Introduction to Programming Lecture 12

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- Course Schedule : Wednesday 14h00 - 15h30 Campus Kirchberg B21
- Course Website : sites.google.com/site/andrewyarmola/itp-uni-lux (https://sites.google.com/site/andrewyarmola/itp-uni-lux)
- Office Hours : Thursday 16h00 - 17h00 Campus Kirchberg G103 and by appointment.

There will not be a Project 2, we will just have joint Final Projects

I've decided to skip Project 2. In effect, we will a Project 1 as a midterm and the Final Project as our final.

Last homework due Friday Dec 23

`scipy` package

The `scipy` package is composed of many useful task-specific modules

- `scipy.cluster` - Vector quantization / Kmeans
- `scipy.constants` - Physical and mathematical constants
- `scipy.fftpack` - Fourier transform
- `scipy.integrate` - Integration routines
- `scipy.interpolate` - Interpolation
- `scipy.io` - Data input and output
- `scipy.linalg` - Linear algebra routines
- `scipy.ndimage` - n-dimensional image package
- `scipy.odr` - Orthogonal distance regression
- `scipy.optimize` - Optimization
- `scipy.signal` - Signal processing
- `scipy.sparse` - Sparse matrices
- `scipy.spatial` - Spatial data structures and algorithms
- `scipy.special` - Any special mathematical functions
- `scipy.stats` - Statistics

Fast Fourier Transforms

Recall that Fourier analysis is a method for expressing a function as a sum of periodic components, and for recovering the function from those components. This process can also be done with discrete data.

Given a function $g : [a, b] \to \mathbb{C}$ and a sampling interval Δt, we can build a sequence $g_n = g(a + n\Delta t)$ for $n = 0, \dots N - 1$. The **discrete Fourier transform** of $\{g_n\}$ is a sequence $\{G_k\}$ such that

$$g_n = \frac{1}{N} \sum_{k=0}^{N-1} G_k \exp\left(2\pi i \frac{nk}{N}\right) = \frac{1}{N} \sum_{k=0}^{N-1} G_k \left(\cos\left(2\pi \frac{nk}{N}\right) + i \sin\left(2\pi \frac{nk}{N}\right)\right)$$

As you can see, the goal is to express g_n in terms of periodic components. With respect to the original interval $[a, b]$ then "true" frequency is given by $f_k = \frac{k}{N\,\Delta t} = \frac{k}{b-a}$ Hz.

It is not hard to show that

$$G_k = \sum_{n=0}^{N-1} g_n \exp\left(-2\pi i \frac{nk}{N}\right)$$

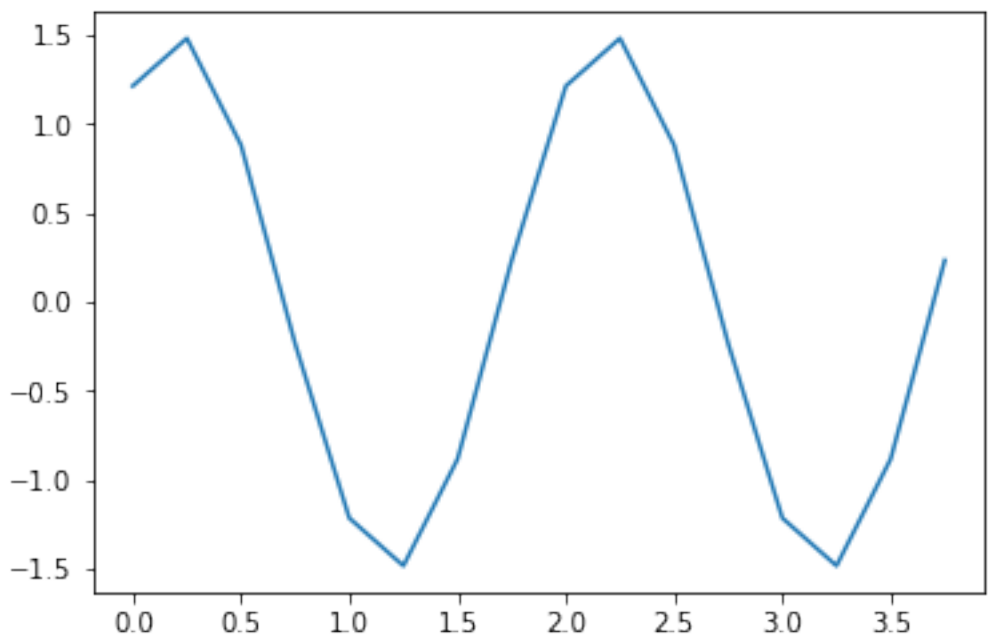
Let $g = [g_0, \dots, g_{N-1}]$. In `scipy.fftpack`, the function `scipy.fftpack.fft(g)` returns the list

$$[G_0, G_1, \dots, G_{\lfloor N/2 \rfloor}, G_{1-\lfloor N/2 \rfloor}, G_{2-\lfloor N/2 \rfloor}, \dots, G_{-1}]$$

So the first half of the list is the "positive" frequencies and the second half is the "negative" frequencies. If g is real valued, then DFT is symmetric under complex conjugation (i.e. $G_k = \overline{G_{-k}}$)

In [1]:

```
import numpy as np
from matplotlib import pyplot as plt
%matplotlib inline

time_step = 0.25
amplitude = 1.5
shift = -0.2
period = 2. # 0.5 Hz
time_vec = np.arange(0, 4, time_step)
sig = amplitude * np.cos(2 * np.pi / period * (time_vec+shift))

plt.plot(time_vec, sig)
plt.show()
```



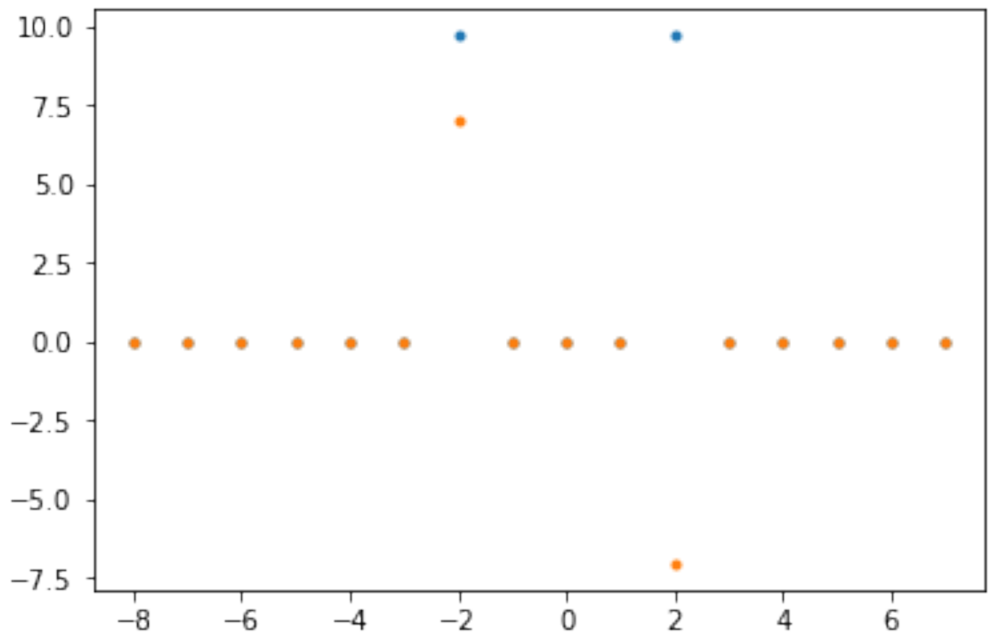
In [2]:

```
from scipy import fftpack

sig_ft = fftpack.fft(sig)

print(sig_ft)
```

```
[ 1.05471187e-15+0.00000000e+00j -3.63808984e-15+1.96454098e-15j
  9.70820393e+00-7.05342303e+00j  2.83291731e-15-1.61783758e-15j
  0.00000000e+00-7.21644966e-16j  1.23319834e-16-2.42982234e-15j
  8.88178420e-16-1.33226763e-15j  1.57003111e-15+2.64377801e-16j
  2.49800181e-15+0.00000000e+00j  1.57003111e-15-2.64377801e-16j
  8.88178420e-16+1.33226763e-15j  1.23319834e-16+2.42982234e-15j
  0.00000000e+00+7.21644966e-16j  2.83291731e-15+1.61783758e-15j
  9.70820393e+00+7.05342303e+00j -3.63808984e-15-1.96454098e-15j]
```

In [3]:

```
# gives k/(n * dt)
n = sig.size
freq_values = fftpack.fftfreq(n, time_step)
k_values = freq_values * (n * time_step)

plt.plot(k_values, sig_ft.real, '.')
plt.plot(k_values, sig_ft.imag, '.')
plt.show()
```



We can clearly see that $k = \pm 2$ (i.e. $2/(4-0) = 0.5$ Hz) is our frequency. Further, we can recover :

In [4]:

```
import cmath

freq = 1/2.
g2 = sig_ft[2]
#print(abs(g2))
# these take a little math using the inverse
print("Amplitude/Power :", 2*abs(g2)/n)
print("Phase : ", (1/freq) *
      cmath.log(g2/abs(g2))/(2*np.pi*1.j))
```

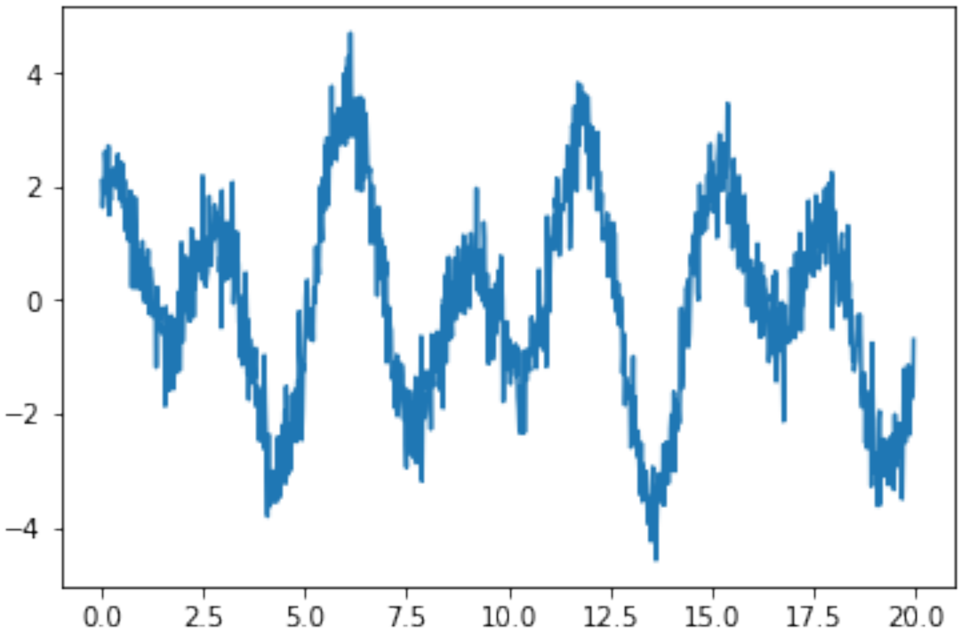
```
Amplitude/Power : 1.5000000000000002
Phase :  (-0.20000000000000018+4.417437057588218e-17j)
```

Remark : Notice that **multiplying** a Fourier coefficient by $\exp(2\pi i p)$ corresponds to shifting the phase of that frequency.

Removing noise from signal

In [5]:

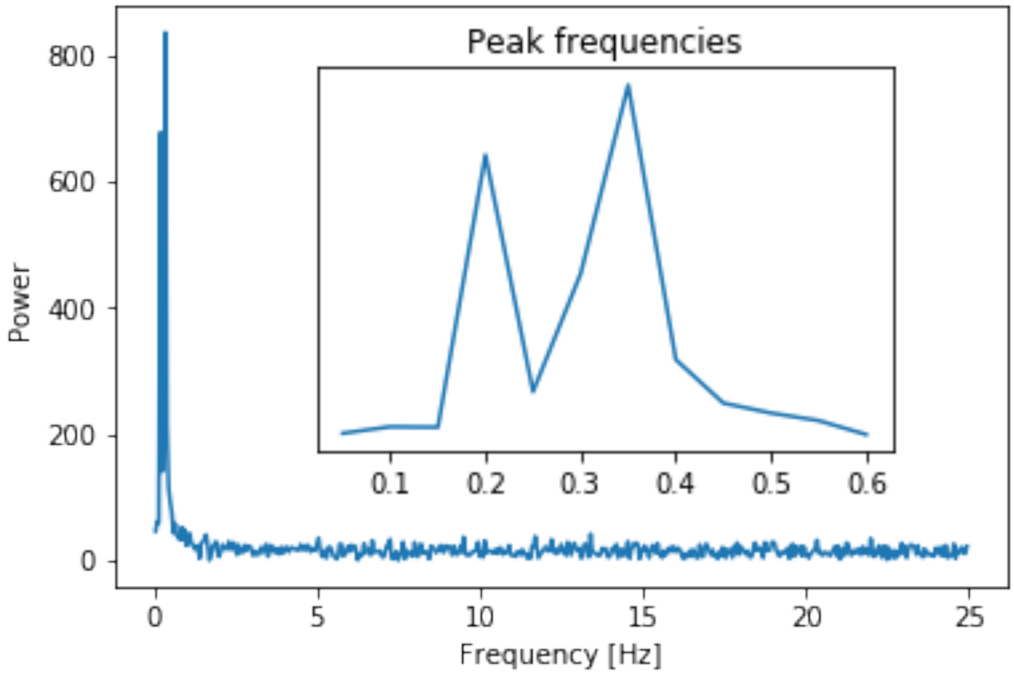
```
import numpy as np
from matplotlib import pyplot as plt

time_step = 0.02
time_vec = np.arange(0, 20, time_step)
p1 = 5. # freq 0.2 Hz
p2 = 3. # freq 0.3333 Hz
a1 = 1.5
a2 = 2.
sig = a1*np.sin(2 * np.pi / p1 * time_vec) + \
      a2*np.cos(2 * np.pi / p2 * time_vec) + \
      0.5 * np.random.randn(time_vec.size)

plt.plot(time_vec, sig)
plt.show()
```



The observer doesn’t know the signal frequencies, only the sampling time step of the signal `sig`. The signal is supposed to come from a real valued function so the Fourier transform will be symmetric. The `scipy.fftpack.fftfreq()` function will generate the sampling frequencies and `scipy.fftpack.fft()` will compute the fast Fourier transform :

In [6]:

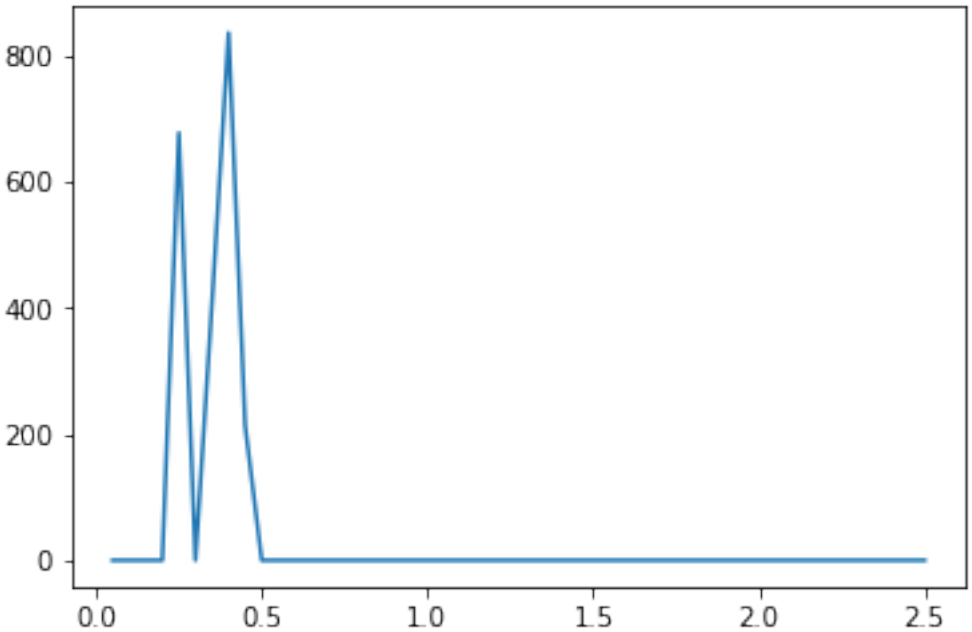
```
from scipy import fftpack

sample_freq = fftpack.fftfreq(sig.size, d=time_step)
sig_fft = fftpack.fft(sig)

# we only care about positive frequencies
pidxs = np.where(sample_freq > 0)
freqs = sample_freq[pidxs]
power = np.abs(sig_fft)[pidxs]

plt.clf()
plt.plot(freqs, power)
plt.xlabel('Frequency [Hz]')
plt.ylabel('Power')
axes = plt.axes([0.3, 0.3, 0.5, 0.5])

plt.title('Peak frequencies')
plt.plot(freqs[:12], power[:12])
plt.setp(axes, yticks=[])

plt.show()
```



We can now clear the noise from the Fourier transform :

In [7]:

```
sig_fft[np.abs(sig_fft) < 200] = 0.
plt.plot(freqs[:50], np.abs(sig_fft)[:50])
plt.show()
```



The resulting filtered signal can be computed by the scipy.fftpack.ifft() function :

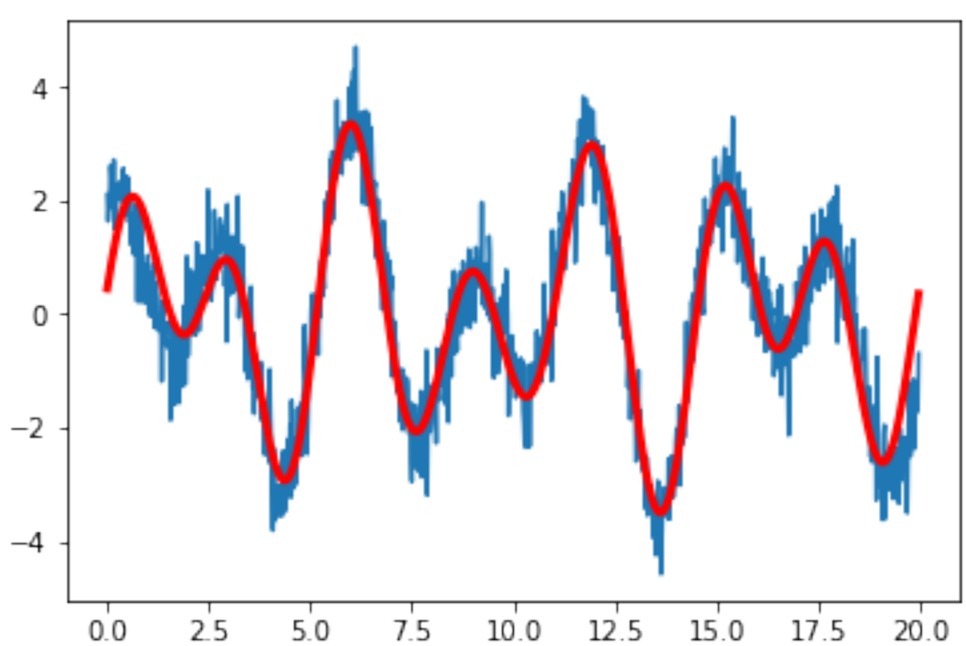
In [8]:

```
main_sig = fftpack.ifft(sig_fft).real
```

Let's see whhat we got :

In [9]:

```
plt.clf()
plt.plot(time_vec, sig)
plt.plot(time_vec, main_sig, color='red', linewidth=3.0)
plt.show()
```



2D Fourier transform and images

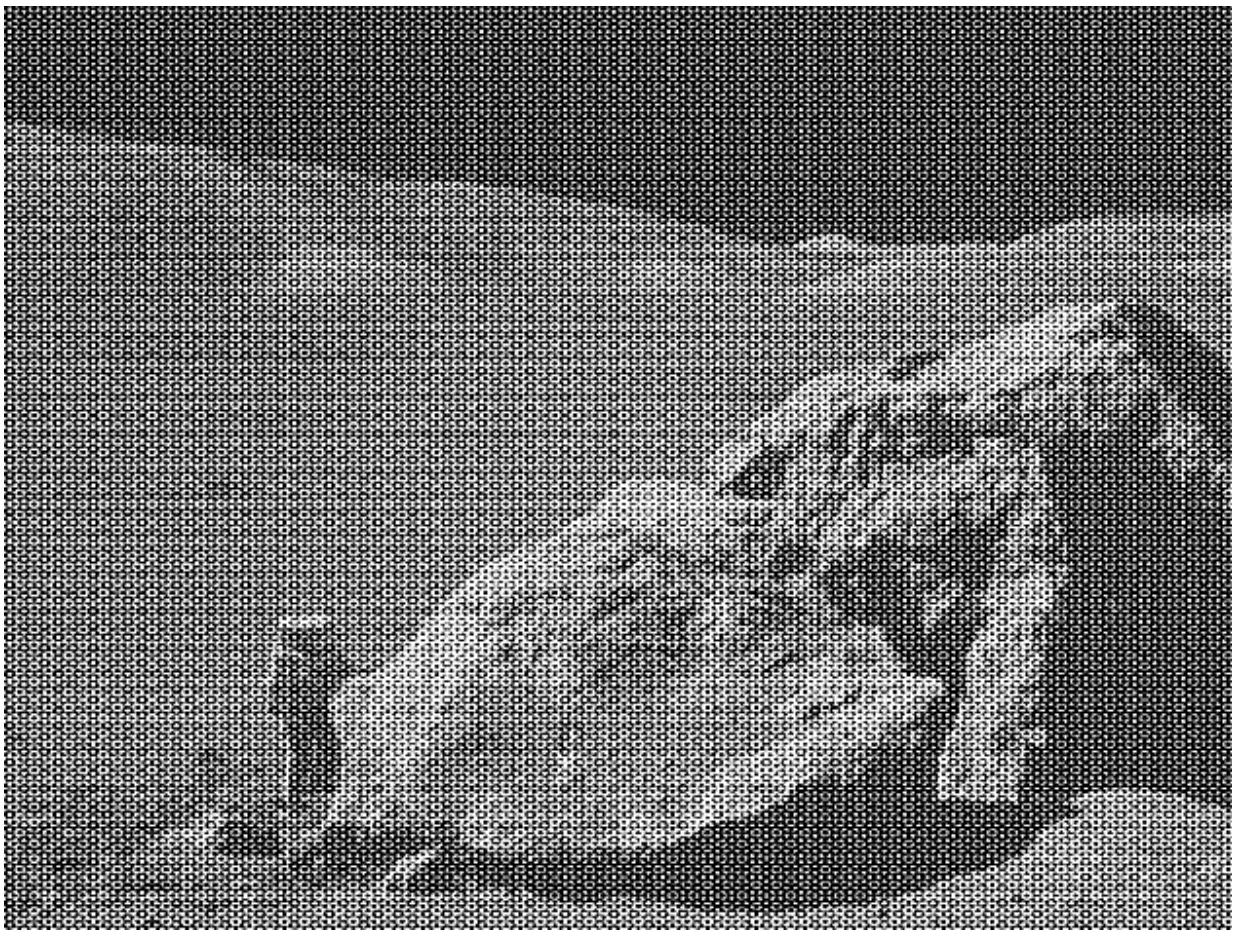
By using higher dimensional analogs of periodic functions, one can also define and n-dimensional Fourier transform. For $n = 2$ and $g : [a, b]^2 \to \mathbb{C}$, one has

$$G_{k,l} = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} g_{m,n} \exp\left(-2\pi i \left(\frac{mk}{M} + \frac{nl}{N}\right)\right)$$

Removing periodic noise from image

Consider the following image with lots of periodic noise.

Note : This is just a demo, this is not a good way to remove noise in general.

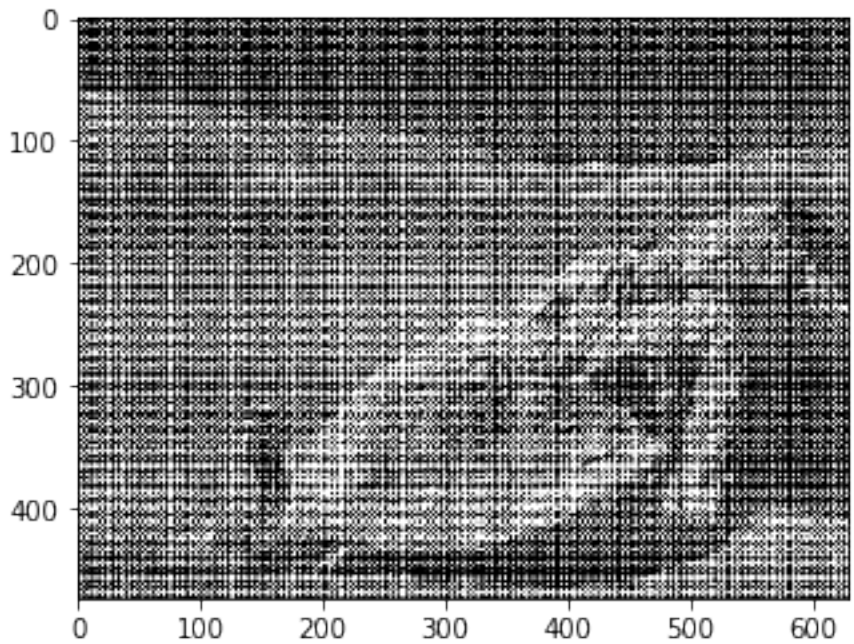


We would like to remove the noise and get a clear picture.

In [10]:

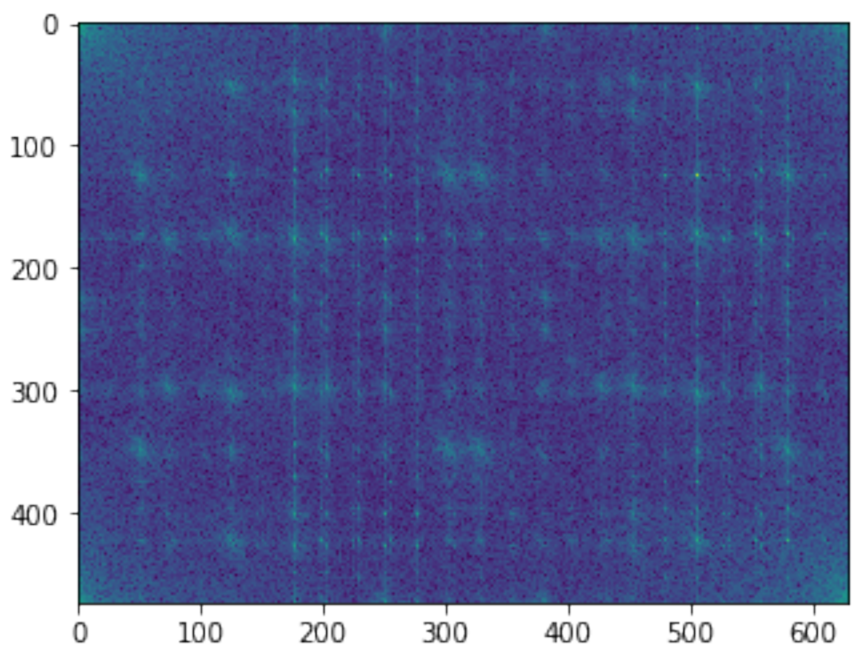
```
img = plt.imread('moonlanding.png')
# this is a black and white image
# as there is no 3rd dimension
print(img.shape)
plt.imshow(img, cmap="gray")
plt.show()
```

(474, 630)



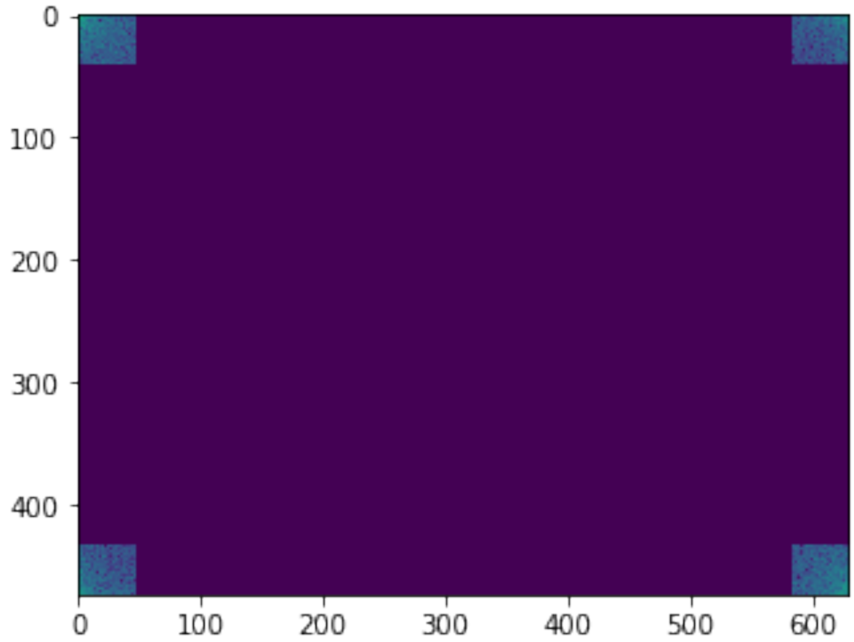
In [11]:

```
# let's see what we get for fft
img_ft = fftpack.fft2(img)
power = np.abs(img_ft)
plt.imshow(np.log(5+power))
plt.show()
```



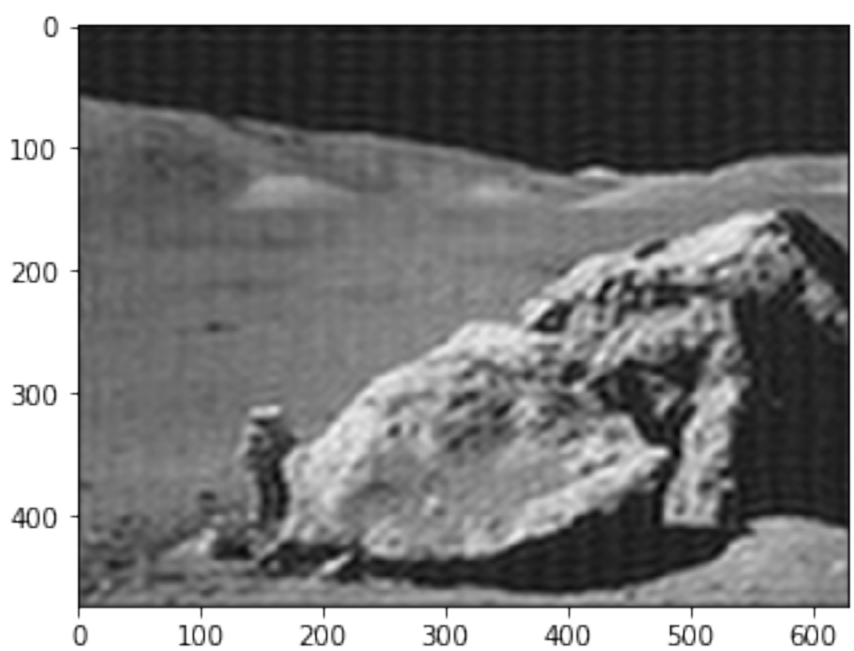
In [12]:

```
r_keep = 42
c_keep = 48
new_ft = img_ft.copy()
new_ft[r_keep : -r_keep, :] = 0.
new_ft[:, c_keep : -c_keep] = 0.

plt.imshow(np.log(5+np.abs(new_ft)))
plt.show()
```



In [13]:

```
new_img = fftpack.ifft2(new_ft).real

plt.imshow(new_img, cmap = 'gray')
plt.show()
```

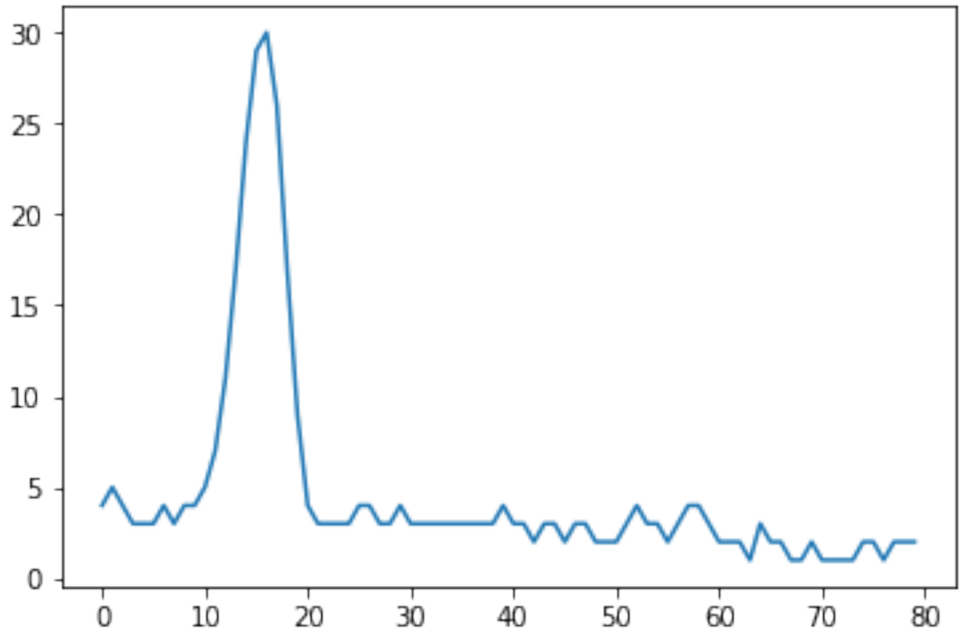


Basic fitting

Given some data and a model with a fixed number of parameters, you might want to find the best (least squares) fit of your data. You can do this using `scipy.optimize.curve_fit`.

In [14]:

```
# we can load data!
data = np.load('waveform_1.npy')
x_vals = np.arange(len(data))

plt.plot(x_vals, data)
plt.show()
```



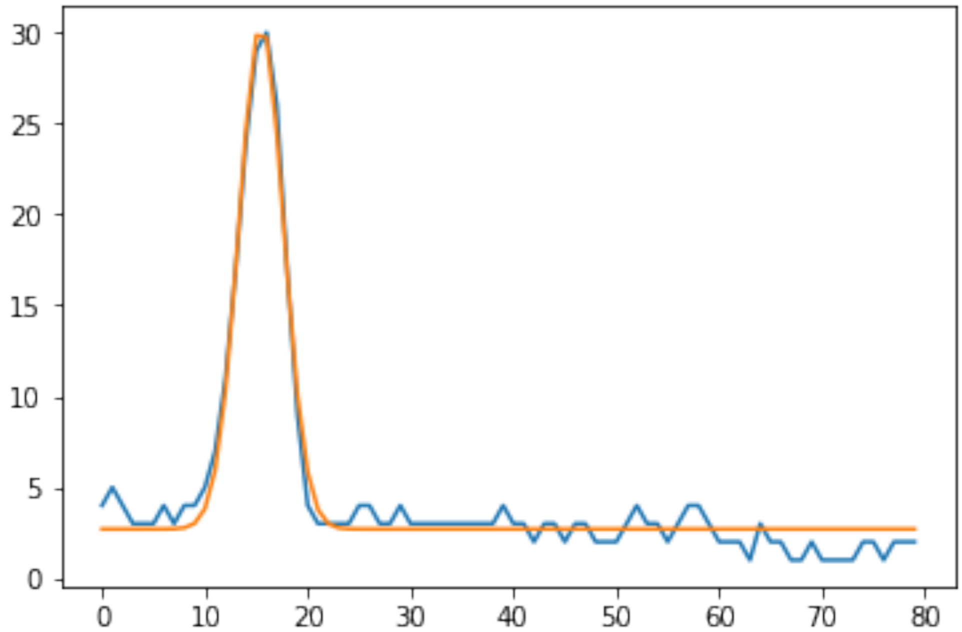
Imagine we want to fit this is a Gaussian model

$$B + A \exp\left(-\left(\frac{t-\mu}{\sigma}\right)^2\right)$$

In [15]:

```
from scipy import optimize

def g(t, b, a, m, s) :
    return b + a * np.exp(-((t-m)/s)**2)

guess = [3., 20., 15., 3.]
params, params_covar = optimize.curve_fit(g, x_vals, data, guess)

print(params)

plt.plot(x_vals, data)
plt.plot(x_vals, g(x_vals, *params))
plt.show()
```

```
[ 2.70363498 27.82022611 15.47923812  3.05635768]
```



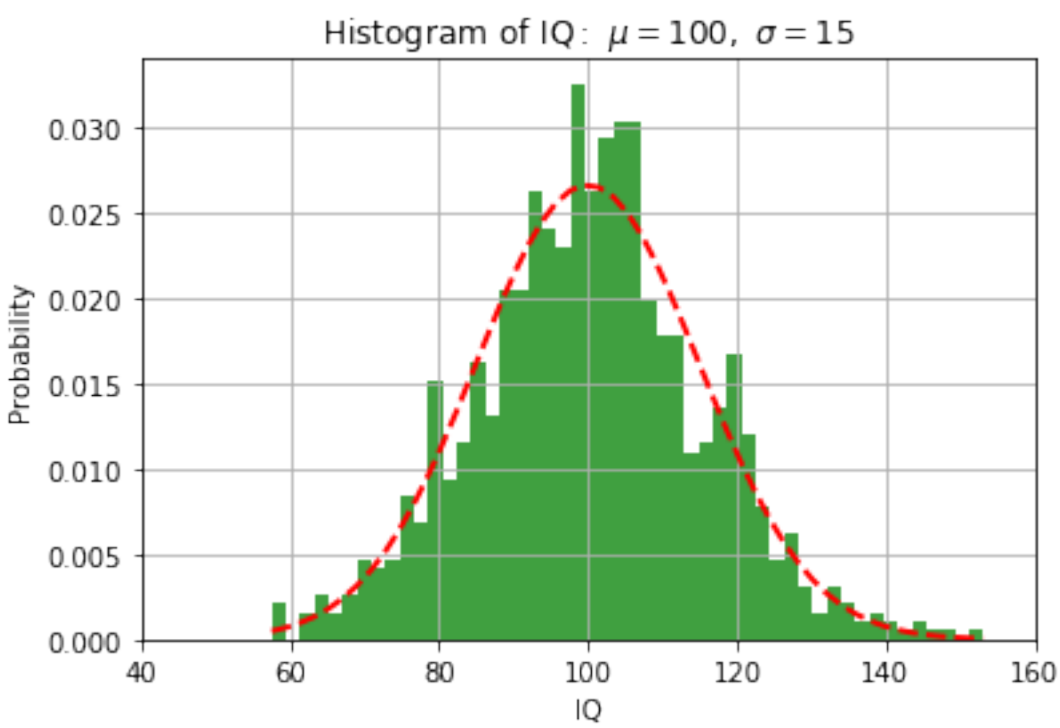
Statistics

Statistics in `python` can be done using `scipy` + `matplotlib` (mostly basic tools), `pandas` + `statsmodels` + `seaborn` (similar but simpler than R), `PyMC` (Bayesian statistical models).

Histograms

In [18]:

```
from scipy import stats

mu, sigma = 100, 15
x = mu + sigma*np.random.randn(1000)

# the histogram of the data
n, bins, patches = plt.hist(x, bins = 50, density=1,
                            facecolor='green', alpha=0.75)
# add a 'best fit' line
y = stats.norm.pdf(bins, mu, sigma)
plt.plot(bins, y, 'r--', linewidth=2.)

plt.xlabel('IQ')
plt.ylabel('Probability')
plt.title(r'$\mathrm{Histogram\ of\ IQ:}\ \mu=100,\ \sigma=15$')
plt.axis([40, 160, 0, 0.034])
plt.grid(True)

plt.show()
```



To get the actual histogram data, we use `numpy.histogram`.

In [19]:

```
hist, bins = np.histogram(x, bins = 50, normed = 1)
```

Basic stats

`scipy.stats` contains most random variables you would want. See the documentation for a complete list.

It also provides some statistical functions. Here are a few examples :

In [20]:

```
# describe
stats.describe(x)
```

Out[20]:

```
DescribeResult(nobs=1000, minmax=(57.56814319725335, 153.093107105
54172), mean=100.3763916464132, variance=230.6844608494801, skewne
ss=0.06859400014818551, kurtosis=0.1259294447202115)
```

In [21]:

```
# Bayesian confidence intervals for the mean, var, and std
stats.bayes_mvs(x)
```

Out[21]:

```
(Mean(statistic=100.3763916464132, minmax=(99.58564138656014, 101.
16714190626625)),
 Variance(statistic=231.1472180427589, minmax=(214.64662962813247,
248.70065470247985)),
 Std_dev(statistic=15.199714660201353, minmax=(14.650823513650433,
15.770245866900105)))
```

In [22]:

```
# percentiles
print(stats.scoreatpercentile(x,90))
```

```
120.15351557513775
```

In [23]:

```
# chisquare
stats.chisquare(hist, stats.norm.pdf(bins[:-1], mu, sigma))
```

Out[23]:

```
Power_divergenceResult(statistic=0.03414551564829696, pvalue=1.0)
```

In [24]:

```
# T-test
a = np.random.normal(0, 1, size=100)
b = np.random.normal(1, 1, size=10)
stats.ttest_ind(a, b)
```

Out[24]:

```
Ttest_indResult(statistic=-3.05538603064692, pvalue=0.002831966139
512011)
```

Example with `pandas` and `seaborn`

See the SciPy Lecture notes for more detailed introduction.

We will work with a dataset of wages based on gender and education.

In [26]:

```
import pandas

# EDUCATION: Number of years of education (columns 0)
# SEX: 1=Female, 0=Male (column 2)
# WAGE: Wage (dollars per hour) (columns 5)

data = pandas.read_csv('wages.txt', skiprows=27,
                       skipfooter=6, sep=None,
                       header=None,
                       names=['education', 'gender', 'wage'],
                       usecols=[0, 2, 5],
                       engine = 'python')

print(type(data))
data[:8]
```

```
<class 'pandas.core.frame.DataFrame'>
```

Out[26]:

	education	gender	wage
0	8	1	5.10
1	9	1	4.95
2	12	0	6.67
3	12	0	4.00
4	12	0	7.50
5	13	0	13.07
6	10	0	4.45
7	12	0	19.47

In [27]:

```
# Convert genders to strings (this is
# particulary useful so that the
# statsmodels formulas detects that
# gender is a categorical variable)
# Look up how numpy.choose works.
# You can also do this using vectorized maps.

data['gender'] = np.choose(data.gender, ['male', 'female'])

# Notice that data['gender'] and data.gender
# are same thing
data[:8]
```

Out[27]:

	education	gender	wage
0	8	female	5.10
1	9	female	4.95
2	12	male	6.67
3	12	male	4.00
4	12	male	7.50
5	13	male	13.07
6	10	male	4.45
7	12	male	19.47

In [28]:

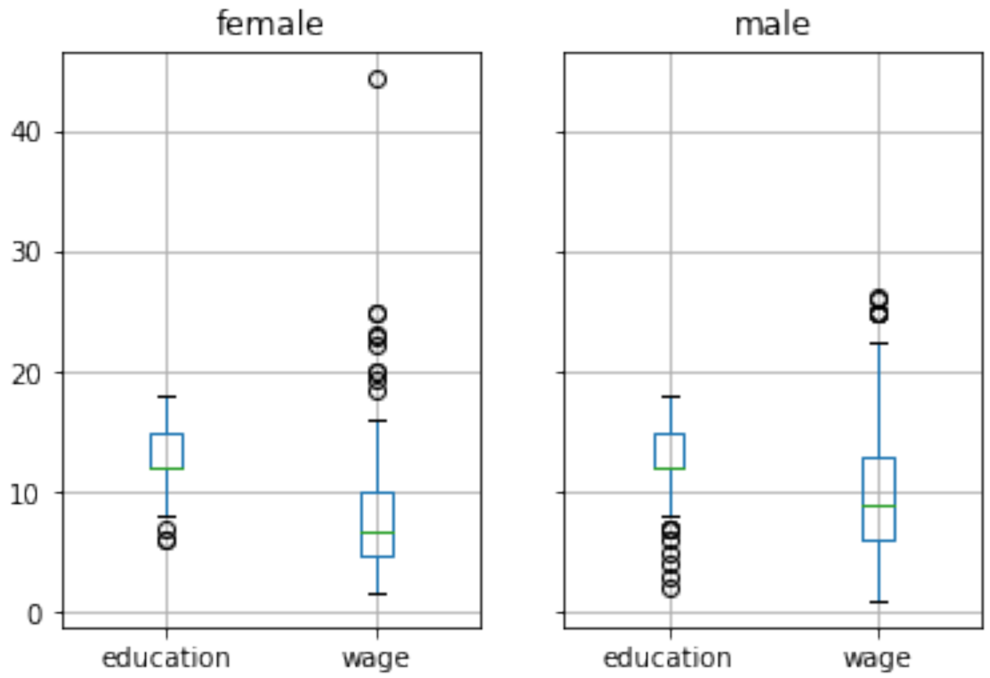
```
# We can do grouping
groupby_gender = data.groupby('gender')
# we can get the gender means
groupby_gender.mean()
```

Out[28]:

	education	wage
gender		
female	13.024490	7.878857
male	13.013841	9.994913

In [29]:

```
_ = groupby_gender.boxplot(column=['education','wage'],
                           return_type='dict')
```



In [30]:

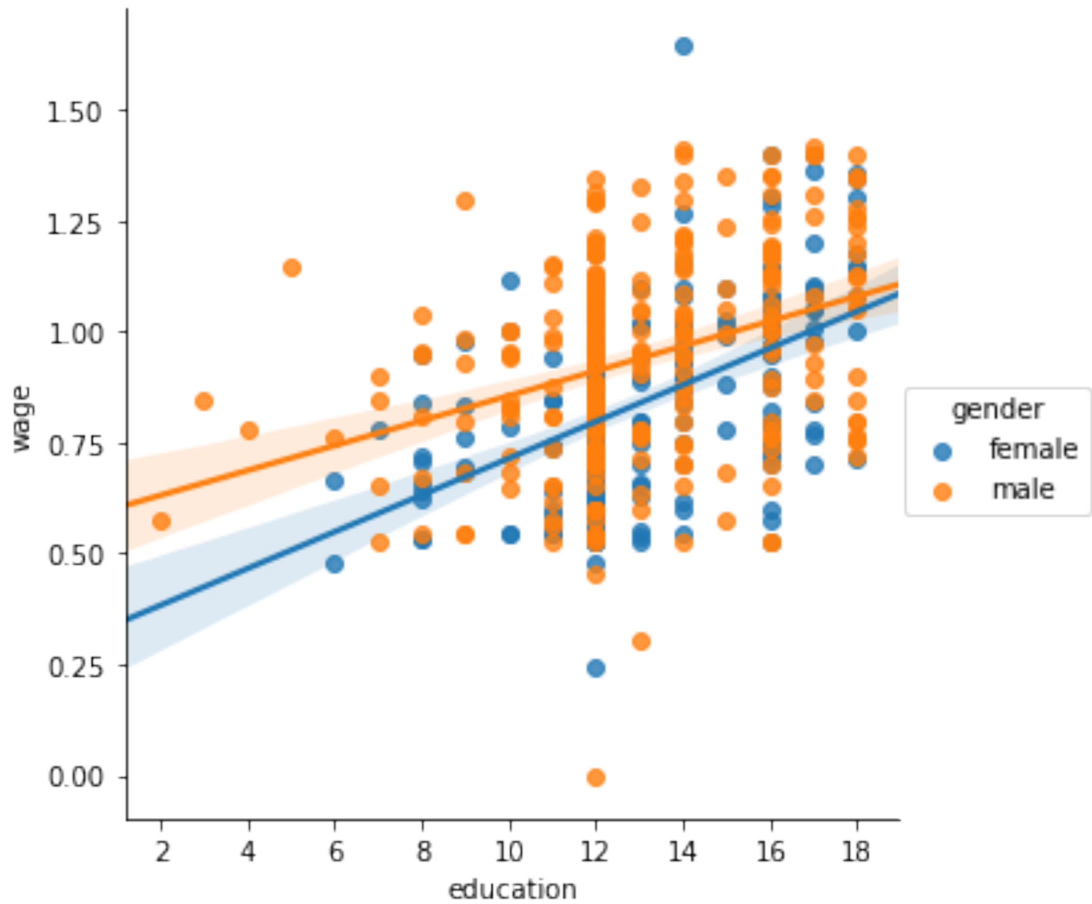
```
# Log-transform the wages, because they typically are increased with
# multiplicative factors
data['wage'] = np.log10(data['wage'])
```

In [32]:

```python
# simple plotting
import seaborn

# Plot 2 linear fits for male and female.
seaborn.lmplot(y='wage', x='education', hue='gender', data=data)
```

Out[32]:

```
<seaborn.axisgrid.FacetGrid at 0x1068c27f0>
```

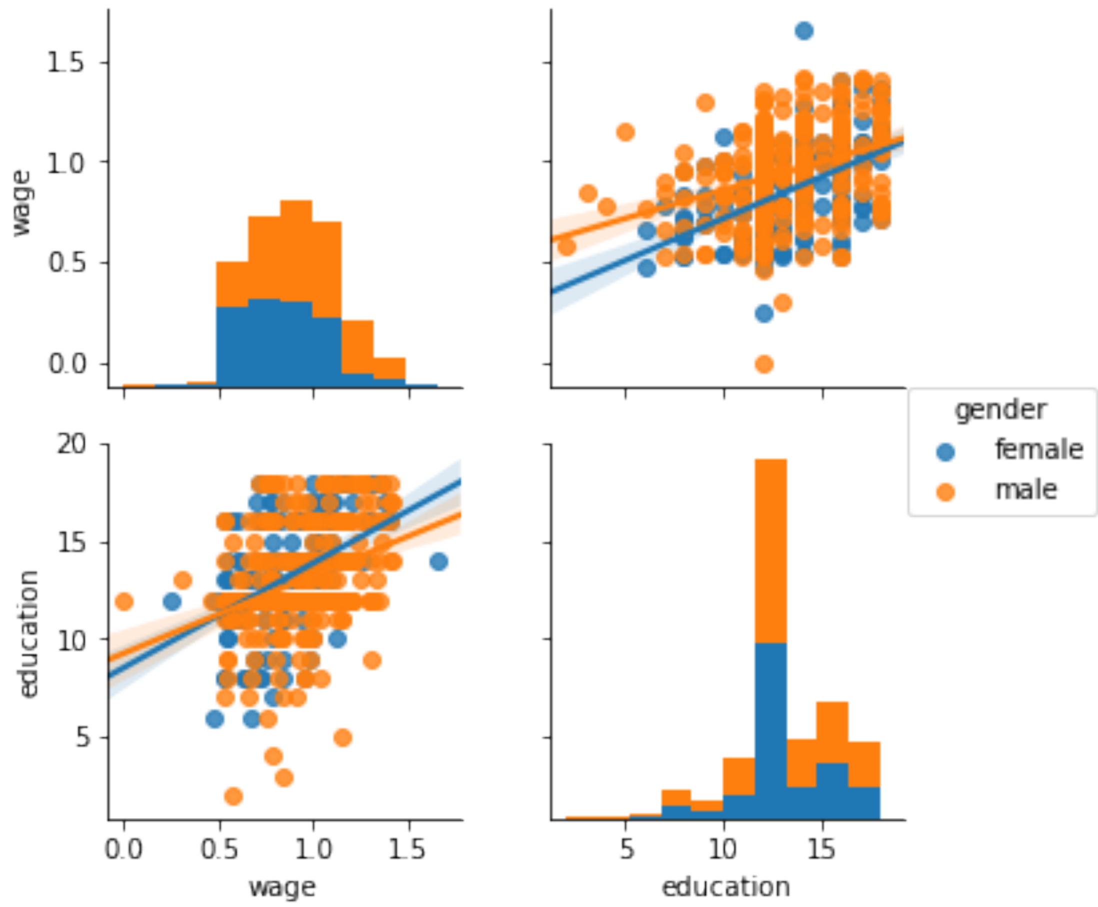


In [33]:

```
# Several plots at once
seaborn.pairplot(data, vars=['wage', 'education'],
                     kind='reg', hue='gender')
```

Out[33]:

```
<seaborn.axisgrid.PairGrid at 0x1c1cfae940>
```



In [34]:

```
# Ordinary Least Squares
import statsmodels.formula.api as sm

# Assume what wage depends linearly on
# education, gender, and gender*education
# i.e. does more education benefit males
# more than females?
form = 'wage ~ education + gender + education * gender'

model = sm.ols(formula = form, data=data).fit()

print(model.summary())
```

```
                            OLS Regression Results
==================================================================
============
Dep. Variable:                   wage   R-squared:
0.198
Model:                            OLS   Adj. R-squared:
0.194
Method:                 Least Squares   F-statistic:
43.72
Date:                Sun, 06 May 2018   Prob (F-statistic):
2.94e-25
Time:                        01:05:24   Log-Likelihood:
88.503
No. Observations:                 534   AIC:
-169.0
Df Residuals:                     530   BIC:
-151.9
Df Model:                           3
Covariance Type:            nonrobust
==================================================================
==========================
                              coef    std err          t      P>|
t|      [0.025      0.975]
------------------------------------------------------------------
--------------------------
Intercept                   0.2998      0.072      4.173      0.0
00       0.159       0.441
gender[T.male]              0.2750      0.093      2.972      0.0
03       0.093       0.457
education                   0.0415      0.005      7.647      0.0
00       0.031       0.052
education:gender[T.male]   -0.0134      0.007     -1.919      0.0
56      -0.027       0.000
==================================================================
============
Omnibus:                        4.838   Durbin-Watson:
1.825
Prob(Omnibus):                  0.089   Jarque-Bera (JB):
5.000
Skew:                          -0.156   Prob(JB):
0.0821
Kurtosis:                       3.356   Cond. No.
194.
==================================================================
============

Warnings:
[1] Standard Errors assume that the covariance matrix of the error
s is correctly specified.
```

In [35]:

```
model.pvalues
```

Out[35]:

```
Intercept                      3.512968e-05
gender[T.male]                 3.092571e-03
education                      9.767921e-14
education:gender[T.male]       5.553993e-02
dtype: float64
```

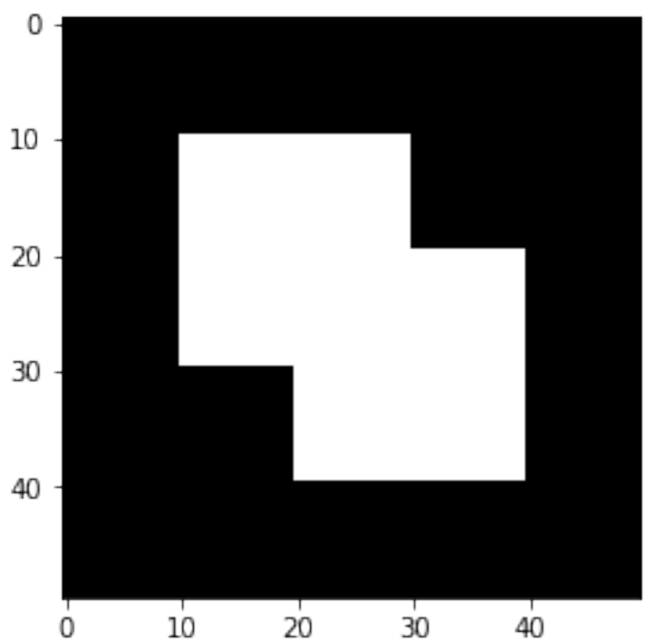
Mathematical Morphology

Mathematical morphology in our setting is the study of shapes in an image. In essense, these are tools that let you grow (i.e. dilate) a region, shrink (i.e. erode) a region, or ther operations.

In the image above, the starting shape is dark blue, the shape after the operation if light blue (with the exception of closing, where the result is the union).

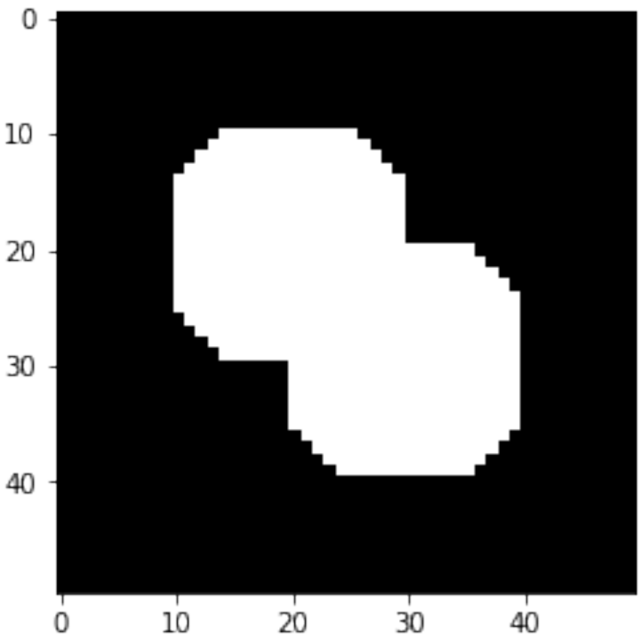
In [36]:

```
a = np.zeros((50, 50))
a[10:30, 10:30] = 1
a[20:40, 20:40] = 1

plt.grid(False)
plt.imshow(a, cmap='gray', interpolation='nearest')
plt.show()
```



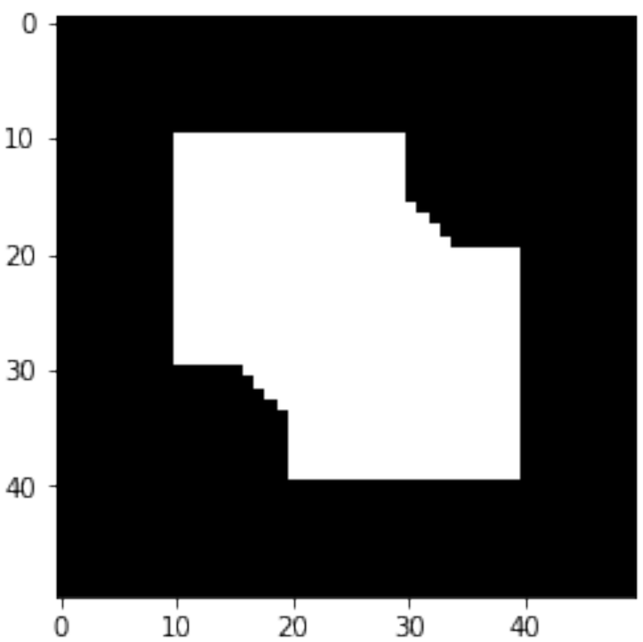
In [37]:

```
from scipy import ndimage

opened_mask = ndimage.binary_opening(a, iterations=4)
plt.grid(False)
plt.imshow(opened_mask, cmap='gray', interpolation='nearest')
plt.show()
```



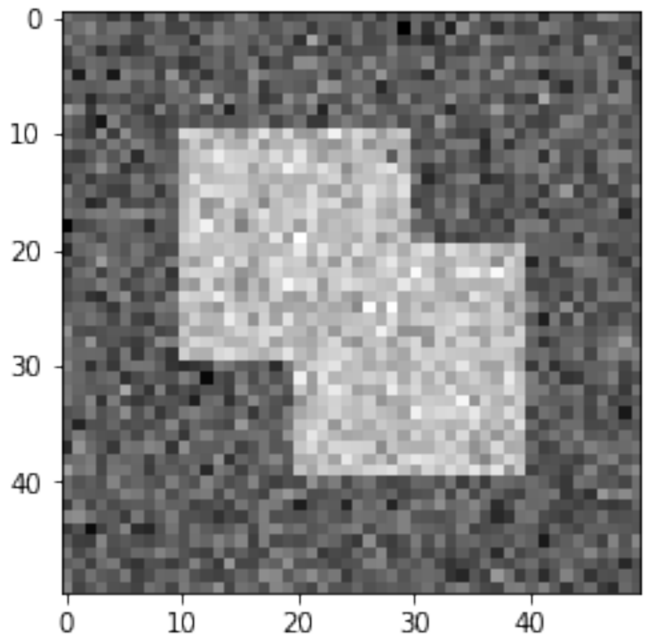
In [38]:

```
closed_mask = ndimage.binary_closing(a, iterations=4)
plt.grid(False)
plt.imshow(closed_mask, cmap='gray', interpolation='nearest')
plt.show()
```



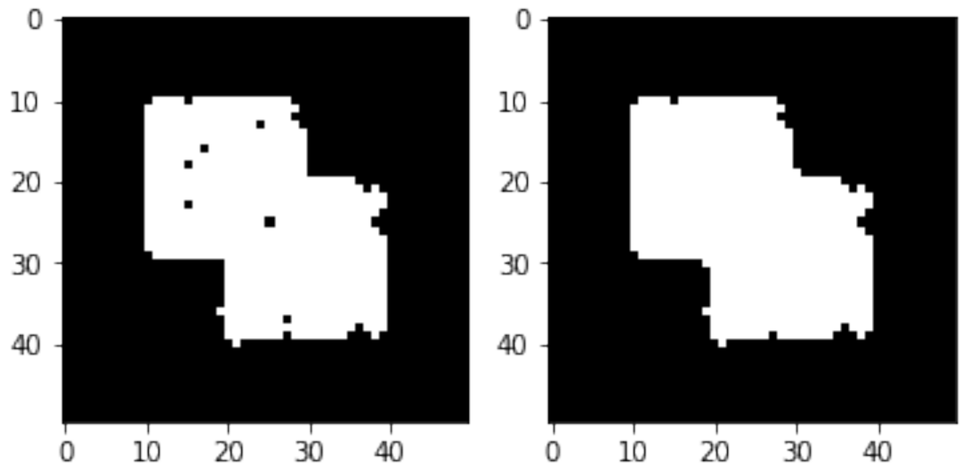
In [39]:

```
a += 0.25 * np.random.standard_normal(a.shape)


plt.grid(False)
plt.imshow(a, cmap='gray', interpolation='nearest')
plt.show()
```



In [40]:

```
mask = a>=0.5
opened_mask = ndimage.binary_opening(mask)
closed_mask = ndimage.binary_closing(opened_mask)

fig = plt.figure()
fig.add_subplot(1,2,1)
plt.grid(False)
plt.imshow(opened_mask, cmap='gray', interpolation='nearest')
fig.add_subplot(1,2,2)
plt.grid(False)
plt.imshow(closed_mask, cmap='gray', interpolation='nearest')
plt.show()
```



Gaussian blur using Fourier transforms

We will blur this image of an elephant

To do this, we will use a 2D Fourier transform to compute the convolution

$$f \star g(t) = \int f(z)\, g(t-z) dz$$

Let $\hat{f}$ denote the Fourier transform of f. Then,

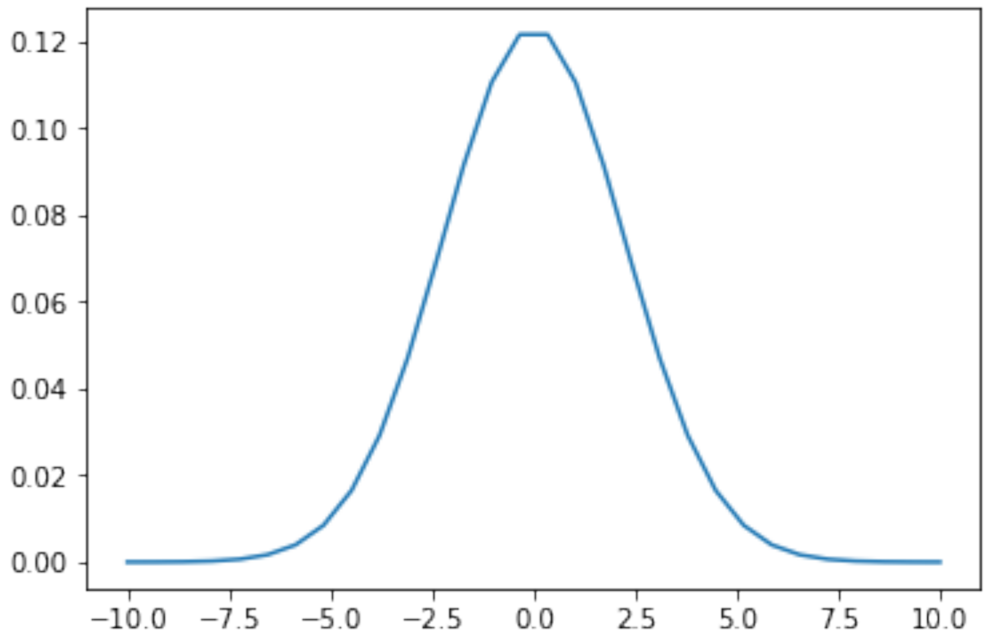
$$\widehat{f \star g} = \hat{f}\hat{g}$$

If g is a Dirac delta at a point p, then $f \star g(t) = f(t-p)$. If g is a normal distribution, then $f \star g(t)$ is "blurred" (i.e. locally averaged) value of $f(t)$ near t.

In [45]:

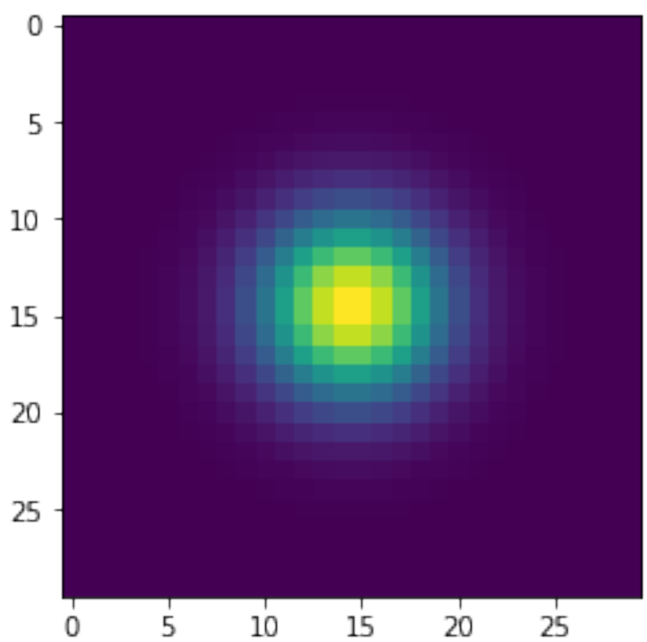
```
# read image
img = plt.imread('elephant.png')
print(img.shape)

# prepare an 1-D Gaussian convolution kernel
t = np.linspace(-10, 10, 30)
bump = np.exp(-0.1*t**2)
bump /= np.trapz(bump) # normalize the integral to 1

plt.plot(t,bump)
plt.show()
```

```
(200, 300, 3)
```



In [46]:

```
# make a 2-D kernel out of it
kernel = bump[:, np.newaxis] * bump[np.newaxis, :]

plt.imshow(kernel)
plt.show()
```



In [47]:

```
shift = np.zeros(img.shape[:2])
shift[-15,-15] = 1
```

In [48]:

```
# padded fourier transform, with the same shape as the image
kernel_ft = fftpack.fft2(kernel, shape=img.shape[:2], axes=(0, 1))
shift_ft = fftpack.fft2(shift, shape=img.shape[:2], axes=(0, 1))

# convolve
img_ft = fftpack.fft2(img, axes=(0, 1))
new_img_ft = kernel_ft[..., np.newaxis] * img_ft
new_img_ft *= shift_ft[...,np.newaxis]
new_img = fftpack.ifft2(new_img_ft, axes=(0, 1)).real

# clip values to range
new_img = np.clip(new_img, 0., 1.)

# plot output
plt.clf()
plt.grid(False)
plt.imshow(new_img)
plt.show()
```