IST557 Homework 1

Andrew Sugarman

today

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1 Problem 1: Expected value of a sum of normal random variables

We are given: $y \sim \mathcal{N}(\mu, \sigma^2)$ and $y \in \mathbb{R}$. To solve the expectation $E[y_1 + y_2 + y_3 + ...y_N]$ given $N \in \mathbb{R}$, we can use the linearity of expectation:

$$E[y_1 + y_2 + y_3 + ...y_N] = E[y_1] + E[y_2] + E[y_3] + ...E[y_N]$$

Since y is a normally distributed random variable with mean μ, this expectation solves to be: $N * \mu$.

2 Problem 2: Conditional and Marginal Distribution of a Gaussian Heirarchical Time series

Refs: `https://www.otexts.com/fpp2/hts.html` `https://en.wikipedia.org/wiki/Autoregressive_model`

1. The conditional distribution is normally distributed. This hierarchical time series is autoregressive, and the y_0 instance is distributed normal.

2. This time series depends only on the value of the immediately prior condition (y_{t-1}).

3 Problem 3: Gradient Descent Optimizer

Steps to write grad descent optimizer for pdf of students t distribution

- to effectively write an optimizer, we need to define a convex function
- with a convex function, we can pursue a gradient of 0
- the function given is the pdf of the student's t distribution, which maps from real space to probability space (0-1)
- to make the function convex and monotonic we can take the negative log likelihood of it
- taking the logarithm of this function simplifies it:
  - the first term separates out to a constant and you are left with: $log(studt) = const + ((v + 1)/2) * log(1 + ((b^2)/v))$
- next, the gradient must be calculated by calculating the partial derivative with respect to b, followed by calculating the partial derivative with respect to v (constant)