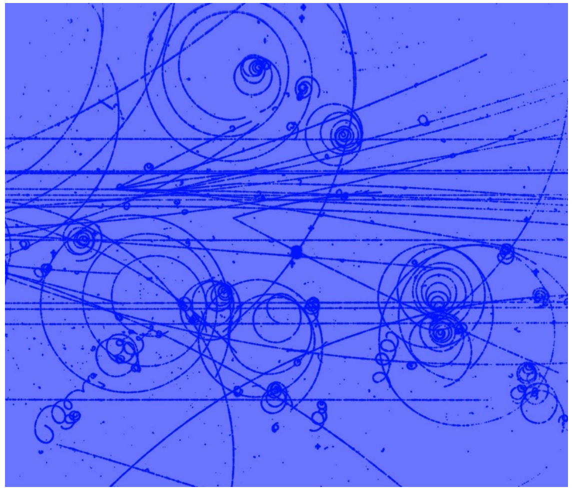
Announcements



Quiz:
- Pick up quiz after class if you have not yet. Next quiz today.

Homework:
Homework 1 grades posted on gradescope. Reach out to Alejandro (with me in cc) if you have questions. Solutions posted on D2L later today.
Third HW posted on D2L after class today: Q1/Q2 can be done now

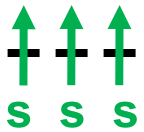
Midterm: Friday, Feb 21 in class
Will cover through “bound states”.
Equation sheet: 1 letter-sized (8 ½ by 11 inches) page front and back, handwritten

Paper:
Topic due Monday Feb 16 at 3pm. Fill out this google form before then:
https://forms.gle/MmCk8NtrMm7RdfLC7

Office hours: next week on Wednesday 4-5pm, not on Friday

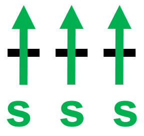
Baryon Bound States

Omega baryon is made of 3 s quarks. But what about the Pauli exclusion principle?



Baryon Bound States & Color

Omega baryon is made of 3 s quarks. But what about the Pauli exclusion principle?



(***r***, ***g***, ***b***)

All three quarks must be in different color states.

Total wave function Ψ includes a color component:

$$\Psi = \underbrace{\Psi_{\text{space(r)}}}_{\text{symmetric}} \; \underbrace{\Psi_{\text{spin}} \; \Psi_{\text{flavor}}}_{\text{symmetric}} \; \underbrace{\Psi_{\text{color}}}_{\text{anti-symmetric}}$$

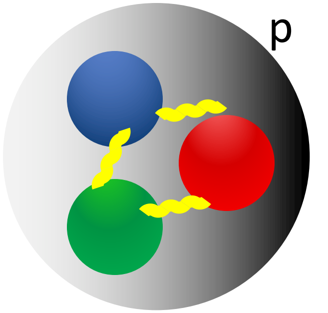
Every naturally occurring particle is a color singlet. For baryons the color state is:

$$\Psi_{\text{color}} = \frac{1}{\sqrt{6}}\left[\; r_1 g_2 b_3 + g_1 b_2 r_3 + b_1 r_2 g_3 - r_1 b_2 g_3 - b_1 g_2 r_3 - g_1 r_2 b_3 \;\right]$$

-> Same for all baryons

Reminder: Color

Color : New degree of freedom for quarks



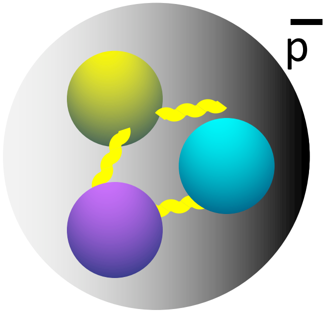
(a) Any quark can exist in three different color states (red, green, blue).

(b) Each of states is characterized by two conserved color charges (color isopin I_3^C, and color hyper charge Y^C)

Quarks	I_3^C	Y^C	Antiquarks	I_3^C	Y^C
r	1/2	1/3	$\bar{r}$	−1/2	−1/3
g	−1/2	1/3	$\bar{g}$	1/2	−1/3
b	0	−2/3	$\bar{b}$	0	2/3

these charges do not depend on flavor of quarks (u,d,c, …).

(c) Color confinement :
only states with zero color charges are observable as free particles (color singlets).



Why are there 8 gluons?

There are three colors, which allow us to write down the following combinations:

$r\bar{b}$ $\quad b\bar{g}$ $\quad g\bar{r}$

$r\bar{g}$ $\quad b\bar{r}$ $\quad g\bar{b}$

$$(r\bar{r} - g\bar{g}) / \sqrt{2}$$

$$(r\bar{r} + g\bar{g} - 2b\bar{b}) / \sqrt{6}$$

$$(r\bar{r} + g\bar{g} + b\bar{b}) / \sqrt{3}$$

Why are there 8 gluons?

What are 8 color states for gluons ?

$r\bar{b}$ $b\bar{g}$ $g\bar{r}$

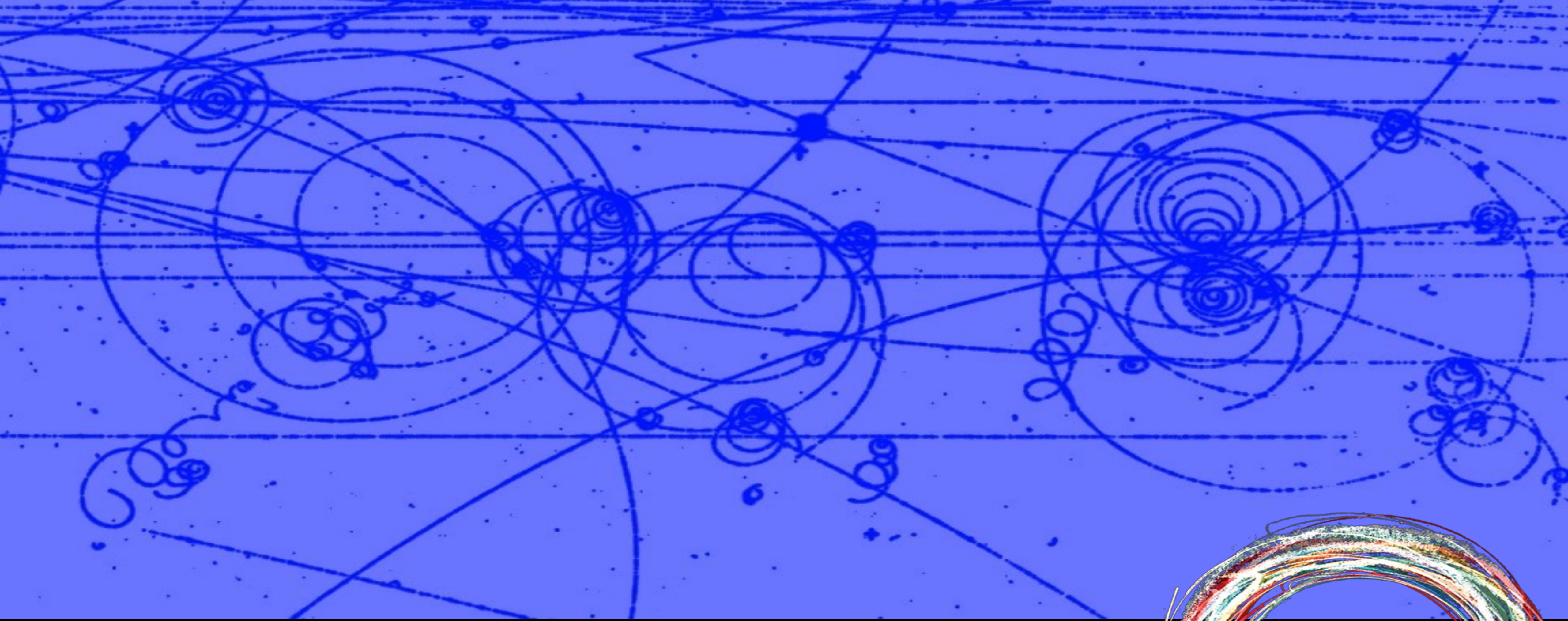
$r\bar{g}$ $b\bar{r}$ $g\bar{b}$

$$(r\bar{r} - g\bar{g}) / \sqrt{2}$$

$$(r\bar{r} + g\bar{g} - 2b\bar{b}) / \sqrt{6}$$

$$\cancel{(r\bar{r} + g\bar{g} + b\bar{b}) / \sqrt{3}}$$

The ninth one has the combination r+g+b, which is color neutral
- Would be a colorless gluon (color singlet) that does not feel the strong interaction
 -> Photon equivalent of the strong interaction



LECTURE 7
FEYNMAN RULES

PHY 493/803

Recap / Up Next

Last time:

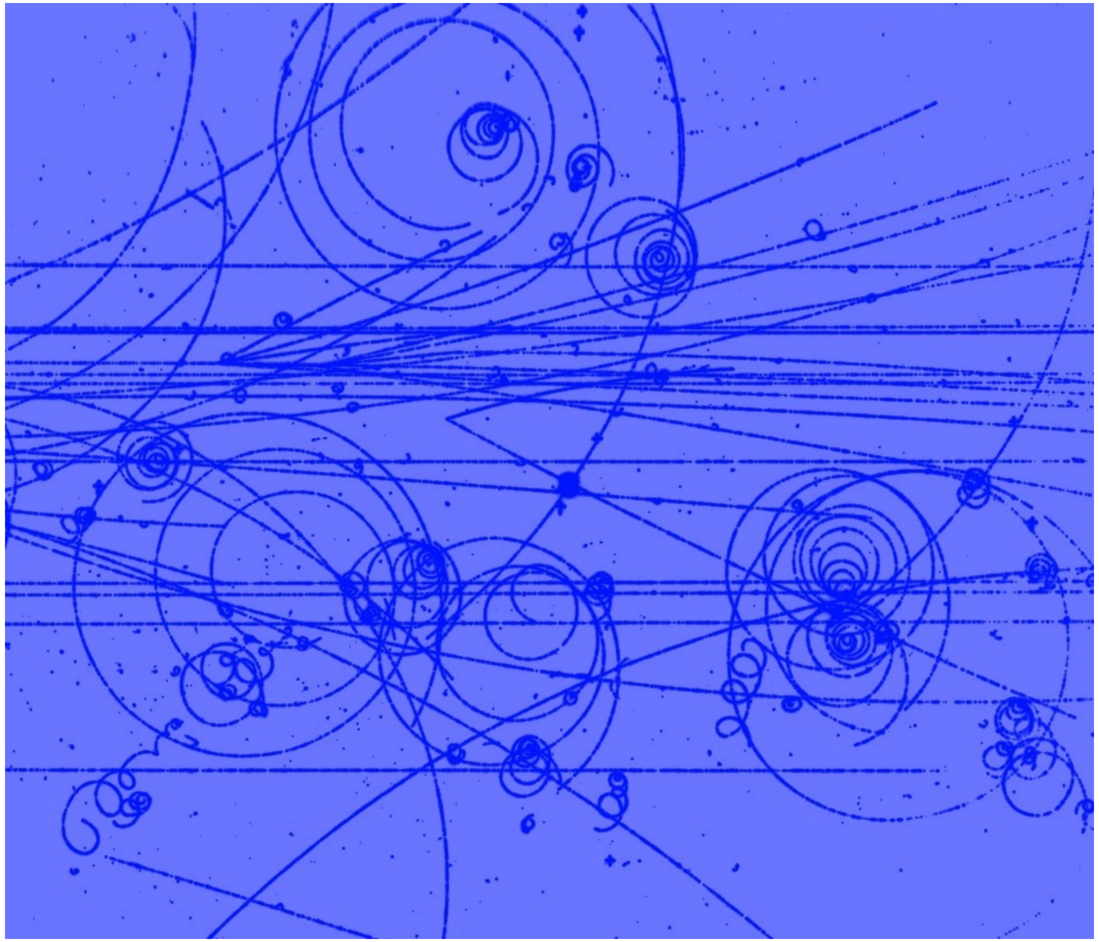
Bound States

- Hydrogen
- ‘Onium
- Mesons/Baryons

This time:

Feynman Calculus

- Decays/Scattering
- The Golden Rule
- Feynman Rules



Observables

To learn anything about particle interactions, we have to observe something.

There are three basic categories:

A) Bound states & their spectra (this was last class)

B) **When particles are "Left Alone", they can:**

1) Do nothing
2) Decay
3) Eventually find another particle and go to category C.

C) **When particles encounter another particle, they can:**

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Decay rate (Γ): Describes how quickly particles disappear.

Lifetime (τ): Describes how long particles stick around on average.

$$\tau = 1/\Gamma$$

Decay Rate and Lifetime

Particles have no concept of history

The probability a particle will decay in a fixed period is constant.

A population of decaying particles can be described as a function of time.

N = Number of particles

$$\begin{aligned}\Delta N(t) &= N(t_a) - N(t_b) \\ &= p\,N(t_b)\,(t_a - t_b)\end{aligned}$$

Also write in terms of probability, given the difference in time between a and b

Written differentially, we have: $$\mathrm{d}N = -\Gamma N \mathrm{d}t$$

Γ is the decay rate, the probability per unit time for a decay to occur
Units: decay rate in 1/s
In natural units: decay rate in eV

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Written differentially, we have: $$\mathrm{d}N = -\Gamma N \mathrm{d}t$$

The number of particles *N* left over after time *t* is given by: $$\boxed{N(t) = N_0 e^{-\Gamma t} = N_0 e^{-t/\tau}}$$

Decay rates & rules

Generally, all particles are unstable and can decay. Rough rules:

1) Energy is conserved: the final state cannot have more total mass
2) If there is not a lower energy/mass state, the particle cannot decay.
3) The decay must satisfy all conservation rules. eg, charge conservation.

Decay rates & rules

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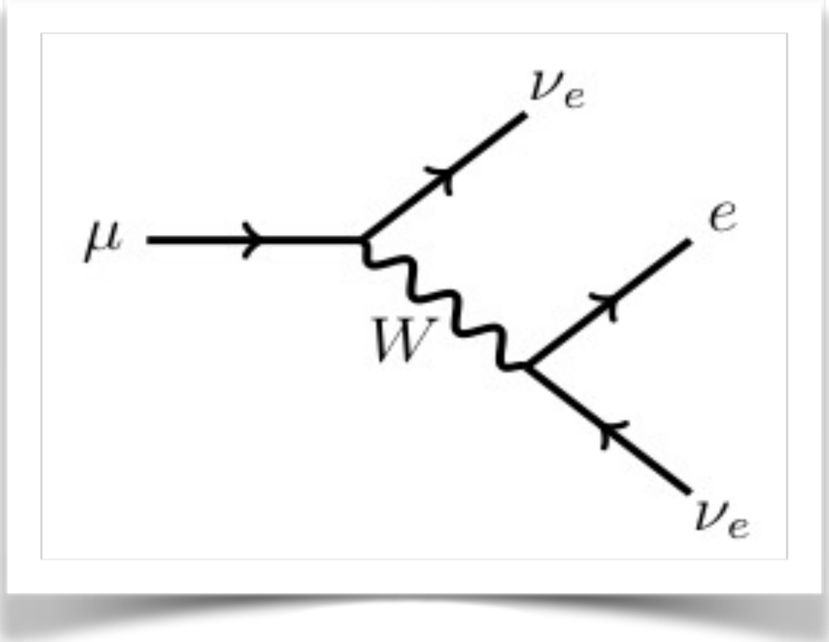
Consider muon decay:

- The final state mass is much smaller than the initial state mass: 0.5 MeV vs 106 MeV
- The W boson must be virtual (M_W~80 GeV)
- The final state momentum is:
  - Shared among 3 particles
  - Determined by the difference in mass between initial and final states.



W boson decays

- Particles can decay in many ways, not just one, and allowed decays happen
- Example: a W boson can decay to a fermion-antifermion pair

$$W^+ \to e^+ + \nu_e$$
$$W^+ \to \mu^+ + \nu_\mu$$
$$W^+ \to \tau^+ + \nu_\tau$$
$$W^+ \to u + \bar{d}$$
$$W^+ \to c + \bar{s}$$

- (Note: W boson cannot decay to $t + \overline{b}$ because the mass of the top quark is larger than the mass of the W boson)

Summing Decay Rates

What happens when a particle has more than one path to decay?

The total decay rate is the sum of individual decay rates.

$$W^+ \to e^+ + \nu_e$$
$$W^+ \to \mu^+ + \nu_\mu$$
$$W^+ \to \tau^+ + \nu_\tau$$
$$W^+ \to u + \bar{d}$$
$$W^+ \to c + \bar{s}$$

To know the decay rate for the W, need to know ALL the decay modes and rates:

$$\mathrm{d}N_W = -\left(\Gamma_{e\nu} + \Gamma_{c\bar{s}} + \cdots\right) N_W \mathrm{d}t$$

$$\Gamma_{tot} = \sum \Gamma_i$$

Careful: the decay rates sum, not the lifetimes

Branching Fraction/Ratio

The fraction of a particle's decays to a particular final state is referred to as the "branching fraction" or "branching ratio"

Calculated as:

$$BR(W \rightarrow e\nu) = \frac{\Gamma_{W\rightarrow ev}}{\Gamma_{\text{tot}}}$$

W^+ DECAY MODES

W^- modes are charge conjugates of the modes below.

	Mode		Fraction (Γ_i/Γ)
Γ_1	$\ell^+\nu$	[a]	(10.86 ± 0.09) %
Γ_2	$e^+\nu$		(10.71 ± 0.16) %
Γ_3	$\mu^+\nu$		(10.63 ± 0.15) %
Γ_4	$\tau^+\nu$		(11.38 ± 0.21) %
Γ_5	hadrons		(67.41 ± 0.27) %

Lifetime

- Lifetime τ describes how long particles live on average before they decay

- Relationship to decay rate: $\tau = 1/\Gamma \quad \Gamma = 1/\tau$

- Units of lifetime in seconds, or 1/eV

- Number of particles left after time t:

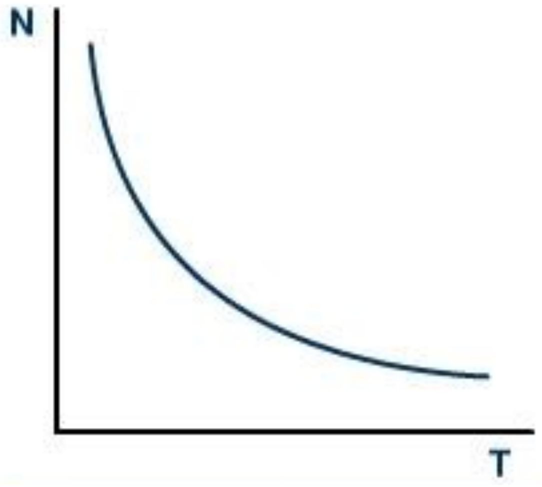
$$N(t) = N_0 e^{-\Gamma t} = N_0 e^{-t/\tau}$$

Exponential decay

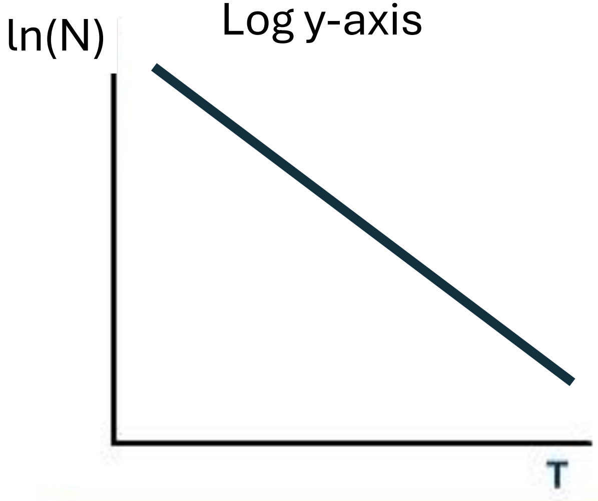
- $N(t) = N_0 e^{-\Gamma t} = N_0 e^{-t/\tau}$
- $\tau = 1/\Gamma$
- With many allowed decays: $\tau = 1/\Gamma_{\text{tot}}$

Sometimes easier to measure slope of line on a log plot than on a linear axis

Linear y-axis



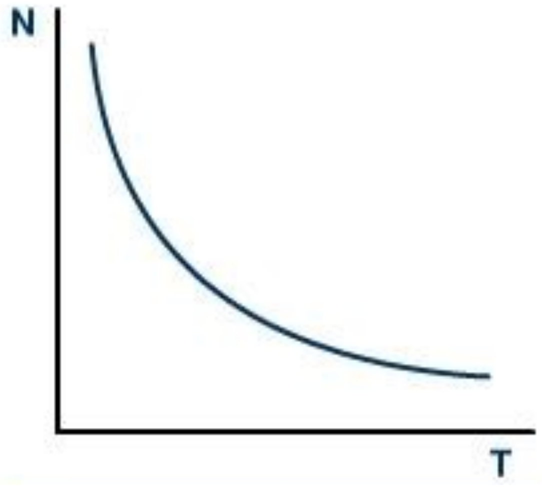
Log y-axis



Exponential decay plots

- $N(t) = N_0 e^{-\Gamma t} = N_0 e^{-t/\tau}$
- $\tau = 1/\Gamma$
- With many allowed decays: $\tau = 1/\Gamma_{tot}$

Linear y-axis



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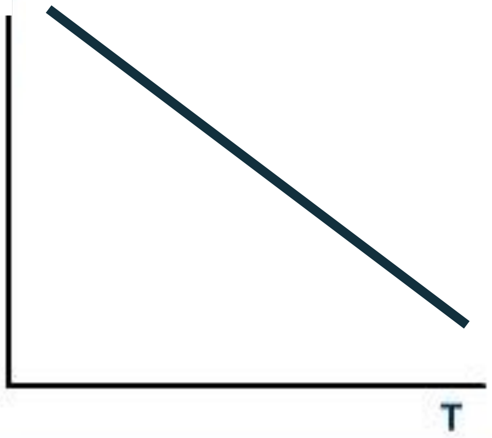
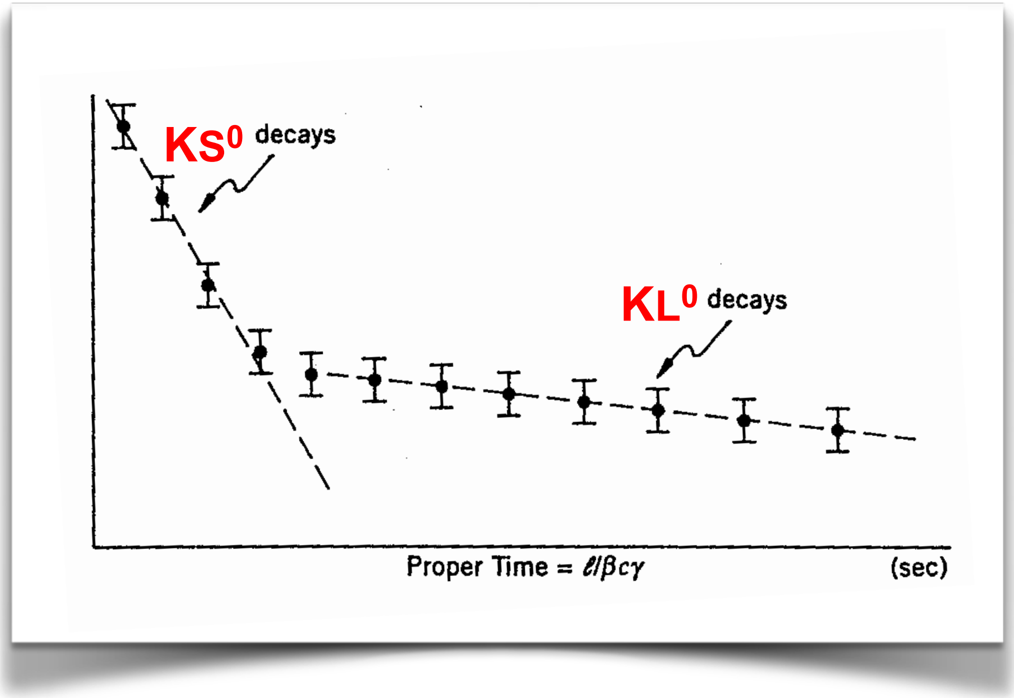
ln(N)

Log y-axis



Example Measurement



Decay & Production

Particle decay and particle production are intrinsically related

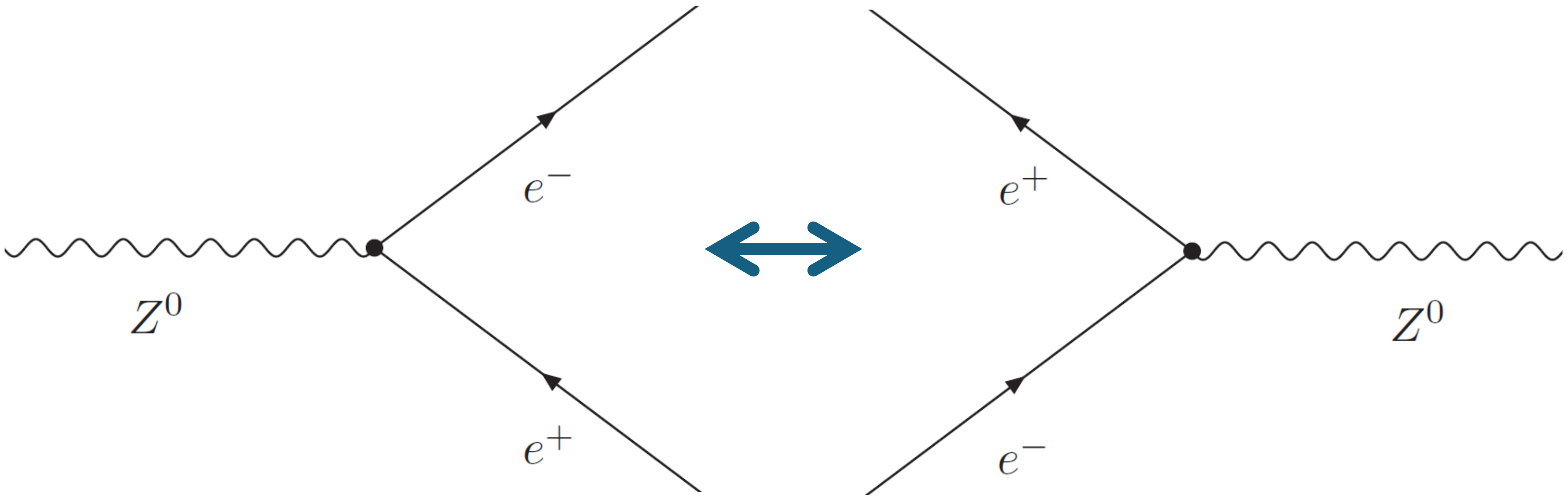
Related to CPT symmetry and rotation of Feynman diagrams

The decay rate for a process is also referred to as the “width” of the production rate “peak”.

Example: Z boson coupling to electron-positron pair

The coupling is the same whether a Z decays to e+e- or whether an e+e- collision produces a Z

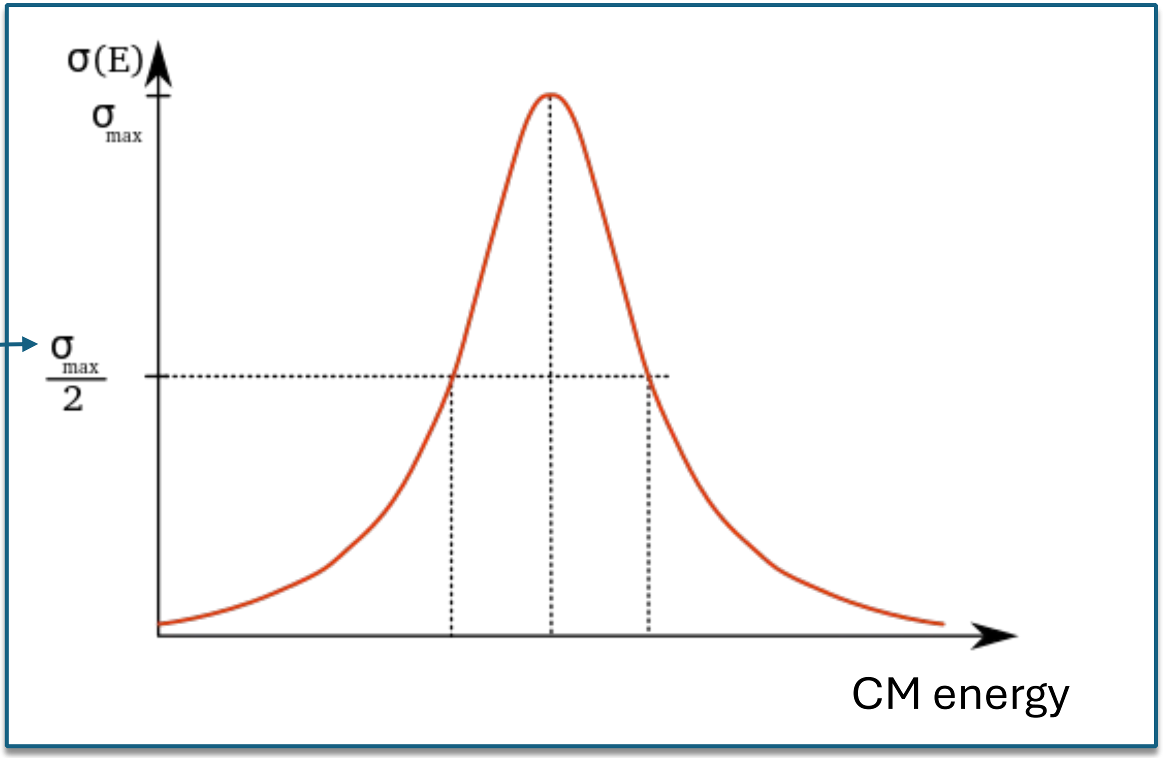
Probability the process happens is the same in either direction!



Decay & Production

- Scan through the energy of the two beams colliding (CM energy) reveals a peak structure for production of a particle -> Measure number of particles produced, proportional to probability of production

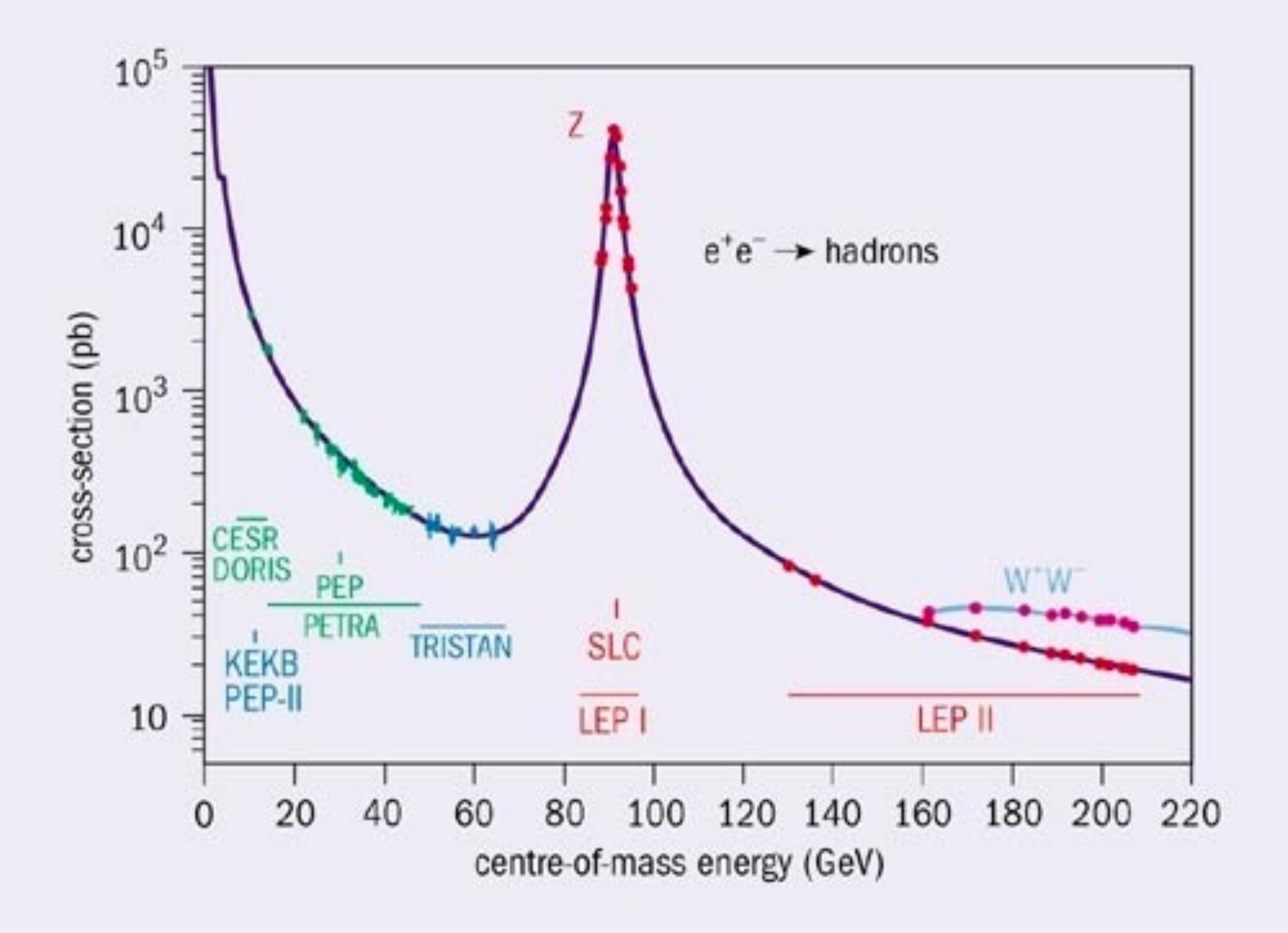
Decay rate



Production rate depends on all ways particles can be produced/decay

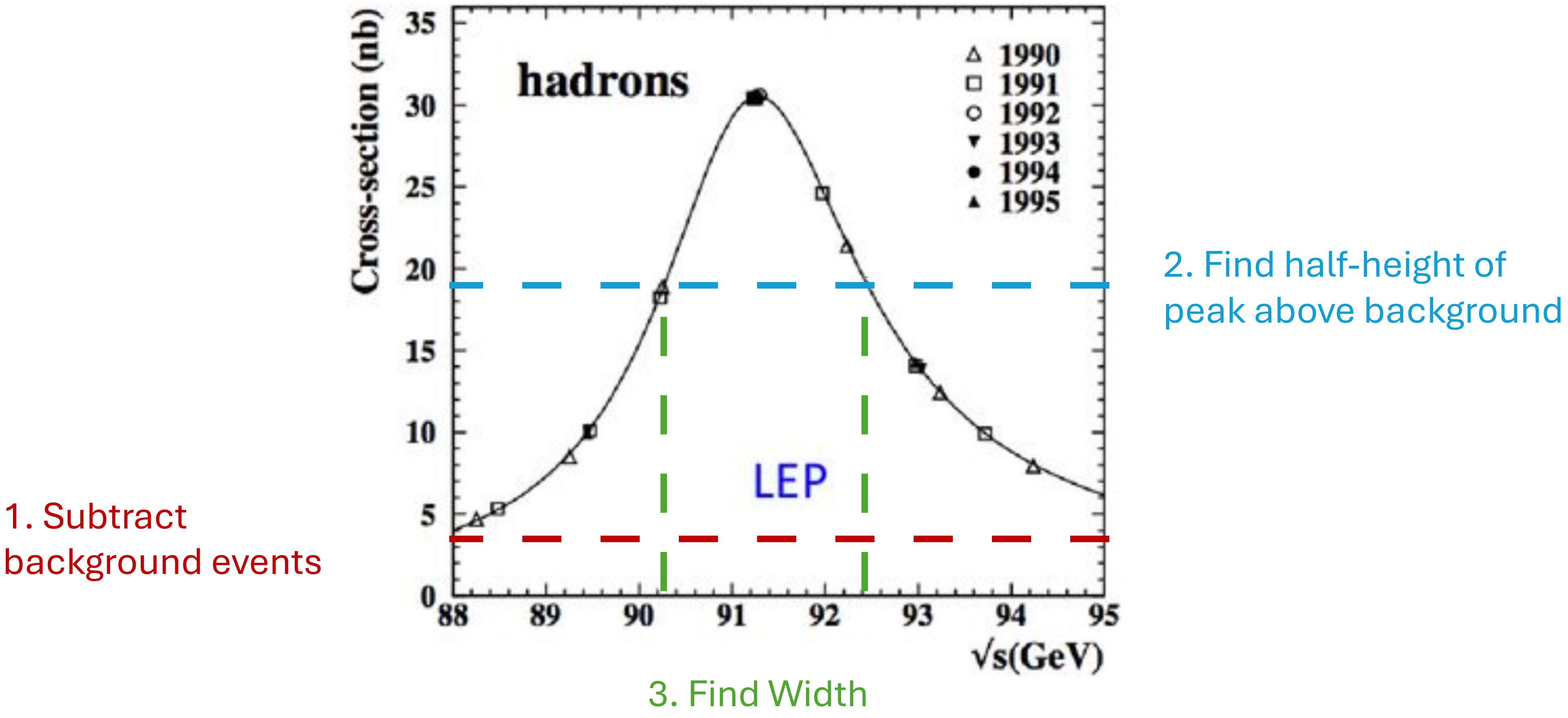
Example: Z boson production and decay

- Precision scan at e+e- colliders:



Example: Z boson production

- Z boson scan at LEP e+e- collider:
- Z boson total decay rate: 2.5 GeV



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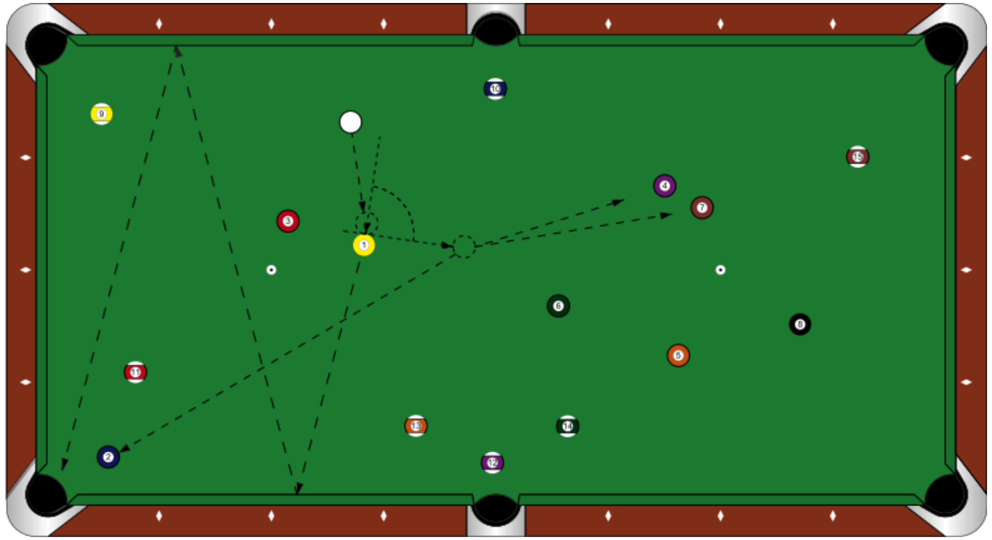
Cross section (σ): Describes the probability for an interaction to occur.

Differential cross section ($\mathrm{d}\sigma/\mathrm{d}X$): Describes the probability for an interaction with a particular final state.

Classical Scattering

We need to connect the idea of a unit area (classical) and the probability for an interaction to occur.

Classically, the cross section is inherently related to the size of an object.



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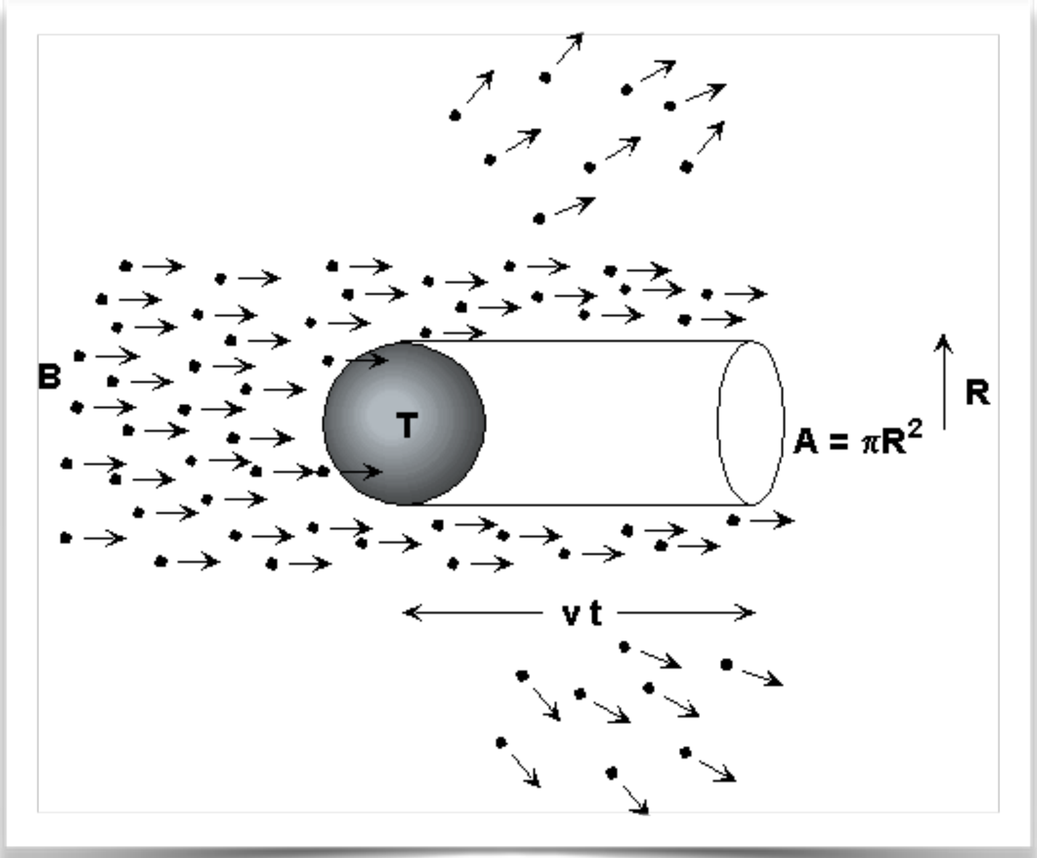
In the classical sense, scattering occurs when the particles in a beam overlap their path with the target's cross section



Quantum Mechanical Scattering

- “Soft” scattering -> the closer together, the greater the deflection
- Interaction rate depends on particles that are scattering as well as the target
- Scattering can be elastic (e+p->e+p) or inelastic (e+p -> e+p+π0, etc)
    - Each process has its own exclusive interaction rate (cross section)
- Total cross section is the sum of all the possible interactions

$$\sigma_{\text{tot}} = \sum_{i=1}^{n} \sigma_i$$

- Requires: description of available phase space and specifics of the interaction (vertex element/coupling and kinematics)

The Golden Rule

Fermi's Golden Rule states:

The transition rate for a system depends on two fundamental quantities

1) The "Matrix Element" or "Transition Amplitude"
2) The final state phase space

$$W = \frac{2\pi}{\hbar} \left|\mathcal{M}_{if}\right|^2 \rho(E)$$

The transition rate (decays or interactions)

The Matrix Element.

Dynamic information

Phase Space or Density of States