Assignmnet 2 Part 1 b) - Andrew Paul 100996250

The second section of this assignment is similar to the first section but different boundary conditions are given to make the solution a bit more complex. The same finite differences method is used as in part 1 a), but now the boundaries of the y axis are set to have a voltage of zero and the boudndaries of the x axis have a voltage of V0.

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clear

% Set the length and width of the grid and the voltage V0

L = 3;
W =2;
V0 = 1;

% Spacing and number of points of grid

dx = 0.1;
dy = 0.1;
nx = L/dx;
ny = W/dy;

% Parameters of Laplace equation

p1 = 1/(dx^2);
p2 = 1/(dy^2);
p3 = -2*(1/dx^2 + 1/dy^2);

% Generate "G" Matrix

G = zeros(nx*ny,nx*ny);

for i = 2:nx-1
    for j = 2:ny-1
        n = i + (j-1)*nx;
        nym = i + (j-2)*nx;
        nyp = i + j*nx;
        nxm = (i-1) + (j-1)*nx;
        nxp = (i+1) + (j-1)*nx;

        G(n,n) = p3;
        G(n,nxm) = p1;
        G(n,nxp) = p1;
        G(n,nym) = p2;
        G(n,nyp) = p2;
    end
end

% Generate "F" Matrix
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```
F = zeros(nx*ny,1);

for i = 1:nx
    n = i;
    G(n,n) = 1;

    n1 = i + (ny-1)*nx;
    G(n1,n1) = 1;
end

for j = 1:ny
    n = 1 + (j-1)*nx;
    G(n,n) = 1;
    F(n) = V0;

    n1 = nx + (j-1)*nx;
    G(n1,n1) = 1;
    F(n1) = 1;
end

% Solution for F matrix is zero at all corners

F(1) = 0;
F(1 + (ny-1)*nx) = 0;
F(nx) = 0;
F(nx + (ny-1)*nx) = 0;

% Finding solution and reshaping the transpose

A = G\F;
solution = reshape(A,[],ny)';

xrange = linspace(0,L,nx);
yrange = linspace(0,W,ny);

figure(1)
surf(xrange,yrange,solution)
xlabel('x')
ylabel('y')
zlabel('Voltage')
title('Numerical Finite Differences Solution')

analytic = zeros(ny,nx);
steps = 200;

% Create replicas of x and y axes to generate analytic solution

xnew = repmat(linspace(-L/2,L/2,nx),ny,1);
ynew = repmat(linspace(0,W,ny),nx,1)';

% Loop through analytical series solution in steps of 2

for n = 1:2:steps
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    analytic = analytic + 1./n.*cosh(n.*pi.*xnew./W)./cosh(n*pi*L/
W).*sin(n.*pi.*ynew./W);
end

analytic = analytic*4*V0/pi;

figure(2)
surf(xrange,yrange,analytic)
xlabel('x')
ylabel('y')
zlabel('Voltage')
title('Analytical Finite Differences Solution')
```

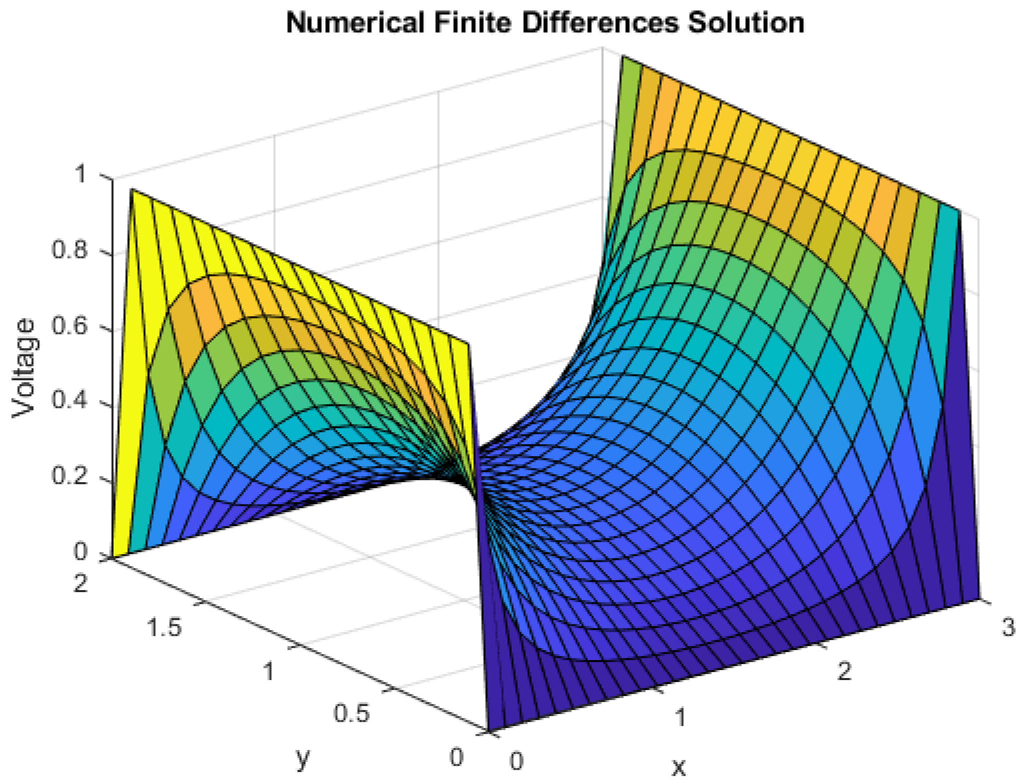

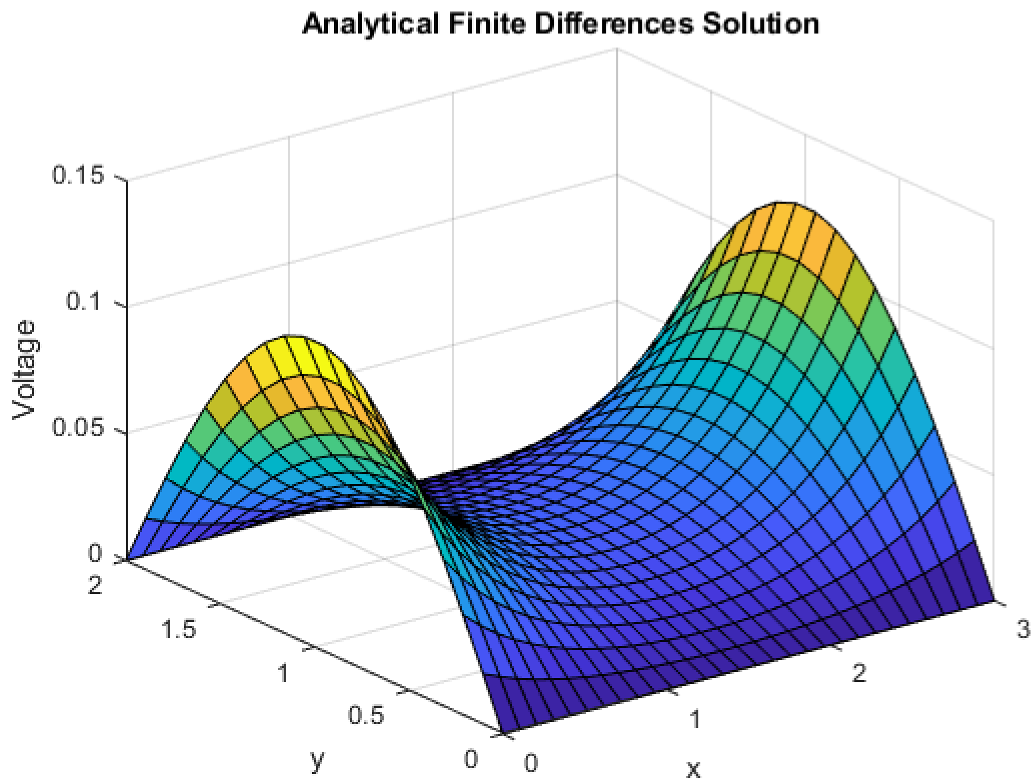

After further analyzing the analytical solution it was found that the series seems to converage after the first 5 or 6 iterations and shows little change as the series continutes.

It is clear that the analytical solution is much easier to impliment as it only requires a few lines of code. The situation used for this assignmnet is relatively basic and more complicated solutions would require a much more complex series solution in order to accomodate different boundary conditions.

As for the mesh size, the mesh used in the plots shown are for a dx and dy value of 0.1 but when a larger dx and dy value were used it was found that the error was much greater. The error was determined by taking an average of the difference between the numerical and analytical solutions for the total number of itterations.

To conclude, the anayltical solution is a good option for simple cases but the numerical solution will be more accurate and potentially easier to impliment when facing complicated systems.

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