

1 Nash Equilibrium Strategy Profile

Theorem 1.1 (LCP Nash Equilibrium). *LCP has a unique Nash equilibrium strategy profile in which the bettor's strategy is monotone-admissible (up to measure zero sets of hands for each player). This strategy profile is given by:*

$$\begin{aligned}
x_0 &= \frac{3t^2(t-1)}{r^3 + t^3 - 7} \\
x_1 &= \frac{-2r^3 + 3r^2 + t^3 - 1}{r^3 + t^3 - 7} \\
x_2 &= \frac{r^3 + t^3 - 1}{r^3 + t^3 - 7} \\
x_3 &= \frac{r^3 - 3r + t^3 - 4}{r^3 + t^3 - 7} \\
x_4 &= \frac{r^3 + 3r^2 - 6r + t^3 - 4}{r^3 + t^3 - 7} \\
x_5 &= \frac{r^3 + t^3 + 3t^2 - 7}{r^3 + t^3 - 7} \\
b_0 &= \frac{t^3}{r^3 + t^3 - 7} \\
b(s) &= \frac{t^3(s+1)^3 - (3s+1)}{(r^3 + t^3 - 7)(s+1)^3} \\
c(s) &= \frac{r^3 + t^3 - 1 + s(r^3 + t^3 - 7)}{(s+1)(r^3 + t^3 - 7)} \\
v(s) &= \frac{r^3 + t^3 - 1 + (r^3 + t^3 - 7)(2s^2 + 4s + 1)}{2(r^3 + t^3 - 7)(s^2 + 2s + 1)}
\end{aligned}$$

where $r = L/(1+L)$ and $t = 1/(1+U)$.

Remark: The change of variables to (r, t) significantly simplifies the expressions compared to the original (L, U) formulation. This transformation reveals underlying symmetries and makes many properties more transparent, as we will see in the analysis of game value and parameter effects.

Refer back to section ?? for an explanation of how these values fit together to actually form the strategy profile.

A proof of this theorem can be found in appendix ??.

This solution is more interpretable in graphical form. Figure 1 shows the strategy profile for various values of L and U ranging from very lenient ($L = 0, U = 10$) to very restricted ($L = 0.5, U = 1$). The more lenient bet size limits

model something closer to NLCP, while the more restricted bet size limits model something closer FBCP with a fixed bet size. Indeed, we see that the strategy profile of for $L = 0, U = 10$ looks qualitatively similar to the strategy profile of NLCP - we will show in section ?? that the strategy profile approaches the Nash equilibrium of NLCP as L and U approach 0 and ∞ , respectively, and that the strategy profile approaches the Nash equilibrium of FBCP as L and U approach some fixed value s from either side.

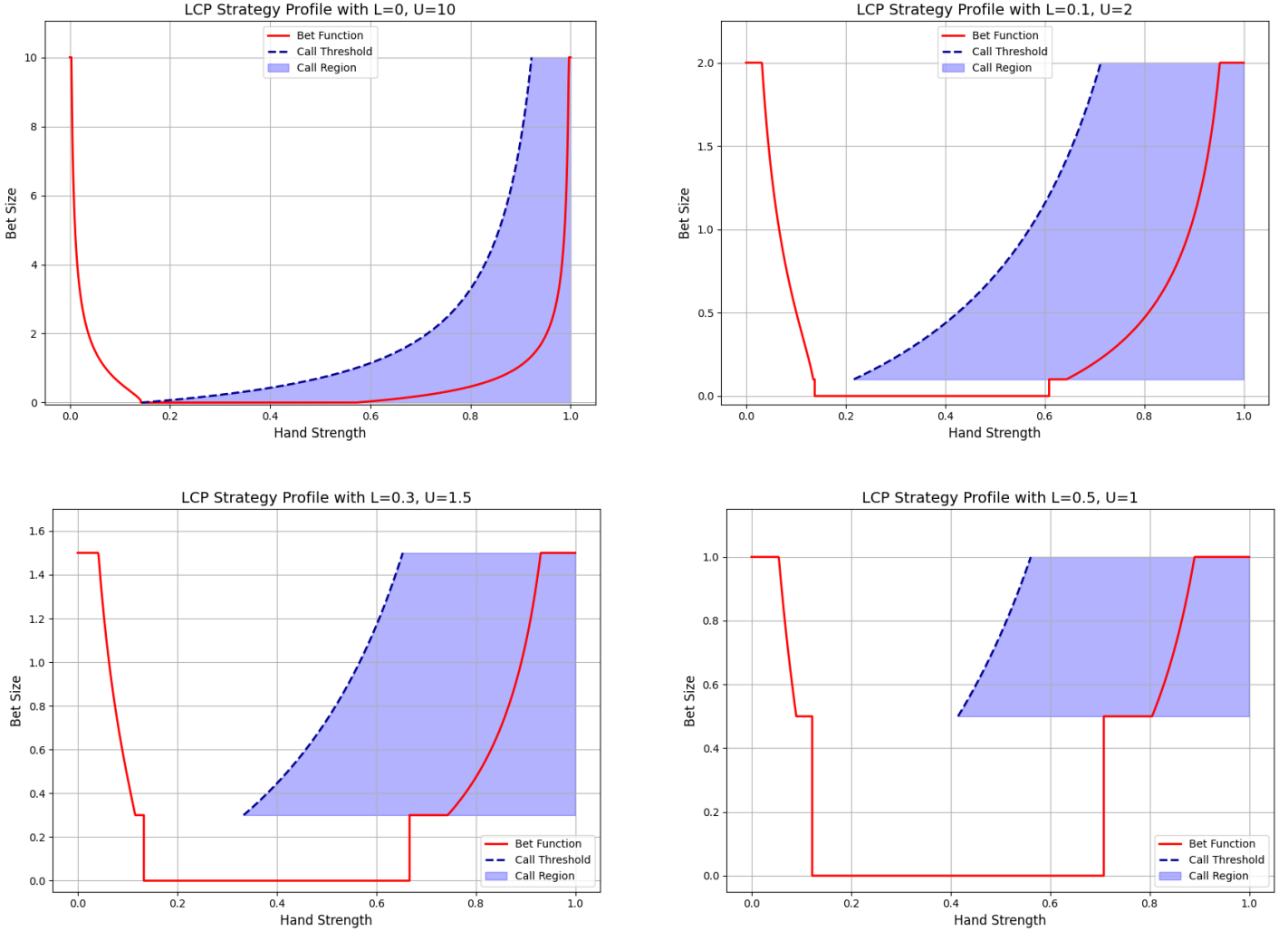


Figure 1: Nash equilibrium strategy profiles for different values of L and U , from very lenient to very restricted bet sizes. The bet function maps hand strengths to bet sizes, while the call function gives the minimum calling hand strength for a given bet size. The shaded regions represent the hand strengths for which the caller should call a given bet size.