# Problem 1

**Kittel & Kroemer, Chapter 4, problem 6 [Pressure of thermal radiation]**

## (a): 1 point

Show for a photon gas that:

$$p = -(\partial U/\partial V)_\sigma = -\sum_j s_j \hbar (d\omega_j/dV),$$

where $s_j$ is the number of photons in the mode $j$.

## (b): 1 point

Show for a photon gas that:

$$d\omega_j/dV = -\omega_j/3V.$$

## (c): 1 point

Show for a photon gas that:

$$p = U/3V.$$

Thus the radiation pressure is equal to $\frac{1}{3}\times$(energy density).

# Problem 2

**Kittel & Kroemer, Chapter 4, problem 11 [Heat capacity of solids in high temperature limit]: 4 points**

Show that in the limit $T \gg \theta$ the heat capacity of a solid goes towards the limit $C_V \to 3Nk_B$, in conventional units. To obtain higher accuracy when $T$ is only moderately larger than $\theta$, the heat capacity can be expanded as a power series in $1/T$, of the form

$$C_V = 3Nk_B \times \left[1 - \sum_n a_n/T^n\right].$$

Determine the first nonvanishing term in the sum. Check your result by inserting $T = \theta$ and comparing with Table 4.2.
(Hint: you will want to Taylor expand the integrand $x^3/(e^x - 1)$. Also, in conventional units, $C_V = (\partial U/\partial T)_V$).

# Problem 3

**Kittel & Kroemer, Chapter 4, problem 13 [Energy fluctuations in a solid at low temperatures]: 4 points**

Consider a solid of $N$ atoms in the temperature region in which the Debye $T^3$ model is valid. The solid is in thermal contact with a heat reservoir. Use the results on energy fluctuations from Chapter 3 to show that the root mean square fractional energy fluctuation $\mathcal{F}$ is given by

$$\mathcal{F}^2 = \langle(\epsilon - \langle\epsilon\rangle)^2\rangle/\langle\epsilon\rangle^2 \approx \frac{0.07}{N}\left(\frac{\theta}{T}\right)^3.$$

Suppose that $T = 10^{-2}$ K, $\theta = 200$ K, and $N \approx 10^{15}$ for a particle 0.01 cm on a side; then $\mathcal{F} \approx 0.02$. At $10^{-5}$ K the fractional fluctuation in energy is of the order of unity for a dielectric particle of volume 1 cm$^3$.

# Problem 4

**Kittel & Kroemer, Chapter 4, problem 17 [Entropy and occupancy]: 4 points**

We argued in this chapter that the entropy of the cosmic black body radiation has not changed with time because the number of photons in each mode has not changed with time, although the frequency of each mode has decreased as the wavelength has increased with the expansion of the universe. Establish the implied connection between entropy and occupancy of the modes, by showing that for one mode of frequency $\omega$ the entropy is a function of the photon occupancy $\langle s\rangle$ only:

$$\sigma = \langle s+1\rangle \ln\langle s+1\rangle - \langle s\rangle \ln\langle s\rangle.$$

It is convenient to start from the partition function.

# Problem 5

**Kittel & Kroemer, Chapter 4, problem 18 [Isentropic expansion of photon gas]**

Consider the gas of photons of the thermal equilibrium radiation in a cube of volume $V$ at temperature $\tau$. Let the cavity volume increase; the radiation pressure performs work during the expansion, and the temperature of the radiation will drop. From the result for the entropy we know that $\tau V^{1/3}$ is constant in such an expansion.

## (a): 2 points

Assume that the temperature of the cosmic black-body radiation was decoupled from the temperature of the matter when both were at 3000 K. What was the radius of the universe at that time, compared to now? If the radius has increased linearly with time, at what fraction of the present age of the universe did the decoupling take place?

## (b): 2 points

Show that the work done by the photons during the expansion is

$$W = (\pi^2/15\hbar^3 c^3) V_i \tau_i^3 (\tau_i - \tau_f)$$

The subscripts $i$ and $f$ refer to the initial and final states.