1. (3pt x 8)

Decays of fundamental particles (particles with no internal structure) that are on-shell (mass corresponds to their rest mass) must strictly follow conservation rules. One of the following decays is allowed and the rest violate at least one conservation rule. For each process, list all violated conservation rules or identify it as allowed and state what interaction is responsible. Consider: energy conservation, charge conservation, lepton number and flavor, baryon number, quark flavor.

(a) $\gamma \rightarrow e^- + \mu^+$

(b) $W^+ \rightarrow t + \overline{b}$

(c) $Z^0 \rightarrow \mu^+ + \mu^+$

(d) $t \rightarrow W^+ + b$

(e) $W^- \rightarrow e^- + \nu_e$

(f) $\gamma \rightarrow \tau^- + \tau^+$

(g) $b \rightarrow c + e^-$

(h) $\overline{b} \rightarrow Z + \overline{s}$

There can be multiple violations in each example, so watch out.

(a) Clearly a violation of lepton flavor. Also, wouldn't an on-shell photon have no energy to contributre to the electron and antimuon mass?

(b) The on-shell W^+ boson has less mass than a top quark, so this cannot occur. I think weak interactions don't care about quark flavor conservation

(c) Charge and lepton flavor violation.

(d) Acceptable. Quark flavor is fine due to weak force

(e) Lepton flavor violation. Would be fine if positron or antineutrino.

(f) Energy violation again with an on-shell photon.

(g) quark flavor violation, no weak forces seen here. Also lepton flavor

(h) energy violation, Z is much heavier than $\overline{b}$

Let's identify each of these particles. Mass in GeV/c^2

γ photon mass:0 charge:0
e^- electron mass:0.000511
μ^+ antimuon mass:0.106

W^+ W boson mass:80.39 charge:+1
t top quark mass:173.1 charge:+2/3
$\overline{b}$ anti-bottom quark mass:4.18 charge: +1/3

Z^0 Z Boson mass:91.19

b bottom quark mass:4.18 charge: -1/3

W^- W boson mass:80.39 charge:-1
ν_e electron neutrino mass:small, charge:0

τ^- tau mass:1.777 charge:-1
τ^+ tau mass:1.777 charge:+1

c charm quark mass:1.28 charge:+2/3

$\overline{s}$ antistrange quark mass: 0.1 charge:+1/3

2. (3pt x 6)

Decays of hadrons (particles made up of quarks) can proceed via many different processes. Evaluate the following reactions and determine which are physically possible. Draw Feynman diagrams at the quark level for the reactions that are allowed. For those that are forbidden, what conservation laws are violated? If the reactions are allowed, which interactions (strong, weak, and electromagnetic forces) should be involved? Please find more information on these particles in the PDG:
https://pdg.lbl.gov/2024/listings/contents_listings.html,
https://pdg.lbl.gov/2024/tables/contents_tables.html.

(a) $\Omega^- \to \Xi^- + \pi^-$

(b) $\Sigma^+ \to \pi^+ + \pi^0$

(c) $\pi^0 \to \mu^+ + e^- + \bar{\nu}_e$

(d) $D^- \to K^+ + \pi^- + \pi^-$

(e) $p + \bar{p} \to \pi^- + \pi^+$

(f) $p \to e^+ + \gamma$

Incomplete, sorry!

(a) charge violation -1 to -2.

(b) This doesn't work out, where does the strange quark in Σ^+ has to go somewhere

(c) This fails due to lepton flavor. The neutral pion has no lepton flavor, so there is a net gain of one -1 muon flavor.

(d)

(e) This one works. Since the π^+ pion is $u\overline{d}$ and the proton is uud, there is a $u\overline{u}$ annihilation and the remaining quarks are split up into their pions. The annihilation creates virtual particles (gluons) which the remaining particles absorb. Gluons are necessary to keep the chromodynamics in order.

(f) quark violation, lepton flavor violation. Where do the quarks go?

Hadron descriptions

Ω^- sss 1.6 GeV/c^2
Ξ^- dss 1.3 GeV/c^2
π^- $\overline{u}d$ .14 GeV/c^2

Σ^+ uus 1.2 Gev/c^2
π^0 $u\overline{u}$ or $d\overline{d}$ .13 GeV/c^2

3. (20pt)
What would be the approximate counting rate observed in the Rutherford scattering of 10 MeV alpha particles off gold foil at an angle of $\pi/4$ in the laboratory? Assume an incident flux of 10^5 alpha particles per second on the foil, a foil of 0.1 cm thickness, and a detector of transverse area 1 cm x 1 cm placed 100 cm from the interaction point, the atomic (mass) number of gold of 79 (197), and the density of gold of 19.7 g/cm^3.

This question asks what the odds are that helium atoms hit gold atoms and scatter at a $45\degree$ angle. Hyperphysics has a pretty good walkthrough of this problem

The scattering of alpha particles from nuclei can be modeled from the Coulomb force and treated as an orbit. The scattering process can be treated statistically in terms of the cross-section for interaction with a nucleus which is considered to be a point charge Ze. For a detector at a specific angle with respect to the incident beam, the number of particles per unit area striking the detector is given by the Rutherford formula:

$$N(\theta) = \frac{N_i n L Z^2 k^2 e^4}{4r^2 KE^2 \sin^4(\theta/2)}$$

$N_i =$ *number of incident alpha particles*
$n =$ *atoms per unit volume in target*
$L =$ *thickness of target*
$Z =$ *atomic number of target*
$e =$ *electron charge*
$k =$ *Coulomb's constant*
$r =$ *target-to-detector distance*
$KE =$ *kinetic energy of alpha*
$\theta =$ *scattering angle*

Since this is the rate per unit area, we can either assume the angle is constant across the 1cm x 1cm detector, or do some calculus. I choose the former.

We'll have $N(\theta)$ calculate the rate of particles per unit area rather than the count. Then we have

$$\text{gold molar mass} = 196.967\ g/mol$$
$$19.7\ g/cm^3\ /\ 196.967\ g/mol\ =\ 0.1 mol/cm^3$$
$$n = 0.1 * 6.023e23\ \text{atoms}/cm^3$$

In [38]:

```
from scipy.constants import e, epsilon_0, pi, N_A
from numpy import sin

Ni = 1e5
n = 19.7/196.967*N_A #atoms / cm3
L = 0.1 # cm
Z = 79
k = 1/(4*pi*epsilon_0)
r = 100 # cm
KE = 10 * e * 1e6 #MeV to J
theta = pi/4

N = Ni * n * L * Z**2 * k**2 * e**4 / ( 4 * r**2 * KE**2 * (sin(theta/2))**4 )
N
```

Out[38]:

```
9.085792346300234e-05
```

That's much smaller than one. Maybe a flux of 10^5 isn't really a big number. Since our unit area is 1cm x 1cm, this is also the rate which the detector sees. It comes out to one particle every ~3 hours.