PHY 491, Fall 2024 - Homework 9

DUE: Monday 11/4/2024, 11:59pm

Problem 4.1 Consider a two-dimensional triangular lattice described by the two primitive vectors (in an orthogonal coordinate system)

$$\vec{a}_1 = a(1,0); \quad \vec{a}_2 = (\frac{1}{2}, \frac{\sqrt{3}}{2})$$

4.1.1 Find the two primitive lattice vectors $\vec{b}_1, \vec{b_2}$ describing the reciprocal lattice. (2 Points)

4.1.2 Sketch the reciprocal lattice and construct the area of the 1st Brillouin zone. (3 Points)

4.1.3 Calculate the area of the 1st Brillouin zone. (2 Points)

Problem 4.2 Consider a simple square lattice in two dimensions.

4.2.1 Show that the kinetic energy of a free electron at a corner of the first Brillouin zone is higher than that of an electron at the midpoint of a side face of the zone by a factor of 2. (5 Points)

4.2.2 What is the corresponding factor for a simple cubic lattice (three dimensions)? (4 Points)

4.2.3 The Fermi energy for a non-interacting ensemble of identical spin-1/2 fermions in a three-dimensional system is given by

$$E_F = \frac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$$

with fermion mass m_e and electron density (electrons per unit cell volume) n. Calculate E_F for the lattice in 4.2.2. (2 Points)

4.2.4 What bearing might the result of 4.2.3 have on the conductivity of divalent metals? (2 points)