PHY 481 - Fall 2024
Homework 11

Due Saturday, November 23, 2024

Preface

In Homework 11 you will get some practice with current densities and calculating magnetic field.You will also 3D plot in Python the trajectory of the charged particle in a magnetic field.

1 Current Densities

We are going to be working with "current densities" for the rest of the term. Let's practice writing down some current densities. These are important for understanding books and papers where folks often use these forms as shorthand to describe what they are talking about.

1. A sphere (radius R, total charge Q uniformly distributed throughout the volume) is spinning at angular velocity $\omega\hat{z}$ about its center, which is at the origin. What is the volume current density $\mathbf{J}(r, \theta, \phi)$ at any point (r, θ, ϕ) in the sphere? (Don't forget direction too!)

2. A very thin DVD has been rubbed so that it has a fixed, constant, uniform surface electric charge density σ everywhere on its top surface. It is spinning at angular velocity ω about its center (which is at the origin). What is the magnitude of surface current density $\mathbf{K}$ at a distance r from the center? What is the **volume** current density $\mathbf{J}$ in cylindrical coordinates? (This may be a little tricky, since the disk is "very thin," there will be a δ function. *Hint: write down a formula for* $\rho(s, \phi, z)$ *first. And, remember that* $\mathbf{J}$ *should be a vector!)*

3. A very thin plastic ring (radius R) has a constant linear charge density, and total charge Q. The ring spins at angular velocity ω about its center, which is the origin. What is the current I, in terms of given quantities? What is the volume current density $\mathbf{J}$ in cylindrical coordinates? (This may be a little tricky, since the ring is "very thin," there will be some δ functions. *Hint: write down a formula for* $\rho(s, \phi, z)$ *first. And, remember that* $\mathbf{J}$ *should be a vector!)*

2 Magnetic field of distributed currents

In the previous problem, we had a DVD (radius R) with a fixed, constant, uniform surface electric charge density σ everywhere on its top surface (figure below). It was spinning at angular velocity ω about its center (the origin). You should find the current density $\mathbf{K}$ at a distance s from the center as $\mathbf{K} = \sigma\omega s\hat{\phi}$

1. Write $\vec{r}$, $\vec{r'}$, $\vec{\mathscr{r}}$, $\hat{\mathscr{r}}$, and da' using the cylindrical coordinates s, ϕ, and z. Write $\mathbf{K} \times \vec{\mathscr{r}}$ in cylindrical coordinates using the fact that $\hat{\phi} \times \hat{z} = \hat{s}$ and $\hat{\phi} \times \hat{s} = -\hat{z}$

2. To calculate the magnetic field, we will use the equation 5.42 in Griffiths:
   $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{K}(\mathbf{r'})\times\vec{\mathscr{r}}}{\mathscr{r}^3}da'$
   Using the result from the previous part, construct the integral. What happens to the part of the integral with $\hat{s} = cos\phi\,\hat{x} + sin\phi\,\hat{y}$? Solve the integral with $\hat{z}$ to express $\mathbf{B}(z)$.

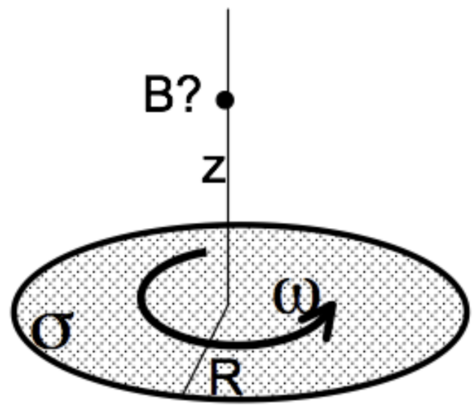

Figure 1: Spinning disk

3. Does your answer to part 2 seem reasonable?
   Please check it, with units, and some limiting behaviors (e.g. what do you expect if $R \to 0$? $\omega \to 0$? $z \to \infty$?) For this last one, don't just say "it goes to zero. This is a dipole, so B should go to 0 like $1/z^3$. (Right?) Show that it does!

3 Magnetic field of a bent wire

An infinitely long wire has been bent into a right angle turn, as shown. The "curvy part" where it bends is a perfect quarter circle, radius R. Point P is exactly at the center of that quarter circle. A steady current I flows through this wire.

Find $\mathbf{B}$ at point P (magnitude and direction) (*Hint: No need to derive any formulas "from scratch" if you can get them from Griffiths examples!*)

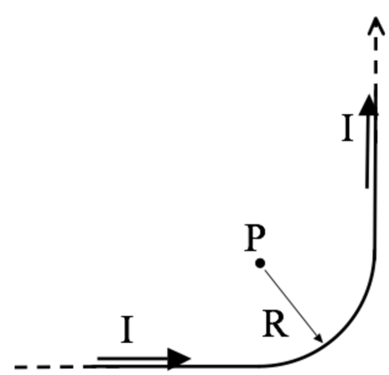

Figure 2: Bent wire

4 Python : Modeling the motion of a charged particle in a magnetic field

We have shown that the motion of a charged particle in a constant magnetic is a circular when the initial velocity is perpendicular to the field. In this problem, you will complete the code in a Jupyter notebook to model the motion of a proton in a magnetic field. Download the template HW11_MotionOfChargeInMagneticField.ipynb from D2L.

1. Your first task is to read through the code and complete the integration loop to compute the trajectory of the proton and plot it in 3D. (You might need to look up how to construct a 3D plot.) For this first case, you should expect a simple circular orbit because the proton starts it's motion moving perpendicular to the magnetic field.

2. Once you have your code working for part 1, change it to give the proton a component of velocity along the direction of the magnetic field. What does the resulting motion look like? Explain qualitatively why it should look like that.