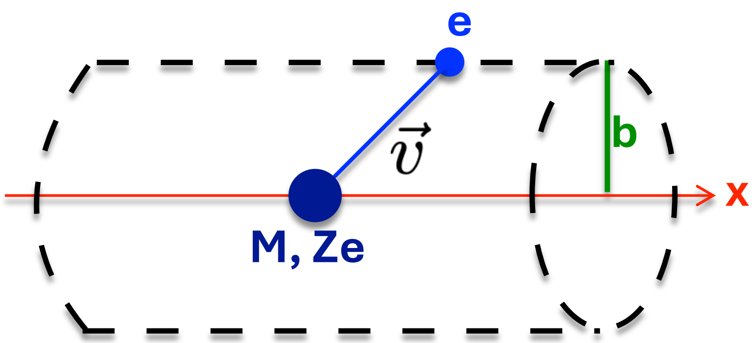
$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

$$\propto \frac{1}{\beta^2} \cdot \ln(\text{const} \times \beta^2 \gamma^2)$$

Bethe-Bloch — Classical Derivation

Consider a particle with charge Ze and velocity v moving through a medium with electron density n.

Electrons in the medium are considered free and initially at rest.



Interaction of a heavy charged particle with the electrons of an atom inside medium.

Momentum transfer perpendicular to the particle's moving direction: ie. Integrate over particle path

$$\Delta p_{\perp} = \int F_{\perp} dt = \int F_{\perp} \frac{dt}{dx} dx = \int F_{\perp} \frac{dx}{v}$$

Momentum transfer parallel to the particle's moving direction averages out!

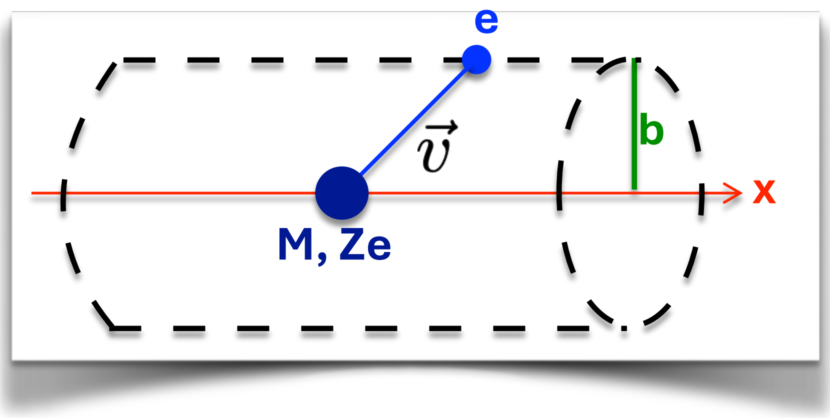
$$\Delta p_{||} = 0$$

Bethe-Bloch classical derivation

Consider a particle with charge Ze and velocity v
moving through a medium with electron density n.
Energy transfer to one electron from Gauss' law: $\int \vec{E} \cdot d\vec{A} = Q/\epsilon_0 = 4\pi Q$

$$\int \vec{E} \cdot d\vec{A} = \int \vec{E}_2 \cdot d\vec{A}_2 = \int E_\perp dA = \int E_\perp 2\pi b dx$$

E = Electric field, A = area, assume cylindrical shape



And therefore $\int E_\perp 2\pi b dx = 4\pi Ze$ or $\int E_\perp dx = \frac{2Ze}{b}$

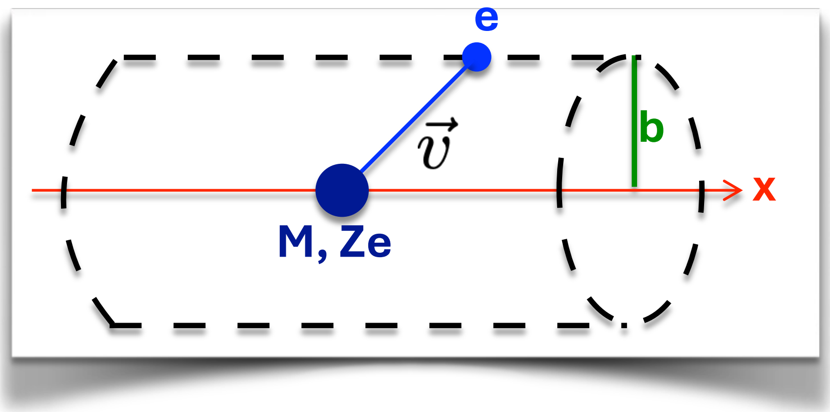
No need to solve this integral.

Instead, compute force on one electron in the medium:

$$F_\perp = eE_\perp$$

So, $\int F_\perp dx = e\int E_\perp dx = \frac{2Ze^2}{b}$

Bethe-Bloch classical derivation



The force on one electron is $F_\perp = eE_\perp$

The momentum transfer for one electron is $\Delta p_\perp = \int F_\perp dt$

Express the integral over time as $dt = dx/v$ and assume momentum transfer is small, velocity unchanged, then

$$\Delta p_\perp = \int F_\perp \frac{dx}{v} = e\int E_\perp \frac{dx}{v} = \frac{2Ze^2}{bv}$$

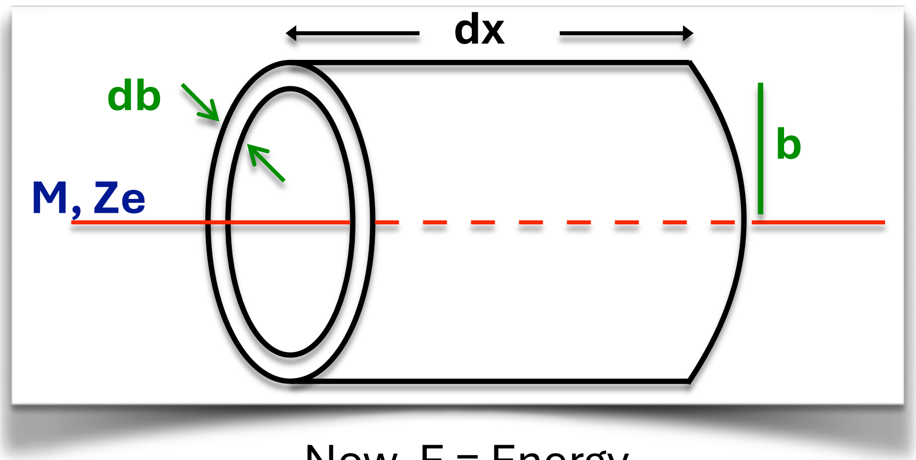
This is the momentum transfer to one electron.
Now consider the target material

Bethe-Bloch — Classical Derivation

Energy transfer (E) onto a single electron for impact parameter b:

$$\Delta E(b) = \frac{\Delta p^2}{2m_e} \quad \text{(Classical)}$$

Substitute from prev. slide $= \dfrac{\left(\frac{2Ze^2}{bv}\right)^2}{2m_e} = \dfrac{4Z^2e^4}{2b^2v^2m_e}$



Now, E = Energy

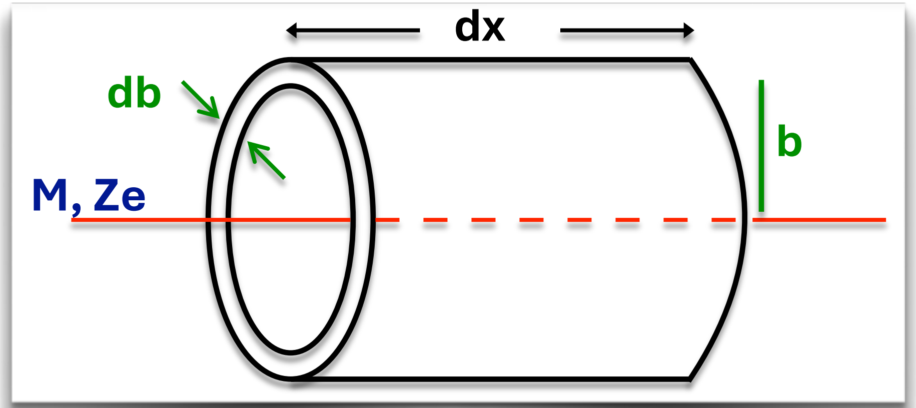
Number of electrons in a cylindrical barrel with thickness db: $N = n_e(2\pi b)dbdx$

Energy loss - *dE* per path length dx for electrons at a distance between b and b+db in medium with electron density n_e:

$$-dE(b) = \frac{\Delta p^2}{2m_e} n_e\,(2\pi b)\,db\,dx = \frac{4n_eZ^2e^4}{2b^2v^2m_e}(2\pi b)\,db\,dx = \frac{4\pi n_eZ^2e^4}{m_ev^2}\frac{db}{b}dx$$

Bethe-Bloch — Classical Derivation

Express energy loss per length as $-\dfrac{dE}{dx}$
Want to integrate over db to find the total energy loss -
But this diverges for b→0!



$$-dE(b) = \frac{\Delta p^2}{2m_e} n_e \, (2\pi b)\, db\, dx = \frac{4 n_e Z^2 e^4}{2b^2 v^2 m_e} (2\pi b)\, db\, dx = \frac{4\pi n_e Z^2 e^4}{m_e v^2} \frac{db}{b} dx$$

So we integrate over the "relevant" range [b_{min}, b_{max}]:

$$-\frac{dE}{dx} = \frac{4\pi n_e Z^2 e^4}{m_e v^2} \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \frac{4\pi n_e Z^2 e^4}{m_e v^2} \ln \frac{b_{\max}}{b_{\min}}$$

Bethe-Bloch — Classical Derivation

We need to determine the relevant range [b_{min}, b_{max}]:

b_{min}: Lower limit is the electron de Broglie wavelength (quantum effects)

Or equivalently, base lower limit on the Heisenberg uncertainty principle

$$b_{\text{min}} = \lambda_e = \frac{h}{p} = \frac{2\pi\hbar}{\gamma m_e v}$$

(Can't get infinitely close to interacting particle)

Bethe-Bloch — Classical Derivation

We need to determine the relevant range [b_{min}, b_{max}]: In a classical picture

b_{max}: Interaction time (b/v) must be shorter than the period of the electron orbit ($\gamma\langle\nu_e\rangle$) to guarantee the relevant energy transfer (adiabatic invariance – v is fast enough that particle sees electrons in a snapshot of time).
$\langle\nu_e\rangle$ is the average revolution frequency of the electron.

$$b_{\text{max}} = \frac{\gamma v}{\langle\nu_e\rangle}$$

Substitute b(max) and b(min):

$$-\frac{dE}{dx} = \frac{4\pi n_e Z^2 e^4}{m_e c^2 \beta^2} \ln \frac{m_e c^2 \beta^2 \gamma^2}{2\pi\hbar \langle\nu_e\rangle}$$

We can recognize/rewrite some of these pieces of the equation:

Electron density : $n = N_A \cdot \rho \cdot Z/A$

Effective ionization potential (“liberation energy”): $I \sim h\langle\nu_e\rangle$

Bethe Bloch Equation

$$-\frac{dE}{dx} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]\times\rho$$

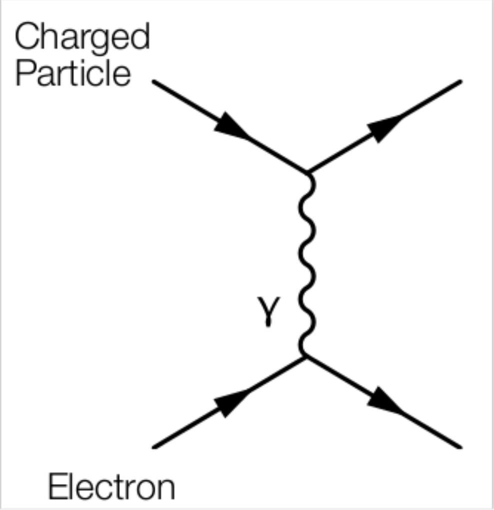
$K = N_Ae^2/\epsilon_0$ = 0.307 MeV cm^2 / g

$$T_{\max} = 2m_ec^2\beta^2\gamma^2/(1+2\gamma m_e/M+(m_e/M)^2)$$

z : charge of incident particle
M : mass of incident particle

Z : charge number of medium
A : atomic mass of medium

I : mean excitation energy of medium
δ : density correction [dielectric screening for highly relativistic particles]



Valid for $0.5<\beta\gamma<500$
$M > m_\mu$

(Not for electrons)

Bethe Bloch Equation

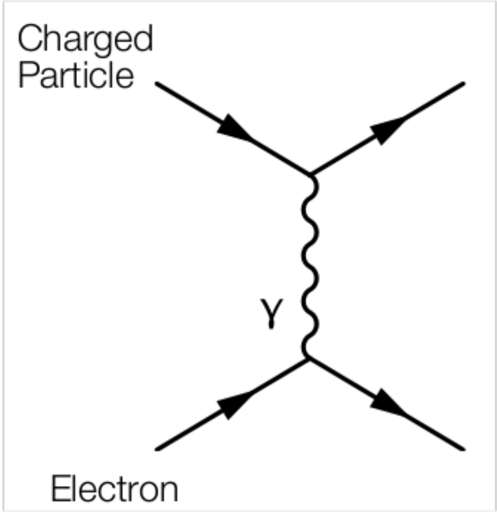
$$-\frac{dE}{dx} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]\times\rho$$

Bethe-Bloch formula gives the linear stopping power in MeV/cm

The PDG review gives mass stopping power,

$$-\langle\frac{dE}{dX}\rangle = -\frac{dE}{\rho dx}$$

...and the units are MeV cm^2 / g



Work backwards from deposited energy to the energy of a particle (eventually it will have deposited all of its energy)

Valid for $0.5<\beta\gamma<500$
$M > m_\mu$