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What's the global minimum of the unknown function?
An experiment in evolutionary computation

Introduction

This paper describes an experiment in evolutionary computation. Evolution is the process by which the mechanisms of variation, competition, and inheritance are applied to replicating agents. This results in the selection of advantageous mutations which can accumulate into significant changes over many generations (Darwin 1859). Evolution need not be limited to a biological medium, and can be facilitated by a computer. I conducted several experiments to assess how computational evolution would be affected by parent/child ratios, mutation rates, death and aging, and sexual reproduction.

Method

I developed an evolutionary program to find the global minimum of an unknown function. This function performed a calculation using five double variables. The object of selection that the program acted upon was a double array containing five numbers. The program randomly seeded the initial population by creating objects with random double values between -128–127. This range was determined by trial and error on my part until I found a range sufficient to produce interesting results.

At each generation, the population would be sorted based upon the result returned when their arrays were passed to the unknown function. Objects producing smaller results were considered fitter and sorted first. Beginning with the fittest, each object would be copied to create a number of children until the required number of parents had reproduced. Each child would be a near perfect copy of their parent, except that one of their five values would be randomly reassigned within the limitations of a mutation range. The next generation would then begin, and sorting would reoccur.

The evolutionary process implemented by this program was modifiable based upon the values assigned to eight global variables. These variables determined:

1. The number of parents that would be selected at each generation.
2. The number of children that each selected parent would have.
3. The number of generations for which the process would run *(default: 50)*.
4. The maximum number of generations for which an object could live *(default: infinite).*
5. The starting numerical range within which one of the child's values would differ from the parent's *(default: 2.56).*[1]
6. Whether this mutation range could be changed through evolution *(default: no).*
7. Whether objects reproduced through recombination *(default: no)*.
8. The probably that a selected object would reproduce (setting this low would enable some less efficient objects to reproduce) *(default: 100%)*.

I experimented with many different combinations for the above settings. The more interesting experiments are discussed below. Every experiment was automatically repeated 100 times, and the mean minimum result was calculated. I evaluate the efficiency of different settings by comparing these averages and their standard deviations.

[1] This default value is 1% of the initial starting range for object values.

Experiments

1. <u>Parent/child ratios</u>

Hypothesis: Maximising the number of parents will result in the best evolutionary outcomes, as this will increase the total diversity of the population.

Allowing for 1 parent and 1 child object at each generation, and using all other default settings, the average minimum result found after 100 experiments was 22.7090 (s=0.0798). These objects consume memory which is a limited resource, so I was initially interested in whether having more parents or children in the population would result in better evolutionary outcomes. I ran several experiments with differing numbers of parents and children, all other settings were the defaults. The following table records the results:

Table 1: Ratios of parents and children

#	Number of parents	Number of children per parent[2]	Average minimum[3]
1	1000	1	21.2696 (s=1.3087)
2	500	3	19.4685 (s=2.1825)
3	250	7	17.3845 (s=2.9824)
4	100	19	16.0750 (s=3.5645)
5	50	39	16.3435 (s=3.3651)
6	25	79	16.3316 (s=3.5994)
7	10	199	15.5840 (s=3.6472)
8	5	399	16.4581 (s=3.5090)
9	2	999	16.3240 (s=3.3428)
10	1	1999	16.3469 (s=3.7803)

My initial hypothesis was incorrect. Having more parents does not improve the evolutionary process as having a population with less than 95% children inhibits evolution. However, over 95%, the results are fairly similar so it's unclear as to the ideal ratio of parents to children. Based on this learning, my remaining experiments were all conducted with 100 parents having 20 children each.

2. <u>Ageing</u>

Hypothesis: Death by old age will enhance the evolutionary process, so long as objects do not die too young, as killing older objects will eliminate those that have settled into local minimums.

I was further interested in how age and death would impact evolution. I counted an object's age as the number of generations that it had survived for. An object was removed from the population if it survived until the maximum age. I ran several experiments with varying maximum ages:

Table 2: Maximum ages

#	Maximum age	Average minimum
11	1	15.4913 (s=3.1251)
12	2	16.0852 (s=3.5563)
13	5	15.6348 (s=3.6836)

[2] Each experiment in table 1 has a total population of exactly 2000 objects. For example, in experiment 8, 25 parents are selected to produce 79 children each. This means that the total population is 25 + (25 * 79).
[3] This is the mean average minimum result across 100 experiments with these settings.

14	10	16.0728 (s=3.6355)
15	25	15.4372 (s=3.5506)
16	nil (objects are immortal)	16.0377 (s=3.4357)

This data does not confirm my hypothesis. There appears to be no trend affecting evolutionary outcomes based upon the allowed maximum age. However, such a trend may occur if the mutation range is altered.

3. Mutation range

Hypothesis: Where the maximum age is low, a higher mutation range should inhibit evolution.

I proceeded to investigate the degree to which mutation range would enhance evolution. To illustrate, where the mutation range is 2.0, a value of 4, if mutated, would randomly change within the range of 3–5. The default mutation range was 1% of the starting range for object values. This seems sensible, given that natural evolution acts upon "infinitesimally small variations" (Darwin 1859). A high mutation range should inhibit evolution if it causes accumulated genetic information to disappear too quickly. This would certainly be the case if the maximum age was set to kill objects after only a few generations. I conducted several experiments with different mutation ranges and no maximum age (immortality), and repeated those experiments with a maximum age of 1 (instant death).

Table 3: Mutation ranges				
#	**Mutation range**	**Average minimum with immortality**	**#**	**Average minimum with instant death**
17	1.0	21.9731 (s=0.7145)	26	21.5968 (s=0.7848)
18	2.0	17.9753 (s=2.8537)	27	17.6230 (s=2.7691)
19	4.0	10.9218 (s=1.6142)	28	12.0850 (s=1.3487)
20	8.0	10.1767 (s=0.0016)	29	12.2263 (s=1.1533)
21	16.0	10.1784 (s=0.0023)	30	13.9604 (s=1.8369)
22	32.0	10.1855 (s=0.0060)	31	16.1788 (s=2.7026)
23	64.0	10.2151 (s=0.0252)	32	19.0964 (s=2.7958)
24	128.0	10.2843 (s=0.0615)	33	20.9494 (s=2.4447)
25	256.0	10.5729 (s=0.2428)	34	21.9607 (s=1.6602)

The results confirm my hypothesis. The optimal mutation range in the instant death case seems to be 4. In the immortality case, there is a very minor trend toward inefficiency at mutation ranges above 8. This is likely due to the fact that it is harder to move toward optimality when the scope for mutation is high, even though successful objects don't die. In both cases, low mutation ranges were inefficient. This is likely due to the fact that very small changes will not allow objects to quickly evolve.

4. Evolving mutation range

Hypothesis: In cases where the minimum result is small, objects will evolve a low mutation range to preserve their genetic code.

Based on the above learnings, I was interested in how the mutation range would change if it was an inherited value. In my previous experiments, mutation range was a global constant. I added mutation range as a gene within the objects. This gene would be inherited by children, but would mutate to a value within a range of 0.5–1.5 of the parent's mutation range. This enabled mutation range to evolve along with an object's other values. I repeated some of the above experiments with a variable mutation range:

Table 4: Evolving mutation ranges

#	Starting mutation range	Average minimum with immortality	#	Average minimum with instant death
35	1	10.1745 (s=0.0000)	38	10.1745 (s=0.0000)
36	2	10.1745 (s=0.0000)	39	10.1745 (s=0.0000)
37	4	10.1745 (s=0.0000)	40	10.1745 (s=0.0000)

In all of the above cases, the average evolved mutation range was 0.0000. This confirmed my hypothesis. I was particularly surprised that when these same experiments were run with a fixed mutation range (see table 3), they did not find such a low minimum. I did not predict that a variable mutation range would enhance the efficiency of the evolutionary process. It is likely that, in the case of low starting ranges, the range evolves upward to ensure more rapid evolution before it evolves downward to preserve optimal genetic information.

5. <u>Recombination</u>

Hypothesis: Evolution by recombination will not enhance evolutionary progress in the simulation.

I was also interested in how sexual selection (Fisher 1930) or recombination would affect the outcome. I wrote code to allow evolution to occur by crossover. Where an object was selected for reproduction, it would be randomly paired with another object so selected. The offspring would inherit either of the parents' values at equal probability, subject to the usual mutations.

Sexual reproduction is very rare in nature. It occurs in some animals and plants. However, the vast majority of organisms from the phylogenetic tree of life reproduce asexually. Given that sexual reproduction has only evolved in a minority of cases in nature, I predicted that it would not enhance evolution in most cases. I ran several experiments, both with and without sexual reproduction, varying only the number of generations that were executed, all other settings were default.

Table 5: Sexual reproduction comparison

#	Number of generations	Asexual reproduction	#	Sexual reproduction
41	5	21.9065 (s=0.7609)	46	14.3005 (s=1.1450)
42	10	21.6802 (s=0.8652)	47	10.2965 (s=0.2067)
43	20	20.6841 (s=1.7238)	48	10.1757 (s=0.0000)
44	30	19.7822 (s=2.1234)	49	10.1747 (s=0.0000)
45	40	17.7813 (s=3.0387)	50	10.1745 (s=0.0000)

The effect of sexual reproduction was phenomenal. The sexual strategy was able to evolve objects after five generations that were more efficient than the asexual strategy could evolve after 40. This not only disconfirmed my hypothesis, it is positive evidence that sexual reproduction can lead to advantages outside of the natural animal and plant kingdoms. On reflection, sex is likely rare in nature due to its cost. An organism must invest in finding a mate and, in the case of a female, produce a nutrient rich egg. The sexual strategy was provided without cost to the digital objects. It therefore resulted in greater benefit than it does in most natural contexts.

6. <u>The 80/20 rule</u>

Hypothesis: The 80/20 rule will enhance the evolutionary process as it will increase the amount of diversity in the population.

My final experiment was to test the 80/20 rule. This is the rule of thumb that the top 20% of objects should have an 80% chance of reproducing, whereas the bottom 80% of objects should have a 20% chance of reproducing. Up until now, objects were sorted by fitness and the top 100 were always selected to parent the next generation. Introducing the 80/20 rule would allow for less efficient objects to reproduce. Again, I tested this rule by applying varying percentages of reproductive probability to the top 20%, allocating the remaining probability to the bottom 80%.

Table 6: The 80/20 rule

#	Probability of top 20% reproduction	Average minimum
51	100%	15.9327 (s=3.3236)
52	90%	16.1439 (s=3.5083)
53	80%	16.1670 (s=3.6107)
54	70%	16.2804 (s=3.3847)
55	60%	16.5400 (s=3.3458)

There is a minor trend toward higher minimums as the probability that efficient objects will reproduce is decreased. This disconfirms my hypothesis. The 80/20 rule does not enhance evolutionary progress. Better progress is achieved if the most efficient objects are always selected.

Discussion and reflection

My experiments have identified the optimal conditions for calculating the global minimum of the unknown function. These conditions are:

- At least 95% of the population are children
- A starting mutation range of 4 that varies with evolution
- Objects do not die of old age
- Reproduction occurs by crossover
- The 80/20 rule is not applied.

I ran one final experiment. This time applying all of these optimal settings. Every one of the 100 executions found the same global minimum, identical to the fifteenth decimal place. This was 10.174460498602658. The standard deviation was 0 and the evolved mutation range was practically 0. I conclude that this is the global minimum of the unknown function.[4]

It was interesting that many of my hypotheses were disconfirmed. These were mostly based upon my knowledge of how evolution has operated in the natural world. Evolution in a digital medium appears to operate differently. This may be partly due to pseudo random number generation.

[4] Incidentally, if I run the program with these settings for several thousand generations, it occasionally finds a minimum that is 2 x 10^-15 lower than the global minimum. In this case, the objects are likely adapting to rounding errors that result from Java's method for calculating doubles.

Genetic mutations in the natural world are the product of molecular and quantum effects, which are genuinely random. Mutations in a digital medium are not.

There are many other evolutionary trends that could be tested via computation. Indeed, there is scope to test my own theories regarding the evolution of religious psychology within our species (Mahoney 2015, 2011, 2008; Mahoney & Bulbulia 2008). I will certainly be pursuing this opportunity in future.

Bibliography

Darwin, C. (1859). *On the origin of the species by natural selection.*

Fisher, R. A. (1999). *The genetical theory of natural selection: a complete variorum edition*. Oxford University Press.

Mahoney, A. (2015). The evolutionary psychology of theology. In Slone, J. & Van Slyke, J., *The Attraction of Religion: A new evolutionary psychology of religion,* 189-210. London, UK: Bloomsbury Academic.

Mahoney. A. (2011). *The evolutionary origin and biological utility of supernatural expertise.* PhD thesis. Victoria University library. Retrieved 13 January, 2018 from http://researcharchive.vuw.ac.nz/bitstream/handle/10063/1638/thesis.pdf?sequence=1

Mahoney, A. (2008). Theological expressions as costly signals of religious commitment. In Bulbulia, J., Sosis. R., Harris, E., Genet, R., Genet, C., & Wyman, K., *The evolution of religion: Studies, theories, & critiques,* 161-166. Santa Margarita, CA: Collins Foundation Press.

Mahoney, A. & Bulbulia, J. (2008). Religious Solidarity: The Hand Grenade Experiment. *Journal of Cognition and Culture,* 8 (3), 295-320.