PHY493/803, Intro to Elementary Particle Physics
Homework 7 – Due April 25th

Please clearly state any assumptions, show all your work, number the equations, and indicate logical connections between the lines.

1. (10 + 5 + 5 pts) Draw the lowest order diagrams for elastic neutrino-electron scattering for the following (Griffiths 11.3):
   a. Electron neutrinos: $\nu_e + e^- \rightarrow \nu_e + e^-$
   b. Muon neutrinos: $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$
   c. Tau neutrinos: $\nu_\tau + e^- \rightarrow \nu_\tau + e^-$

   Hint: is there a difference with regards to electron neutrinos and muon and tau neutrinos?

2. (20 pts) Supernova SN1987A occurred in the Large Magellanic Cloud 1.7 x 10^5 light years from earth. Neutrinos from this explosion, with energies ranging from 20-30 MeV, were detected within a 10 second time interval. What upper bound on the neutrino mass does this imply? Assume all the neutrinos were produced at the same instant. (Griffiths 11.5b)
   Hint: Use the approximation that the velocity of an ultra-relativistic particle with mass m and energy E is approximately:

$$v \approx c\left[1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2\right]; \frac{1}{v} \approx \frac{1}{c}\left[1 + \frac{1}{2}\left(\frac{mc^2}{E}\right)^2\right]$$

3. (20 pts) By making the appropriate changes in the calculated lifetime of the muon, determine the lifetime of the tau lepton, pretending that its decay is purely leptonic. Also assume the muon mass can be neglected relative to the mass of the tau. Compare your result to the experimental value. Why might it be different? (Griffiths 9.5)

   The formula for the muon lifetime is:

$$\tau = \frac{1}{\Gamma} = \left(\frac{M_W}{m_\mu g_w}\right)^4 \frac{12\hbar(8\pi)^3}{m_\mu c^2}$$

4. (5+5+5 pts) **Required only for PHY 803 students. Extra credit for PHY493 students.** Consider two flavor neutrino oscillations:
   a. In the limit $|\Delta m^2| \gg (E/L)$, draw a sketch of $\sin^2(1.27\ \Delta m^2 L/E)$ as a function of L in the volume of a detector for a fixed energy E. That is, draw the $\sin^2$ function from L to L+ΔL, where ΔL is the length of the detector.

   b. In the same limit $|\Delta m^2| \gg (E/L)$, show that a measurement of the survival probability $P_{surv} = P(\nu_e \rightarrow \nu_e)$ determines the neutrino mixing angle to be $\sin^2 2\theta = 2(1 - P_{surv})$.

c. In the other limit, when $|\Delta m^2| \ll (E/L)$, you can use the small angle approximation ($\sin\Delta = \Delta$). Show that a given measurement of the survival probability $P_{surv} = P(\nu_e \rightarrow \nu_e)$ determines the neutrino mixing to be $\sin^2 2\theta = C\left(\frac{1}{\Delta m^2}\right)^2$, with a constant of proportionality $C = (1 - P_{surv})(E/1.27L)^2$.