1 Appendix

1.1 Model

1.1.1 Model Definition

Sawmill Firms

Table 1: Notation for Sawmill Firms

	Symbol	Description
Variables	Y^W	Lumber output
	K	Capital input to sawmill firms
	L	Labour input to sawmill firms
Prices	r	Rental rate of capital
	w	Wage rate of labour
	P^W	Price of lumber
Parameters	A^S	Technology parameter for sawmill firms
	α	Capital share parameter, $\alpha \in (0, 1)$

Production Function:

$$Y_t^W = A^S K_t^\alpha L_t^{1-\alpha} \tag{1}$$

Budget Constraint:

$$TC = r_t K_t + w_t L_t \tag{2}$$

Construction Firms

Table 2: Notation for Construction Firms

	Symbol	Description
Variables	Y^F	Final output produced by construction firms
	W	Lumber input used in construction
	Ψ	Alternative inputs to lumber
Prices	$\mathcal{P}^W$	Effective price of lumber
	P^Ψ	Price of alternative inputs
Parameters	A^C	Technology parameter for construction firms
	θ	Input share or scaling parameter in construction production
	ϕ	Elasticity of substitution between lumber and alternatives

Production

$$Y_t^F = A^C \left(\theta\, W_t^\phi + (1-\theta)\, \Psi_t^\phi\right)^{1/\phi}. \tag{3}$$

Cost

$$TC_t^C = \mathcal{P}_t^W\, W_t + P_t^\Psi\, \Psi_t. \tag{4}$$

Effective price of lumber

$$\mathcal{P}_t^W = (1-\omega)\, P_t^W. \tag{5}$$

Government

Table 3: Notation for Government

	Symbol	Description
Variables	G	Government expenditure on lumber tax credits
Policy Instruments	T	Lump-sum tax levied on households
	ω	Tax credit rate applied to lumber inputs

Lumber tax credit expenditure

$$G_t = \omega\, P_t^W\, W_t. \tag{6}$$

Government budget constraint

$$G_t = T_t. \tag{7}$$

Households

Table 4: Notation for Households

	Symbol	Description
Variables	C	Consumption of the final good
	L	Labour supplied by the household
Parameters	γ	Preference weight on leisure in utility

Preferences

$$U = \log(C_t) + \gamma \log(1 - L_t). \tag{8}$$

Budget constraint

$$C_t + K_{t+1} = w_t L_t + r_t K_t - T_t. \tag{9}$$

External Demand

Table 5: Notation for External Demand

	Symbol	Description
Variables	X	External demand for lumber

1.1.2 Market Clearing Conditions

Table 1: Notation for Market Clearing Conditions

	Symbol	Description
Markets	Y^W	Lumber market clearing output
	Y^F	Final goods market clearing output
	L	Labour market clearing quantity

Market clearing in each sector requires:

$$Y^W = W + X, \tag{10}$$

$$Y^F = C, \tag{11}$$

$$L = L^D. \tag{12}$$

1.1.3 First Order Conditions

Households

The household chooses $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize (8) subject to (9). The first-order conditions are:

$$\frac{1}{C_t} = \lambda_t, \tag{13}$$

$$\frac{\gamma}{1 - L_t} = \lambda_t w_t, \tag{14}$$

$$\lambda_t = \beta \lambda_{t+1}(r_{t+1} + 1 - \delta). \tag{15}$$

These conditions imply the Euler equation:

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}}(r_{t+1} + 1 - \delta). \tag{16}$$

Sawmill Firms

Sawmill firms choose K_t and L_t to maximize profits given prices. The first-order conditions are:

$$(1-\alpha)P_t^W A^S K_t^{\alpha} L_t^{-\alpha} = w_t, \tag{17}$$

$$\alpha P_t^W A^S K_t^{\alpha-1} L_t^{1-\alpha} = r_t. \tag{18}$$

Construction Firms

Construction firms choose W_t and Ψ_t to minimize costs subject to (3). The first-order conditions are:

$$\theta A^C \left(\theta W_t^{\phi} + (1-\theta)\Psi_t^{\phi}\right)^{\frac{1}{\phi}-1} W_t^{\phi-1} = \mathcal{P}_t^W, \tag{19}$$

$$(1-\theta) A^C \left(\theta W_t^{\phi} + (1-\theta)\Psi_t^{\phi}\right)^{\frac{1}{\phi}-1} \Psi_t^{\phi-1} = P_t^{\Psi}. \tag{20}$$