lattice plane: A plane containing (at least) three non-colinear lattice points

family of lattice planes: infinite set of equally spaced lattice planes such that all lattice points are included in one of the planes

Claim: A family of lattice planes is orthogonal to reciprocal lattice vector $\vec{G}$ and the spacing between neighbouring planes is

$$d = \frac{2\pi}{|\vec{G}_{min}|}$$

$\vec{G}_{min}$ = shortest recip. lattice vector in direction $\vec{G}$

if $\vec{G}$ is not $\vec{G}_{min}$

$(246) \to (123)$

Miller indices

$$\vec{G} = h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3 = (hkl)$$

assume $\vec{b}$s orthogonal:

$$d = \frac{2\pi}{\sqrt{h^2 b_1^2 + k^2 b_2^2 + l^2 b_3^3}}$$

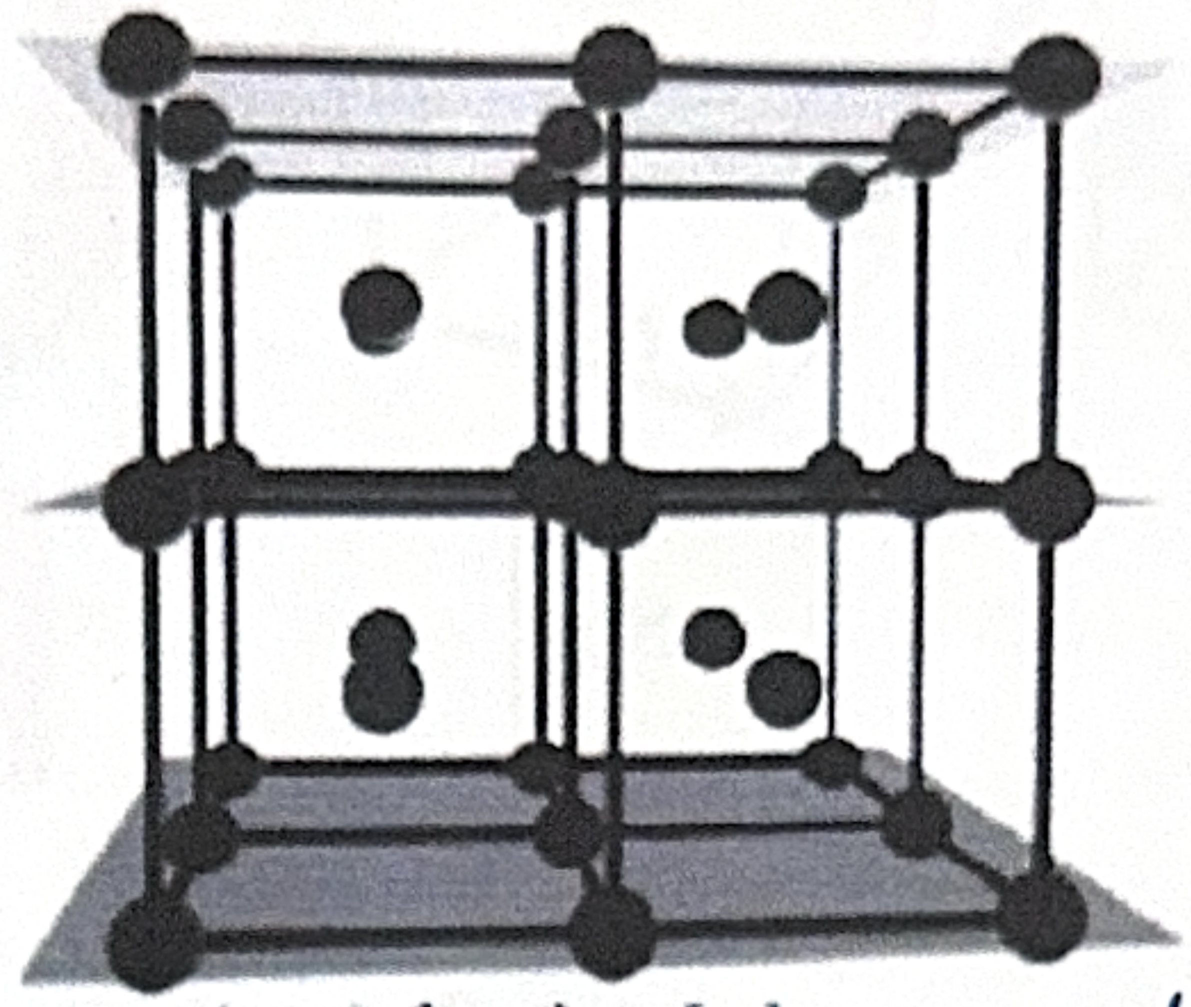
for cubic:

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

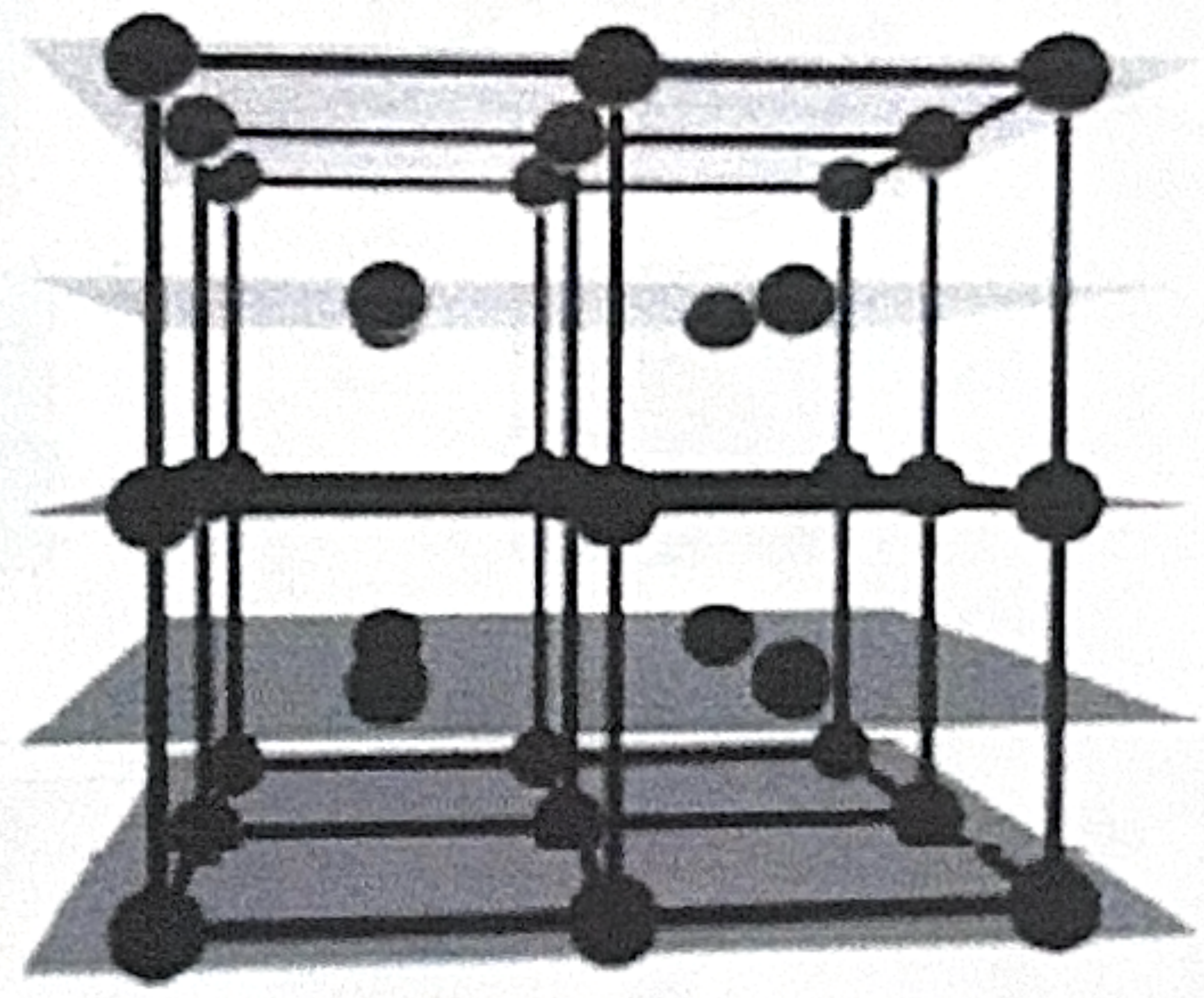
equivalent sets of Miller indices:

$\{hkl\}$ = equiv sets of (hkl)

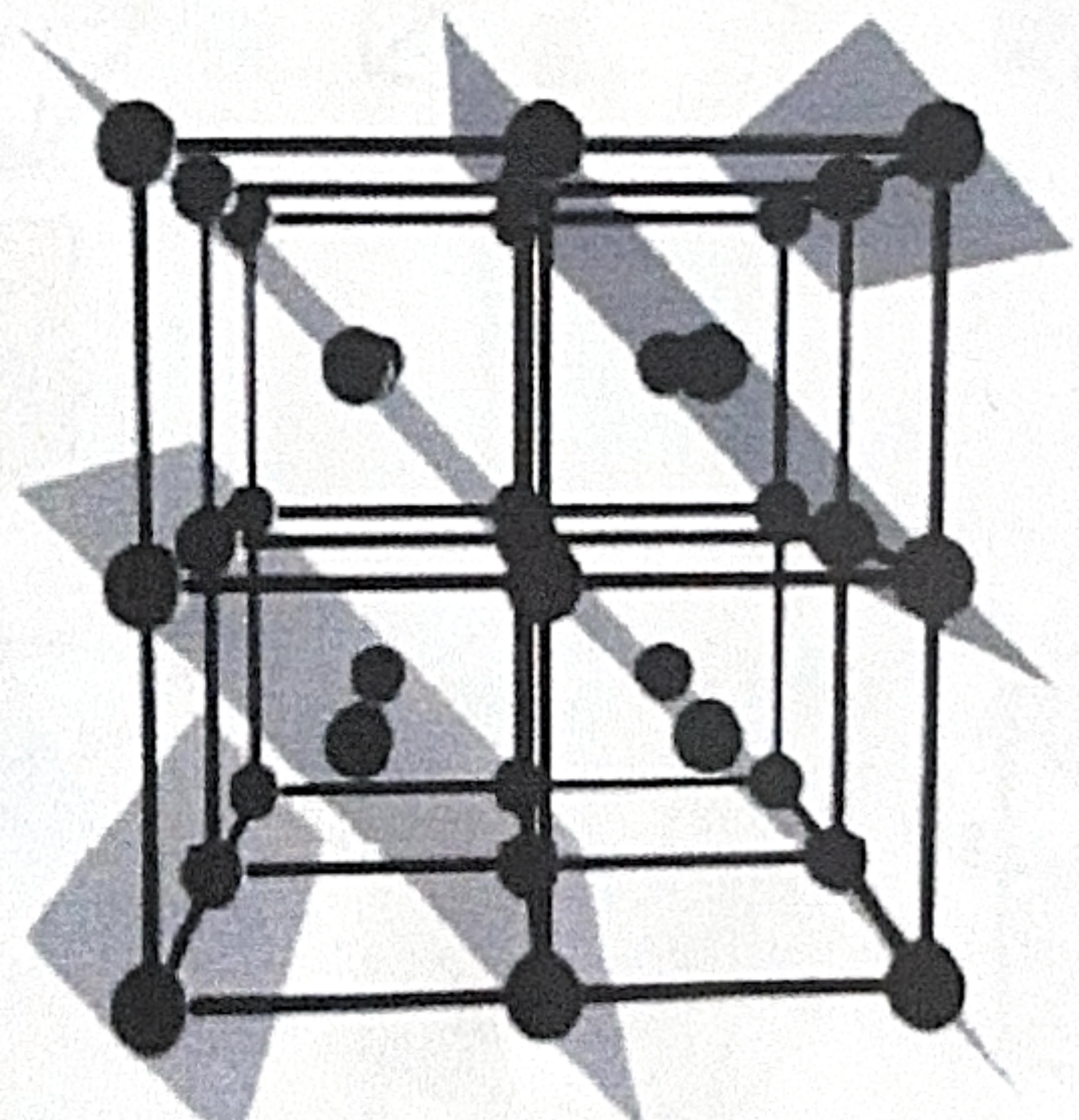
$$\{100\} = \underbrace{\begin{matrix} (100) & (\bar{1}00) \rightarrow (-1\ 0\ 0) \\ (010) & (0\bar{1}0) \\ (001) & (00\bar{1}) \end{matrix}}_{N = 6 \text{ (multiplicity)}}$$



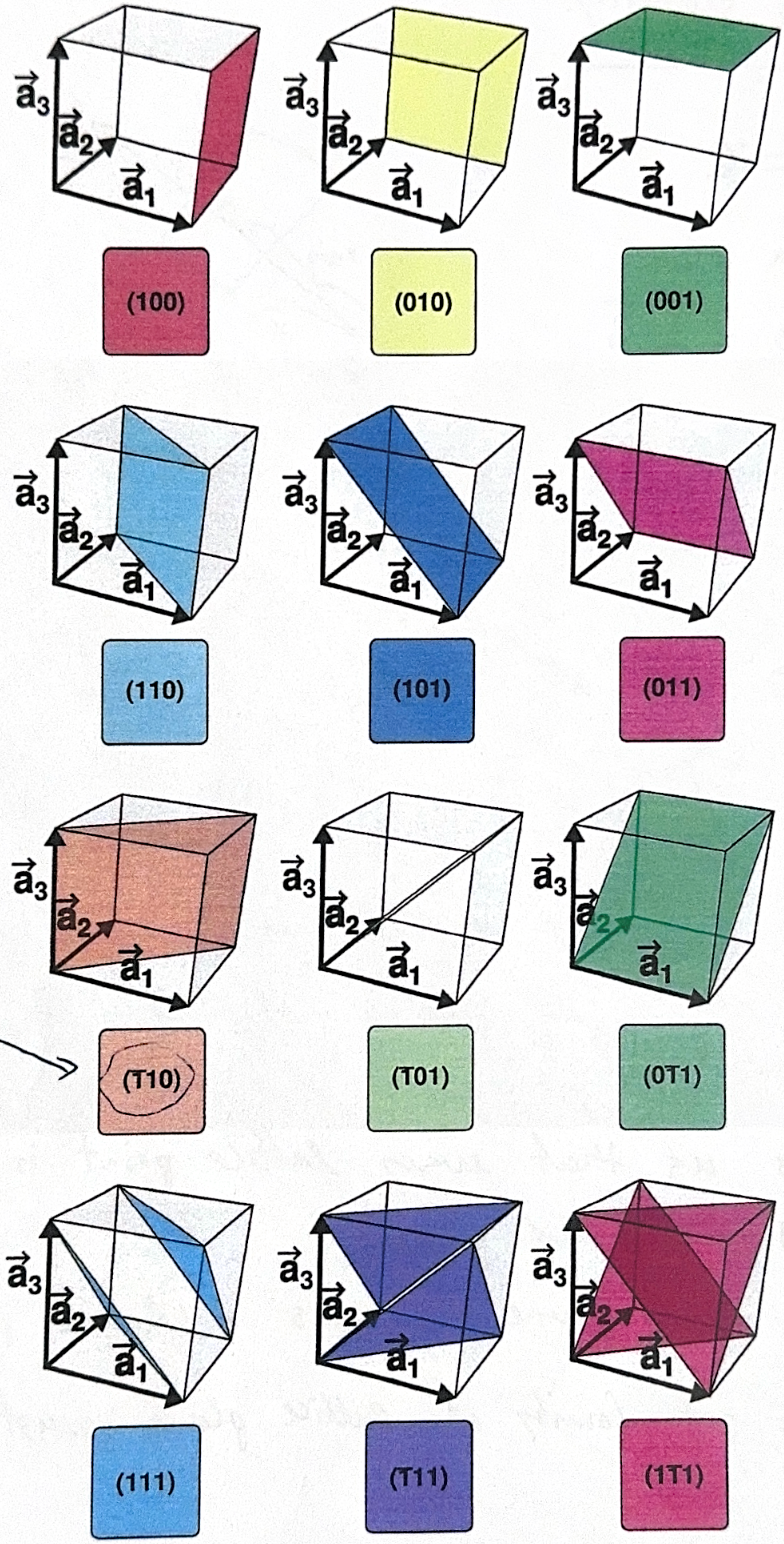
(010) ~~family of planes~~ not a family
(not all lattice points included)



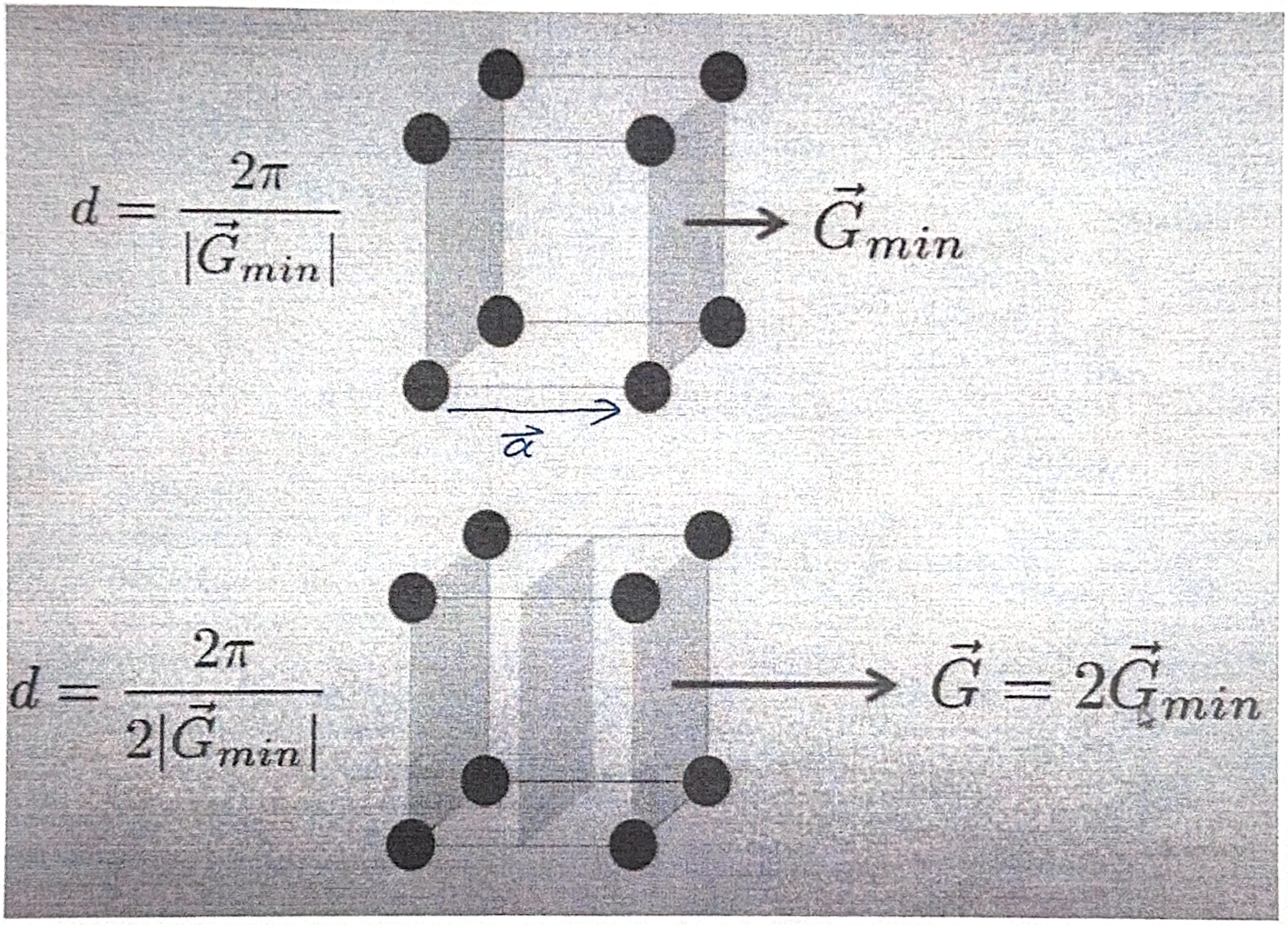
(020) family of lattice planes



(110) family of lattice planes

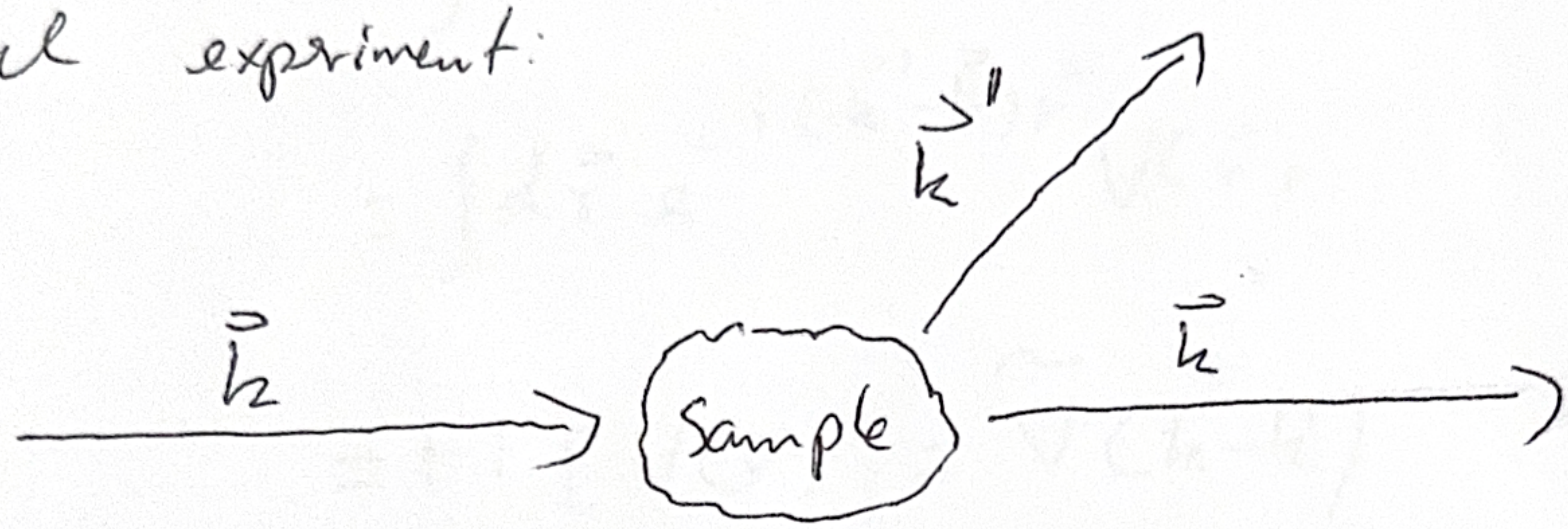


$(-110) \rightarrow (\bar{1}10)$


$d = \frac{2\pi}{|\vec{G}_{min}|}$
$\vec{G}_{min}$
$\vec{a}$
$d = \frac{2\pi}{2|\vec{G}_{min}|}$
$\vec{G} = 2\vec{G}_{min}$

Scattering Experiments

general experiment:



want to observe atoms ($\sim Å$). Use waves on same length scale

- X-rays ←
- neutrons
- electrons

atoms in sample (crystal) provide potential $V(\vec{r})$ that scatters wave

$$\Gamma(\vec{k}', \vec{k}) = \frac{2\pi}{\hbar} |\langle k' | V(\vec{r}) | k \rangle|^2 \underbrace{\delta(E - E')}_{\text{elastic scattering}}$$

Fermi's Golden Rule

scattering rate

elastic scattering $E = E' \rightarrow |\vec{k}| = |\vec{k}'|$

assume plane waves:

$$\langle k' | V(\vec{r}) | \vec{k} \rangle = \int d\vec{r}\; e^{-ik'\vec{r}}\, V(\vec{r})\, e^{ik\vec{r}}$$

$$= \int d\vec{r}\; e^{-i(\vec{k}' - \vec{k})\vec{r}}\, V(\vec{r})$$

$$= FT[V(\vec{r})] = \tilde{V}(k-k')$$