Homework 3

1) $e^+ e^- \to 2\pi$. EM process.

a. What are possible final states?

Initial charge is 0 $\to$ final charge must be 0

So:

$\pi^+\pi^-$ with isospin $|1,1\rangle|1,-1\rangle$

or

$\pi^0\pi^0$ with isospin $|1,0\rangle|1,0\rangle$

b. EM does not preserve isospin $\to$ photon can have isospin 0 or 1.

Find decay amplitudes for both isospin states $|0,0\rangle$ and $|1,0\rangle$.

Write particles corresponding to isospin states for each coefficient.

Need isospin 1 table from CG.

$\to$ find $|0,0\rangle$ and $|1,0\rangle$ states

$\to$ lower right box, 2/3 columns

$\to$ Read down columns to find state combinations + remember square root

$$|1,0\rangle = \sqrt{1/2}\,|1,1\rangle|1,-1\rangle + 0 + -\sqrt{1/2}\,|1,-1\rangle|1,1\rangle$$

$$|0,0\rangle = \sqrt{\tfrac{1}{3}}\,|1,1\rangle|1,-1\rangle - \sqrt{1/3}\,|1,0\rangle|1,0\rangle + \sqrt{1/3}\,|1,-1\rangle|1,1\rangle$$

1) Cont.

c. One of these decays is not allowed.
Which one + why?

$\gamma \to \pi^0 \pi^0$ or $\gamma \to \pi^+ \pi^-$

Charge conjugation of $\gamma = -1$
of $\pi^0 = +1$

$\to \gamma \to \pi^0 \pi^0$ is not allowed.

π^+ / π^- are not eigenstates of $\hat{C}$, so this decay does not concern charge conjugation.

2) Start with 1 million μ

a. At rest, how many still around 2.2×10^{-5} s later?

$$\tau_\mu = 2.2 \times 10^{-6} s$$

$$N = N_0 e^{-t/\tau}$$

$$= 1 \times 10^{6} e^{-(2.2 \times 10^{-5} / 2.2 \times 10^{-6})}$$

$$= 10^{6} e^{-10}$$

$$= 45.8 \quad \text{or} \sim 46 \mu$$

b. Assuming $E_\mu = 1 GeV$ and $m_\mu = 100 MeV$ how many still there 2.2×10^{-5} s later?

Lorentz boost: $\gamma = E/m$

$$= \frac{1000 MeV}{100 MeV} = 10$$

$$\rightarrow N = N_0 e^{-t/\tau}$$

$$= 10^{6} \cdot e^{(-2.2 \times 10^{-5} / 2.2 \times 10^{-5})}$$

$$= 10^{6} \cdot e^{-1}$$

$= 368{,}000 \mu \rightarrow$ almost 3 orders of magnitude more

2) c. $E_\mu = 10 \text{ GeV}$ $m_\mu = 100 \text{ MeV}$

→ How many 2.2×10^{-5} s later?

$$\gamma = \frac{E}{m} = \frac{10 \text{ GeV}}{100 \text{ MeV}} = 100$$

$$N = N_0 e^{-t/\tau}$$

$$= 10^6 e^{(-2.2 \times 10^{-5} / 2.2 \times 10^{-4})}$$

$$= 10^6 \times e^{-0.1}$$

$$= 905{,}000 \ \mu$$

3) $A + B \to A + B$

a. Cross section in CM frame:

$$\frac{d\sigma}{d\Omega} = \frac{S|\mathcal{M}|^2}{(8\pi)^2(E_1+E_2)^2} \frac{|p_f|}{|p_i|}$$

Assuming recoil of target can be neglected, lab frame is approximately CM frame

$\rightarrow V_B \sim 0 \rightarrow \beta_B \sim 0$

$\rightarrow$ Magnitude of momentum of A unchanged

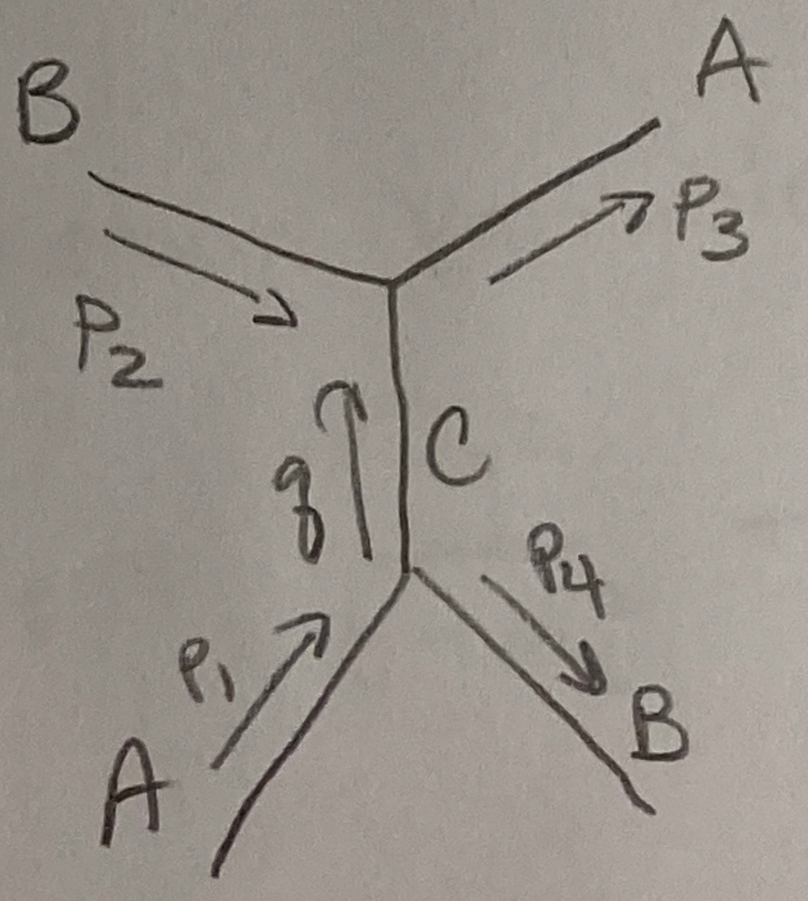
$|p_f| = |p_i|$

and $E_1 + E_2 = E_A + m_B = m_B$ (since $E_1 << m_B$)

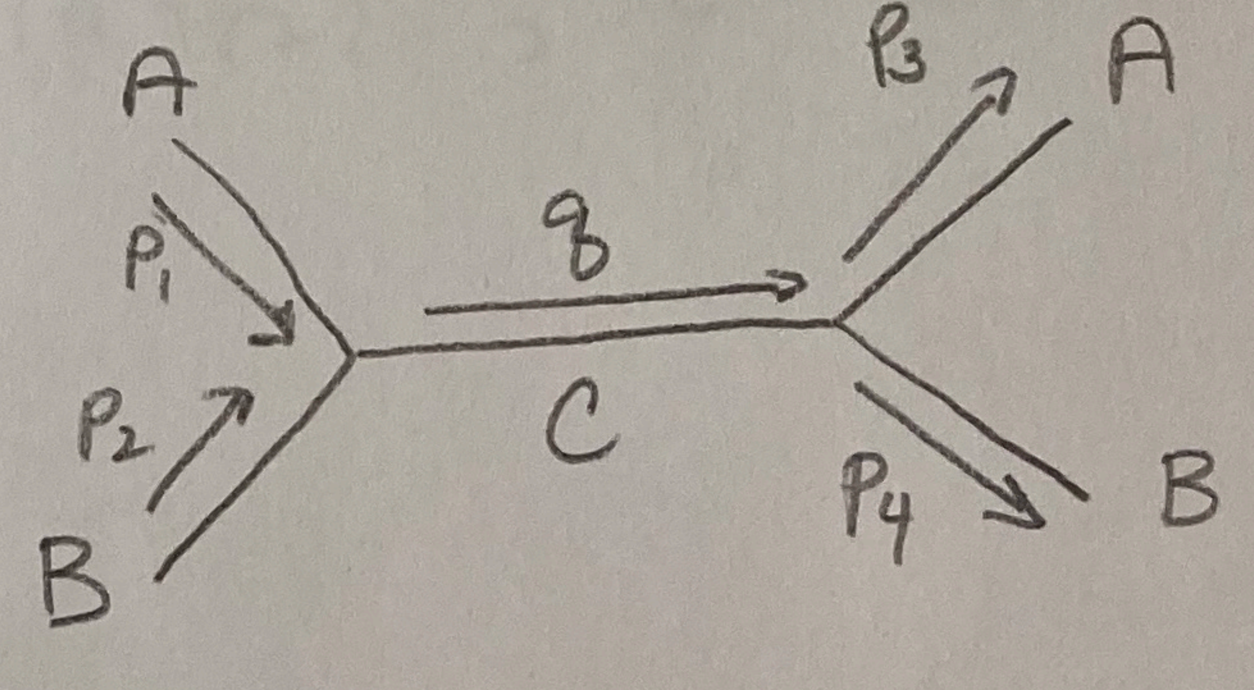
$$\rightarrow \frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{(8\pi)^2 m_B^2}$$

b. Lowest order diagrams in ABC theory

1.



2.



$\rightarrow$ Final state particles are not identical

$\Rightarrow$ no u-channel diagram

3) c. Calculate Scattering amplitude in ABC thry

1. $$\mathcal{M}_1 = i\int (ig)^2 \left(\frac{i}{q^2 - m_c^2}\right)(2\pi)^4 \delta(p_1 - p_4 - q)(2\pi)^4 \delta(p_2 + q - p_3) \times \frac{d^4 q}{(2\pi)^4}$$

$$= \frac{g^2}{(p_1 - p_4)^2 - m_c^2}$$

2. $$\mathcal{M}_2 = i\int (ig)^2 \left(\frac{i}{q^2 - m_c^2}\right)(2\pi)^4 \delta(p_1 + p_2 - q)(2\pi)^4 \delta(q - p_3 - p_4) \times \frac{d^4 q}{(2\pi)^4}$$

$$= \frac{g^2}{(p_1 + p_2)^2 - m_c^2}$$

$$\mathcal{M}_{tot} = \mathcal{M}_1 + \mathcal{M}_2$$

$$= \frac{g^2}{(p_1 - p_4)^2 - m_c^2} + \frac{g^2}{(p_1 + p^2)^2 - m_c^2}$$

Mandlestam variables: $s = (p_1 + p_2)^2$, $t = (p_1 - p_4)^2$

$$\rightarrow \mathcal{M}_{tot} = \frac{g^2}{t - m_c^2} + \frac{g^2}{s - m_c^2}$$

3) d. Differential cross section

→ assume $E_1 << m_B$, $m_B >> m_A, m_C$

$$s = (p_1 + p_2)^2 = (E_1 + m_B)^2 - |p_1|^2 = m_A^2 + m_B^2 + 2E_1 m_B \approx m_B^2$$

$$t = (p_1 - p_4)^2 = (E_1 - m_B)^2 - |p_1|^2 = m_A^2 + m_B^2 - 2E_1 m_B \approx m_B^2$$

$$\rightarrow \mathcal{M}_{tot} = \frac{g^2}{m_B^2 - m_c^2} + \frac{g^2}{m_B^2 - m_c^2} = \frac{2g^2}{m_B^2} \text{, using } m_B >> m_c$$

$$\rightarrow \frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}^2|}{(8\pi)^2 m_B^2} = \left(\frac{2g^2}{m_B^2}\right)^2 \cdot \frac{1}{(8\pi m_B)^2} = \frac{4g^4}{m_B^4 (8\pi m_B)^2} = \frac{g^4}{16\pi^2 m_B^6}$$

e. Total cross section:

→ Notice differential cross section has no angular dependence

$$\int \frac{d\sigma}{d\Omega} d\Omega = 4\pi$$

$$\rightarrow \sigma = \frac{g^4}{4\pi m_B^6}$$

4) a. In center of momentum frame, with $m_A = m_B = m$

$m_c = 0$.

Initial state particles:

A: $p_1 = (E, p_1)$

B: $p_3 = (E, p_3)$

Final state particles:

A: $p_2 = (E, -p_1)$

B: $p_4 = (E, -p_3)$

Calculate mandlestam variables:

$$s = (p_1 + p_2)^2$$
$$= (2E, 0)^2$$
$$= (2E)^2 - 0^2$$
$$= 4E^2$$

$$t = (p_1 - p_4)^2$$
$$= (0, (\vec{p}_1 + \vec{p}_3))^2$$
$$= -(\vec{p}_1 + \vec{p}_3)^2$$
$$= -(\vec{p}_1^{\,2} + \vec{p}_3^{\,2} + 2\vec{p}_1 \cdot \vec{p}_3)$$

then $m_A = m_B \rightarrow p_1 = -p_2$
$= p_3 = -p_4$

$$\rightarrow t = -(-2\vec{p}_1^{\,2} + 2|p_1|^2 \cos\theta)$$
$$= -2|p_1|^2 (1 + \cos\theta)$$
$$= -4|p_1|^2 \cos^2(\theta/2)$$

Calculate matrix element:

$$\mathcal{M}_{tot} = \frac{g^2}{t - m_c^2} + \frac{g^2}{s - m_c^2}, \quad m_c = 0$$

$$= \frac{g^2}{4E^2} + \frac{-g^2}{4|p_1|\cos^2(\theta/2)}$$

In COM frame: $m_A = m_B \rightarrow |p_f| = |p_4|$, $|p_i| = |p_1| = |p_4|$

and $E_1 + E_2 = 2E$

4) a. cont

$$\frac{d\sigma}{d\Omega} = \frac{S|\mathcal{M}|^2}{(8\pi)^2(E_1+E_2)^2}\frac{|p_f|}{|p_i|}$$

$$= \frac{1}{(8\pi)^2(2E)^2}\left(\frac{g^2}{4E^2} - \frac{g^2}{4|p_i|\cos^2(\theta/2)}\right)^2$$

$$= \frac{g^4}{(64\pi E)^2}\left(\frac{1}{E^2} - \frac{1}{|p_i|^2\cos^2(\theta/2)}\right)^2 \rightarrow \text{this is ok as final answer}$$

$$= \frac{g^4}{(64\pi E)^2}\left(\frac{E^2\tan^2(\theta/2)+m^2}{E^2(E^2-m^2)}\right)^2$$

b. Total cross section: $\theta: 0 \rightarrow \pi$

- Integral over $d\Omega$: $d\Omega = \sin\theta\, d\theta\, d\varphi$, sin($\theta$) multiplication doesn't matter here
- $\tan^2(\theta/2) \rightarrow \infty$ for $\theta = \pi/2$ $\rightarrow$
- $\frac{1}{\cos^2(\theta/2)} \rightarrow \infty$ for $\theta = \pi$
- Difference from 4e: not assuming $m_B >> m_A$, but $m_A = m_B$
- Lab frame calculation with equal masses is also divergent
- When $m_A = m_B$ this is like $AA \rightarrow AA$ scattering: $\sigma \rightarrow \infty$ for $\theta = \pi$