The Electron

Experiment Four

Physics 192
Michigan State University

Before Lab

- Carefully read the entire lab guide
- Try a simulation of a charged particle moving in a magnetic field at https://ophysics.com/em8.html
- Attempt the theory questions

Experiment Overview

The goal of this two-week lab is to determine the charge-to-mass ratio of the electron, which is a fundamental constant of the universe. You will directly observe the action of the electromagnetic force as you manipulate a beam of electrons in a special type of cathode ray tube.

- Practice using common electronic components
- Interpret a basic circuit diagram and implement the design
- Measure the charge-to-mass ratio of the electron
- Calculate the mass of an individual electron

1 Electrons

In the spectroscopy experiment, a simplified model of atomic structure was introduced. It was shown that electrons bound to atoms can move between discrete energy states, emitting or absorbing light in the process. Macroscopic observations of the emitted light yielded information about the atomic composition. In this experiment, our investigation will probe an even smaller scale by studying *free* electrons. By liberating electrons from atomic bonds, we can determine their most fundamental properties.

The electron is considered to be an elementary particle, meaning it is indivisible with no known substructure. For this reason, the magnitude of the electron charge is defined as the **elementary charge unit**, denoted by the letter e, with a defined value of about 1.6022×10^{-19} C. Because the electron is a fundamental building block of matter, high-precision measurements of its properties are essential to chemistry, engineering, and physics.

Quantity	Symbol	Value	Unit
Charge	$-e$	-1.6022×10^{-19}	C
Mass	m	9.1094×10^{-31}	kg
Ratio	$-e/m$	-1.7588×10^{11}	C/kg

Table 1: The approximate electron charge and mass to be used as reference values

2 The Lorentz Force

The charge of a particle describes how it will respond to electromagnetic fields. If a particle with charge q is placed in a static electric field $\vec{E}$, it will experience an **electric force**

$$\vec{F}_E = q\vec{E}. \tag{1}$$

Notice that the electric force is oriented in the direction of the electric field either parallel or antiparallel depending on the sign on the charge. Equation 1 shows that the electric force vector *is exactly* the electric field vector multiplied by a scalar (the charge). Whether the particle is stationary or moving at a constant velocity, the electric force is determined solely by the charge and the properties of the field at its position. The same cannot be said about a charge moving in a magnetic field. The **magnetic force** is

$$\vec{F}_B = q\vec{v} \times \vec{B} \tag{2}$$

where $\vec{v}$ is the particle's velocity and $\vec{B}$ is the magnetic field at some location. A vector formed by a cross product is always perpendicular to the two generating vectors (in this case, $\vec{v}$ and $\vec{B}$). Evidently, the magnetic force *never* points in the direction of the magnetic field. The dependance of $\vec{F}_B$ on both the magnitude and direction of the velocity leads to some peculiar trajectories.

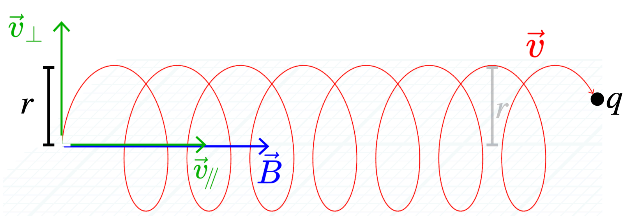


Figure 1: Charged particles circulate around magnetic field lines.

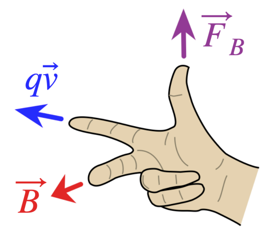


Figure 2: The right-hand rule is used to determine the direction of the magnetic force.

Figure 1 shows a red path indicating the trajectory of a charge moving at an oblique angle through a magnetic field. The spiraled path is characteristic of the magnetic force. The velocity can be expressed as $\vec{v} = \vec{v}_\perp + \vec{v}_\parallel$ identifying the components perpendicular and parallel to the blue magnetic field vector, $\vec{B}$.

- The perpendicular component, $\vec{v}_\perp$, contributes to the magnetic force, driving the circular orbit around the $\vec{B}$ vector. The direction of the force $\vec{F}_B$ is radially inward towards $\vec{B}$, which can be verified using the right-hand rule (Figure 2).
- The parallel component, $\vec{v}_\parallel$, is unaffected, so the particle continues to move in the direction of B, tracing out a helical path
- If the magnetic field is uniform, the magnitude of the magnetic force is constant, maintaining a circular orbit about $\vec{B}$ with radius R.

The **Lorentz force**[1] is the combined effect of electric and magnetic fields and is also referred to as the electromagnetic force:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right). \tag{3}$$

3 Cyclotron Motion

The circular trajectory around the magnetic field shown in Figure 1 is referred to as **cyclotron motion**. Michigan State University had a cyclotron particle accelerator called the National Superconducting Cyclotron Laboratory (NSCL). The NSCL exploited the nature of the Lorentz force to accelerate heavy ions to extremely high energies for collisions. The collisions could be tailored for applications ranging from the creation of rare isotopes for medical imaging to simulating the earliest stages of the evolution of the universe. MSU is now home to a new accelerator facility called the Facility for Rare Isotope Beams (FRIB).

[1] Some texts refer to the magnetic force alone as the Lorentz force.

Imagine a car driving around a circular racetrack at a constant speed. The friction between the tires and the track provides a centripetal force allowing the car to maintain its circular trajectory. A similar situation is shown in Figure 3, in which a beam of electrons circulates around a uniform magnetic field. The centripetal force that maintains the circular trajectory is not friction here, but rather the magnetic force. In any case, the **centripetal force** necessary to keep an object of mass m moving along a circular path is

$$F_c = m\frac{v^2}{r} \tag{4}$$

where v is the velocity and r is the radius of the circle. Consider a single electron labeled as e$^-$ in Figure 3. The electron has a charge $q = -e$ and it moves with velocity $\vec{v}$. The picture shows a magnetic field vector labeled as $\vec{B}$ pointing out of the page, but it is important to understand that the magnetic field is uniformly spread throughout the region shown. This results in a magnetic force

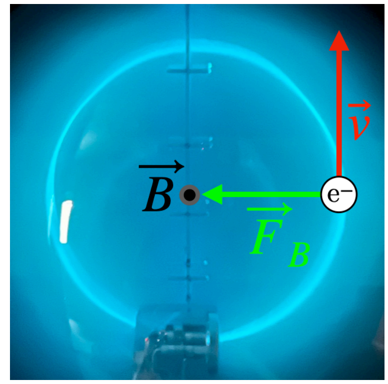


Figure 3: A beam of negatively charged electrons circulating around a uniform magnetic field.

$$\vec{F}_B = -e\vec{v} \times \vec{B},$$

which is always directed radially inward, forcing the electron to follow a circular path. The magnitude of the force vector is

$$F_B = evB\sin\theta$$

where θ is the angle between the $\vec{v}$ and $\vec{B}$. If the magnetic field is oriented to be perpendicular to the velocity, say, by moving around some magnets, then $\theta = 90°$, which results in a force of magnitude

$$F_B = evB \qquad (\text{for } \vec{v} \perp \vec{B}) \tag{5}$$

exerted on the electron. Remember, the magnetic force *is* the centripetal force responsible for sustaining circular motion, so $F_c = F_B$. Setting Equations 4 and 5 equal gives

$$m\frac{v^2}{r} = evB.$$

This equation can be rearranged to obtain a very useful relationship called the **cyclotron radius**

$$r = \frac{mv}{eB}, \tag{6}$$

which is the radius of the circular path taken by the electron. You should pause at this point to appreciate the significance of this result; it suggests an experimental technique for determining a relationship between the charge and mass of an *individual* electron. The ratio m/e can be ascertained by sending a beam of electrons through a region with a fixed magnetic field and measuring the radius of the circular path. The practice is not limited to electrons, but rather any charged particle. It is the operating principle of a mass spectrometer, a tool used throughout the field of chemistry.

4 Electron Gun

The goal of this lab is to make use of Equation 6 and determine the charge-to-mass ratio of the electron, e/m. This first step is to produce a beam of free electrons, which may be accomplished using a device called an electron gun. The electron gun used in this experiment operates via thermionic emission, meaning the electrons are liberated thermally by a high-temperature filament. A schematic view of the device is shown in Figure 5.

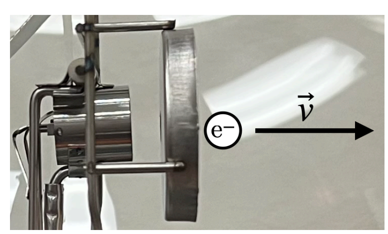


Figure 4: The electron gun used in this lab generates a continuous beam.

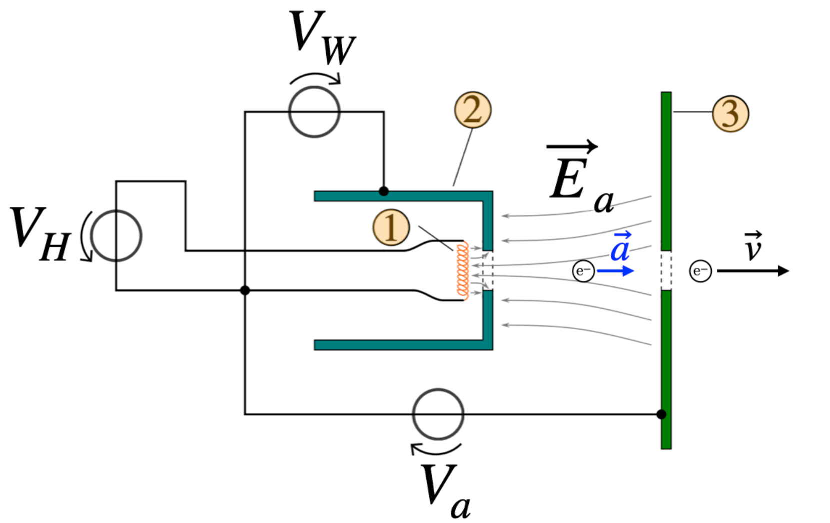


Figure 5: A schematic view of a thermionic electron gun with the three voltages sources required for operation.

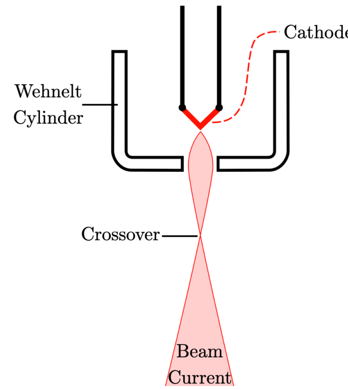


Figure 6: An alternative depiction showing the focusing effect of the Wehnelt cylinder

The device has three primary components labeled in Figure 5, each of which requires a different DC voltage source:

1. A filament similar to that in an incandescent light bulb is heated by a voltage V_H. At high temperatures, electrons near the surface of the filament are ejected from the material, leaving the filament with a net positive charge. For this reason, the filament is called a hot **cathode**.

2. The cathode is enclosed by a **Wehnelt cylinder**, a cylindrical shell with a narrow aperture positioned very close to the cathode. Applying a negative voltage V_W relative to the filament creates a repulsive electric field that suppresses electron emission nearly everywhere except near the aperture. Figure 6 shows how the repulsive field pinches the beam, producing a focusing effect.

3. The **anode** is positioned on the right and is held at a high positive voltage V_a relative to the filament. This generates a strong electric field similar in character to a parallel plate capacitor. It is nearly constant in the region labeled $\vec{E}_a$ in the schematic and effectively zero elsewhere. There is a small hole in the center of the anode that allows electrons to escape. The freed electrons move with a fixed velocity determined entirely by the **acceleration voltage** V_a.

The electric field between the cathode and anode does work $W = eV_a$ on each electron, producing electrons with well-defined kinetic energies of

$$\frac{1}{2}mv^2 = eV_a. \tag{7}$$

This result gives a relationship between the experimental acceleration voltage and the electron velocity. Rearranging Equation 7 gives

$$v = \sqrt{\frac{2eV_a}{m}}. \tag{8}$$

This relationship is useful in practice because the velocity cannot be easily measured, but the voltage is set by the experimenter. It can be combined with the cyclotron radius (Equation 6) to obtain an even more useful equation:

$$r^2 = \frac{m}{e}\frac{2V_a}{B^2}. \tag{9}$$

All of this footwork has yielded an equation that relates the mass and charge of an individual electron to macroscopic quantities we can measure in a lab: V_a, B, and r.

5 Teltron Tube

A **teltron tube** consists of a simple electron gun housed in a glass bulb filled with a low pressure inert gas, such as helium or neon. Some electrons in the beam collide with and ionize the gas, causing the emission of light when ions recombine. This makes the electron beam visible as shown in Figure 7, in which a teltron tube is placed in a uniform magnetic field. The electrons in the beam circulate around the magnetic field as described in Section 3. The radius of the illuminated circular path can be easily measured in a lab. A specific acceleration voltage is set by the experimenter, and if the strength of the magnetic field is known, the radius of the path can be used to obtain the electron charge-to-mass ratio, e/m.

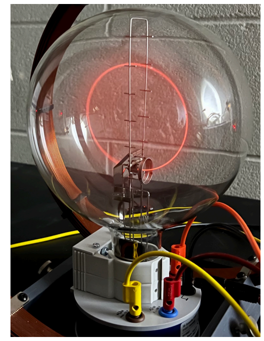


Figure 7: A teletron tube in a uniform magnetic field.

6 Helmholtz Coil

We can achieve cyclotron motion in the lab using our electron gun if we are able to generate a uniform magnetic field. This is best accomplished using a specific electromagnet configuration called a **Helmholtz coil**. The design is shown in Figure 8, consisting of two axially-aligned wire loops carrying a current I. If the loops are separated by distance equal to the loop radius, then the magnetic field generated is remarkably uniform near the center. The red box in Figure 9 shows the uniformity of the field between the two loops.

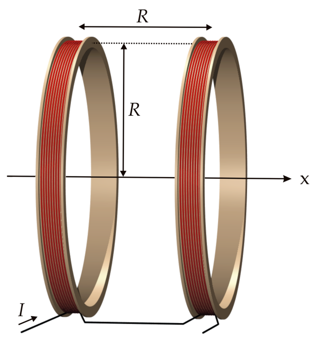


Figure 8: A Helmholtz coil carrying a current I.

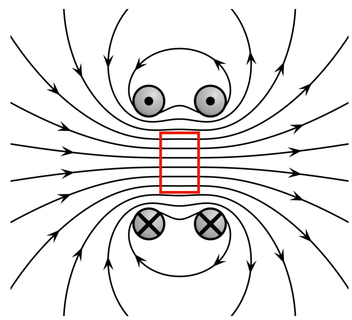


Figure 9: The magnetic field of a Helmholtz coil.

Determining the magnetic field at the midpoint between two wire loops can be done using the Biot-Savart law. At the midpoint, the horizontal components of the magnetic field cancel, producing a field that points directly along the central axis with magnitude

$$B_0 = \frac{\mu_0 N I}{R}\left[\frac{1}{1+\left(\frac{d}{2R}\right)^2}\right]^{3/2} \tag{10}$$

where

- $\mu_0 = 4\pi \times 10^{-7}$ H/m is the vacuum permeability
- N is the number of turns in each loop (our setup has $N = 124$)
- I is the current passing through the coil
- R is the loop radius
- d is the separation between loops

While this expression only gives the value of the field at the midpoint, it is a good approximation of the field throughout the plane between the two loops. The field strength in the midplane is illustrated in Figure 11.

Measuring from the middle of the central axis, the magnetic field falls off slowly with the distance labeled r. The red point indicates the distance $r = 0.5R$, which is half of the loop radius. At this distance, the magnetic field differs from Equation 10 by only a few percent. Beyond this point, the difference between the true field, B, and the axial field B_0, becomes significant.

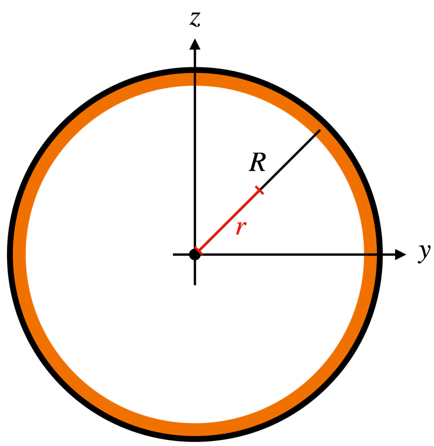


Figure 10: Looking down the central axis

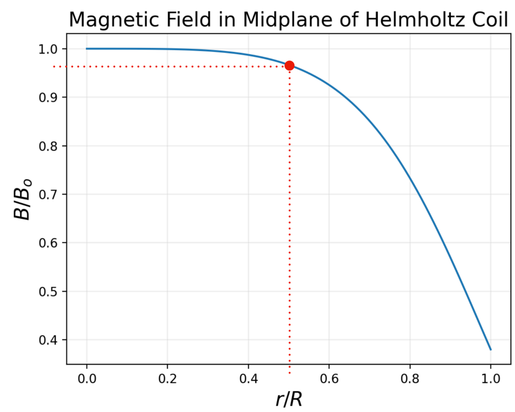
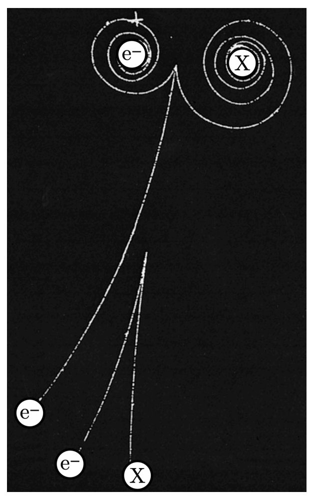


Figure 11: The magnetic field strength in the plane shown to the left.

7 Theory Questions

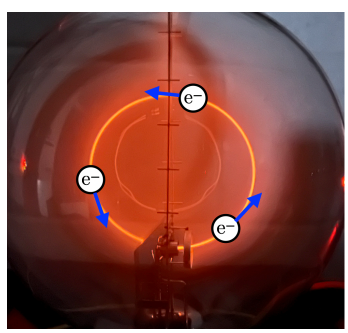
1. The picture to the right shows an image from a type of particle detector called a bubble chamber. High energy collisions can create particle-antiparticle pairs (matter and antimatter). The collisions occur in a chamber filled with a special fluid in a uniform magnetic field. Newly created charged particles produce a stream of bubbles as they move through the fluid. In the picture, the spiral trajectories indicate the creation of an electron and another particle. As the particles move through the chamber, they slow down from collisions within the fluid causing an inward spiral.



   (a) (1 pt) What force is responsible for the spiral trajectories?

   (b) (1 pt) What causes the particles labeled X to spiral in a direction opposite from the electrons? Can you guess the name of particle X?

2. On the right is an electron beam confined to a circular orbit using a Helmholtz coil. The blue arrows show the velocity of the electrons at the indicted points.



   (a) (1 pt) Sketch the path of the electrons and draw the magnetic force vector at each point.

   (b) (1 pt) In which direction does the magnetic field point?

   (c) (1 pt) If the magnetic field is suddenly turned off, will the electrons continue to move in a circle until another force is applied?

3. (1 pt) Use Equation 8 to substitute the electron velocity out of the cyclotron radius equation (6). Simplify your answer to arrive at Equation 9. **Show your work**.

4. (2 pt) In the lab, you have the ability to measure r, V_a, and indirectly, B. Define substitutions x and y such that you can write your answer to the previous equation as $y = (e/m)x$.

5. (1 pt) Equation 10 gives the magnetic field at the midpoint between two axially-aligned wire loops separated by a distance d. A Helmholtz coil has $d = R$ where R is the loop radius. Use this fact to simplify Equation 10. You will use this to calculate the field strength in your Helmholtz coil during the experiment.

6. (1 pt) An electron moves with velocity $\vec{v}$ in a Helmholtz coil at a distance $r = 0.8R$ from the midpoint. At this distance, the magnetic field has a reduced value of $B = \beta B_0$. Use Figure 11 to estimate the value of β at this point. Based on the plot, do you think the approximation $B \approx B_0$ when $r = 0.3R$ is valid?

8 Experimental Apparatus

Each mechanism described in the preceding sections can be combined to build an experiment for determining the charge-to-mass ratio of an electron. The primary components of the system are shown in Figure 12.

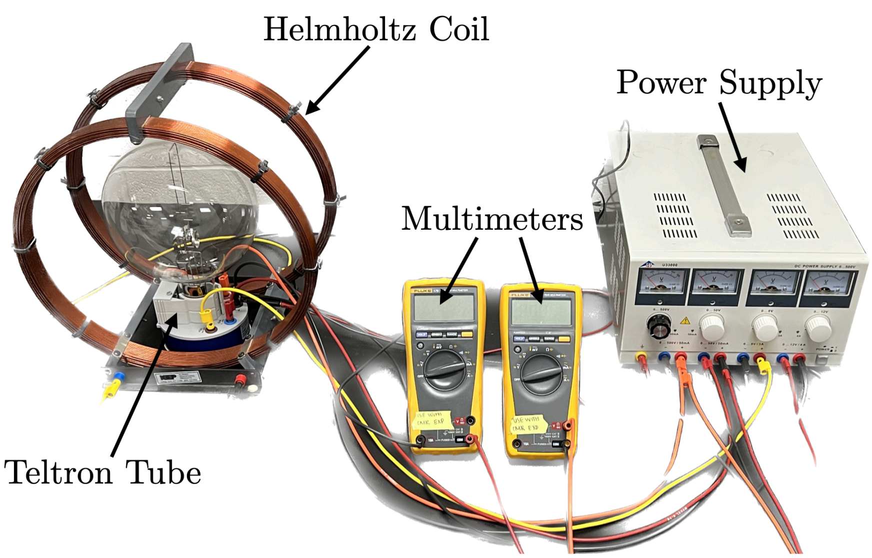


Figure 12: The e/m experimental apparatus.

8.1 Digital Multimeter

A digital multimeter (DMM) is an electronic instrument used to measure electrical parameters like voltage, current, and resistance in a circuit. The Fluke 179 DMM is the model used in this lab. The dial in the center can be rotated to select different measurement features. A yellow button on the top right side is used to toggle between AC/DC signials.

- A voltage measurement looks at the difference between two sides of a circuit element, requiring the multimeter to be wired in parallel as shown in Figure 13.
- A current measurement looks at the *flow* through a circuit, requiring the multimeter to be wired in series as shown in Figure 13.

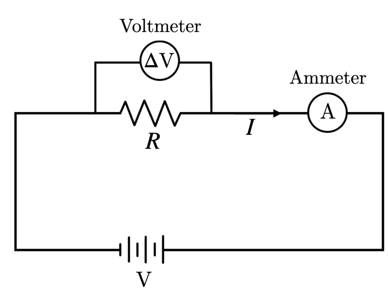


Figure 13: Voltage and current measurements in a circuit.

- **All sources in this experiment operate on direct current.** Always ensure that "DC" is shown on the right side of the display for proper use.
- There are four input terminals on the bottom of the DMM. **Never use the 400 mA input.** There should be tape over this terminal to prevent accidental use. This input is for measuring small currents. The currents measured in this lab exceed the maximum value for this circuit and will blow the fuse in the DMM.
- The "10 A" terminal is the input for measuring electric current on the Amp scale.
- The terminal labeled with "V Ω" is used for measuring both voltage and resistance. The measurement type is selected using the mode selection dial.
- The terminal labeled "COM" is common (return) input.

8.2 Power Supply

The Electron Gun section (4) explains that three voltages are required to generate the focused beam of electrons. A fourth source is required to drive the Helmholtz coils. In this experiment, you will use a single power supply with four channels to operate the system. The channels are ordered from left to right as shown in Figure 14. Each channel has a different range of voltages indicted on the scale and above the input and return terminals.

Important precaution: always check that all knobs are rotated counterclockwise until they stop before turning the device on. This ensures each channel is OFF before power is supplied to a circuit.

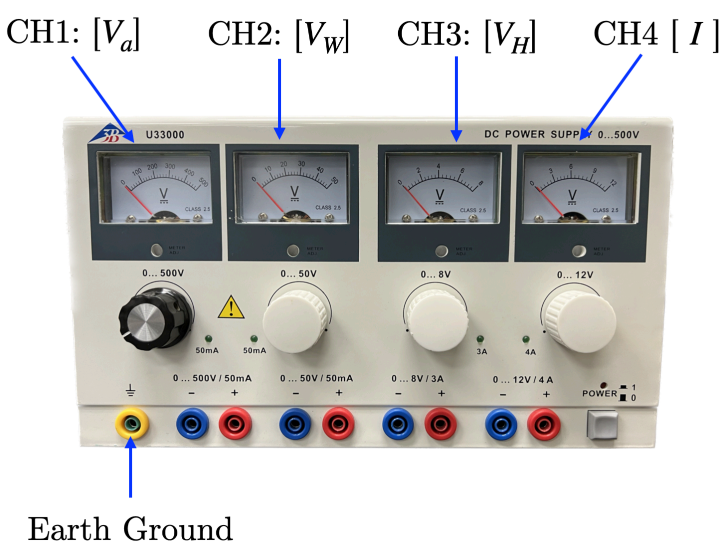


Figure 14: The four channel 3B Scientific U33000 power supply.

CH1: this is the channel used for the acceleration voltage, V_a. It is a high voltage source, operating from 0-500 V. **Never exceed 260 V in this experiment**. This Black knob indicates a fine adjustment, which is needed to span the large range of possible voltages.

CH2: this channel is used for the Wehnelt cylinder, providing the voltage V_W that focuses the beam.

CH3: this channel supplies the voltage V_H to heat the cathode (electron source) inside of the teltron tube. In most cases, this need not exceed 6 V.

CH4: this channel provides a voltage that drives the current in the Helmholtz coil, providing control of the magnetic field.

9 Constructing the Circuits

In order to perform your experiment, you need to wire the circuits to operate the electron gun and Helmholtz coil. You have all components necessary at your bench, including a multiple cables with *banana* style connectors. The connectors plug directly into the terminals on each component, and they can be linked together in series and parallel. The color of the cable has no effect on its operation.

Understanding the circuits in the apparatus is best done be considering each loop independently. Work your way from left to right using the diagram below. You may find it simpler to wire the circuits without multimeters, adding them in the proper orientation (series or parallel) once you have formed the loop.

- Depending on your equipment, you may have leads labeled with "U" instead of "V" for voltages.
- **Do not try to tighten or loosen the teltron tube (bulb) in the base.**
- **DO NOT TURN ON YOUR POWER SUPPLY UNTIL YOUR TA HAS CHECKED YOUR CIRCUIT**.
- In order to maximize the lifetime and quality of the teltron tubes, try not to leave them running for extended periods of time while answering questions or working on your report.

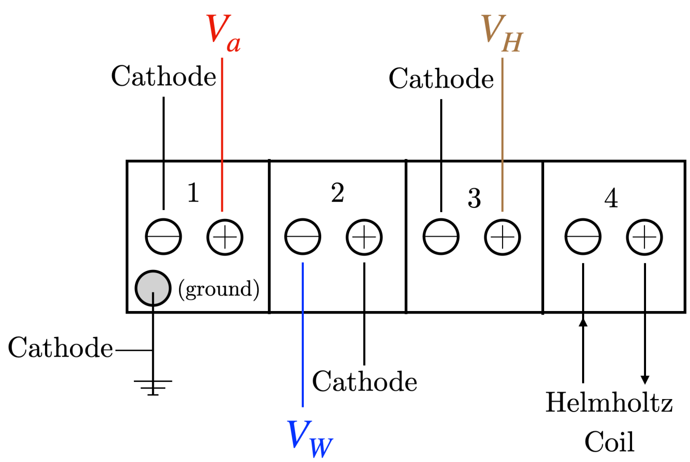


Figure 15: A simplified input and output diagram for each channel on the power supply.

All diagrams and sketches in your lab report can be done by hand. You cannot copy-paste pictures from the lab guide. It is important that you work out your own diagrams to build an understanding of your system.

1. (2 pt) Explain how a teltron tube works. Describe the electron gun, including an explanation of the cathode, anode, Wehnelt cylinder, and the necessary voltages for operating the device.
2. (1 pt) Sketch a diagram of a Helmholtz coil, including a rough sketch of the magnetic field. Explain how the device works and its role in the experiment.
3. (1 pt) Explain how you will use your power supply to operate the teltron tube and Helmholtz coil.
4. (3 pt) Sketch a simplified diagram of your apparatus, including the wires connecting each component. It should be drawn such that another student could interpret and build each circuit.
5. Use the banana cables to construct each circuit using Figure 15 as a guide.
6. (1 pt) You will use one multimeter to measure the acceleration voltage, V_a. On which channel should this be connected? Should it be wired in series or parallel? Add the multimeter to your circuit.

7. (1 pt) You will use another multimeter to measure the current in the Helmholtz coil. On which channel should this be connected? Should it be wired in series or parallel? Add the multimeter to your circuit.
8. (2 pt) Once your circuit is complete, have your TA come check your system and show you how to use a third multimeter for precisely measuring V_W and V_H. Take a picture of you apparatus and include it in your report.
9. (1 pt) Measure the radius and separation between the wire loops in your Helmholtz coil

10 Charge-to-Mass Ratio Experiment

Once your TA has approved your apparatus, you can begin your experiment to determine the charge-to-mass ratio of the electron. An rough outline of the procedure for operating your apparatus is as follows:

- Turn on the power supply and both multimeters. Set the multimeters to the appropriate mode for measuring their respective channels. Attach the third multimeter to CH3 to measure V_H with a higher precision than what is capable with the analog scale on the power supply.
- Increase the heater voltage to about 5.5 V. With your bench as dark as possible, look near the back of the cathode. You should see the filament begin to glow red as it is heated. Heating may take close to 1 minute. If the filament is still not visible, slightly increase V_H. **Do not exceed 6.5 V or the filament can burn out**.
- Increase the acceleration voltage (CH1) to around 150 V. Monitor the voltage on the multimeter as you turn the dial. You should start to see a fuzzy beam of electrons emerging from the aperture in the anode.
- Use CH4 to produce a current in the Helmholtz coil. The effect should be immediately obvious as the electrons respond to the magnetic field. If your beam bends downward, the direction of current in the coil must be reversed. You should observe cyclotron motion. By adjusting CH4, you can directly tune the orbit radius (Equation 6).
- Increase the voltage applied to the Wehnelt cylinder (CH2) and observe the effect. Tune this value to produce a tightly focused beam.
- Spend some time adjusting the voltages and observed the effects. Try rotating the teltron tube by a few degrees in each direction and watch how the trajectory of the beam changes. *Do not tighten or loosen the bulb in the base.*
- Do your best to form a bright, narrow beam that forms a closed circle.

1. (1 pt) If the Wehnelt voltage is increased too high, the beam vanishes. What is responsible for this phenomenon?
2. (1 pt) Which voltage in your system determines the kinetic energy of the electrons in the beam?
3. (1 pt) Does increasing the current in the Helmholtz coil increase the magnetic field strength? What effect does this have on the speed of the electrons?

In the theory section, it was shown that the equation

$$r^2 = \frac{m}{e}\frac{2V_a}{B^2} \tag{11}$$

relates the charge and mass to quantities we can control in the lab. You will use this equation to model your experiment. Housed inside of the teltron tube is a measurement scale that looks like a ladder. The perpendicular rungs are spaced 2 cm apart as shown in Figure 16. This allows you to directly measure the beam radius.

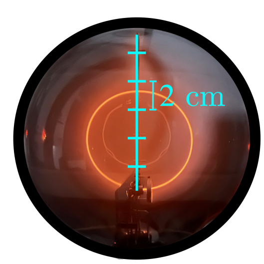


Figure 16

4. (1 pt) By tuning the magnetic field strength, you can precisely adjust the beam radius. As shown in Figure 11, the magnetic field is very uniform for $r < 0.5R$. For this reason, we can approximate the magnetic field experience by the electron beam to be that directly at the midpoint between the loops, $B \approx B_0$. Write the equation for B using your result from theory Question 5. Each wire loop has N=124 turns. Plug in μ_0, N, and R and express B as a function of I (the current).

5. (1 pt) With your beam nicely focused and following a circular path, record $V_H \pm \delta V$ and $V_W \pm \delta V$. Leave these values fixed for the remained of the lab.

6. Set $V_a = 150$ V. Using the rungs on the measurement scale as a reference, record the current, I, for at least 6 different values of r.

7. (5 pt) Use a curve fitting technique to determine e/m. You need to make use of curve.fit Equation 11, and your measurements of I and r. You can use the linearization method (theory Question 4) or you can choose a different function to fit to your data. Include error bars, axes labels, and a title including the the acceleration voltage used. **Important: show how your error bars were determined**.

8. (5 pt) Repeat the measurements for $V_a = 200$ V and make another plot with V_a in the title.

9. (5 pt) Repeat the measurements for $V_a = 250$ V and make another plot with V_a in the title.

10. (3 pt) Calculate your best estimate of the charge-to-mass ratio and its uncertainty statistically using the three values obtained via curve fitting. Remember, the uncertainty in the mean of a quantity is the standard error.

11. (2 pt) Calculate the percent error using $e/m = 1.7588 \times 10^{11}$ C/kg as the reference value. Comment on the accuracy of your result. What could be the largest sources of error?

12. (1 pt) A relatively simple bench-top experiment called Millikan Oil Drop Experiment can be performed to measure the elementary charge unit e. Combined with the results from this experiment, the mass of an individual electron can be determined. Assume you performed the oil drop experiment and obtained $e = 1.6022 \times 10^{-19}$ C $\pm$ 5%. Combine this number with your result to calculate $m \pm \delta m$ for an individual electron.

13. (1 pt) Write a short summary of this lab.

Extra credit (2 pt): determine which of your three measurements yielded the most *precise* result quantitatively by calculating the fractional uncertainties in e/m.