Bayesian causal inference: day 2 practical

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Yesterday, we studied the Lalonde experimental dataset from Dehejia and Wahba (1991) and Lalonde (1986). Today we will analyze a dataset with the same treated individuals combined with 2490 controls from the nonexperimental Panel Study of Income Dynamics (PSID), consisting of male household heads from 1975 to 1978 under the age of 55 who did not classify themselves as retired. The objective is to apply ideas from the lectures today to investigate whether we can replicate the experimental results by analyzing this dataset as if it came from an observational study. Again, it is recommended to read the first two sections of Imbens and Xu (2024) if you haven't already.

The skeleton code is found in `day2_practical.R`. We will need the `BART` R package for today's practical. Please install this package if you haven't already. Like yesterday, we will start by computing the difference-in-means estimator with the experimental data:

```
diff_means <- mean(ldw[ldw$treat == 1,Y]) - mean(ldw[ldw$treat == 0,Y])
```

The value of this estimator will act as our experimental benchmark to be compared with the results from our observational analyses.

We will run BART probit-link regression to fit the propensity score model on the observational data `ldw_psid`. As a default, we discard 2000 burn-in samples before obtaining 5000 posterior samples.

```
## run probit-link BART regression to fit propensity score model
Mskip <- 2000 # number of burn-in samples
M <- 5000 # number of posterior samples
pi_post <- pbart(x.train = ldw_psid[,covar], y.train = ldw_psid[,treat], ndpost = M,
nskip = Mskip, printevery = 500L)
```

Next, we extract the posterior draws of the propensity score:

```
pi_draws <- pi_post$prob.train
```

This gives us the posterior draws of $\pi(X_i)$ for each of the data covariate values X_i.

Now we proceed to fit the outcome regression model $\mu(t,x) = \mathbb{E}[Y \mid T = t, X = x]$ using BART. To compute the average treatment effects later, we need posterior draws of $\mu(1,X_i)$ and $\mu(0,X_i)$ for each X_i. So we start by preparing the test data:

```
## prepare test data for fitting BART outcome regression model
test_dat_tr <- cbind(rep(1, nrow(ldw_psid)), ldw_psid[,covar])
test_dat_co <- cbind(rep(0, nrow(ldw_psid)), ldw_psid[,covar])
names(test_dat_tr)[1] <- "treat"
names(test_dat_co)[1] <- "treat"
test_dat <- rbind(test_dat_tr, test_dat_co)
```

Then we fit the BART outcome regression model and extract the posterior samples for $\mu(1, X_i)$ and $\mu(0, X_i)$:

```
## fit BART outcome regression model
mu_post <- wbart(x.train = ldw_psid[,c(treat,covar)], y.train = ldw_psid[,Y],
x.test = test_dat, ndpost = M, nskip = Mskip, printevery = 500L)

## extract posterior samples for mu
mu_draws_tr <- mu_post$yhat.test[,1:n]
mu_draws_co <- mu_post$yhat.test[,(n+1):(2*n)]
```

The above routine should take 1-2 minutes.

(i) Use mu_draws_tr and mu_draws_co to obtain the marginal posterior draws of the CATE estimand

$$\chi_{CATE} = \frac{1}{n}\sum_{i=1}^{n} \mu(1, X_i) - \mu(0, X_i)$$

from Hill (2011). Compute the posterior mean and the 95% central credible interval (i.e., the 2.5% and 97.5% quantiles of the draws).

Now we obtain samples from the one-step posterior for the average treatment effect. Recall that the one-step corrected parameter for the ATE takes the form

$$\tilde{\chi}_{ATE} = \tilde{P}\left[\frac{T(Y-\mu(1,\cdot))}{\pi} - \frac{(1-T)(Y-\mu(0,\cdot))}{1-\pi} + \mu(1,\cdot) - \mu(0,\cdot)\right],$$

where $\tilde{P}$ is drawn from the Bayesian bootstrap posterior. In practice, we sample $\tilde{\chi}_{ATE}$ via Algorithm 1, where $\pi^{(b)}$ and $\mu^{(b)}$ denote the posterior draws of π and μ respectively. Following conventional practice, we "trim" the posterior draws of π away from 0 and 1 to stabilize estimation in the presence of inverse weighting. Algorithm 1 is implemented in the script.

(ii) Compute the posterior mean and the 95% central credible interval for $\tilde{\chi}_{ATE}$. Compare the results to those of χ_{CATE} and the difference-in-means estimator.

The next section of the script uses the posterior mean of the propensity score (pi_postmean) as a point estimate to evaluate the overlap in the dataset. The following commands generate two histogram plots:

```
# overlap plot for propensity score
plot_hist(ldw_psid_prop, "pi_est", treat, odds = FALSE, breaks = 50,
          density = TRUE, main = "", xlim = c(0,1), ylim = NULL)

# overlap plot on log odds scale
plot_hist(ldw_psid_prop, "pi_est", treat, odds = TRUE, breaks = 50,
          density = TRUE, main = "", xlim = NULL, ylim = NULL)
```

Algorithm 1: One-step posterior sampling for the ATE

for $b \leftarrow 1$ *to* M **do**

Draw $(W_1^{(b)}, \dots, W_n^{(b)})$ from $\text{Dir}(n; 1, \dots, 1)$;

$\pi^{(b)} \leftarrow \min(\pi^{(b)}, 0.9)$;

$\pi^{(b)} \leftarrow \max(\pi^{(b)}, 0.1)$;

Evaluate $\tilde{\chi}_{ATE}^{(b)} = \sum_{i=1}^n W_i^{(b)} \left[\frac{T_i(Y_i - \mu^{(b)}(1, X_i))}{\pi^{(b)}(X_i)} - \frac{(1-T_i)(Y_i - \mu^{(b)}(0, X_i))}{1 - \pi^{(b)}(X_i)} + \mu^{(b)}(1, X_i) - \mu^{(b)}(0, X_i) \right]$;

end

Return $\{\tilde{\chi}_{ATE}^{(1)}, \dots, \tilde{\chi}_{ATE}^{(M)}\}$.

The first plot compares the densities of the estimated propensity score values for treated and controls. The second plot does the same thing but on the log-odds scale (i.e. $\hat{\pi} \mapsto \log(\hat{\pi}/\{1 - \hat{\pi}\})$).

(iii) Use these plots to argue that the average treatment effect on treated (ATT) $\mathbb{E}[Y^1 - Y^0 \mid T = 1]$ is in fact a more appropriate estimand than the ATE.

(iv) Write a routine to obtain marginal posterior draws of the CATT estimand

$$\chi_{CATT} = \frac{1}{n_t} \sum_{i:T_i=1} \mu(1, X_i) - \mu(0, X_i)$$

from Hill (2011), where $n_t = \sum_{i=1}^n T_i$. Compute the posterior mean and the 95% central credible interval.

(v) The one-step posterior parameter for the ATT has the following form:

$$\tilde{\chi}_{ATT} = \tilde{P} \left[\frac{(T - \pi)(Y - \mu(0, \cdot))}{1 - \pi} \right].$$

Write a routine to draw posterior samples for $\tilde{\chi}_{ATT}$. Compute the posterior mean and the 95% central credible interval and compare the results to those of χ_{CATT} and the difference-in-means estimator.

(vi) As described in the lectures, Hahn et al. (2020) proposed to include an estimate of the propensity score as an additional splitting covariate for the BART outcome regression model. Using `pi_postmean` (or a different propensity score estimate if you prefer, such as logistic regression), implement this idea and obtain the results for the new posteriors for χ_{CATT} and $\tilde{\chi}_{ATT}$.

References

R. Dehejia and S. Wahba. Causal Effects in Nonexperimental Studies: Reevaluating the Evaluation of Training Programs. *Journal of the American Statistical Association*, 94(448):1053–1062, 1991.

P. Hahn, J. Murray, and C. Carvalho. Bayesian regression tree models for causal inference: regularization, confounding, and heterogeneous effects. *Bayesian Analysis*, 15:965–1056, 2020.

J. Hill. Bayesian nonparametric modeling for causal inference. *Journal of Computational and Graphical Statistics*, 20:217–240, 2011.

G. Imbens and Y. Xu. Lalonde (1986) after nearly four decades: lessons learned. *arXiv preprint arXiv:2406.00827*, 2024.

R. Lalonde. Evaluating the Econometric Evaluations of Training Programs with Experimental Data. *The American Economic Review*, 76:604–620, 1986.