▶ Q1

(a) Possible outcomes with net zero charge:

$$\pi^0\pi^0$$
$$\pi^+\pi^-$$

From the quark contents of these pions (1/2 isospin), we have these

$$\pi^0 : \ |1,0\rangle$$
$$\pi^+ : \ |1,1\rangle$$
$$\pi^- : \ |1,-1\rangle$$

(b) Decay amplitudes

The photon has isospin $|J,M\rangle$ which correspond to the top bar. We use the 1x1 table since each pion $l=1$.

We can find the probability of decaying to m_1, m_2 using the table

For photon isospin $|1,0\rangle$:

state	m_1, m_2	amplitude
$\|\pi^+\pi^-\rangle$	$+1,-1$	$\sqrt{1/2}$
$\|\pi^0\pi^0\rangle$	$0,0$	0
$\|\pi^-\pi^+\rangle$	$-1,+1$	$\sqrt{1/2}$

For photon isospin $|0,0\rangle$:

state	m_1, m_2	amplitude
$\|\pi^+\pi^-\rangle$	$+1,-1$	$\sqrt{1/3}$
$\|\pi^0\pi^0\rangle$	$0,0$	$-\sqrt{1/3}$
$\|\pi^-\pi^+\rangle$	$-1,+1$	$\sqrt{1/3}$

(c) Can photons decay into $\pi^0\pi^0$ in nature? Both $\pi^0\pi^0$ and $\pi^+\pi^-$ are eigenstates of C, only with different eigenvalues.

Since only $\pi^+\pi^-$ has the same eigenvalue as $C\gamma = -1\gamma$, γ cannot pair-produce $\pi^0\pi^0$ and only isospin $|1,0\rangle$ is valid

▶ Q2

(a) $N(t) = N_0 e^{-t/\tau}$

Muon half life: 1.56 $\mu s = 1.56 \times 10^{-6} s$. That means 14 half-lives will pass. Only 56 muons will remain.

(b) $E = \gamma m \rightarrow \gamma = E/m = 10$

One second for the particle is ten seconds for us.

$$\Delta t' = \gamma \Delta t$$
$$2.2 \times 10^{-5}/10 = 2.2 \times 10^{-6}$$

Now this is 1.4 half-lives. 376k muons remain

(c) $\gamma = 100$. 91% or 907k of the muons remain

▶ Q3

(a) Griffiths does most of the heavy lifting in 6.2.2

Since m_b is very large, the lab frame and CM frame are the same and we can use the formula from Griffiths

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2} \frac{|p_f|}{|p_i|}$$

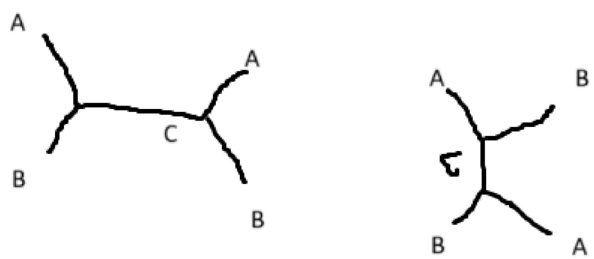
Elastic scattering conserves momentum $|p_f| = |p_i|$, $S = 1$, and the condition $E_2 = m_b$ and $m_b \gg E_1$ means $(E_1+E_2)^2 = (E_1+m_b)^2 \approx m_b^2$

These conditions give us our result ($c = \hbar = 1$)

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(m_b)^2} \frac{|p_i|}{|p_i|} = \frac{|\mathcal{M}|^2}{(8\pi m_b)^2}$$

Note: Why are lab frame and CM frame the same? We often use $\sum \vec{p}_i = 0$ for CM frame, but it is equivalent to say the center of mass doesn't move. The fact that B doesn't recoil after collision with A indicates the center of mass isn't really changing

(b)



Since the particles in and out are distinguishable, we don't have to worry about a u-channel. Momentum can be defined by particle in -and particle out- momenta

(c)
$\mathcal{M} = i(\text{vertex factors})(\text{propogator factors})(\text{momentum conservation})$

We'll use $p_{Ai},\ p_{Bi}$ for incoming particles and p_{Af}, p_{Bf} for outgoing. Remember the extra 2π factors, only one set is removed after integration

1st diagram:

$$\mathcal{M}_1 = i\int \frac{d^4 p_c}{2\pi^4}$$
$$\times(-ig)(-ig)\left(\frac{i}{p_C^2 - m_C^2}\right)(2\pi\delta(p_{Ai}+p_{Bi}-p_C))^4(2\pi\delta(p_C - p_{Af} - p_{Bf}))^4$$
$$p_C \to p_{Ai} + p_{Bi}$$
$$\mathcal{M}_1 = \frac{(2\pi)^4 g^2}{(p_{Ai}+p_{Bi})^2 - m_C^2}\delta^4(p_{Ai}+p_{Bi}-p_{Af}-p_{Bf})$$
$$= \frac{g^2}{s - m_C^2}$$

Rereading the lecture slides, it looks like we drop the extra $(2\pi)^2\delta^4$ term.

The second diagram is similar, only with momentum swapped around. The delta functions are now

$$\delta^4(p_{Ai}+p_C-p_{Bf})$$
$$\delta^4(p_{B_i}-p_C-p_{Af})$$
$$p_C \to p_{Bf} - p_{Ai}$$
$$\to \delta^4(p_{B_i}+p_{A_i}-p_{Bf}-p_{Af})$$

Note that the only change is in the propogator factor

$$\mathcal{M}_2 = \frac{g^2}{t - m_C^2}$$

(d) m_a and m_c small imply that m_b dominates in the squared terms. In (e) we'll need more factors of $1/m_B$. Remember that squaring a 4-vector returns it's total energy.

$$s = p_{Ai}^2 + p_{Bi}^2 + 2p_{Ai}\cdot p_{Bi} = E_A^2 + E_B^2 + 2E_A E_B$$
$$s \sim 0^2 + m_B^2 + 2(0)(m_B) = m_B^2$$
$$t = p_{Bf}^2 + p_{Ai}^2 - 2E_A E_B \sim m_B^2$$

These terms both collapse down to approximately m_B^2 under the right conditions

$$|\mathcal{M}_{\text{tot}}|^2 = \frac{g^4}{(s-m_C^2)^2} + \frac{g^4}{(t-m_C^2)^2} + 2\frac{g^4}{(s-m_C^2)(t-m_C^2)}$$
$$= 4\frac{g^4}{m_B^4}$$

The m_B term 'eats' all the smaller masses when the denominators are squared. $s \times m_C^2$ and $t \times m_C^2$ both go to zero since m_C is very small.

The final result for differential cross section thus is

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi m_B)^2}\frac{4g^4}{m_B^4}$$
$$= \frac{g^4}{16\pi^2 m_B^6}$$

(e) Our differential cross section doesn't rely on θ or ϕ, so this is

$$\int \frac{d\sigma}{d\Omega} d\Omega = 4\pi \frac{d\sigma}{d\Omega} = \frac{g^4}{4\pi m_B^6}$$

This worked out pretty well after I remembered we were working with 4-vectors instead of 3-vectors

▶ Q4

This result will be somewhat similar to our answer in the slides, I hope

(a)

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{(16\pi E)^2}$$
$$s = E^2 + E^2 + 2E^2 = 4E^2$$
$$t = E^2 + E^2 - 2E^2 = 0???$$

Wait, doest that mean $\mathcal{M}_2 \sim \frac{1}{0}$? Not quite.

$p_{Bf} \cdot p_{Ai}$ depends on angles, since momentum can shift between A and B, even if total momentum is conserved $p_{Bi} \cdot p_{Ai} = E^2$

According to our slides, $t = -2p^2(1 - \cos\theta)$

$$t^2 = 4E^4 \left(\cos^2(\theta) - 2\cos(\theta) + 1\right)$$
$$st = -8E^2(1 - \cos(\theta))$$

$$|\mathcal{M}_{\text{tot}}|^2 = \frac{g^4}{s^2} + \frac{g^4}{t^2} + 2\frac{g^4}{st}$$
$$= \frac{g^4}{16E^4} + \frac{g^4}{4p^2(1-\cos\theta)^2} - \frac{g^4}{4E^2p^2(1-\cos\theta)}$$
$$= \frac{g^4\left[p^2(1-\cos\theta) - 2E^2\right]^2}{16E^4p^2(1-\cos(\theta))^2}$$

I'm a bit worried that we're not using the hint, and expanding this out won't necessarily simplify things.

$$\frac{d\sigma}{d\Omega} = \frac{1}{(16\pi E)^2}\left[\frac{g^4\left[p^2(1-\cos\theta) - 2E^2\right]^2}{16E^4p^2(1-\cos(\theta))^2}\right]$$

(b) It appears as though this value diverges, which is unfortunate since I didn't expect it to. The $1 - cos(\theta)$ term seen multiple times in $\mathcal{M}$ results in infinite values at certain angles. Perhaps this means the particles cannot scatter when $\theta = 0$, so the cross section explodes.

3e didn't have this issues since one particle was truely stationary.