1 Nash Equilibrium Strategy Profile

The full solution process is detailed in Appendix ??. Here we present the final solution, which was obtained by solving for c(s) in terms of x_2 , then using this to solve for v(s), and finally solving for b(s) up to a constant of integration. The resulting system of 7 equations in 7 unknowns was solved symbolically using Mathematica and simplified by finding common subexpressions $A_0, A_1, A_2, A_3, A_4, A_5$.

Theorem 1.1 (Nash Equilibrium Strategy Profile). The unique admissible Nash equilibrium strategy profile for Limit Continuous Poker with minimum bet size L and maximum bet size U is given by:

$$x_{0} = \frac{3(L+1)^{3}U}{A_{4}}$$

$$x_{1} = \frac{3A_{0}LU + A_{0}U - L^{3} - 3L^{2}}{A_{4}}$$

$$x_{2} = \frac{A_{5}}{A_{4}}$$

$$x_{3} = \frac{A_{2}L^{3} + 3A_{2}L^{2} + 3L\left(5U^{3} + 15U^{2} + 15U + 4\right) + 4U^{3} + 12U^{2} + 12U + 3}{A_{4}}$$

$$x_{4} = \frac{3A_{1}L^{2} + A_{2}L^{3} + 3A_{2}L + 4U^{3} + 12U^{2} + 12U + 3}{A_{4}}$$

$$x_{5} = \frac{3A_{3}L^{2} + 3A_{3}L + A_{3} + L^{3}\left(6U^{3} + 18U^{2} + 15U + 2\right)}{A_{4}}$$

$$b_{0} = -\frac{(L+1)^{3}}{A_{4}}$$

$$b(s) = b_{0} - \frac{(1+3s)(x_{2}-1)}{6(1+s)^{3}}$$

$$c(s) = \frac{x_{2} + s}{s+1}$$

$$v(s) = \frac{x_{2} + s}{2(s+1)^{2}}$$

where the common subexpressions are:

$$A_0 = U^2 + 3U + 3$$

$$A_1 = 7U^3 + 21U^2 + 21U + 6$$

$$A_2 = 6U^3 + 18U^2 + 18U + 5$$

$$A_3 = 7U^3 + 21U^2 + 18U + 3$$

$$A_4 = 3A_1L^2 + 3A_1L + A_1 + A_2L^3$$

$$A_5 = 3A_0L^2U + 3A_0LU + A_0U - L^3$$

Proof given in Appendix ??. Refer to section ?? for an explanation of how these values fit together to actually form the strategy profile.

This solution is more interpretable in graphical form. Figure 1 shows the strategy profile for various values of L and U ranging from very lenient (L=0,U=10) to very restricted (L=0.5,U=1). The more lenient bet size limits model something closer to NLCP, while the more restricted bet size limits model something closer FBCP with a fixed bet size. Indeed, we see that the strategy profile of for L=0,U=10 looks qualitatively similar to the strategy profile of NLCP - we will show in section ?? that the strategy profile approaches the Nash equilibrium of NLCP as L and U approach 0 and ∞ , respectively, and that the strategy profile approaches the Nash equilibrium of FBCP as L and U approach some fixed value s from either side.

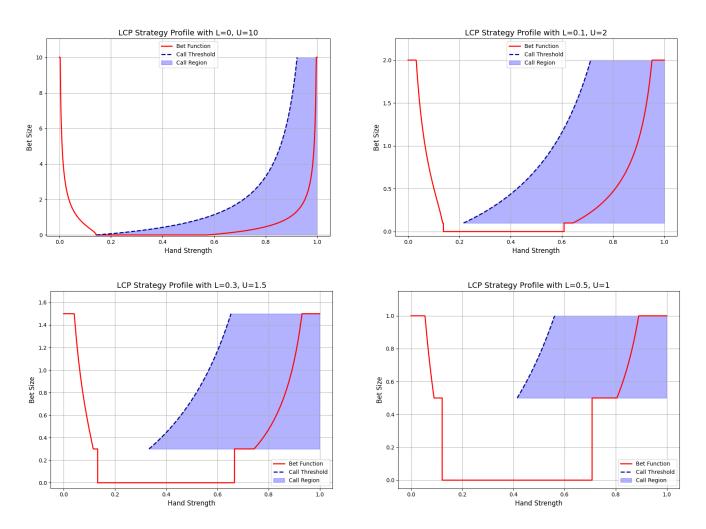


Figure 1: Nash equilibrium strategy profiles for different values of L and U, from very lenient to very restricted bet sizes. The bet function maps hand strengths to bet sizes, while the call function gives the minimum calling hand strength for a given bet size. The shaded regions represent the hand strengths for which the caller should call a given bet size.