

# Constraints on Assembly Bias from Galaxy Clustering.

Andrew R. Zentner<sup>1</sup>, Andrew P. Hearin<sup>2</sup>, Frank C. van den Bosch<sup>3</sup>,  
Antonio Villareal<sup>1</sup>, Johannes U. Lange<sup>3</sup>, P. Rogers Nelson<sup>4</sup>

<sup>1</sup>*Department of Physics and Astronomy & Pittsburgh Particle Physics, Astrophysics, and Cosmology Center (PITT PACC), University of Pittsburgh, Pittsburgh, PA 15260*

<sup>2</sup>*Yale Center for Astronomy & Astrophysics, Yale University, New Haven, CT*

<sup>3</sup>*Department of Astronomy, Yale University, P.O. Box 208101, New Haven, CT*

<sup>4</sup>*Paisley Park, Chanhassen, MN*

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## ABSTRACT

We constrain the newly-introduced Decorated HOD model using SDSS DR7 measurements of projected galaxy clustering,  $w_p$ , and number density,  $n_d$ , made from r-band luminosity-threshold samples. The Decorated HOD is a model for the galaxy–halo connection that augments the traditional Halo Occupation Distribution (HOD) by allowing for the possibility of *assembly bias*: galaxy luminosity may be correlated with dark matter halo properties besides virial mass  $M_{\text{vir}}$  alone. We demonstrate that it is not possible to rule out assembly bias using DR7 measurements of  $w_p$  and  $n_d$ . Moreover, galaxy samples  $M_r < -20, -20.5$  favor strong levels of central galaxy assembly bias: high-concentration halos are more likely to host a central galaxy relative to low-concentration halos of the same  $M_{\text{vir}}$ . We rule out zero assembly bias with high significance for these samples. Satellite assembly bias only becomes significant for the faintest sample we study,  $M_r < -19$ . We find no evidence for assembly bias in the  $M_r < -21$  sample. Because the Decorated HOD subsumes the traditional HOD, and additionally because of errors made in the original analysis of these samples, our results differ from and supersede existing HOD constraints from low-redshift clustering measurements.

## 1 INTRODUCTION

For more than a decade, halo occupation modeling has been used to interpret large-scale structure measurements and exploit these measurements to constrain galaxy formation models and cosmology (e.g., Yang et al. 2003; Tinker et al. 2005; Zehavi et al. 2005; Porciani & Norberg 2006; van den Bosch et al. 2007; Zheng et al. 2007; Conroy & Wechsler 2009; Yang et al. 2009; Zehavi et al. 2011; Guo et al. 2011; Wake et al. 2011; Yang et al. 2011, 2012; Leauthaud et al. 2012; Rodríguez-Puebla et al. 2012; Tinker et al. 2013; Cacciato et al. 2013; More et al. 2013; Guo et al. 2014; Zu & Mandelbaum 2015). The key assumptions underlying halo occupation modeling are: (1) all galaxies reside in dark matter halos that are biased tracers of the density field; and (2) galaxies occupy halos as a function of halo masses only. It is now well known that halo bias depends upon halo properties other than mass (e.g. Gao et al. 2005; Wechsler et al. 2006; Gao & White 2007; Zentner 2007; Dalal et al. 2008; Lacerna & Padilla 2011), an effect called halo assembly bias. If galaxies occupy halos as a function of properties other than halo mass, then standard halo occupation methods will be subject to a systematic error due to galaxy assembly bias.

Several of us have previously shown that this error can be significant in an analysis of galaxy clustering and can bias inferences about many aspects of galaxy evolution (Zentner et al. 2014). Consequently, we have developed halo occupation models that enable galaxies to occupy halos in a manner that depends upon several halo properties (Hearin et al. 2016). In this paper, we revisit the interpretation of luminosity-dependent galaxy clustering in the Sloan Digital Sky Survey (SDSS) Data Release 7 (DR7) data, analyzed previously by Zehavi et al. (2011), in the context both standard halo occupation models and these new models.

The ultimate goal of halo occupation modeling is to characterize the galaxy–dark matter connection by constraining the probability distribution  $P(X_g|Y_h)$ . Here  $X_g = \{x_1, x_2, \dots, x_n\}$  is a set of  $n$  galaxy properties, and  $Y_h = \{y_1, y_2, \dots, y_m\}$  a set of  $m$  halo properties. Accurate knowledge of this probability distribution yields invaluable insight regarding the processes of galaxy formation, and allows one to make accurate predictions for the clustering of galaxies as function of any property  $x_i$ . Most halo occupation studies to date have restricted themselves to only one halo property, namely halo mass. This

includes Halo Occupation Distribution (HOD) models, which aim to constrain  $P(N|M_{\text{vir}})$ , the probability that a halo of mass  $M$  contains  $N$  galaxies of some specified properties (e.g., Kauffmann et al. 1997; Jing et al. 1998; Benson 2001; Berlind & Weinberg 2002), and Conditional Luminosity Function (CLF) models, which aim to constrain  $\Phi(L|M)$ , the luminosity (or stellar mass) function of galaxies in haloes of mass  $M$  (e.g., Yang et al. 2003; van den Bosch et al. 2003; Cooray 2006). The HOD and CLF models are mainly used in combination with the halo model, which describes the non-linear matter field in terms of halo building blocks (e.g., Seljak 2000; Cooray & Sheth 2002), to predict the clustering of galaxies as function of luminosity or stellar mass. Although entirely analytical, and therefore fast, the downside of this method is that it is extremely challenging to construct a model that is accurate at the few percent level. The main issues are halo exclusion (see Smith et al. 2007; van den Bosch et al. 2013), scale dependence of the halo bias (see Tinker et al. 2005), and the halo assembly bias mentioned above.

These issues are trivially avoided by directly populating dark matter haloes in numerical simulations with galaxies. This can be done using either a parameterized model for the halo occupation statistics (e.g., Yang et al. 2004; Moster et al. 2010; Behroozi et al. 2010, 2013) or by rank-order matching galaxy properties (i.e., luminosity or stellar mass) to halo properties (i.e., halo mass). The latter method is known as subhalo abundance matching (SHAM). Although many SHAM studies use halo mass as the halo property of choice (e.g., Vale & Ostriker 2004, 2006; Conroy et al. 2007; Conroy & Wechsler 2009), a number of studies have used alternative halo properties, such as the maximum circular velocity,  $V_{\text{max}}$ , or its peak value over the halo’s history,  $V_{\text{peak}}$  (e.g. Tasitsiomi et al. 2004; Conroy et al. 2006; Marín et al. 2008; Reddick et al. 2013).

Since, at fixed halo mass,  $V_{\text{max}}$  is correlated with halo concentration, and thus with halo assembly time, the resulting occupation models have assembly bias build in (see Zentner et al. 2014); i.e., at fixed halo mass, galaxy properties will be correlated with halo assembly time. Lehmann et al. (2015) took this one step further, and explored SHAM models in which the halo property varies between  $M$  and  $V_{\text{max}}$  via a continuously-valued parameter. Using SDSS clustering measurements to constrain their model, they conclude that at fixed halo mass galaxy luminosity reveals a significant dependence on halo concentration. Since the latter is strongly correlated with halo formation time (e.g., Wechsler et al. 2002; Zhao et al. 2009; Ludlow et al. 2013), their conclusions argue for non-zero assembly bias in the galaxy population.

Additional support for the presence of assembly bias has come from a variety of studies that have shown, or suggested, that the large-scale environment (i.e., clustering strength) of dark matter haloes *of fixed mass* is correlated with the star formation rate of their central galaxies (Yang et al. 2006; Blanton & Berlind 2007; Wang et al. 2008; Tinker et al. 2012; Wang et al. 2013; Hearin & Watson 2013; Hearin et al. 2013, 2014; Watson et al. 2015). However, Lin et al. (2015) claim that most of these differences in clustering strength can be explained as due to differences in halo mass or contamination by satel-

lite galaxies, and therefore argue that there is no significant evidence for assembly bias in galaxy-scale halos (see also Paranjape et al. 2015). On the other hand, Miyatake et al. (2016) and More et al. (2016) recently presented strong evidence for the presence of assembly bias in massive clusters; in particular, they find that clusters with a more concentrated distribution of satellite galaxies are less strongly clustered than their equal-mass counterparts whose satellite galaxies are less centrally concentrated. To summarize, while there is strong evidence for the presence of assembly bias on the scale of clusters, there is not consensus yet as to whether assembly bias plays also a role in less massive haloes.

Our work is important for several reasons. Our work is a re-analysis of the SDSS DR7 data that overcomes shortcomings of previous analyses. In particular, we use direct population of galaxies in a cosmological simulation so that delicate issues present in analytic modeling, such as scale-dependent halo bias, are treated exactly. The simulation we use is based on the latest Planck cosmological parameters, updating previous work. Furthermore, many important differences between this work and the previous analysis by Zehavi et al. (2011) result from the fact that the Monte Carlo Markov Chains used by Zehavi et al. were too small and had therefore not converged (Z. Zheng & I. Zehavi, private communication). Therefore, our analysis *supersedes previous analyses even in the case of standard halo occupation models*.

Furthermore, our work demonstrates explicitly that significant assembly bias in  $M_r$ -selected samples from SDSS DR7 cannot be ruled out based on a standard analysis of galaxy clustering only. In fact, in agreement with Lehmann et al. (2015), we find that several samples *favor galaxy assembly bias to a degree that is statistically significant*. As demonstrated by Zentner et al. (2014), this conclusion could have important consequences for the interpretation of both extant and forthcoming data.

Our paper is organized as follows. In Section 2, we discuss our implementation of halo occupation models and our assumptions in our parameter inference analysis. We present results for both standard halo occupation analysis and analysis in the context of models with galaxy assembly bias in Section 3. We summarize our results and draw conclusions in Section 4.

## 2 METHODS

### 2.1 Halotools Implementation of HOD Models

To generate predictions for galaxy clustering, we directly populate dark matter halos with mock galaxies using **Halotools**. We explore halo occupation distribution models (HOD) models in this work (e.g. Seljak 2000; Ma & Fry 2000; Scoccimarro et al. 2001; Berlind & Weinberg 2002), though other techniques that can be used to interpret such data, such as the conditional luminosity function (CLF, e.g., Yang et al. 2003; van den Bosch et al. 2013), exist. In this section, we review the “standard” HOD model used in the present work which assumes that there is no galaxy assembly bias. We refer to such a model as “standard” because all HOD analyses of

galaxy clustering to date have assumed no galaxy assembly bias. In the following section, we describe the Decorated HOD model described in Hearin et al. (2016). In both cases, we will only review the salient features of our methodology briefly; interested readers can always refer to `halotools.readthedocs.io` and Hearin et al. (2016) for further details.

### 2.1.1 Simulation

All of our analyses are based on the Bolshoi-Planck simulation (Riebe et al. 2011). Bolshoi-Planck was run with cosmological parameters based on Planck Collaboration et al. (2014),  $\Omega_\Lambda = 0.693$ ,  $\Omega_m = 0.307$ ,  $\Omega_b = 0.048$ ,  $h = 0.7$ ,  $n_s = 0.96$  and  $\sigma_8 = 0.82$ , within a cubic box  $250 \text{ Mpc}/h$  on a side, requiring a particle mass of  $m_p = 1.35 \times 10^8 M_\odot/h$ . Further information about the Bolshoi-Planck simulation is available at <https://www.cosmosim.org>.

We use publicly available<sup>1</sup> dark matter halo catalogs based on the ROCKSTAR halo-finder (Behroozi et al. 2011) and CONSISTENT TREES algorithm (Behroozi et al. 2013). In particular, we use the `halotools.alpha.version2` version of the  $z = 0$  snapshot of the ‘`bolplanck`’ catalog included with `Halotools`. Halos in these catalogs are based on the virial radius density contrast given in Bryan & Norman (1998). When populating this catalog with mock galaxies, we only use present-day host halos with a value of  $M_{\text{peak}}$  that exceeds 300 particles.

### 2.1.2 Occupation statistics

In standard HOD models, central galaxies and satellite galaxies are treated separately, so the model is specified by two probability distributions, one for each type of galaxy. The galaxy-halo connection is specified in terms of  $P(N_{\text{cen}}|M_{\text{vir}})$  and  $P(N_{\text{sat}}|M_{\text{vir}})$ , the probability that a halo of mass  $M_{\text{vir}}$  hosts  $N_{\text{cen}}$  central and  $N_{\text{sat}}$  satellite galaxies, respectively.  $P(N_{\text{cen}}|M_{\text{vir}})$  is typically a nearest-integer distribution, as a host halo has only either zero or one central galaxy. Consequently, the occupation statistics of central galaxies are specified by the first moment of  $P(N_{\text{cen}}|M_{\text{vir}})$ , which we model as

$$\langle N_{\text{cen}} \rangle(M_{\text{vir}}) = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{\log(M_{\text{vir}}) - \log(M_{\text{min}})}{\sigma_{\log M}} \right) \right) \quad (1)$$

For every host halo in the catalog we draw a random number from a uniform distribution  $\mathcal{U}(0, 1)$ ; for a host halo of present-day virial mass  $M_{\text{vir}}$ , a central galaxy is assigned to the halo if the associated random number is less than  $\langle N_{\text{cen}} \rangle(M_{\text{vir}})$ ; halos with random values exceeding  $\langle N_{\text{cen}} \rangle(M_{\text{vir}})$  are left devoid of centrals. The parameter  $\log(M_{\text{min}})$  specifies the halo mass at which the halo has a 50% probability of hosting a central galaxy, while the parameter  $\sigma_{\log M}$  specifies the rate at which  $\langle N_{\text{cen}} \rangle(M_{\text{vir}})$  transitions from zero to unity, with smaller values of  $\sigma_{\log M}$  corresponding to a more rapid transition.

We model the distribution  $P(N_{\text{sat}}|M_{\text{vir}})$  as a Poisson distribution with first moment given by a power law,

$$\langle N_{\text{sat}} \rangle(M_{\text{vir}}) = \left( \frac{M_{\text{vir}} - M_0}{M_1} \right)^\alpha. \quad (2)$$

The parameter  $M_0$  allows the power-law to be truncated more rapidly at low masses and we set  $\langle N_{\text{sat}} \rangle(M_{\text{vir}}) = 0$  for  $M_{\text{vir}} < M_0$ .

The five parameters of the standard HOD models are varied in standard analyses are  $\log(M_{\text{min}})$ ,  $\sigma_{\log M}$ ,  $\alpha$ ,  $\log(M_1)$ , and  $\log(M_0)$ , though, as we show below, central galaxies usually outnumber satellite galaxies in by a factor of several, so  $\log(M_{\text{min}})$  and  $\sigma_{\log M}$  usually vary along a narrow degeneracy that fixes the total galaxy number density to the observed value. There are many particular choices that can be made for the functional forms of  $\langle N_{\text{cen}} \rangle(M_{\text{vir}})$  and  $\langle N_{\text{sat}} \rangle(M_{\text{vir}})$ . We have made choices that mimic the standard SDSS DR7 analysis of Zehavi et al. (2011), to expedite comparisons with their results.

### 2.1.3 Galaxy profiles

Central galaxies in the standard HOD models reside at the halo center, moving with the same velocity as the host halo peculiar velocity. We model the intra-halo spatial distribution of satellite galaxies to be located within  $R_{\text{vir}}$  of the halo center, with a spherically symmetric NFW profile (Navarro et al. 1997). The concentration  $c$  of each halo’s satellite galaxy profile is taken to be the same as the concentration of the dark matter particles in the halo.<sup>2</sup>

We model the radial velocity distribution of satellite galaxies as a Gaussian with first moment equal to the host halo velocity and second moment equal to the solution to the isotropic Jeans equation for an NFW profile (More et al. 2009),

$$\sigma_r^2(\tilde{r}|c) = V_{\text{vir}}^2 \frac{c^2 \tilde{r}(1 + c\tilde{r})^2}{g(c)} \int_{c\tilde{r}}^\infty dy \frac{g(y)}{y^3(1+y)^2}, \quad (3)$$

where  $\tilde{r} = r/R_{\text{vir}}$ ,  $g(x) = \ln(1+x) - x/(1+x)$ , and  $V_{\text{vir}}^2 = GM_{\text{vir}}/R_{\text{vir}}$ . We assume that velocities are isotropic, setting the peculiar velocities in each Cartesian direction according to random draws from the above radial velocity distribution.

### 2.1.4 Predictions for observables

After populating a halo catalog with mock galaxies, we calculate the comoving number density of our mock galaxy sample as  $n_g = N_{\text{gal}}/L_{\text{box}}^3$ . We apply the distant-observer approximation and use the simulation  $z$ -axis as the line-of-sight direction, and the distance between points in the  $xy$ -plane to define the projected distance  $r_p$ . We place mock galaxies into redshift-space by replacing each galaxy’s  $z$ -coordinate with  $z \rightarrow z + V_z/H_0$ .

<sup>2</sup> We set a maximum value of  $c = 25$  to the NFW concentration, because halos with very large values for the concentration tend to be poorly described by an NFW profile, for example due to a recent merger.

<sup>1</sup> <http://www.slac.stanford.edu/~behroozi/BPlanck.Hlists>

We count pairs of points in each of our  $r_p$  bins, rejecting pairs with  $z$ -distance exceeding  $\pi_{\max} = 60$  Mpc/h. We employ the Landy & Szalay (1993) estimator to turn pair counts into a  $w_p$  prediction, as described in detail in `halotools.readthedocs.io`.

## 2.2 HOD with Assembly Bias: The Decorated HOD

In addition to the standard occupation statistics described in the previous section, in this paper we also use the decorated HOD formalism to connect galaxies to dark matter halos in a manner that has simultaneous dependence on both  $M_{\text{vir}}$  and halo concentration. Briefly, we use Equations 1 and 2 as our “baseline” first occupation moments. At fixed  $M_{\text{vir}}$ , halos are divided into one of two categories, those of high- and low-concentration, depending on whether the concentration of the halo places it above or below the rank-order percentile  $f_{\text{split}}$ , which we keep fixed to  $f_{\text{split}} = 0.5$  throughout the paper for simplicity. High-concentration halos have a different first occupation moment relative to low-concentration halos of the same mass,  $\langle N_{\text{gal}} | M_{\text{vir}}, C_{\text{high}} \rangle \neq \langle N_{\text{gal}} | M_{\text{vir}}, C_{\text{low}} \rangle$ . The difference between the first moment of high- and low-concentration halos is modulated by  $\mathcal{A}_{\text{bias}}$ , the novel parameter of the decorated HOD governing assembly bias. Values of  $\mathcal{A}_{\text{bias}} = \pm 1$  correspond to the maximum strength of assembly bias allowable by the constraint that the model preserves the marginalized first moment,  $\langle N_{\text{gal}} | M_{\text{vir}} \rangle$ ; thus regardless of the value of  $\mathcal{A}_{\text{bias}}$ , in the decorated HOD the marginalized first moment of centrals and satellites are *unchanged from the baseline value defined by Equations 1 and 2*. Decorated HOD models all have the same HODs, when averaged over all halos at fixed mass, as standard HOD models. The only change in Decorated HOD models is whether or not an additional property also modulates halo occupation at fixed halo mass. A value of  $\mathcal{A}_{\text{bias}} = 0$  indicates no galaxy assembly bias whatsoever. We refer the reader to Hearin et al. (2016) for further details about the decorated HOD.

In the present work, we fix our model to the simplest class of galaxy assembly bias models, though a recipe for generalizing to more complicated models can be found in Hearin et al. (2016). In particular, we split halos into two populations as specified in the previous paragraph. We then populate halos with satellite galaxies specified by an assembly bias parameter  $-1 \leq \mathcal{A}_{\text{sat}} \leq 1$  and central galaxies with a distinct assembly bias parameter  $-1 \leq \mathcal{A}_{\text{cen}} \leq 1$ . These two additional parameters are varied in our analyses that include assembly bias, making the number of parameters that vary in these analyses seven. As we show below, these additional parameters are in many instances poorly constrained by clustering data of the quality of SDSS DR7 alone, so exploring more complex models of assembly bias does not yet seem justified.

## 2.3 Parameter Inference

To infer parameters for the HOD and Decorated HOD models described in the previous subsections, we performed a Markov Chain Monte Carlo (MCMC) sampling

Parameter	Prior Interval
$\log(M_{\min})$	[9.0, 14.0]
$\sigma_{\log M}$	[0.01, 1.5]
$\log(M_0)$	[9.0, 14.0]
$\log(M_1)$	[10.7, 15.0]
$\alpha$	[0.0, 2.0]
$\mathcal{A}_{\text{cen}}$	[-1.0, 1.0]
$\mathcal{A}_{\text{sat}}$	[-1.0, 1.0]

**Table 1.** Ranges for the priors used in the parameter inference. All prior distributions are uniform over the specified ranges.

of the posteriors using the affine-invariant ensemble sampler of Goodman & Weare (2010) as implemented in the `emcee` software package (Foreman-Mackey et al. 2013). For most cases, we find that  $\sim 3 - 10 \times 10^6$  samples are necessary in order for our chains to converge.

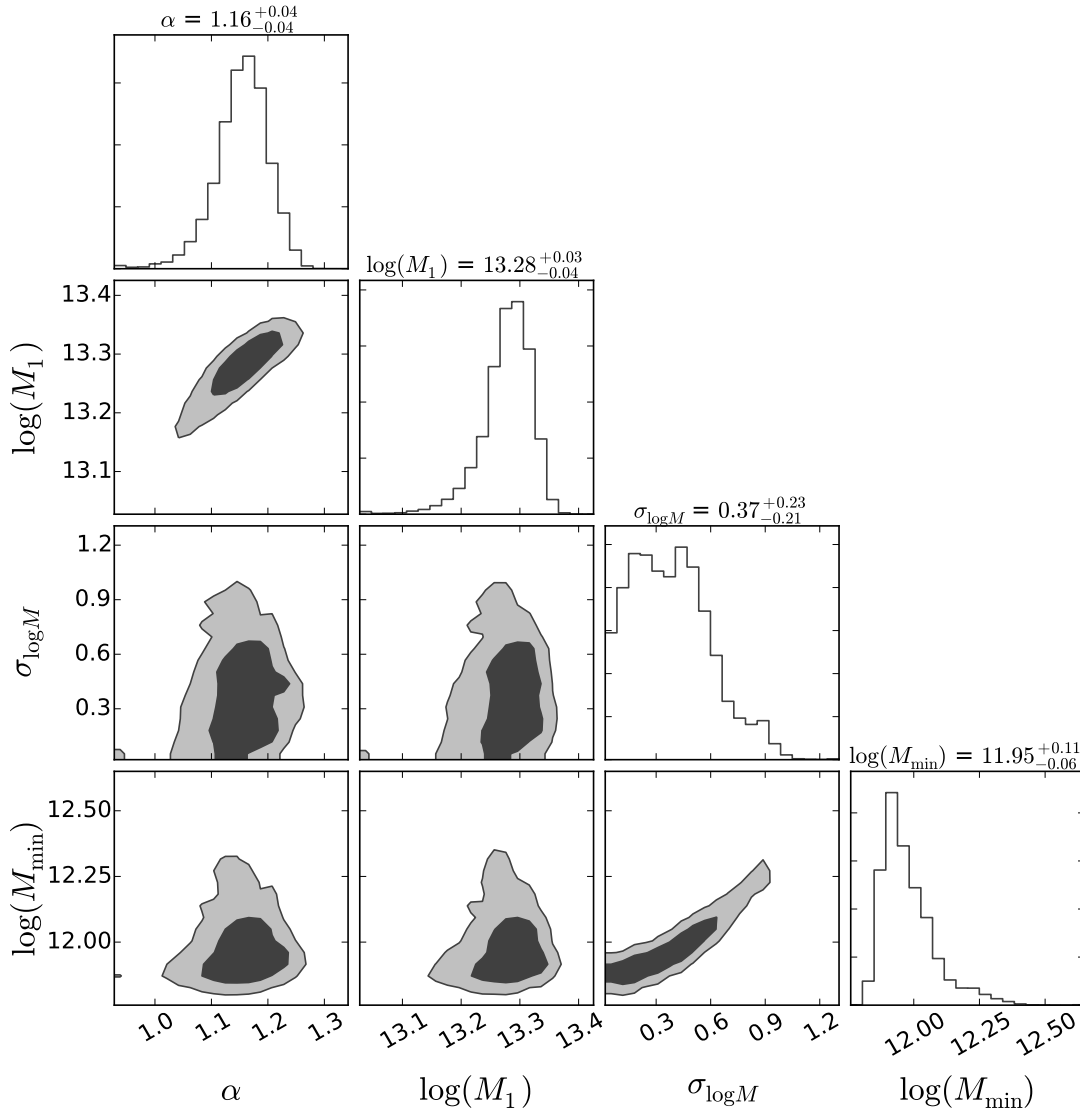
The most important detail of this analysis is the priors on the parameters. In all analyses discussed in this paper, we adopt priors that are uniform distributions over the intervals specified in Table 3.2. In the case of the assembly bias parameters  $\mathcal{A}_{\text{cen}}$  and  $\mathcal{A}_{\text{sat}}$ , the priors represent physical boundaries. These parameters must satisfy  $-1 \leq \mathcal{A}_{\text{cen}, \text{sat}} \leq 1$ . Physical considerations require parameter  $\sigma_{\log M} > 0$ . All other priors have a negligible influence on the posterior aside from  $\log M_0$ . We find that  $\log M_0$  is often very poorly constrained by clustering data.

## 3 RESULTS

We have performed parameter inference analyses in order to infer the underlying HOD of galaxies from the projected galaxy two-point function  $w_p(r_p)$  as described in the preceding section. In this section, we describe the primary results of these analyses. Our marginalized one-dimensional parameter constraints are given in Table 3.1.

### 3.1 Standard Analysis

Prior to discussing our results using models that include assembly bias, we present results of standard HOD analyses that include no model for assembly bias. The results of the standard HOD analyses and all other analyses are shown in the form of marginalized constraints on individual parameters in Table 3.1. We compare our parameter constraints to the standard HOD analysis performed by Zehavi et al. (2011) in Table 3.1 as well. An example of the inferred posteriors for the HOD parameters is shown in Figure 1. The left-hand panels of Figure 2, Figure 3, and Figure 4 show the projected correlation function data along with predictions for  $w_p(r_p)$  from 50 randomly-selected models from the MCMC chains. Note that the significant covariance in the data makes it difficult to determine the quality of fit from visual inspection of these figures.



**Figure 1.** Two-dimensional marginalized constraints on HOD parameters inferred from standard HOD fits to  $w_p(r_p)$  data for the  $M_r < -20$  sample. The HOD parameter  $\log(M_0)$  is extremely poorly constrained by the  $w_p(r_p)$  data and has been omitted. The inner contours contain 68% of the posterior probability while the outer contours contain 95% of the probability. The panels along the diagonal show the one-dimensional, marginalized posteriors on each of these parameters. The values above each panel on the diagonal show the median value for the parameter in our chains along with the 16<sup>th</sup> and 84<sup>th</sup> percentiles.

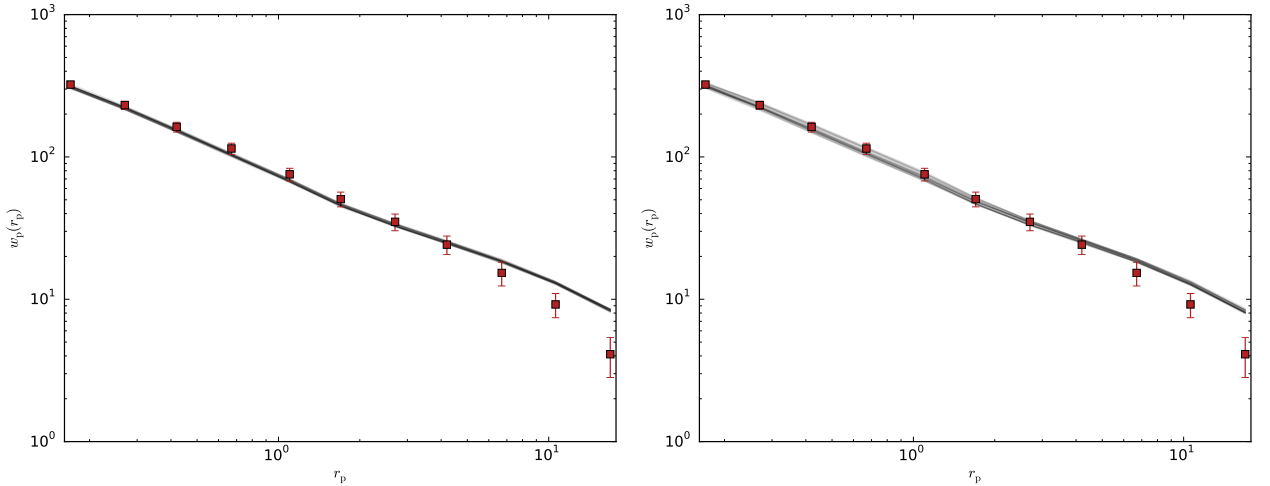
The inferred parameters from our standard analyses differ in several ways from the Zehavi et al. (2011) analysis. Firstly, in our re-analysis of the projected clustering data, we generally find all mass scales to be slightly higher than in the work of Zehavi et al. (2011). This difference is largely due to the slightly different cosmologies adopted in this work. The most important difference are in the values of  $\Omega_M$ , and  $\sigma_8$ . Zehavi et al. (2011) assumed  $\Omega_M = 0.25$  and  $\sigma_8 = 0.8$ , whereas in the present work, we use the BolshoiP simulation in which  $\Omega_M = 0.307$  and  $\sigma_8 = 0.82$ . Slightly larger mass scales are necessary in an analysis with higher  $\Omega_M$  and  $\sigma_8$  in order to maintain galaxy number densities with larger halo number densities.

A second noteworthy difference between the present

work and that of Zehavi et al. (2011) is that we find many parameters to be notably more poorly constrained. At the lower luminosity thresholds, for example, we constrain  $\log(M_{\min})$  and  $\sigma_{\log M}$  with several times lower precision than Zehavi et al. (2011). We do not show our constraints on  $\log(M_0)$  as they are very poor, with 1-sigma constraints  $\gtrsim 1$  dex for all samples. In several cases, the constraint on  $\log(M_0)$  is determined by the prior given in Table 3.2. This is in stark contrast to several of the results of Zehavi et al. (2011). For example, for the threshold sample with  $M_r < -19.5$  ( $M_r < -20.5$ ), Zehavi et al. (2011) quote  $\log(M_0) = 12.23 \pm 0.17$  ( $12.35 \pm 0.24$ ), whereas we infer  $\log(M_0) = 11.38^{+0.95}_{-1.57}$  ( $11.19^{+0.89}_{-1.39}$ ). Examining the form of Eq. (2), it is sensible that the parameter  $\log(M_0)$  should be unconstrained at the lower end, because the

Sample $M_r$	Authors	$\log(M_{\min})$	$\sigma_{\log M}$	$\log(M_1)$	$\alpha$	$A_{\text{cen}}$	$A_{\text{sat}}$	$\chi^2/\text{DoF}$
−21	Zehavi+11	$12.78 \pm 0.10$	$0.68 \pm 0.15$	$13.80 \pm 0.03$	$1.15 \pm 0.06$	—	—	3.1
−21	Zentner+16	$12.92^{+0.07}_{-0.11}$	$0.74^{+0.09}_{-0.16}$	$13.93^{+0.04}_{-0.05}$	$1.23^{+0.10}_{-0.12}$	—	—	1.59
−21	Zentner+16	$12.83^{+0.11}_{-0.09}$	$0.60^{+0.15}_{-0.17}$	$13.93^{+0.05}_{-0.08}$	$1.16^{+0.12}_{-0.14}$	$0.29^{+0.44}_{-0.35}$	$0.08^{+0.49}_{-0.36}$	1.34
−20.5	Zehavi+11	$12.14 \pm 0.03$	$0.17 \pm 0.15$	$13.44 \pm 0.03$	$1.15 \pm 0.03$	—	—	2.7
−20.5	Zentner+16	$12.25^{+0.07}_{-0.03}$	$0.23^{+0.17}_{-0.15}$	$13.59^{+0.02}_{-0.02}$	$1.20^{+0.04}_{-0.04}$	—	—	1.90
−20.5	Zentner+16	$12.32^{+0.13}_{-0.08}$	$0.45^{+0.21}_{-0.25}$	$13.59^{+0.04}_{-0.04}$	$1.14^{+0.05}_{-0.06}$	$> 0.08(90\%)$	$0.22^{+0.40}_{-0.31}$	1.40
−20	Zehavi+11	$11.83 \pm 0.03$	$0.25 \pm 0.11$	$13.08 \pm 0.03$	$1.00 \pm 0.05$	—	—	2.1
−20	Zentner+16	$11.95^{+0.11}_{-0.6}$	$0.37^{+0.23}_{-0.21}$	$13.28^{+0.03}_{-0.04}$	$1.16^{+0.04}_{-0.04}$	—	—	2.19
−20	Zentner+16	$12.23^{+0.33}_{-0.21}$	$0.84^{+0.37}_{-0.31}$	$13.20^{+0.06}_{-0.08}$	$1.05^{+0.06}_{-0.08}$	$> 0.28(99\%)$	$0.01^{+0.32}_{-0.26}$	1.16
−19.5	Zehavi+11	$11.57 \pm 0.04$	$0.17 \pm 0.13$	$12.87 \pm 0.03$	$0.99 \pm 0.04$	—	—	1.00
−19.5	Zentner+16	$11.76^{+0.33}_{-0.11}$	$0.51^{+0.51}_{-0.29}$	$13.05^{+0.04}_{-0.08}$	$1.12^{+0.04}_{-0.07}$	—	—	1.24
−19.5	Zentner+16	$11.80^{+0.36}_{-0.16}$	$0.63^{+0.53}_{-0.37}$	$13.04^{+0.09}_{-0.12}$	$1.06^{+0.07}_{-0.10}$	$> -0.01(84\%)$	$> -0.16(84\%)$	0.69
−19	Zehavi+11	$11.45 \pm 0.04$	$0.19 \pm 0.13$	$12.64 \pm 0.04$	$1.02 \pm 0.02$	—	—	1.8
−19	Zentner+16	$11.72^{+0.33}_{-0.19}$	$0.69^{+0.52}_{-0.46}$	$12.78^{+0.04}_{-0.04}$	$1.03^{+0.04}_{-0.04}$	—	—	2.77
−19	Zentner+16	$11.62^{+0.33}_{-0.13}$	$0.53^{+0.57}_{-0.35}$	$12.83^{+0.06}_{-0.07}$	$1.02^{+0.04}_{-0.04}$	$0.35^{+0.45}_{-0.66}$	$> 0.02(84\%)$	2.01

**Table 2.** Results of standard HOD fits to SDSS DR7  $w_p(r_p)$  as well as fits using a parameterized model of assembly bias. Assembly bias is quantified by the parameters  $A_{\text{cen}}$  ( $A_{\text{sat}}$ ) for central (satellite) galaxies. The secondary property that we assume to determine the galaxy HOD is halo concentration.  $A_{\text{cen,sat}} = 0$  means that there is no assembly bias while  $A_{\text{cen,sat}} = 1$  ( $A_{\text{cen,sat}} = -1$ ) means that galaxy abundance is maximally correlated (anticorrelated) with halo concentration at fixed  $M_{\text{vir}}$ . Thus the  $A_{\text{cen,sat}}$  parameters span the range  $[-1, 1]$ . If the constraints on  $A_{\text{cen}}$  and  $A_{\text{sat}}$  are unspecified in the table, then the model used to interpret the data does not include assembly bias. In our analyses, quoted parameter values with errors correspond to the median value of the parameter and the 16<sup>th</sup> and 84<sup>th</sup> percentiles. In cases for which the posterior is monotonic within the physical parameter space, we show one-sided percentiles.

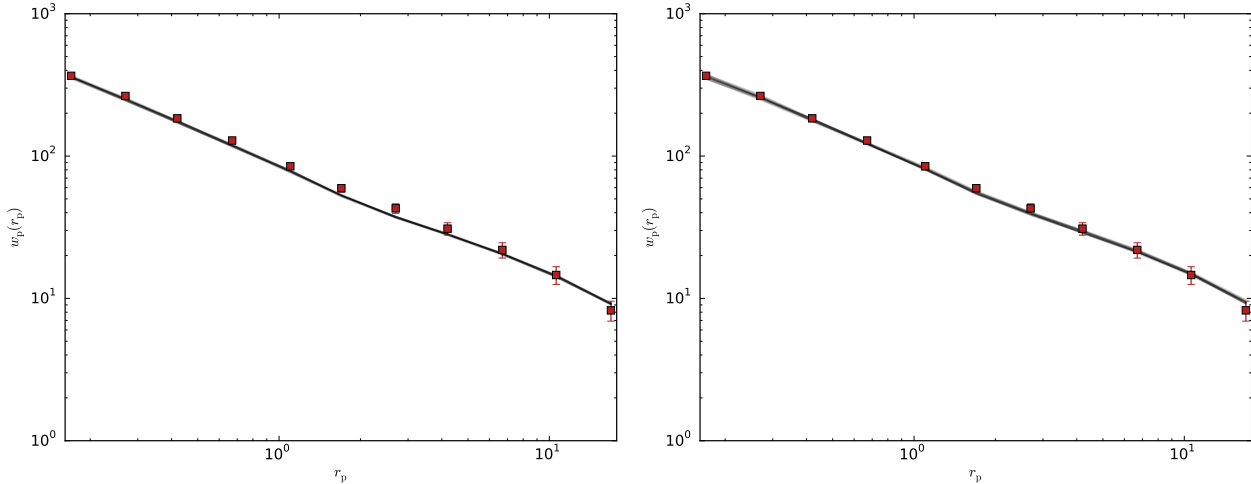


**Figure 2.** **Left:** The  $M_r < -19$  threshold sample projected correlation function with diagonal elements of covariance (points with errorbars). The grey lines are 25 randomly-selected HOD models that yield  $\Delta\chi^2 < 1$  compared to the best-fitting model. **Right:** Same as the left panel but using a fit to a Decorated HOD model that contain parameters to describe the strength of assembly bias.

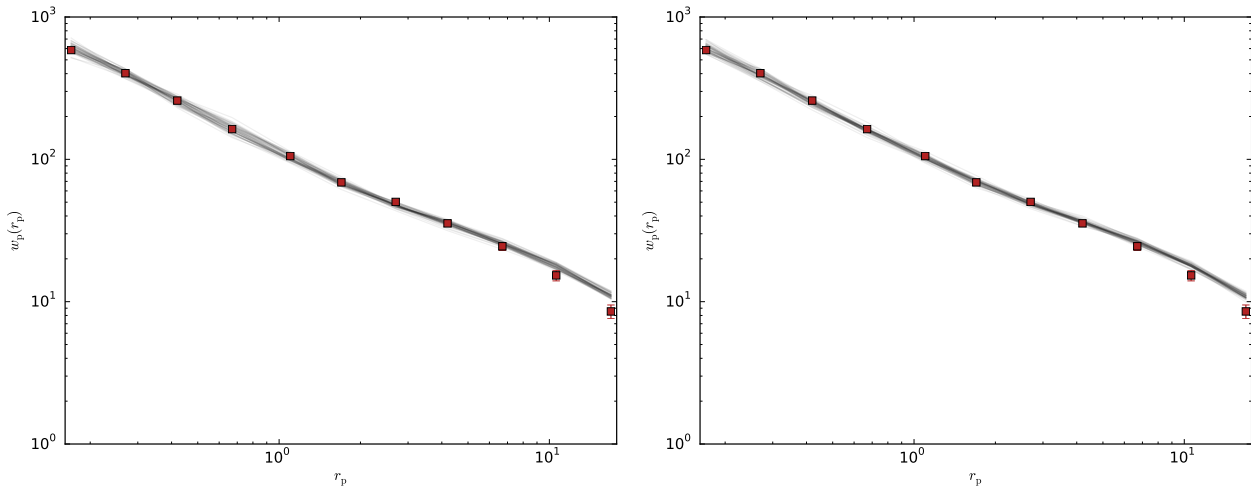
value of  $M_0$  does not alter the predicted satellite number once  $M_0 \ll M_1$ . Therefore, it seems likely that the tighter constraints quoted by Zehavi et al. (2011) must be an error.

Additionally, we have confirmed with a subset of the

authors of Zehavi et al. (2011) that the number of MCMC samples they included in their analysis was insufficient in a number of cases and that this can lead to a significant underestimation of the uncertainties on the inferred parameters, especially  $\log(M_{\min})$  and  $\sigma_{\log M}$  (Z. Zheng & I.



**Figure 3.** The same as Figure 2, but for the  $M_r < -20$  threshold sample.



**Figure 4.** The same as Figure 2, but for the  $M_r < -21$  threshold sample.

Zehavi, private communication). The analysis of Zehavi et al. (2011) used  $10^4$  samples, whereas we find several  $\times 10^6$  samples are often necessary for convergence. Additionally, we have recreated qualitatively similar behavior by considering only small subsets of our full MCMC chains. Consequently, insufficient sampling of the posterior seems to be the likely resolution of the discrepancies between our work and that of Zehavi et al. (2011).

Two degeneracies are manifest in Fig. 1 that are common to all of our analyses. The parameters  $\log(M_1)$  and  $\alpha$  are degenerate with each other and positively correlated. The parameter  $M_1$  is the mass scale at which a halo has one satellite on average, and  $\alpha$  is the power-law index describing the dependence of average satellite number on halo mass. Increasing  $M_1$  *decreases* the number of satellites in massive halos by increasing the mass scale where the power law abundance becomes operative. An increase in  $\alpha$  can partly compensate for an increase in  $M_1$  by increasing the rate at which average satellite number grows with halo mass.

As is evident in Figure 1,  $\log(M_{\min})$  and  $\sigma_{\log M}$  share a relatively narrow degeneracy as well. This degeneracy is largely induced by the measured number density of the sample. Increasing  $\log(M_{\min})$  decreases galaxy number density, but this can be compensated by an increase in  $\sigma_{\log M}$ , which places galaxies in a fraction of the considerably more numerous halos with masses less than  $M_{\min}$ . The consequence is that  $\log(M_{\min})$  and  $\sigma_{\log M}$  are degenerate with each other such that most of the posterior probability lies in a narrow band along which  $\log(M_{\min})$  and  $\sigma_{\log M}$  are positively correlated, as shown in Fig. 1. In the following plots, we suppress the parameter  $\sigma_{\log M}$ , in order to increase the clarity of the plots, because the viable range of  $\sigma_{\log M}$  is determined by this simple degeneracy with  $\log(M_{\min})$ .

The results of this subsection demonstrate that we achieve reasonable fits to projected galaxy clustering data using direct HOD population of a high-resolution numerical simulation of structure formation. These results also update and supersede existing constraints in the litera-

ture in at least three respects. First, we work within the best-fit Planck cosmology. Second, we perform our parameter inference analysis using direct population of halos identified in a numerical simulation of cosmological structure formation (BolshioP). This greatly mitigates modeling uncertainties associated with nonlinear density field evolution, scale-dependent halo bias, halo exclusion, or other effects that have been difficult to incorporate into analytical halo models with high precision. Third, we have explored the posteriors of the parameters with significantly more samples (roughly two orders of magnitude), thereby mitigating errors on inferred parameters and their errors induced by insufficient sampling of the posterior.

### 3.2 Analysis with Decorated HOD

We turn now to a discussion of our parameter inference analysis of projected galaxy clustering in Decorated HOD models that include a treatment of galaxy assembly bias. In this work, we consider only the simplest model of galaxy assembly bias, introducing only two new parameters,  $A_{\text{cen}}$  and  $A_{\text{sat}}$ , that describe the strength of central galaxy and satellite galaxy assembly bias respectively. These parameters are limited to values of  $-1 \leq A_{\text{cen,sat}} \leq 1$ , and  $A_{\text{cen,sat}} = 0$  when there is no galaxy assembly bias. In this work, we use halo concentration as our secondary halo property, so  $A_{\text{cen,sat}} = 1$  ( $A_{\text{cen,sat}} = -1$ ) means that the mean number of galaxies per halo is maximally correlated (anti-correlated) with halo concentration. The model and its implementation in `halotools` is discussed further in Section 2.2 above and in Hearin et al. (2016).

Examples of our fits are given in the right-hand panels of Figure 2, Figure 3, and Figure 4. The general trend that can be gleaned from these figures is that introducing assembly bias improves the ability of the predicted two-point functions to match the measured two-point functions across the transition from the one-halo (highly nonlinear) to two-halo (nearly linear) regimes near  $r_p \sim 2h^{-1}\text{Mpc}$ . This is most apparent for the  $M_r < -20$  threshold sample shown in Fig. 3. Visually, these differences appear to be small; however, Table 3.1 shows that they are statistically important.

The one-dimensional marginalized constraints on all parameters from these analyses are given in the lowest row of each luminosity threshold grouping in Table 3.1. In cases where the posterior on a parameter is monotonic within the physical parameter range, we quote an upper or lower limit on the parameter. One- and two-dimensional visualizations of the posteriors from our analysis are shown in Figure 5, Figure 6, and Figure 7.

Table 3.1 and Figures 5-?? all make several simple, generic points. Introducing additional parameter freedom associated with galaxy assembly bias generally increases the viable parameter space, even for the subset of standard HOD parameters. Constraints on the standard HOD parameters are generally less restrictive. This is exactly what is expected from the introduction of additional parameter freedom.

Focusing attention on the parameters describing

$M_r$ Threshold	$\Delta\text{BIC}$
-21	-0.54
-20.5	1.33
-20	4.56
-19.5	0.26
-19	4.37

**Table 3.** Change to the Bayesian Information Criterion,  $\Delta\text{BIC}$ , after introducing additional parametric freedom to accommodate galaxy assembly bias. Sign convention is such that positive values favor models including assembly bias, negative values favor standard HOD models with no assembly bias parameters. Changes in the Bayesian Information Criterion  $|\Delta\text{BIC}| \geq 5$  strongly favor one model over another.

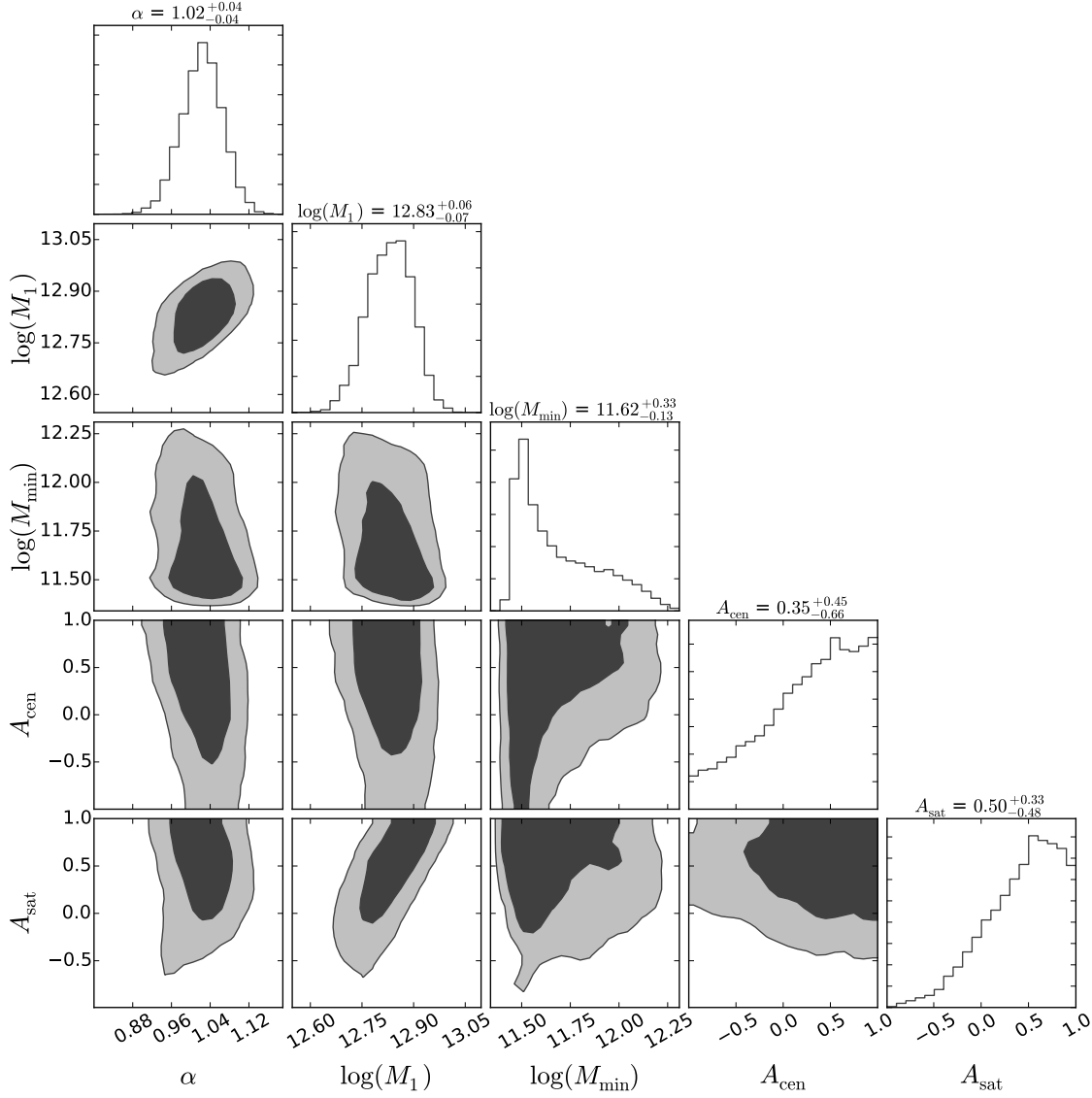
galaxy assembly bias, it is evident that these parameters are often quite poorly constrained by galaxy clustering data. This is important as it implies that galaxy clustering of the precision of SDSS DR7 measurements cannot rule out, or strongly restrict galaxy assembly bias in many cases. Nonetheless, it is apparent that the presence of assembly bias can alter the inferred HOD, or more generally, the inferred relationship between galaxies and halo mass. This is most evident for the  $M_r < -20$  threshold sample, for which there are significant differences in the inferred values of all baseline HOD parameters between models with and without galaxy assembly bias. Other threshold samples exhibit significant changes particularly for  $\alpha$ , and to a lesser degree for  $\log(M_{\text{min}})$  and  $\sigma_{\log M}$ .

Beyond those generic conclusions, a few specific cases are worthy of further examination. Consider the  $M_r < -20$  sample. The inferred value of  $A_{\text{cen}} > 0.28$  at 99% confidence. In this case the data strongly prefer  $A_{\text{cen}} > 0$  and thus strongly prefer galaxies to reside in halos of larger concentration at fixed halo mass. This particular threshold sample is the most significant outlier in this regard. Nevertheless, there are hints of assembly bias in other samples. Satellite galaxies show a marginal preference for occupying halos of higher concentration in the  $M_r < -19$  threshold sample. The  $M_r < -19$  sample exhibits weak preference for a positive correlation of galaxy occupation with halo concentration at fixed mass for both satellite galaxies and central galaxies. Continuing upward with luminosity, the  $M_r < -20.5$  sample exhibits a significant preference for central galaxy assembly bias. Lastly, there is no preference for either central galaxy or satellite galaxy assembly bias for the  $M_r < -21$  threshold sample, for which both  $A_{\text{cen}}$  and  $A_{\text{sat}}$  are consistent with zero within  $1\sigma$ . These data suggest that assembly bias may be present in the real universe.

## 4 CONCLUSIONS

We have re-analyzed the SDSS DR7 measurements of projected galaxy clustering,  $w_p$ , and number density,  $n_g$ , originally published in Zehavi et al. (2011). Our work is





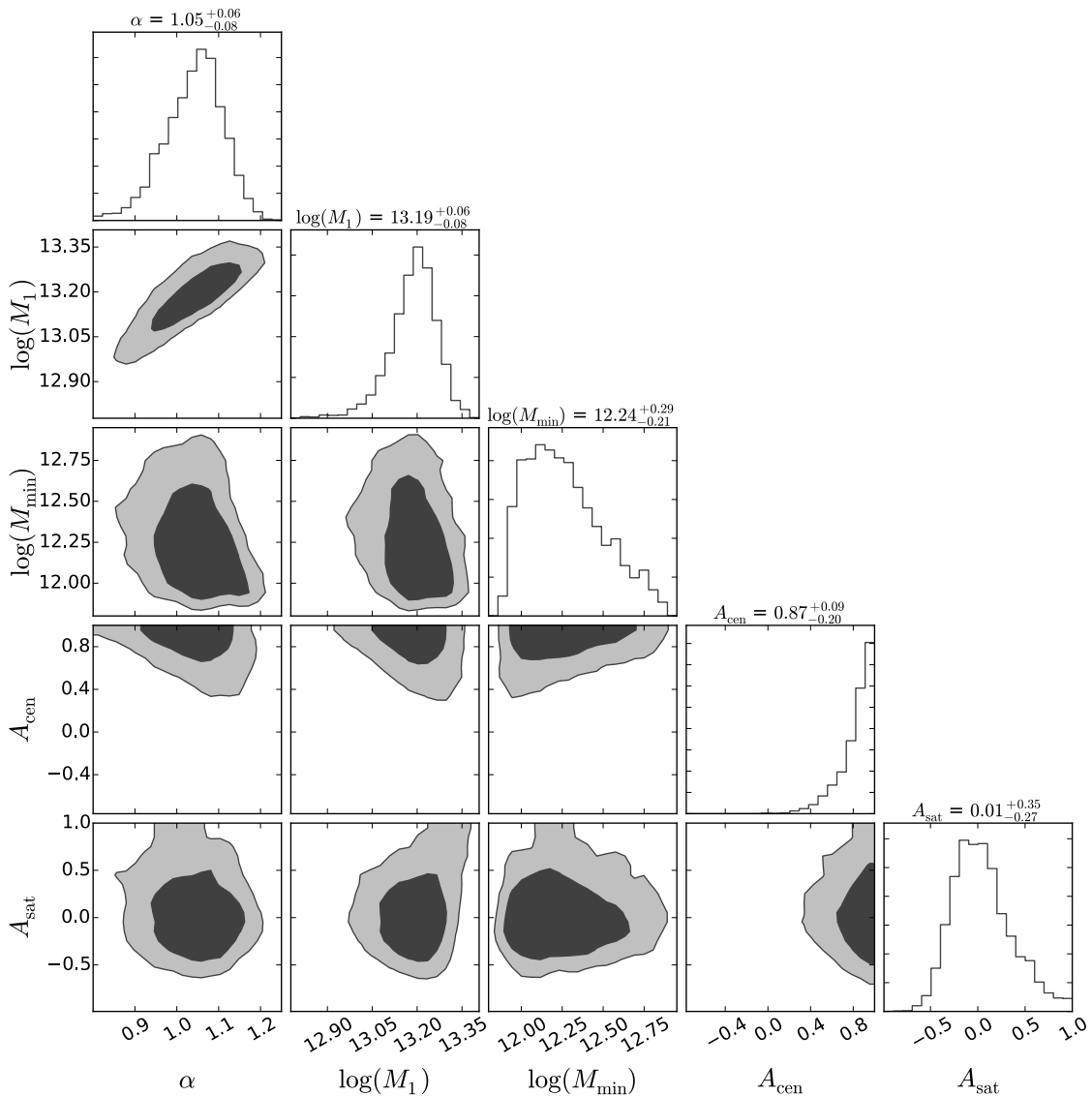
**Figure 5.** Two-dimensional marginalized constraints on decorated HOD parameters inferred from fits to  $w_p(r_p)$  data for the  $M_r < -19$  sample. The contours and histograms along the diagonal panels are as in Fig. 1. The decorated HOD models include a two-parameter model for assembly bias. The HOD parameter  $\log(M_0)$  is extremely poorly constrained by the data and has been suppressed for clarity. Likewise, as in Fig. 1,  $\sigma_{\log M}$  and  $\log(M_{\min})$  share a narrow degeneracy, so we have suppressed  $\sigma_{\log M}$  in order to make constraints on other parameters more easily visible.

especially novel in that we provide the first quantitative constraints on assembly bias derived from the Decorated HOD, an extension to the traditional HOD introduced in Hearin et al. (2016) developed for exactly this purpose. We enumerate our most important conclusions below.

- (i) It is not possible to rule out galaxy assembly bias using SDSS DR7 measurements of  $w_p$  and  $n_g$ .
- (ii) Decorated HOD fits to  $w_p$  and  $n_g$  favor significant levels of assembly bias, particularly in the lower luminosity thresholds we study. Both the  $M_r < -20, -20.5$  samples prefer relatively strong central galaxy assembly bias, while at lower luminosities, the  $M_r < -19$  sample favors satellite assembly bias.
- (iii) Galaxy assembly bias generally weakens for

brighter galaxy samples: Decorated HOD fits to the  $M_r < -21$  sample are consistent with both  $\mathcal{A}_{\text{bias}}^{\text{cens}} = 0$  and  $\mathcal{A}_{\text{bias}}^{\text{sats}} = 0$ . This consistent with the well-established result that *halo assembly bias* weakens with increasing halo mass over the dynamic range relevant to these galaxy samples (see, e.g., Figure 8 of Hearin et al. 2016, and references therein).

- (iv) Our posteriors and best-fit parameters summarized in Table 3.1 supersede the values published in Zehavi et al. (2011), as direct-mock-population together with the Decorated HOD allows us to account for highly significant systematics that have heretofore been neglected from all HOD fits to SDSS data. We note that our findings update the original Zehavi et al. (2011) results *even for our fits in which assembly bias has been*



**Figure 6.** The same as Figure 5, but for the  $M_r < -20$  sample.

fixed to zero, since the original results derive from unconverged MCMC chains that do not sufficiently sample the HOD model posteriors.

We conclude by noting that since  $w_p$  and  $n_g$  are already very well-measured in DR7, it is likely that further improvements on assembly bias constraints at low-redshift will require additional observational measurements, e.g., galaxy–galaxy lensing, group statistics, etc.

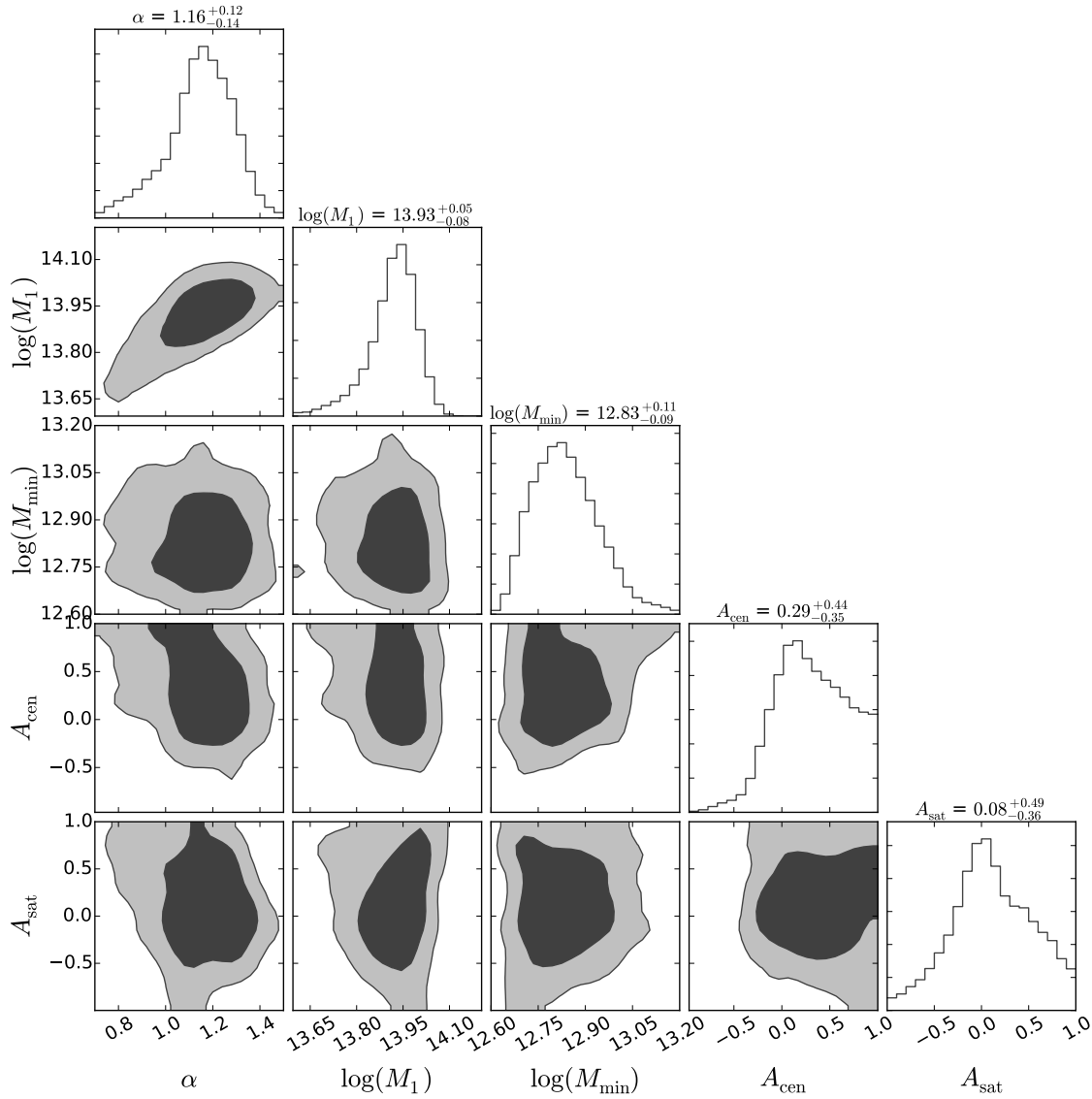
## 5 ACKNOWLEDGEMENTS

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puting Centre (LRZ, [www.lrz.de](http://www.lrz.de)). The Bolshoi simulations have been performed within the Bolshoi project of the University of California High-Performance Astro-Computing Center (UC-HiPACC) and were run at the NASA Ames Research Center. FvdB is supported by the US National Science Foundation through grant AST 1516962. **AZ; add yours as well.**

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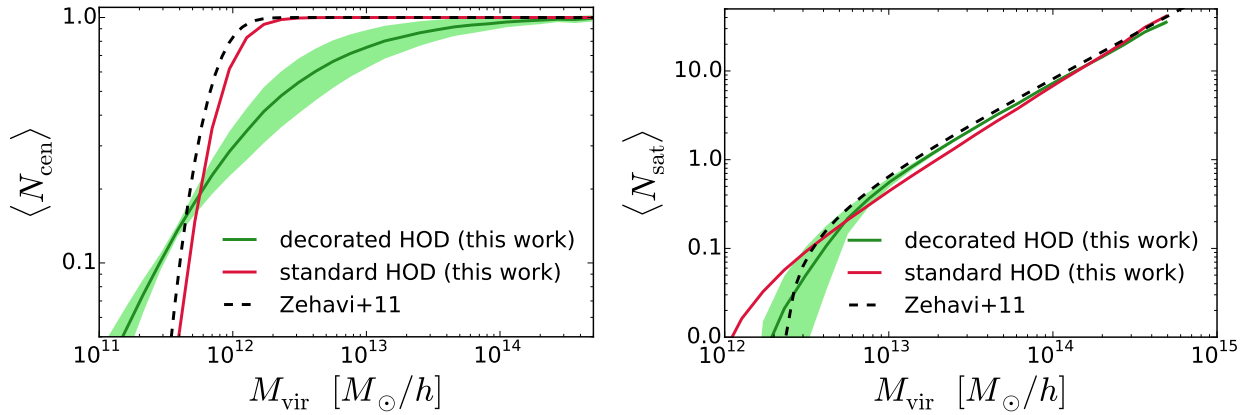
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**Figure 7.** The same as Figure 5, but for the  $M_r < -21$  sample.

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**Figure 8.** We show the first moments of the central and satellite occupation distributions from the best-fit models to the  $M_r < -20$  sample in the left and right panels, respectively. The green curve shows the HOD of our best-fit model that includes the effect of assembly bias; the red curve shows the HOD of the best-fit model in which assembly bias is assumed to be zero; the dashed black curve shows the best-fit HODs from Zehavi et al. (2011). The green error band around the decorated HOD results shows the variance in the HOD from randomly sampling one hundred points in parameter space with  $\chi^2_{\text{model}} - \chi^2_{\text{min}} < 1$ .

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