

1 Nash Equilibrium Strategy Profile

The full solution process is detailed in Appendix ???. Here we present the final solution, which was obtained by solving for $c(s)$ in terms of x_2 , then using this to solve for $v(s)$, and finally solving for $b(s)$ up to a constant of integration. The resulting system of 7 equations in 7 unknowns was solved symbolically using Mathematica and simplified by finding common subexpressions $A_0, A_1, A_2, A_3, A_4, A_5$.

Theorem 1.1 (LCP Nash Equilibrium). *LCP has a unique Nash equilibrium strategy profile in which the bettor's strategy is monotone-admissible (up to measure zero sets of hands for each player). This strategy profile is given by:*

$$\begin{aligned}
 x_0 &= \frac{3(L+1)^3 U}{A_4} \\
 x_1 &= \frac{3A_0 L U + A_0 U - L^3 - 3L^2}{A_4} \\
 x_2 &= \frac{A_5}{A_4} \\
 x_3 &= \frac{A_2 L^3 + 3A_2 L^2 + 3L(5U^3 + 15U^2 + 15U + 4) + 4U^3 + 12U^2 + 12U + 3}{A_4} \\
 x_4 &= \frac{3A_1 L^2 + A_2 L^3 + 3A_2 L + 4U^3 + 12U^2 + 12U + 3}{A_4} \\
 x_5 &= \frac{3A_3 L^2 + 3A_3 L + A_3 + L^3(6U^3 + 18U^2 + 15U + 2)}{A_4} \\
 b_0 &= -\frac{(L+1)^3}{A_4} \\
 b(s) &= b_0 - \frac{(1+3s)(x_2 - 1)}{6(1+s)^3} \\
 c(s) &= \frac{x_2 + s}{s+1} \\
 v(s) &= \frac{x_2 + 2s^2 + 4s + 1}{2(s+1)^2}
 \end{aligned}$$

where the common subexpressions are:

$$\begin{aligned}
A_0 &= U^2 + 3U + 3 \\
A_1 &= 7U^3 + 21U^2 + 21U + 6 \\
A_2 &= 6U^3 + 18U^2 + 18U + 5 \\
A_3 &= 7U^3 + 21U^2 + 18U + 3 \\
A_4 &= 3A_1L^2 + 3A_1L + A_1 + A_2L^3 \\
A_5 &= 3A_0L^2U + 3A_0LU + A_0U - L^3
\end{aligned}$$

Refer back to section ?? for an explanation of how these values fit together to actually form the strategy profile.

This solution is more interpretable in graphical form. Figure 1 shows the strategy profile for various values of L and U ranging from very lenient ($L = 0, U = 10$) to very restricted ($L = 0.5, U = 1$). The more lenient bet size limits model something closer to NLCP, while the more restricted bet size limits model something closer FBCP with a fixed bet size. Indeed, we see that the strategy profile of for $L = 0, U = 10$ looks qualitatively similar to the strategy profile of NLCP - we will show in section ?? that the strategy profile approaches the Nash equilibrium of NLCP as L and U approach 0 and ∞ , respectively, and that the strategy profile approaches the Nash equilibrium of FBCP as L and U approach some fixed value s from either side.

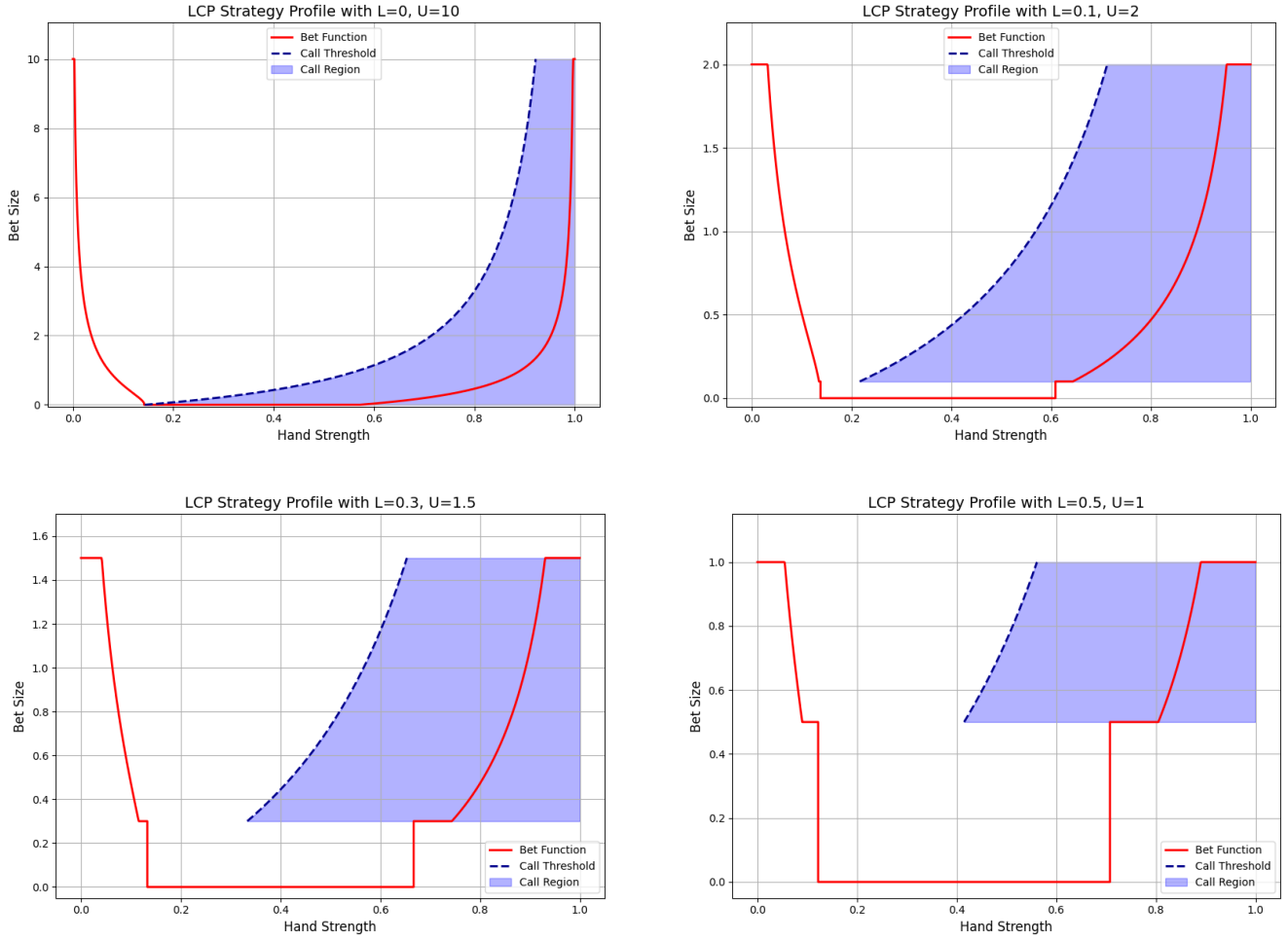


Figure 1: Nash equilibrium strategy profiles for different values of L and U , from very lenient to very restricted bet sizes. The bet function maps hand strengths to bet sizes, while the call function gives the minimum calling hand strength for a given bet size. The shaded regions represent the hand strengths for which the caller should call a given bet size.