

# Announcements

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## Quiz:

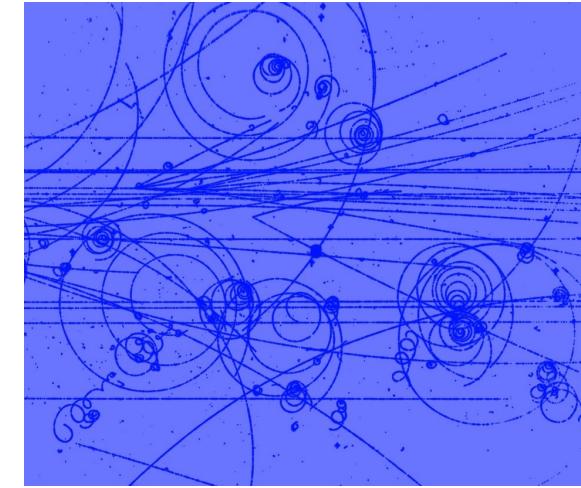
- Pick up quiz after class. **No quiz this week** – midterm instead.

## Homework:

Homework 1 and 2 grades posted on gradescope. Solutions posted on D2L.

Third HW posted on D2L : Q1/Q2 can be done now:

**Now due Friday Feb. 28**



## Midterm next class! Friday, Feb 21

Will cover through “bound states”.

Equation sheet: 1 letter-sized (8 ½ by 11 inches) page front and back, handwritten

**Office hours:** Today, **Wednesday 4-5pm this week**, not on Friday

# Outlines

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Thank you all for submitting your paper topics!

**Due February 28th at the beginning of class, 1-2 pg**

1. Title
2. Abstract
3. Logical structure of the paper
4. One sentence for each section

**Logical structure of paper?**

Lab report format:

Intro, Methods, Results, Discussion, Conclusions

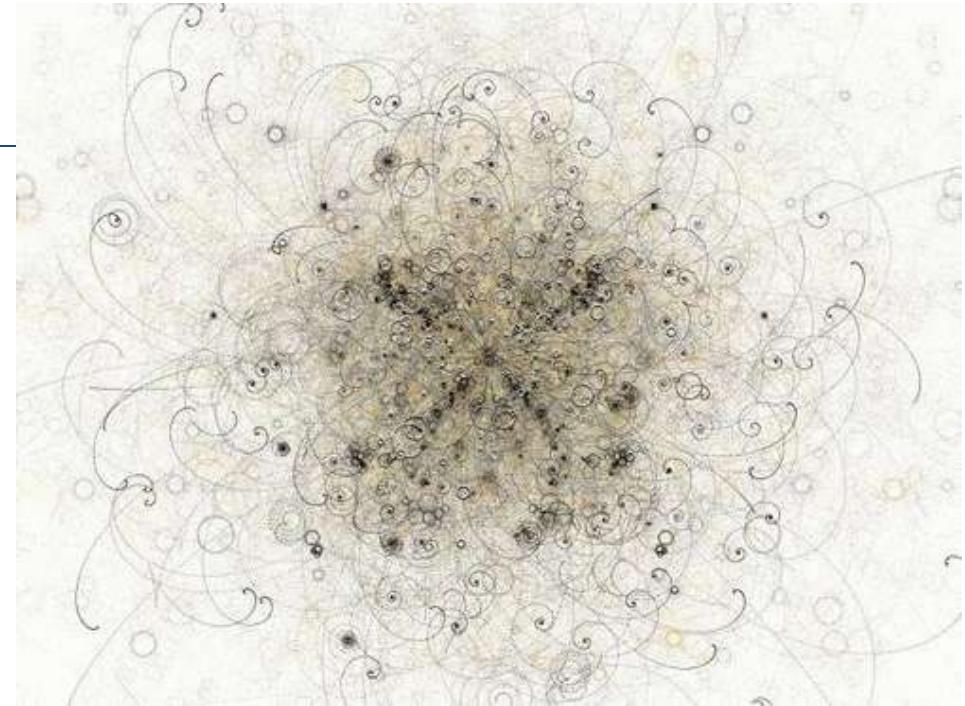
Or another structure that is more useful for the topic that you have chosen

The sentence for each section should help me to follow the flow of the structure for your topic

Submit as pdf on gradescope

Worth 1% of class grade: pass/no-pass depending on if you turn in the above on time

I will return them to you after spring break

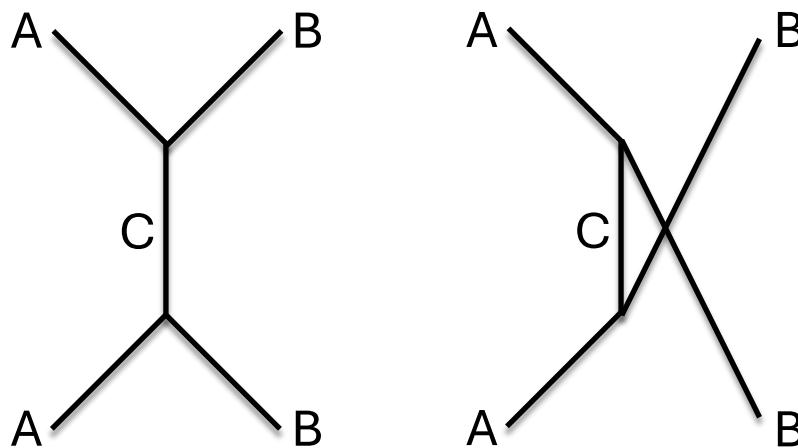


# A+A→B+B Scattering

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Last time:

Calculated differential cross section for A + A → B+B scattering from two Feynman diagrams

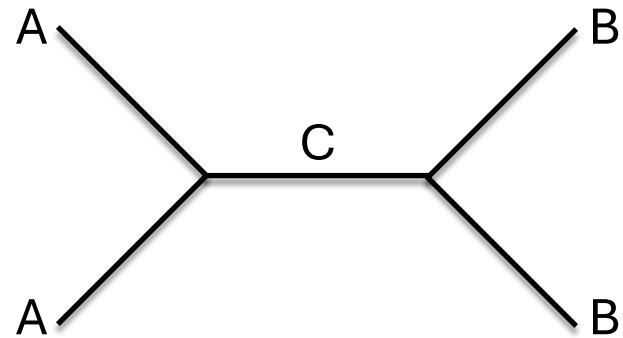


$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \left( \frac{g^2}{16\pi E |p|^2 \sin^2 \theta} \right)^2$$

# $A+A \rightarrow B+B$ Scattering

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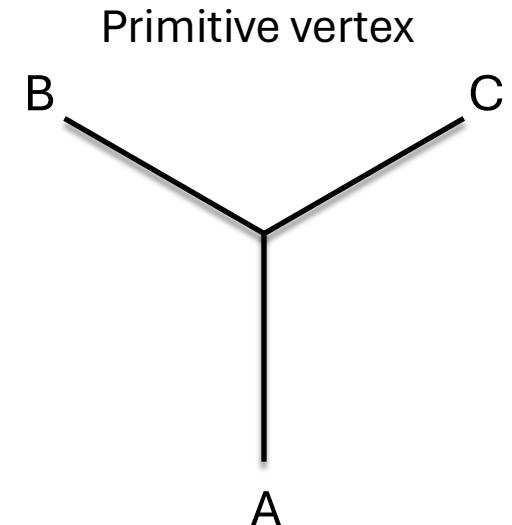
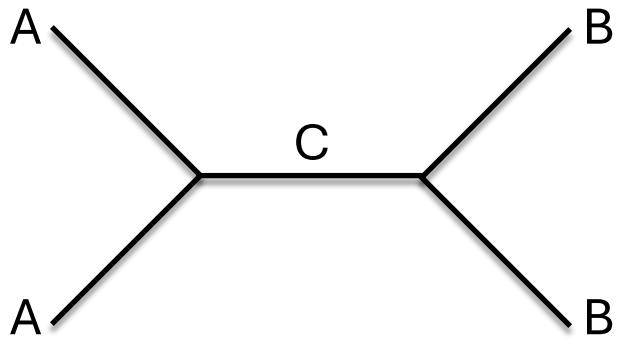
Why didn't we consider this diagram?



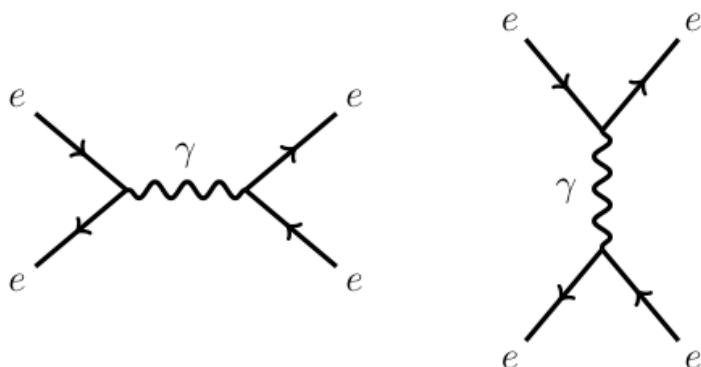
# A+A→B+B Scattering

Why didn't we consider this diagram?

Be careful to not read too deeply into ABC theory!



Answer: There are no AAC or CBB vertices in ABC theory. But this diagram does exist in QED, for example.



# Another question

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Earlier we said cross sections sum:

$$\sigma_{\text{tot}} = \sum \sigma_i$$

And we just said matrix elements sum:

$$\mathcal{M}_{\text{tot}} = \mathcal{M}_1 + \mathcal{M}_2$$

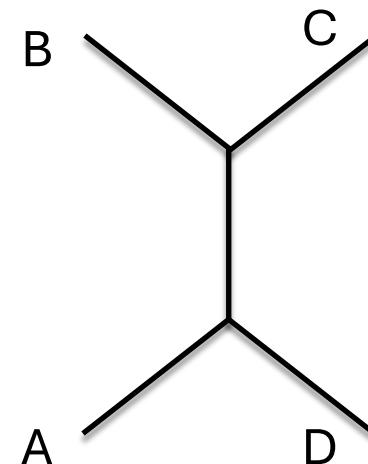
The first statement is only true if there is no interference between the matrix elements!  
(cross section depends on matrix element squared)

$$|\mathcal{M}|^2 = |\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + 2|\mathcal{M}_1 \cdot \mathcal{M}_2|^2$$

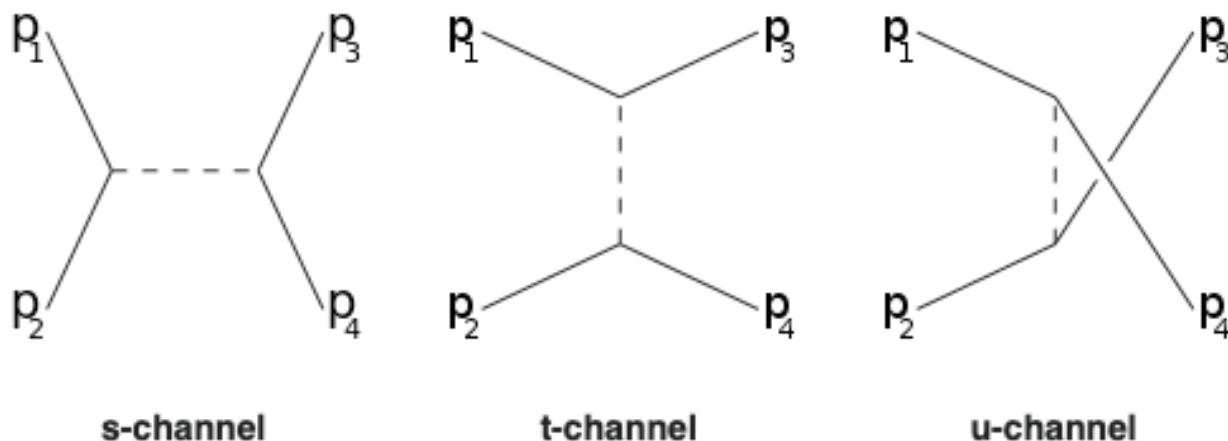
# Mandlestam Variables

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- Relate 4-momentum of incoming/outgoing particles



Sometimes classify Feynman diagrams in similar terms:



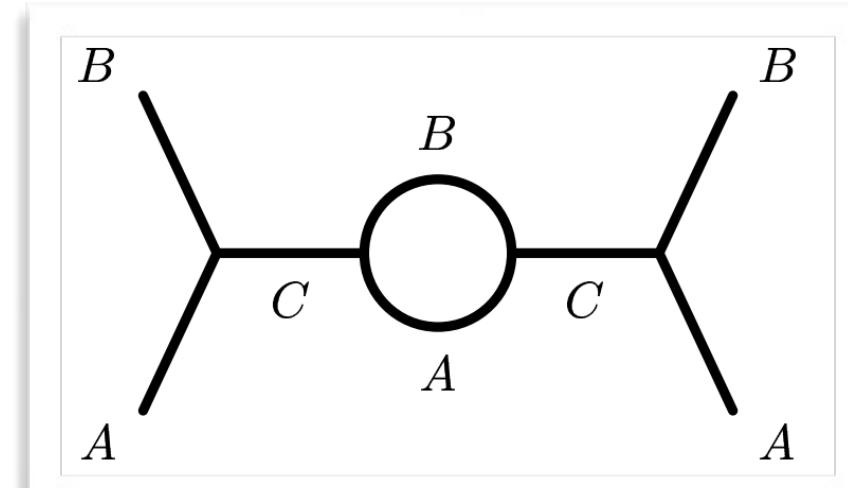
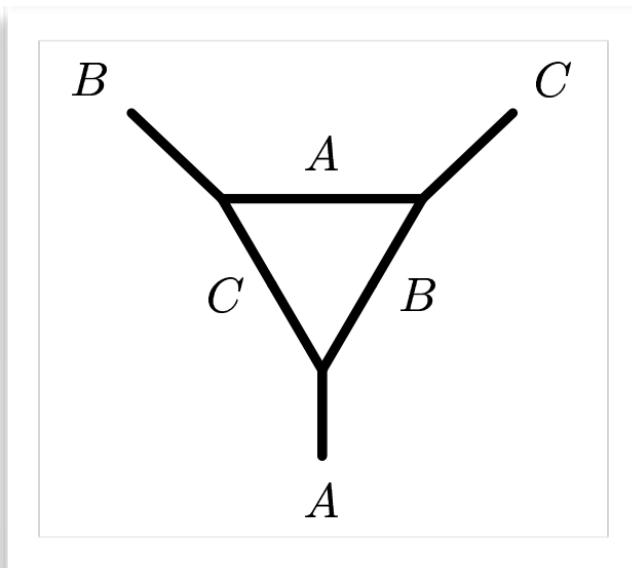
$$s = (p_A + p_B)^2$$
$$t = (p_A - p_c)^2$$
$$u = (p_A - p_D)^2$$

# Higher Order Diagrams

These interference effects frequently (but not always) arise due to higher order diagrams

ABC theory limits the number of such diagrams, so continue to be wary

We're not going to dig deep into this right now, but be aware they exist



# Recap / Up Next

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Last time:

Feynman Calculus

Decays/Scattering

The Golden Rule

Feynman Rules

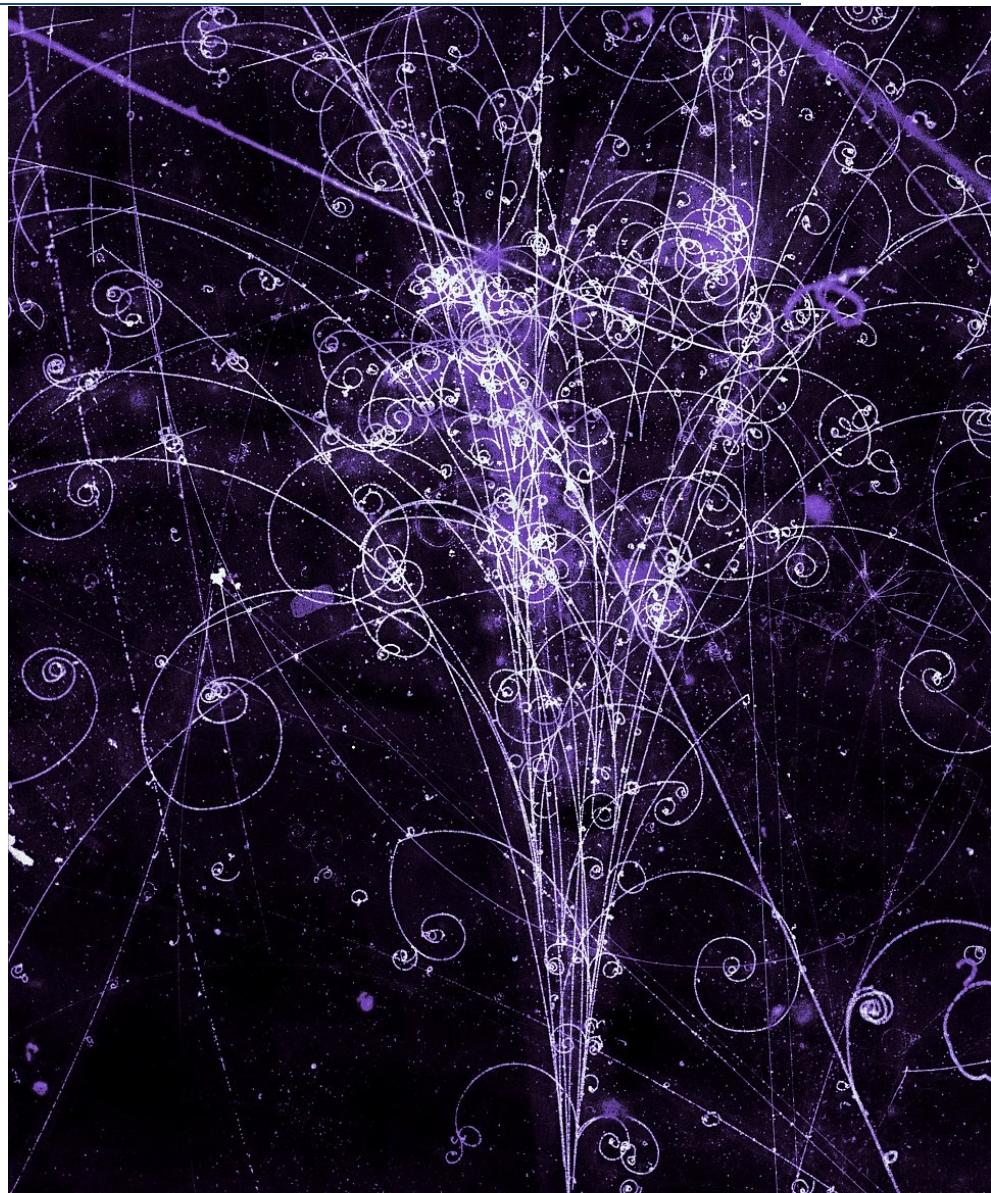
This time:

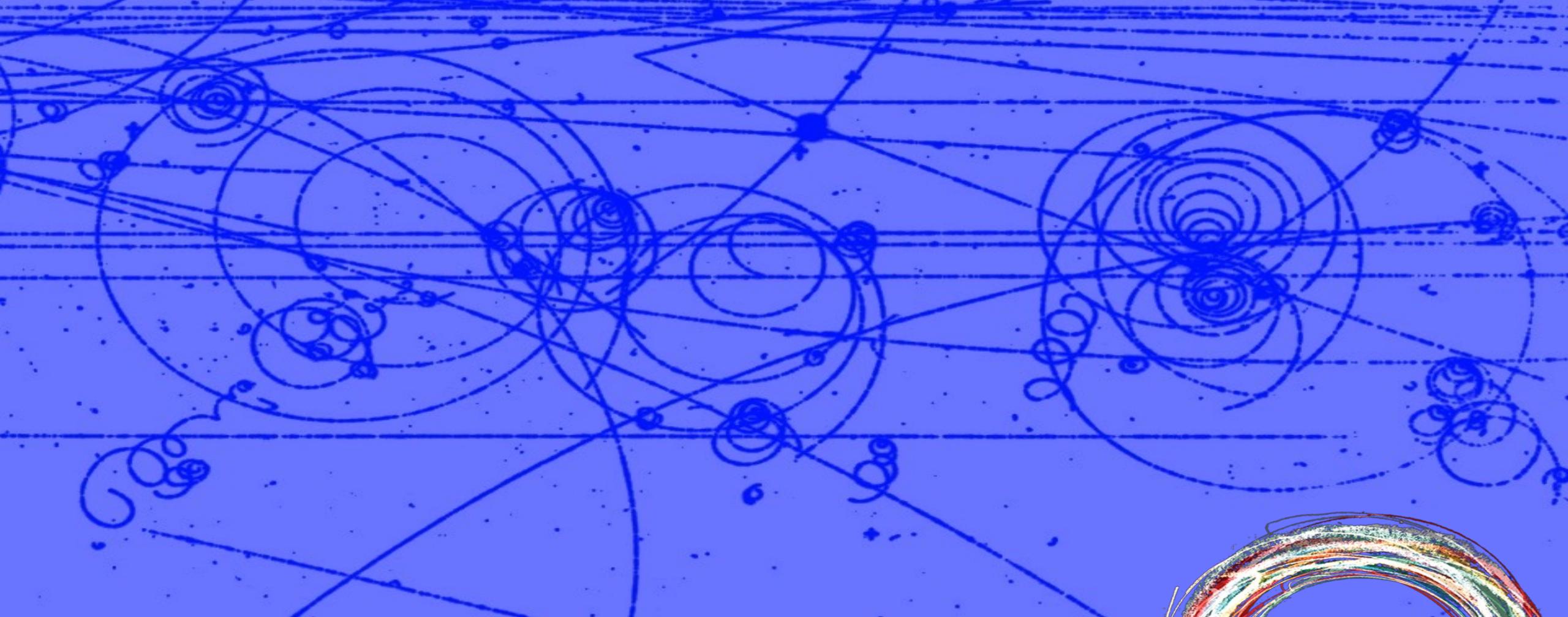
Particle Accelerators

Cyclotrons

Synchrotrons

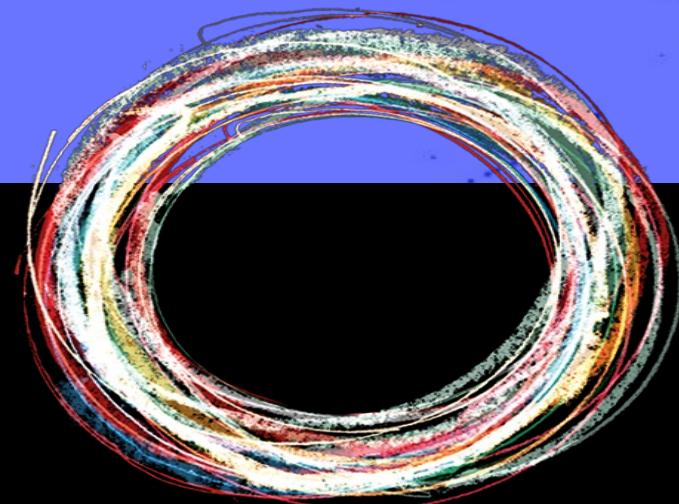
The Large Hadron Collider





# LECTURE 8

# PARTICLE ACCELERATORS



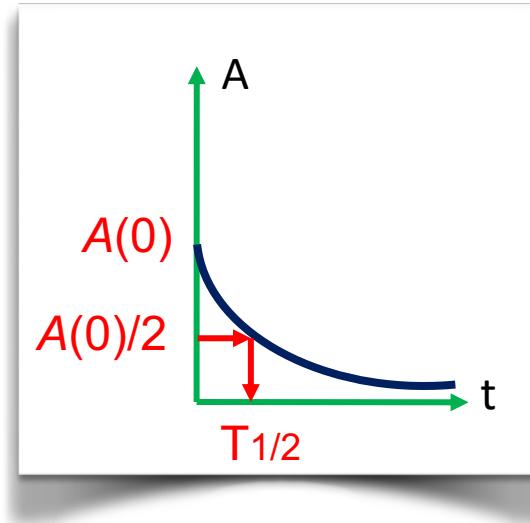
# Experimental Methods

Experiments require clever choices to address a problem.

We can typically measure the rate of decays for radioactive isotopes in the lab setting.

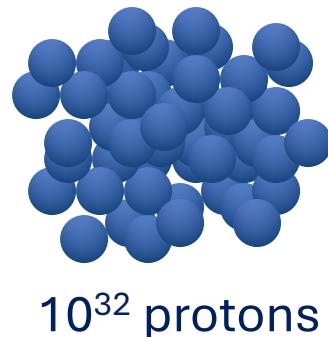
one can observe the activity  $A$  as a function of time

$$A(t) = \lambda N_0 \exp(-\lambda t)$$



But .... decay of the proton : lifetime  $> 10^{32}$  years

- 1 proton To observe one proton decay, could watch & wait for about  $10^{32}$  years, ... or



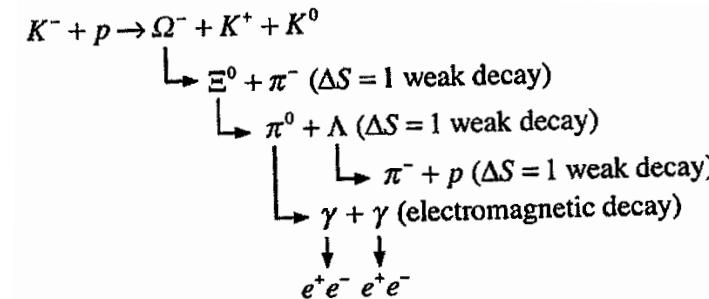
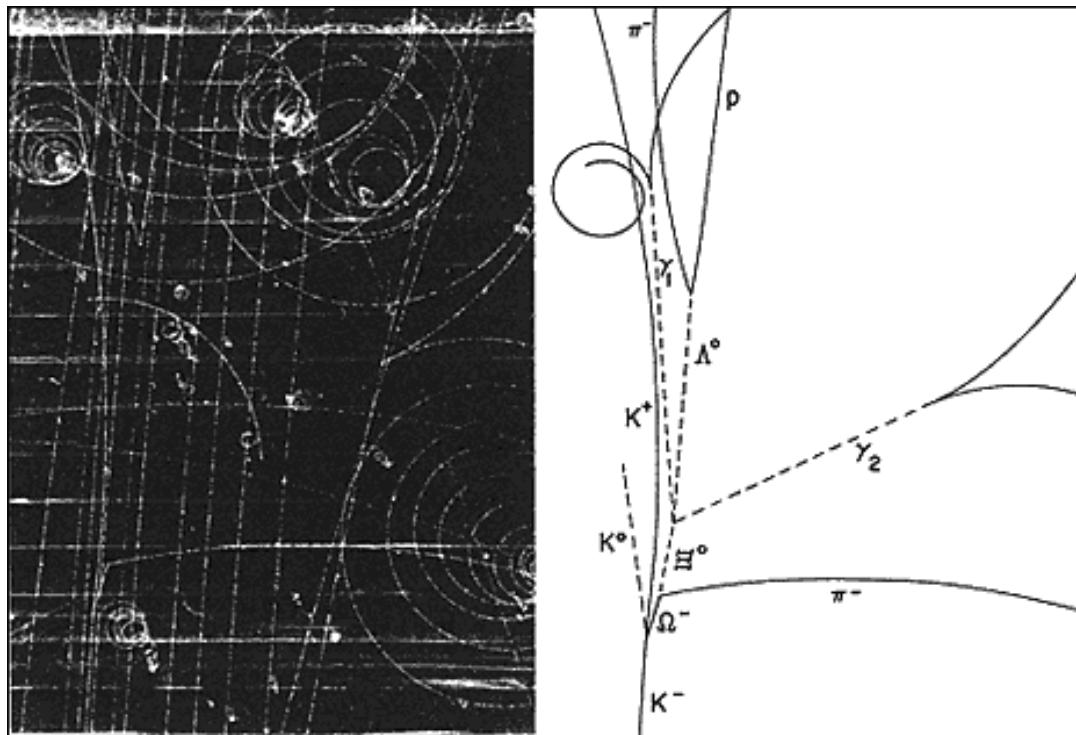
$10^{32}$  protons

Could collect  $10^{32}$  protons ( $10^7$  liters of water!) and observe for at least 1 year...

# Experimental Methods

In "High Energy" physics, the limiting factor is often energy!

Progress was made over the past century by observing cosmic ray interactions using bubble chambers or photographic emulsions.



**Decay of  $\Omega^-$  (sss,  $S=-3$ ) occurs in a sequential process, each of which has  $\Delta S=1$  (at most).**

# Experimental Methods

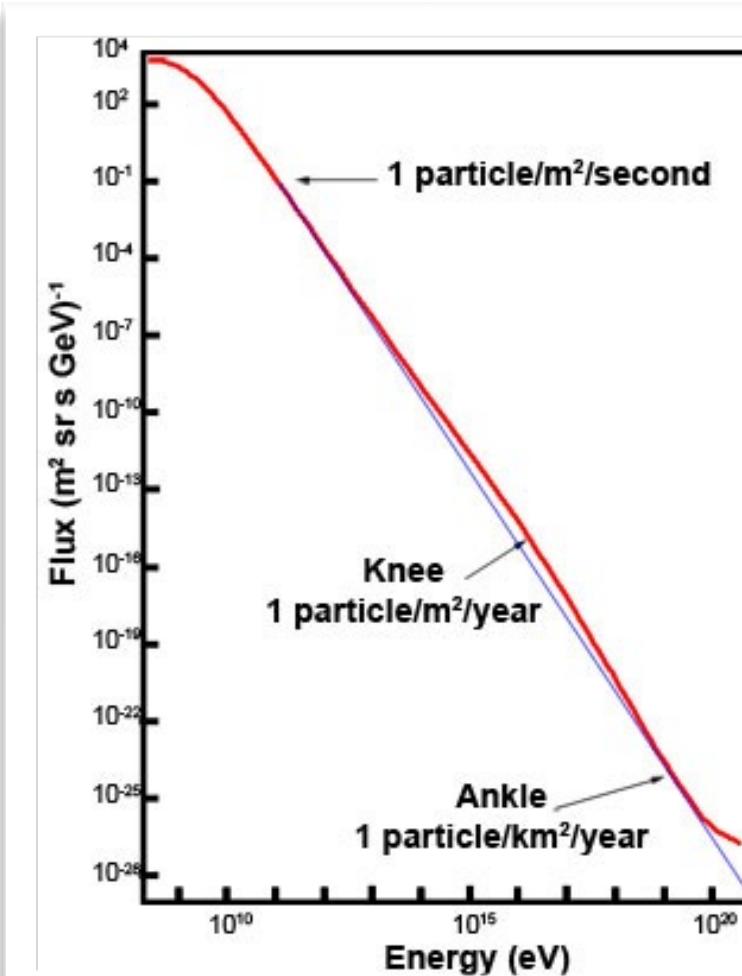
In High Energy Physics, the limiting factor is typically energy!

Cosmic rays are particles accelerated by non-terrestrial sources

- We don't fully understand the acceleration mechanisms
- The energy spectrum is as expected: higher energy particles are rarer.

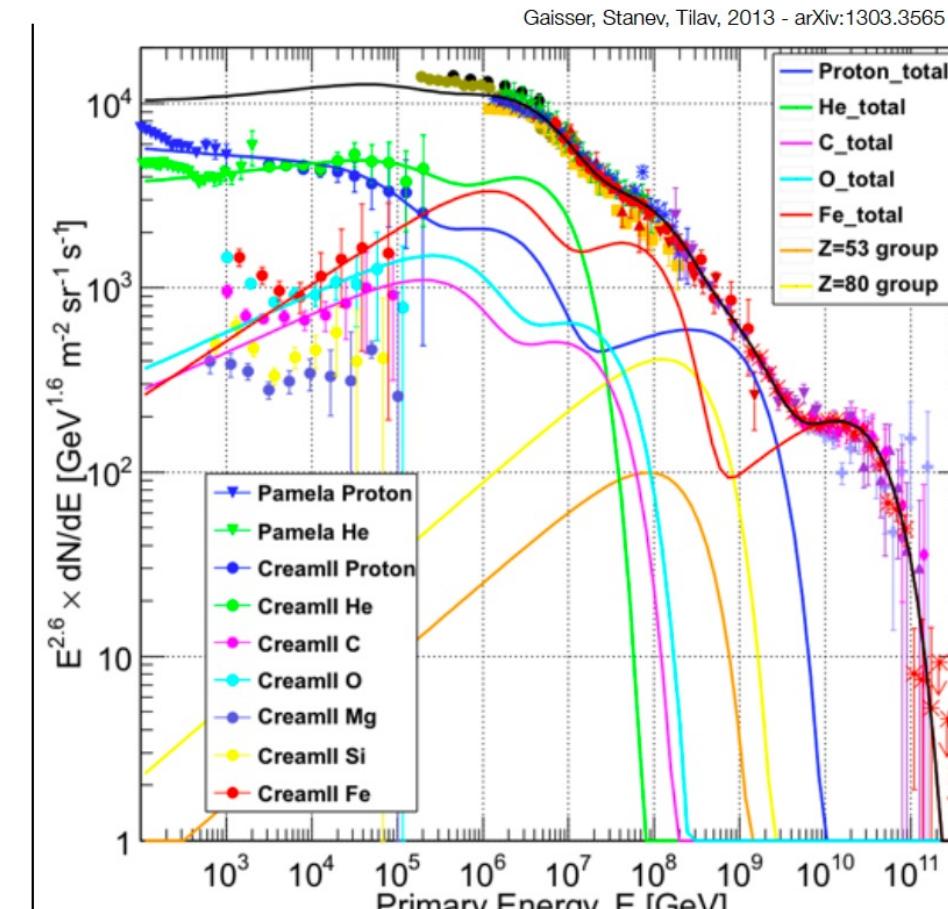
Consider the cosmic ray rates on earth:

- $10^{11}$  eV (100 GeV) @  $1/\text{m}^2/\text{sec}$
- $10^{13}$  eV ( 10 TeV) @  $1/\text{m}^2/\text{hour}$
- $10^{16}$  eV ( 10 PeV) @  $1/\text{m}^2/\text{year}$
- $10^{19}$  eV ( 10 EeV) @  $1/\text{km}^2/\text{year}$



# Cosmic Rays

- Mainly protons (beam) hitting upper atmosphere (target)
  - But not only protons
- To record interactions, place large-volume detectors in upper atmosphere
- Stationary target is not optimal
  - need large beam energy



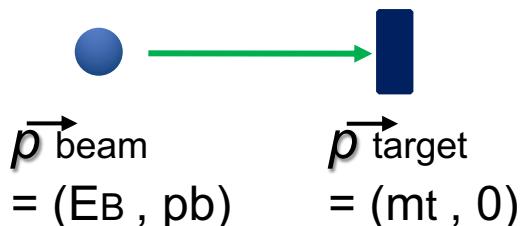
# Fixed Target vs Colliding Beams

To explore the structure of nuclei (nuclear physics) or hadrons (particle physics) requires projectiles whose wave length at least as small as the size of nuclei or hadrons.

To produce new and (more) unstable particles, large **center-of-momentum energies** are required.

$$E_{cm}^2 = (\vec{p}_{beam} + \vec{p}_{target})^2$$

Fixed target

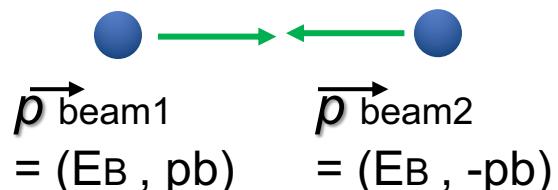


$$\begin{aligned} E_{cm}^2 &= (\vec{p}_{beam}^2 + \vec{p}_{target}^2 + 2\vec{p}_{beam}\vec{p}_{target}) \\ &= (m_b^2 + m_t^2 + 2m_t E_B) \end{aligned}$$

Ecm increases like  $E_B^{1/2}$

Advantage:  
large # targets

Colliding beam



$$\begin{aligned} E_{cm}^2 &= (\vec{p}_{beam1}^2 + \vec{p}_{beam2}^2 + 2\vec{p}_{beam1}\vec{p}_{beam2}) \\ &= (E_B^2 - p_b^2 + E_B^2 - p_b^2 + 2E_B^2 + 2p_b^2) \\ &= (2E_B)^2 \end{aligned}$$

Ecm increases like  $2E_B$

Disadvantage:  
low luminosity

# Cosmic Rays vs. Collider Experiments

- Contrast cosmic rays to a collider like the LHC:

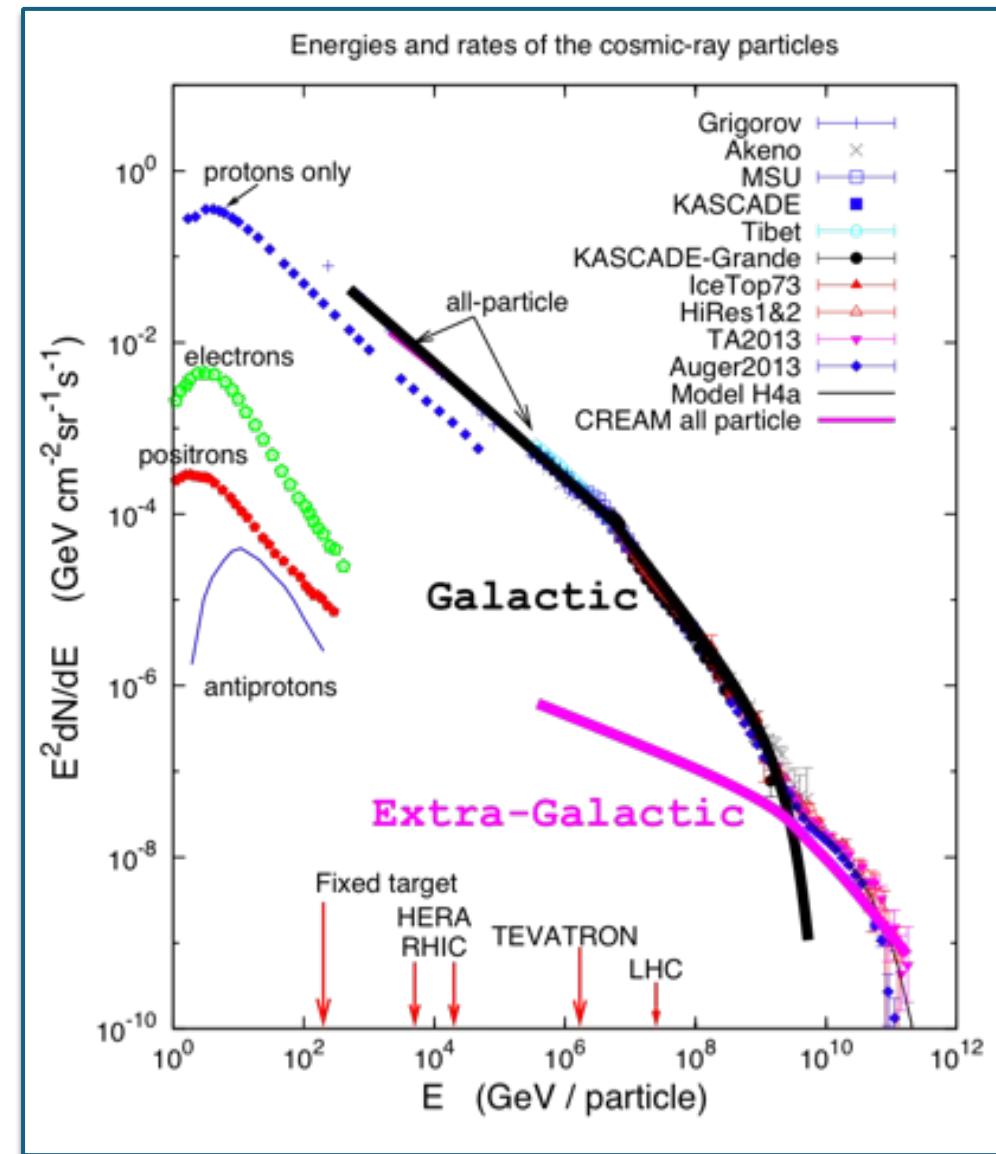
Consider the LHC collision environment:

- 40 MHz collisions
- 13 TeV CoM energy

Cosmic rays @ 13 TeV:  $1/m^2/14$  hours

- $4 \times 10^{10}$  years for 1 week of LHC data

- And every LHC collision happens at the center of a detector



# Accelerators are Everywhere!

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## Discovery Science

- Particle & Nuclear Physics
- Materials science, chemistry, ...



## Medicine

- Cancer therapy (PET)
- Medical radioisotopes



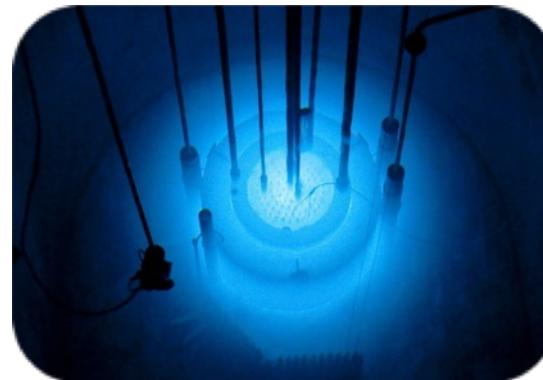
## Industry

- Sterilization
- Ion implantation



## National Security

- Cargo screening
- Radiography



## Energy and Environment

- Accelerator-driven reactors
- Inertial confinement fusion

# Accelerators are Everywhere!

<b><i>Application</i></b>	<b><i>Worldwide Systems (ca. 2010)</i></b>
Ion Implantation	10,000
Electron beam modification	7,000
Electron and X-ray irradiators	2,000
Ion beam analysis and AMS	200
Radioisotope production	600
High energy x-ray inspection	750
Neutron generators	2,000

The most well known category of accelerators – particle physics research accelerators – is one of the smallest in number. The technology for other types of accelerators came from particle physics.

Nuclear and Particle Physics Research	110
<b>Total</b>	<b>~30,000</b>

# Types of Accelerators

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There are many ways to break down different types of accelerators.

## By accelerating voltage

1. DC Accelerators: a static voltage is used for particle acceleration.
2. AC Accelerators: the accelerating field varies with time.

## By architecture

1. Linear Accelerators: particles are accelerated along a straight path.
2. Circular Accelerators: particles follow a curved path that often recycles into itself, creating a “stored” beam.

## By particle type

1. Electron Accelerators: most common, but limited in maximum energy.
2. Hadron Accelerators: less common, but with a higher energy potential.

# Equations of Motion

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We can start by considering the Lorentz Invariant E-p relationship

$$E^2 = p^2 c^2 + m^2 c^4$$

Using the time derivative....

$$E \frac{dE}{dt} = c^2 p \cdot \frac{dp}{dt}$$

Then remember the Lorentz force equation...

$$\frac{d\vec{p}}{dt} = q (\vec{E} + \vec{v} \times \vec{B})$$

We can describe the change in the accelerated particle's acceleration.

$$\frac{dE}{dt} = \frac{qc^2}{E} \vec{p} \cdot (\vec{E} + \vec{v} \times \vec{B})$$

# Linear accelerator

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- In a linear accelerator, the magnetic field  $B$  is zero

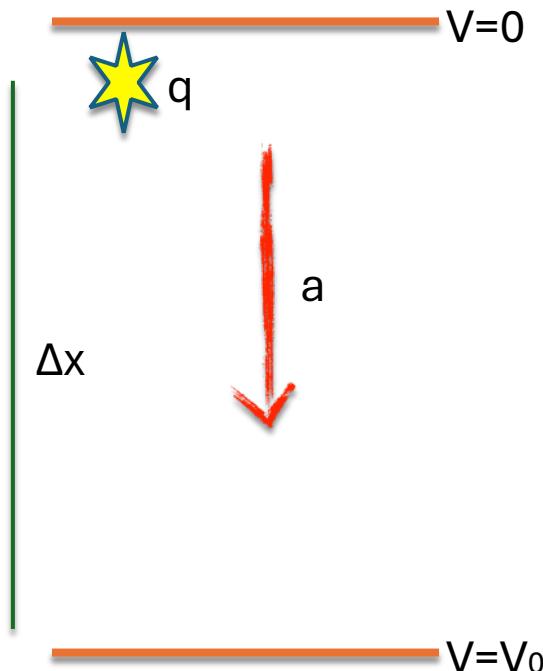
$$\frac{dE}{dt} = \frac{qc^2}{E} \vec{p} \cdot (\vec{E} + \vec{v} \times \cancel{\vec{B}})$$

- An electric field is applied to accelerate charged particles in the direction of motion
  - Linear direction

# What to Accelerate?

If we're going to build an accelerator, work out what it is we want to accelerate first!

Consider the simplest accelerator: electric potential



No matter what we accelerate, the kinetic energy ( $T$ ) is fixed:

$$T = q V = q E \Delta x$$

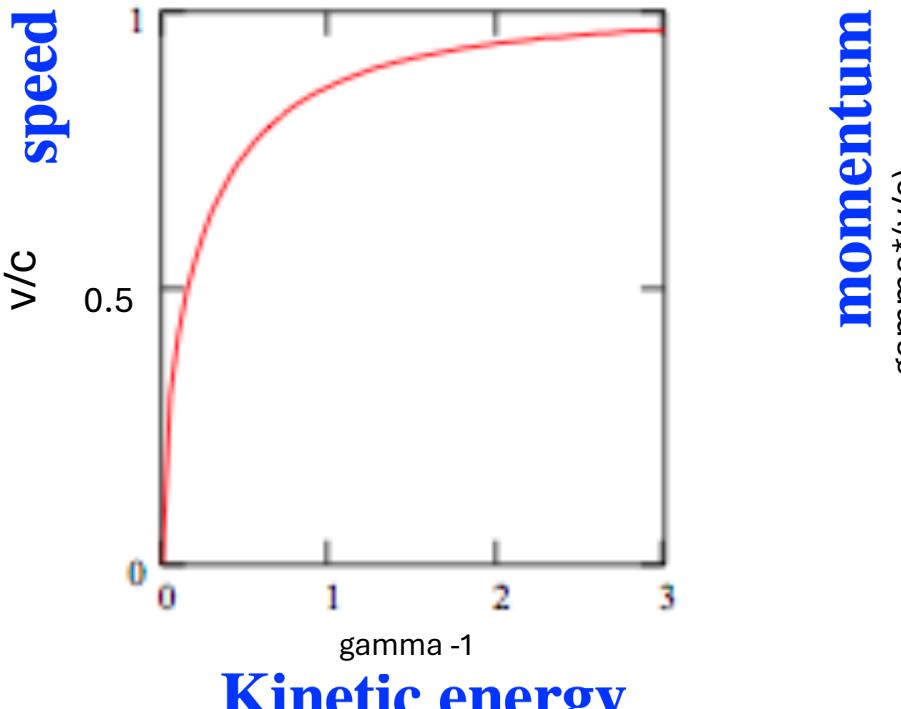
And in relativistic terms  $T = (\gamma - 1)mc^2$

Electrons: stable, relatively large  $q/m$  ratio = easy to accelerate.

Hadrons: can be stable, relatively small  $q/m$  ratio.

Neutral particles: cannot be accelerated 😞

# Accelerating Electrons or Protons



Electron: 0 0.5 1.0 1.5 MeV

Proton: 0 1000 2000 3000 MeV

- Electrons get to relativistic energies much faster than protons

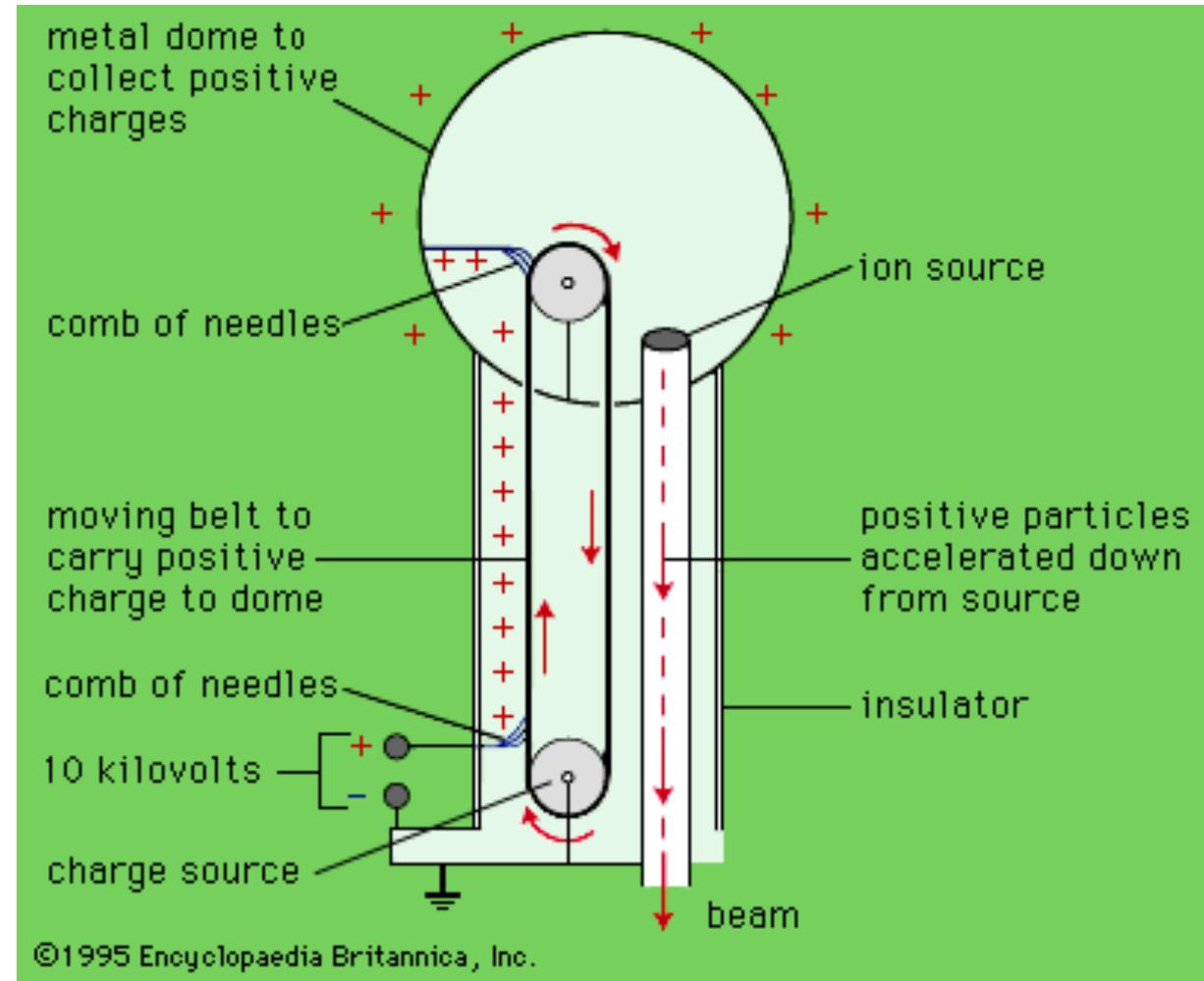
# DC accelerators

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- In a DC accelerator, particles are accelerated through a DC potential
- Challenge is to produce a large potential (voltage)
- AC voltages can be transformed to large values with transformers
- How to produce large DC voltages?

# Van de Graaff generator

- Mechanical belt to transport charges to a metal dome
- Can reach 100 keV
- Small current



# Cockroft-Walton Generator

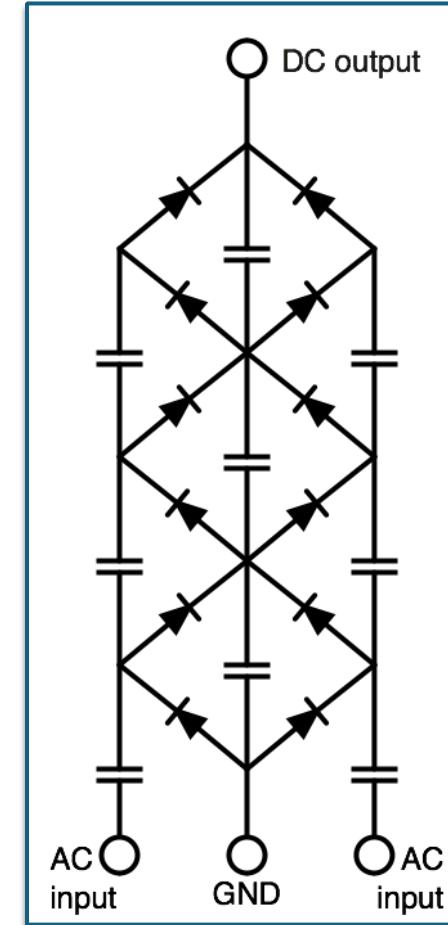
The earliest type of DC accelerator used a Cockroft-Walton generator that produced the high voltages.

- An AC voltage  $V$  is applied at the inputs
- Each stage has capacitors that get charged up
- Each stage provides a voltage increase of  $2 V$ 
  - The first two capacitors charge up to  $V$
- Because of the diodes, the capacitors cannot discharge when the AC voltage goes negative
- A  $N$ -stage Cockroft-Walton gives a DC voltage of  $2N \cdot V$

$N$  = number of capacitor layers

Diodes that allow current only in one direction

capacitors



# Cockroft Walton at Fermilab

Fermilab used a Cockroft-Walton accelerator as the first stage for decades

- 75 kV input AC voltage
- 5 stages
- 750 kV DC output voltage
- To accelerate negative H ions
- Then strip electrons off

