

1 Nash Equilibrium Strategy Profile

Having established the equilibrium structure in Section ?? and derived the indifference equations in Section ??, we now present the complete solution. Before stating the formal result, we first show the equilibrium graphically to convey its qualitative structure.

Figure 1 displays the Nash equilibrium strategy profiles for various betting limits, ranging from lenient ($L = 0, U = 10$) to restrictive ($L = 0.5, U = 1$). Several key features are apparent:

- The bettor partitions hands into three regions: *bluffing* (weak hands bet), *checking* (medium hands pass), and *value betting* (strong hands bet).
- Within each betting region, bet sizes vary continuously with hand strength— weaker bluffs bet larger, stronger value hands bet larger.
- The caller’s threshold $c(s)$ increases with bet size: larger bets require stronger hands to call profitably.
- As limits become more lenient (top-left panel), the profile approaches NLCP; as limits become more restrictive (bottom-right panel), it approaches fixed-bet poker.

The system of equations from Section ?? was solved symbolically using Mathematica (see Appendix ??), yielding closed-form expressions for all threshold values and strategic functions.

Theorem 1.1 (LCP Nash Equilibrium). *The following strategy profile constitutes a Nash equilibrium of LCP. Moreover, it is the unique equilibrium in which the caller uses a monotone strategy and the bettor’s strategy is monotone-admissible (see Section ?? for these refinements). The strategy profile is given by:*

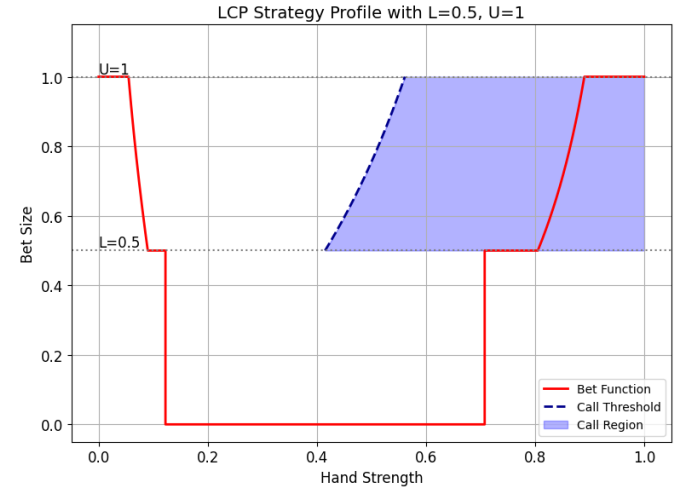
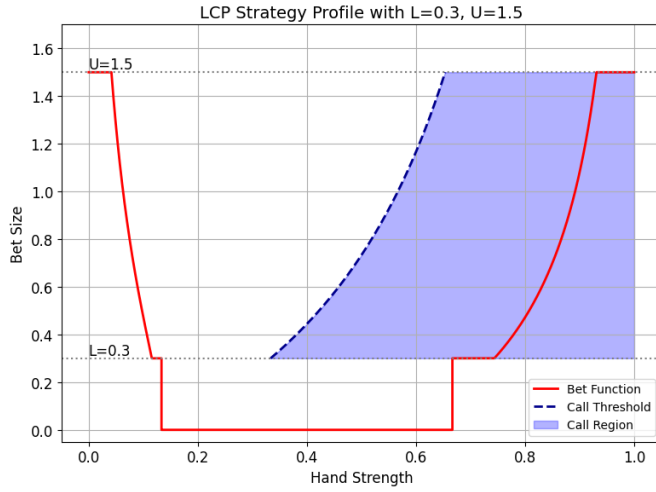
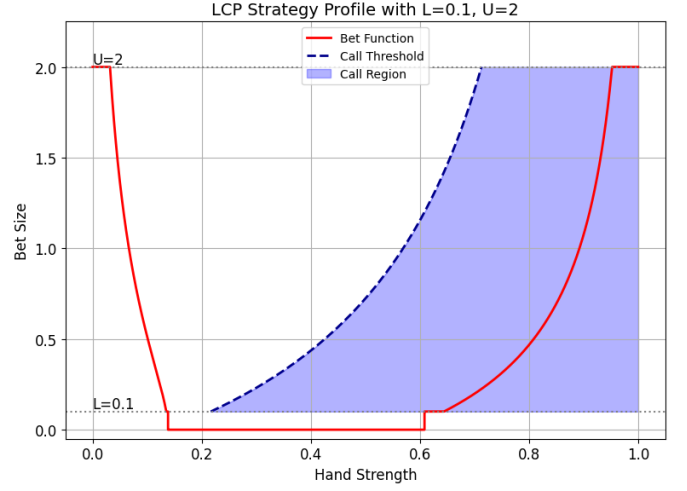
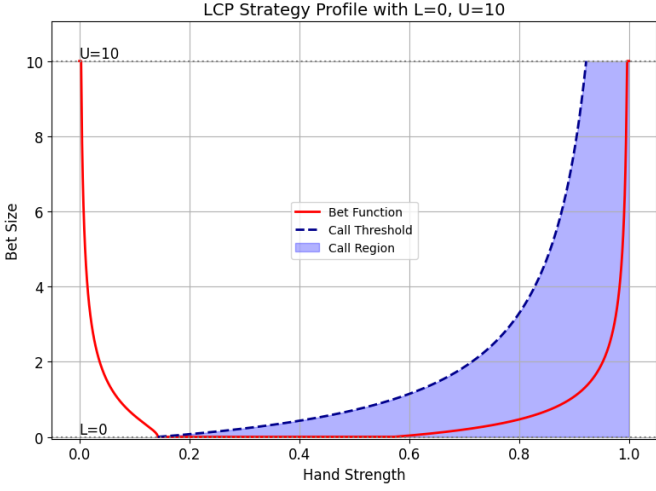


Figure 1: Nash equilibrium strategy profiles for different values of L and U , from lenient to restrictive bet size limits. The bet function maps hand strengths to bet sizes, while the call function gives the minimum calling hand strength for a given bet size. The shaded regions represent the hands for which the caller should call.

$$\begin{aligned}
 x_0 &= \frac{3t^2(t-1)}{r^3+t^3-7} \\
 x_1 &= \frac{-2r^3+3r^2+t^3-1}{r^3+t^3-7} \\
 x_2 &= \frac{r^3+t^3-1}{r^3+t^3-7} \\
 x_3 &= \frac{r^3-3r+t^3-4}{r^3+t^3-7} \\
 x_4 &= \frac{r^3+3r^2-6r+t^3-4}{r^3+t^3-7} \\
 x_5 &= \frac{r^3+t^3+3t^2-7}{r^3+t^3-7} \\
 b_0 &= \frac{t^3}{r^3+t^3-7} \\
 b(s) &= \frac{t^3(s+1)^3-(3s+1)}{(r^3+t^3-7)(s+1)^3}
 \end{aligned}$$

where $r = L/(1 + L)$ and $t = 1/(1 + U)$.

Remark: The change of variables to (r, t) significantly simplifies the expressions compared to the original (L, U) formulation. This transformation reveals underlying symmetries and makes many properties more transparent, as we will see in the analysis of game value and parameter effects.

Refer back to Section ?? for an explanation of how these threshold values and functions fit together to form the complete strategy profile. A proof that this profile constitutes a Nash equilibrium is provided in Appendix ??.

As noted above, the strategy profile approaches NLCP as $L \rightarrow 0$ and $U \rightarrow \infty$, and approaches FBCP as $L, U \rightarrow B$ for any fixed bet size B . We formalize these convergence results in Section ??.