1 Solving LCP

LCP has an infinite class of Nash equilibria, differentiated only by how the bettor sizes their bluffing hands. In this section, we define a way to distinguish between these equilibria. This involves defining a class of *monotone* calling strategies which are in some sense more reasonable than non-monotone strategies, then restricting the bettor's strategy to be admissible (not weakly dominated) against these calling strategies. This turns out to be enough to uniquely determine the Nash equilibrium.

1.1 Monotone Strategies

Definition 1.1 (Monotone Calling Strategy)**.** A calling strategy is *monotone* if it satisfies two conditions:

1. For bet size s and any two hand strengths $y_1 < y_2$, if the caller calls with y_1, they must also call with y_2.

2. For hand strength y and any two bet sizes $s_1 > s_2$, if the caller calls s_2, they must also call s_1.

This should sound intuitive. Violating the first condition on a nonzero-measure set of hands would be strictly dominated. The second condition is more subtle, but arguably realistic and motivated by the notion of pot odds - a larger bet is more risky, so the caller should be more selective about when they call.

Definition 1.2 (Monotone-Admissible Strategy)**.** A betting strategy σ_B is *monotone-admissible* if it is admissible in LCP, restricted to monotone calling strategies. More explicitly, for any betting strategy σ'_B and any monotone calling strategy σ_C, the bettor's expected payoff $\pi_B(\sigma_B, \sigma_C)$

This is useful in distinguishing bettor strategies which differ only in how they bluff. The hand strength of a bluff is irrelevant if the caller plays optimally, but becomes important if the caller deviates to a suboptimal but still monotone strategy. If the caller becomes too loose, i.e. calling too often, then the bettor ends up winning some pots accidentally if they make their smallest bluffs with their strongest bluffing hands.

1.2 Nash Equilibrium Structure

We will now describe the structure of the Nash equilibrium.

1. The caller has a calling threshold $c(s)$ that is non-decreasing and continuous in s, including at endpoints L and U. They call with hands $y \geq c(s)$ and fold with hands $y < c(s)$.

2. The bettor partitions $[0, 1]$ into three regions: bluffing $x \in [0, x_2]$, checking $x \in [x_2, x_3]$, and value betting $x \in [x_3, 1]$.

3. Within the bluffing region, the bettor's partitions into a max-betting region $x \in [x_0, x_1]$, an intermediate region $x \in [x_1, x_2]$, and a min-betting region $x \in [x_2, x_3]$.

4. Within the intermediate bluffing region, the bettor bets according to a continuous, decreasing function $s = b^{-1}(x)$ with endpoints $b^{-1}(x_0) = U$ and $b^{-1}(x_3) = L$.

5. Within the value betting region, the bettor partitions into a min-betting region $x \in [x_3, x_4]$, an intermediate region $x \in [x_4, x_5]$, and a max-betting region $x \in [x_5, 1]$.

6. Within the intermediate value betting region, the bettor bets according to a continuous, increasing function $s = v^{-1}(x)$ with endpoints $v^{-1}(x_3) = L$ and $v^{-1}(x_5) = U$.

Theorem 1.1. *LCP has a Nash equilibrium with the structure above, and this is the unique Nash equilibrium in which the bettor's strategy is monotone-admissible (up to measure zero sets of hands for each player).*