Physics 410 -- Useful Formulas for quiz #5

I. Chemical potential: $\mu_{total} = \mu_{int} + \mu_{ext}$, where μ_{int} depends on the particle density (e.g. through the Ideal Gas relation) and μ_{ext} is a potential energy per particle added to the system).

Diffusive equilibrium between systems A and B occurs when $\mu^A_{total} = \mu^B_{total}$

Chemical potential of ideal gas in 3D: $\mu = \tau \ln\left(\frac{n}{n_Q}\right)$, where n is the concentration of particles and $n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{\frac{3}{2}}$ is the quantum concentration.

II. Grand canonical ensemble: independent variables τ, V, μ

Grand Partition function, also called the "Gibbs sum": $\mathcal{Z} = \sum_{N=0}^{\infty} \sum_{s(N)} e^{[(N\mu - \varepsilon_s)/\tau]}$,

Grand canonical distribution function (probability): $P(N,\varepsilon) = \dfrac{e^{[(N\mu-\varepsilon)/\tau]}}{\mathcal{Z}}$

The numerator of $P(N,\varepsilon)$ is called the "Gibbs factor"

Mean *total* number of particles: $\langle N \rangle = \lambda \dfrac{d(\ln \mathcal{Z})}{d\lambda}$, where $\lambda = e^{\mu/\tau}$ is the absolute activity.

III. Ideal gas: classical regime of Fermi-Dirac and Bose-Einstein distributions

Ideal gas distribution function: $f(\epsilon) = e^{\frac{\mu-\epsilon}{\tau}}$.

IV. Fermi-Dirac distribution function: $f(\epsilon) = \frac{1}{e^{\frac{\epsilon-\mu}{\tau}}+1}$

At $\tau = 0$, $f(\epsilon) = 1$ for $\epsilon < \epsilon_F$ and $f(\epsilon) = 0$ for $\epsilon > \epsilon_F$

Thermal averages via distribution function:
$\langle X \rangle = \sum_{\text{orbitals}} X f(\epsilon) = \int_0^\infty X\, D(\epsilon)\, f(\epsilon)\, d\epsilon$, where $D(\epsilon)$ is the density of states

Total number of particles: $N = \sum_{\text{orbitals}} f(\epsilon) = \int_0^\infty D(\epsilon)\, f(\epsilon)\, d\epsilon$