

# Constraints on Assembly Bias from Galaxy Clustering

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## ABSTRACT

We constrain the newly-introduced Decorated HOD model using SDSS DR7 measurements of projected galaxy clustering,  $w_p$ , and number density,  $n_g$ , made from r-band luminosity-threshold samples. The Decorated HOD is a model for the galaxy–halo connection that augments the traditional Halo Occupation Distribution (HOD) by allowing for the possibility of *assembly bias*: galaxy luminosity may be correlated with dark matter halo properties besides virial mass  $M_{\text{vir}}$  alone. We demonstrate that it is not possible to rule out assembly bias using DR7 measurements of  $w_p$  and  $n_g$ . Moreover, galaxy samples  $M_r < -20, -20.5$  favor strong levels of central galaxy assembly bias: high-concentration halos are more likely to host a central galaxy relative to low-concentration halos of the same  $M_{\text{vir}}$ . We rule out zero assembly bias with high significance for these samples. Satellite assembly bias only becomes significant for the faintest sample we study,  $M_r < -19$ . We find no evidence for assembly bias in the  $M_r < -21$  sample. Because the Decorated HOD subsumes the traditional HOD, and additionally because of errors made in the original analysis of these samples, our results differ from and supersede existing HOD constraints from low-redshift clustering measurements.

## 1 INTRODUCTION

For more than a decade, halo occupation modeling has been used to interpret large-scale structure measurements and exploit these measurements to constrain galaxy formation models and cosmology (e.g., Yang et al. 2003; Tinker et al. 2005; Zehavi et al. 2005; Porciani & Norberg 2006; van den Bosch et al. 2007; Zheng et al. 2007; Conroy & Wechsler 2009; Yang et al. 2009; Zehavi et al. 2011; Guo et al. 2011; Wake et al. 2011; Yang et al. 2011, 2012; Leauthaud et al. 2012; Rodríguez-Puebla et al. 2012; Tinker et al. 2013; Cacciato et al. 2013; More et al. 2013; Guo et al. 2014; Zu & Mandelbaum 2015). The key assumptions underlying halo occupation modeling are: (1) all galaxies reside in dark matter halos that are biased tracers of the density field; and (2) galaxies occupy halos as a function of halo mass  $M_{\text{vir}}$  only.

It is now well known that halo bias depends upon halo properties other than  $M_{\text{vir}}$  (e.g. Gao et al. 2005; Wechsler et al. 2006; Gao & White 2007; Zentner 2007; Dalal et al. 2008; Lacerna & Padilla 2011), an effect called halo assembly bias. If galaxies occupy halos as a function of halo properties other than  $M_{\text{vir}}$ , then standard halo occupation methods will be subject to a systematic error

due to galaxy assembly bias. Several of us have previously shown that this error can be significant in an analysis of galaxy clustering and can bias inferences about many aspects of galaxy evolution (Zentner et al. 2014).

There is increasing observational evidence that galaxy assembly bias is present in the real Universe. Using an approach based on subhalo abundance matching, Lehmann et al. (2015) showed that the clustering of low-redshift galaxies favors models in which stellar mass depends upon a combination of  $M_{\text{vir}}$  and halo concentration. Miyatake et al. (2016) and More et al. (2016) have presented evidence for the presence of assembly bias in massive clusters; in particular, they find that clusters with a more concentrated distribution of satellite galaxies exhibit a weaker large-scale bias relative to clusters of the same mass with satellite galaxies that are less centrally concentrated. Additional support for the presence of assembly bias has come from a variety of studies that have shown, or suggested, that the large-scale environment of dark matter halos of fixed mass is correlated with the star formation rate of their central galaxies (Yang et al. 2006; Blanton & Berlind 2007; Wang et al. 2008, 2013; Hearin et al. 2014).

Despite this growing evidence, the empirical modeling community has not yet reached a consensus on the question of galaxy assembly bias. For example, Lin et al. (2015) claim that some of the above evidence for assembly bias can be explained by differences in halo mass between the galaxy samples under consideration. Paranjape et al. (2015) urges caution that contamination by satellite galaxies can masquerade as a false signal of assembly bias. Tinker et al. (2008) argue that measurements of void statistics place strong bounds on the possible strength of the signal.

In much of the literature on this topic, the character of the supporting evidence suffers from a severe shortcoming. Until very recently, the only models that have been fit to observational clustering measurements have been traditional models in which assembly bias is assumed from the outset to be zero. Models that include assembly bias such as semi-analytic models are typically too computationally expensive for anything besides a “one-off” comparison (e.g., Croton et al. 2007), prohibiting any conclusive statement to be made about the strength of the signal.

Motivated by this shortcoming, we have developed a new class of empirical models that enable galaxies to occupy halos in a manner that has *simultaneous* dependence upon multiple halo properties (Hearin et al. 2016), *including continuously variable levels of assembly bias*. Crucially, our implementation is of sufficient computational efficiency to permit a proper likelihood analysis of the model parameter space. Armed with this new methodology, in this paper we revisit the interpretation of luminosity-dependent galaxy clustering in the Sloan Digital Sky Survey (SDSS) Data Release 7 (DR7) data, analyzed previously by Zehavi et al. (2011), in the context both standard halo occupation models and these new models.

This work presents a re-analysis of the SDSS DR7 data that overcomes numerous shortcomings of previous analyses. Our analysis improvements stem primarily from our directly populating mock galaxy catalogs into a cosmological simulation, whereas previous analyses relied upon analytical fitting functions with parameters calibrated against a suite of simulations. In direct mock population, delicate issues present in analytic modeling, such as scale-dependent halo bias and halo exclusion, are treated exactly. Our approach also provides a natural framework for studying models of assembly bias, since we can use the exact clustering properties of simulated halos as a function of any arbitrary halo properties in the catalog. With direct mock population, systematic uncertainty in the model is limited to the sample variance of the simulated box, errors related to insufficient numerical resolution, and numerical inaccuracies of halo-finding (e.g., over-merging).

In addition to improvements from direct mock population, the simulation we use is based on the latest Planck cosmological parameters (Planck Collaboration et al. 2014), updating previous work. Furthermore, many important differences between this work and the previous analysis by Zehavi et al. (2011) result from the fact that the Monte Carlo Markov Chains used in Zehavi et al. (2011) did not sufficiently sample the posterior distribution

(Z. Zheng & I. Zehavi, private communication). Therefore, *our analysis supersedes previous work even in the case of standard halo occupation models*.

Most importantly, our work demonstrates explicitly that significant assembly bias in  $M_r$ -selected samples from SDSS DR7 cannot be ruled out based on a standard analysis of galaxy clustering only. In fact, in agreement with Lehmann et al. (2015), we find that several samples *favor galaxy assembly bias to a degree that is statistically significant*. As demonstrated by Zentner et al. (2014), this conclusion has important consequences for the interpretation of both extant and forthcoming data.

Our paper is organized as follows. In Section 2, we discuss our implementation of halo occupation models and the parameter inference methodology. We present results from both traditional and assembly-biased halo occupation analyses in Section 3. We summarize our results and draw conclusions in Section 4.

## 2 METHODS

### 2.1 Halotools Implementation of HOD Models

To generate predictions for galaxy clustering, we directly populate dark matter halos with mock galaxies using **Halotools**. We explore halo occupation distribution models (HOD) models in this work (e.g. Seljak 2000; Ma & Fry 2000; Scoccimarro et al. 2001; Berlind & Weinberg 2002), though other techniques that can be used to interpret such data, such as the conditional luminosity function (CLF, e.g., Yang et al. 2003; van den Bosch et al. 2013), exist. In this subsection, we review the “standard” HOD model used in the present work, which assumes that there is no galaxy assembly bias. We refer to such a model as “standard” because all HOD analyses of galaxy clustering to date have assumed no galaxy assembly bias. In the following subsection, we describe the Decorated HOD model described in Hearin et al. (2016). In both cases, we will only review the salient features of our methodology briefly; interested readers can always refer to **halotools.readthedocs.io** and Hearin et al. (2016) for further details.

#### 2.1.1 Simulation

All of our analyses are based on the Bolshoi-Planck (BolshoiP) simulation (Riebe et al. 2011). BolshoiP was run with cosmological parameters based on Planck Collaboration et al. (2014),  $\Omega_\Lambda = 0.693$ ,  $\Omega_m = 1 - \Omega_\Lambda = 0.307$ ,  $\Omega_b = 0.048$ ,  $h = 0.7$ ,  $n_s = 0.96$  and  $\sigma_8 = 0.82$ , within a cubic box 250 Mpc/h on a side, requiring a particle mass of  $m_p = 1.35 \times 10^8 M_\odot/h$ . Further information about the BolshoiP simulation is available at <https://www.cosmosim.org>.

We use publicly available<sup>1</sup> dark matter halo catalogs based on the **ROCKSTAR** halo-finder (Behroozi et al. 2011) and **CONSISTENT TREES** algorithm (Behroozi et al. 2013).

<sup>1</sup> [http://www.slac.stanford.edu/~behroozi/BPlanck\\_HLists](http://www.slac.stanford.edu/~behroozi/BPlanck_HLists)

In particular, we use the `halotools_alpha_version2` version of the  $z = 0$  snapshot of the ‘`bolplanck`’ catalog included with `Halotools`. Halos in these catalogs are based on the virial radius density contrast given in Bryan & Norman (1998). When populating this catalog with mock galaxies, we only use present-day host halos with a value of  $M_{\text{peak}}$  that exceeds 300 particles.

### 2.1.2 Occupation statistics

In standard HOD models, central galaxies and satellite galaxies are treated separately, so the model is specified by two probability distributions, one for each type of galaxy. The galaxy-halo connection is specified in terms of  $P(N_{\text{cen}}|M_{\text{vir}})$  and  $P(N_{\text{sat}}|M_{\text{vir}})$ , the probability that a halo of mass  $M_{\text{vir}}$  hosts  $N_{\text{cen}}$  central and  $N_{\text{sat}}$  satellite galaxies, respectively.  $P(N_{\text{cen}}|M_{\text{vir}})$  is typically a nearest-integer distribution, as a host halo has only either zero or one central galaxy. Consequently, the occupation statistics of central galaxies are specified by the first moment of  $P(N_{\text{cen}}|M_{\text{vir}})$ , which we model as

$$\langle N_{\text{cen}} \rangle(M_{\text{vir}}) = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{\log(M_{\text{vir}}) - \log(M_{\text{min}})}{\sigma_{\log M}} \right) \right) \quad (1)$$

For every host halo in the catalog we draw a random number from a uniform distribution  $\mathcal{U}(0,1)$ ; for a host halo of present-day virial mass  $M_{\text{vir}}$ , a central galaxy is assigned to the halo if the associated random number is less than  $\langle N_{\text{cen}} \rangle(M_{\text{vir}})$ ; halos with random values exceeding  $\langle N_{\text{cen}} \rangle(M_{\text{vir}})$  are left devoid of centrals. The parameter  $\log(M_{\text{min}})$  specifies the halo mass at which the halo has a 50% probability of hosting a central galaxy, while the parameter  $\sigma_{\log M}$  specifies the rate at which  $\langle N_{\text{cen}} \rangle(M_{\text{vir}})$  transitions from zero to unity, with smaller values of  $\sigma_{\log M}$  corresponding to a more rapid transition.

We model the distribution  $P(N_{\text{sat}}|M_{\text{vir}})$  as a Poisson distribution with first moment given by a power law,

$$\langle N_{\text{sat}} \rangle(M_{\text{vir}}) = \left( \frac{M_{\text{vir}} - M_0}{M_1} \right)^\alpha. \quad (2)$$

The parameter  $M_0$  allows the power-law to be truncated more rapidly at low masses and we set  $\langle N_{\text{sat}} \rangle(M_{\text{vir}}) = 0$  for  $M_{\text{vir}} < M_0$ .

The five parameters of HOD models that are varied in standard analyses are  $\log(M_{\text{min}})$ ,  $\sigma_{\log M}$ ,  $\alpha$ ,  $\log(M_1)$ , and  $\log(M_0)$ , though, as we show below, central galaxies usually outnumber satellite galaxies by a factor of several, so  $\log(M_{\text{min}})$  and  $\sigma_{\log M}$  usually vary along a narrow degeneracy that fixes the total galaxy number density to the observed value. There are many particular choices that can be made for the functional forms of  $\langle N_{\text{cen}} \rangle(M_{\text{vir}})$  and  $\langle N_{\text{sat}} \rangle(M_{\text{vir}})$ . We have made choices that mimic the standard SDSS DR7 analysis of Zehavi et al. (2011), to expedite comparisons with their results.

### 2.1.3 Galaxy profiles

Central galaxies in the standard HOD models reside at the halo center, moving with the same velocity as the host halo peculiar velocity. We model the intra-halo spatial distribution of satellite galaxies to be located within  $R_{\text{vir}}$

of the halo center, with a spherically symmetric NFW profile (Navarro et al. 1997). The concentration  $c$  of each halo’s satellite galaxy profile is taken to be the same as the concentration of the dark matter particles in the halo.<sup>2</sup>

We model the radial velocity distribution of satellite galaxies as a Gaussian with first moment equal to the host halo velocity and second moment equal to the solution to the isotropic Jeans equation for an NFW profile (More et al. 2009),

$$\sigma_r^2(\tilde{r}|c) = V_{\text{vir}}^2 \frac{c^2 \tilde{r}(1 + c\tilde{r})^2}{g(c)} \int_{c\tilde{r}}^\infty dy \frac{g(y)}{y^3(1+y)^2}, \quad (3)$$

where  $\tilde{r} = r/R_{\text{vir}}$ ,  $g(x) = \ln(1+x) - x/(1+x)$ , and  $V_{\text{vir}}^2 = GM_{\text{vir}}/R_{\text{vir}}$ . We assume that velocities are isotropic, setting the peculiar velocities in each Cartesian direction according to random draws from the above radial velocity distribution.

### 2.1.4 Predictions for observables

After populating a halo catalog with mock galaxies, we calculate the comoving number density of our mock galaxy sample as  $n_g = N_{\text{gal}}/L_{\text{box}}^3$ , where  $N_{\text{gal}}$  is the total number of galaxies in the mock sample. We apply the distant-observer approximation and use the simulation  $z$ -axis as the line-of-sight direction, and the distance between points in the  $xy$ -plane to define the projected distance  $r_p$ . We place mock galaxies into redshift-space by replacing each galaxy’s  $z$ -coordinate with  $z_{\text{RS}} = z + V_z/H_0$ . We count pairs of points in each of our  $r_p$  bins, rejecting pairs with  $z_{\text{RS}}$ –distance exceeding  $\pi_{\text{max}} = 60$  Mpc/h, which is the same projection depth chosen by Zehavi et al. (2011). We employ the Landy & Szalay (1993) estimator to turn pair counts into a  $w_p$  prediction, as described in detail in `halotools.readthedocs.io`.

## 2.2 HOD with Assembly Bias: The Decorated HOD

In addition to the standard occupation statistics described in the previous section, in this paper we also use the decorated HOD formalism to connect galaxies to dark matter halos in a manner that has simultaneous dependence on both  $M_{\text{vir}}$  and *halo concentration*. Briefly, we use Equations 1 and 2 as our “baseline” first occupation moments. At fixed  $M_{\text{vir}}$ , halos are divided into one of two categories, those of high- and low-concentration, depending on whether the concentration of the halo places it above or below the rank-order percentile  $f_{\text{split}}$ , which we keep fixed to  $f_{\text{split}} = 0.5$  throughout the paper for simplicity. High-concentration halos have a different first occupation moment relative to low-concentration halos of the same mass,  $\langle N_{\text{gal}}|M_{\text{vir}}, c_{\text{high}} \rangle \neq \langle N_{\text{gal}}|M_{\text{vir}}, c_{\text{low}} \rangle$ . The difference between the first moment of high- and low-concentration halos is modulated by  $\mathcal{A}_{\text{bias}}$ , the novel

<sup>2</sup> We set a maximum value of  $c = 25$  to the NFW concentration, because halos with very large values for the concentration tend to be poorly described by an NFW profile, for example due to a recent merger.

parameter of the decorated HOD governing assembly bias. Values of  $\mathcal{A}_{\text{bias}} = \pm 1$  correspond to the maximum strength of assembly bias allowable by the constraint that the model preserves the marginalized first moment,  $\langle N_{\text{gal}} | M_{\text{vir}} \rangle$ ; thus regardless of the value of  $\mathcal{A}_{\text{bias}}$ , in the decorated HOD the marginalized first moment of centrals and satellites are *unchanged from the baseline value defined by Equations 1 and 2*. In other words, *decorated HOD models all have the same HODs, when averaged over all halos at fixed mass, as standard HOD models*. The only change in Decorated HOD models is whether or not an additional property also modulates halo occupation at fixed halo mass. A value of  $\mathcal{A}_{\text{bias}} = 0$  indicates no galaxy assembly bias whatsoever. We refer the reader to Hearin et al. (2016) for further details about the decorated HOD.

In the present work, we fix our model to the simplest class of galaxy assembly bias models, though a recipe for generalizing to more complicated models can be found in Hearin et al. (2016). In particular, we split halos into two populations as specified in the previous paragraph. We then populate halos with satellite galaxies specified by an assembly bias parameter  $-1 \leq A_{\text{sat}} \leq 1$  and central galaxies with a distinct assembly bias parameter  $-1 \leq A_{\text{cen}} \leq 1$ . These two additional parameters are varied in our analyses that include assembly bias, making the number of parameters that vary in these analyses seven. As we show below, these additional parameters are in many instances poorly constrained by clustering data of the quality of SDSS DR7 alone, so exploring more complex models of assembly bias does not yet seem justified in such analyses.

### 2.3 Parameter Inference

We constrain HOD parameters based on SDSS DR7 measurements of the projected galaxy two-point functions,  $w_p(r_p)$ , and galaxy number densities,  $n_g$ , for luminosity threshold samples published in Zehavi et al. (2011). We use the full covariance matrix of the projected correlation function available at [astroweb.cwru.edu/izehavi/dr7\\_covar/table8](http://astroweb.cwru.edu/izehavi/dr7_covar/table8). We assume a likelihood of the form  $\mathcal{L} \propto e^{-\chi^2/2}$ , where

$$\chi^2 = \Delta w_{p,i} [C^{-1}]_{ij} \Delta w_{p,j} + \frac{(n_g^{\text{mock}} - n_g^{\text{meas}})^2}{\sigma_n^2}, \quad (4)$$

$\Delta w_{p,i} = w_p^{\text{mock}}(r_{p,i}) - w_p^{\text{meas}}(r_{p,i})$  is the difference between the projected two-point function predicted by the mock catalog and the measured value in the  $i^{\text{th}}$  separation bin (of 12 bins total),  $C^{-1}$  is the inverse of the covariance matrix of the measurements, and the last term in Eq. (4) is the contribution from the difference between the predicted and measured galaxy number density. The error on the galaxy number density assumes Poisson statistics for both the measured and predicted galaxy number densities.

To infer parameters for the HOD and Decorated HOD models described in the previous subsections, we performed a Markov Chain Monte Carlo (MCMC) sampling of the posterior distribution using the affine-invariant ensemble sampler of Goodman & Weare (2010)

Parameter	Prior Interval
$\log(M_{\text{min}})$	[9.0,14.0]
$\sigma_{\log M}$	[0.01,1.5]
$\log(M_0)$	[9.0,14.0]
$\log(M_1)$	[10.7,15.0]
$\alpha$	[0.0,2.0]
$A_{\text{cen}}$	[-1.0,1.0]
$A_{\text{sat}}$	[-1.0,1.0]

**Table 1.** Ranges for the priors used in the parameter inference. All prior distributions are uniform over the specified ranges.

as implemented in the `emcee` software package (Foreman-Mackey et al. 2013). For most cases, we find that  $\sim 3 - 10 \times 10^6$  samples are necessary in order for our chains to converge.

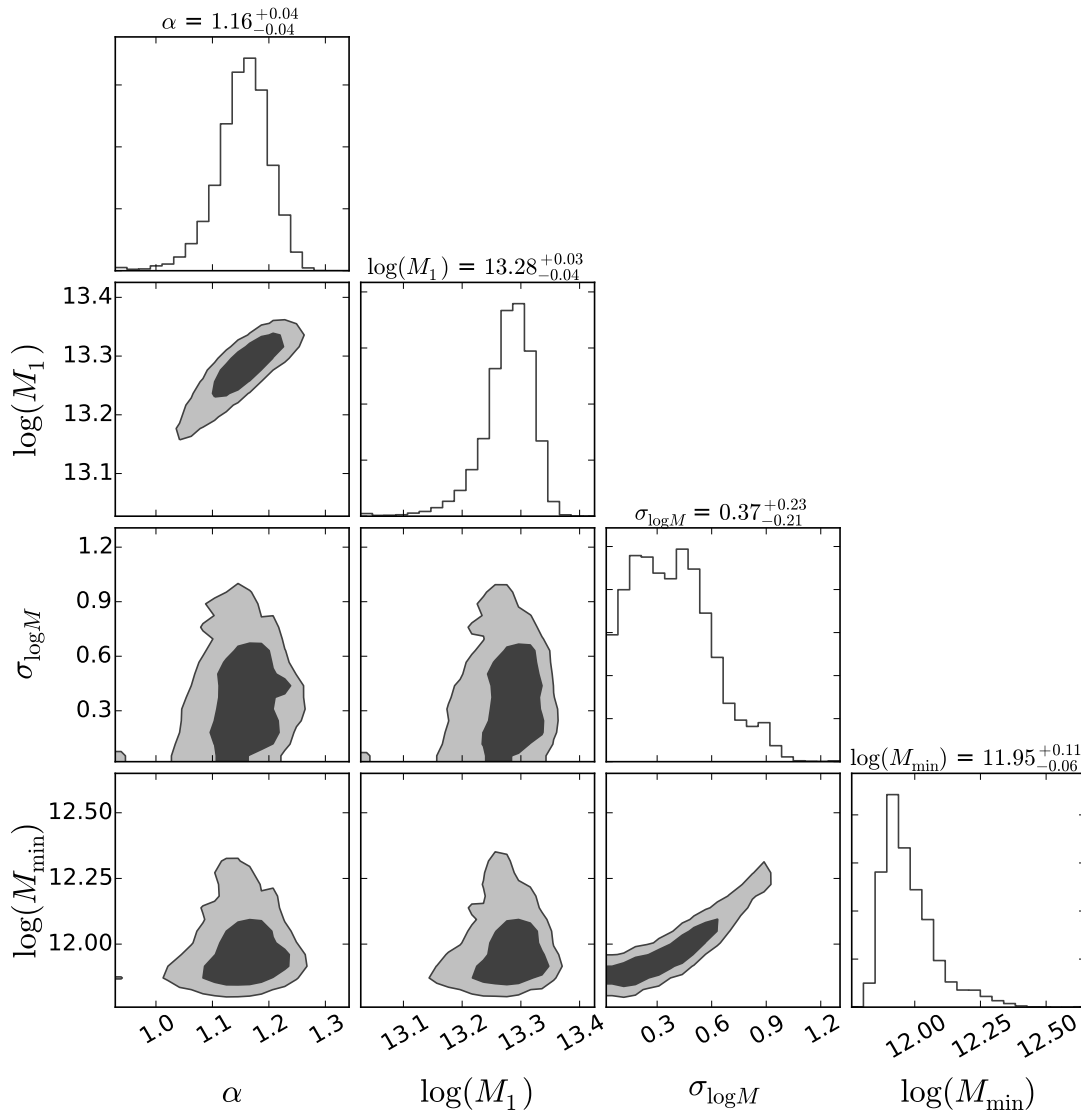
The most important detail of this analysis is the priors on the parameters. In all analyses discussed in this paper, we adopt priors that are uniform distributions over the intervals specified in Table 3.2. In the case of the assembly bias parameters  $A_{\text{cen}}$  and  $A_{\text{sat}}$ , the priors represent physical boundaries. These parameters must satisfy  $-1 \leq A_{\text{cen,sat}} \leq 1$ . Physical considerations require parameter  $\sigma_{\log M} > 0$ . All other priors have a negligible influence on the posterior aside from  $\log M_0$ . We find that  $\log M_0$  is often very poorly constrained by clustering data and priors on  $\log M_0$  can have a non-negligible influence on inferred parameters.

## 3 RESULTS

We have performed parameter inference analyses in order to infer the underlying HOD of galaxies from the projected galaxy two-point function  $w_p(r_p)$  as described in the preceding section. In this section, we describe the primary results of these analyses. Our marginalized one-dimensional parameter constraints are given in Table 3.1.

### 3.1 Standard Analysis

Prior to discussing our results using models that include assembly bias, we present results of standard HOD analyses that include no model for assembly bias. The results of the standard HOD analyses and all other analyses are shown in the form of marginalized constraints on individual parameters in Table 3.1. We compare our parameter constraints to the standard HOD analysis performed by Zehavi et al. (2011) in Table 3.1 as well. An example of the inferred posteriors for the HOD parameters is shown in Figure 1. The left-hand panels of Figure 2, Figure 3, and Figure 4 show the projected correlation function data along with predictions for  $w_p(r_p)$  from 50 randomly-selected models from the MCMC chains. Note that the significant covariance in the data makes it difficult to determine the quality of fit from visual inspection of these figures.



**Figure 1.** Two-dimensional marginalized constraints on HOD parameters inferred from standard HOD fits to  $w_p(r_p)$  data for the  $M_r < -20$  sample. The HOD parameter  $\log(M_0)$  is extremely poorly constrained by the  $w_p(r_p)$  data and has been omitted. The inner contours contain 68% of the posterior probability while the outer contours contain 95% of the probability. The panels along the diagonal show the one-dimensional, marginalized posteriors on each of these parameters. The values above each panel on the diagonal show the median value for the parameter in our chains along with the 16<sup>th</sup> and 84<sup>th</sup> percentiles.

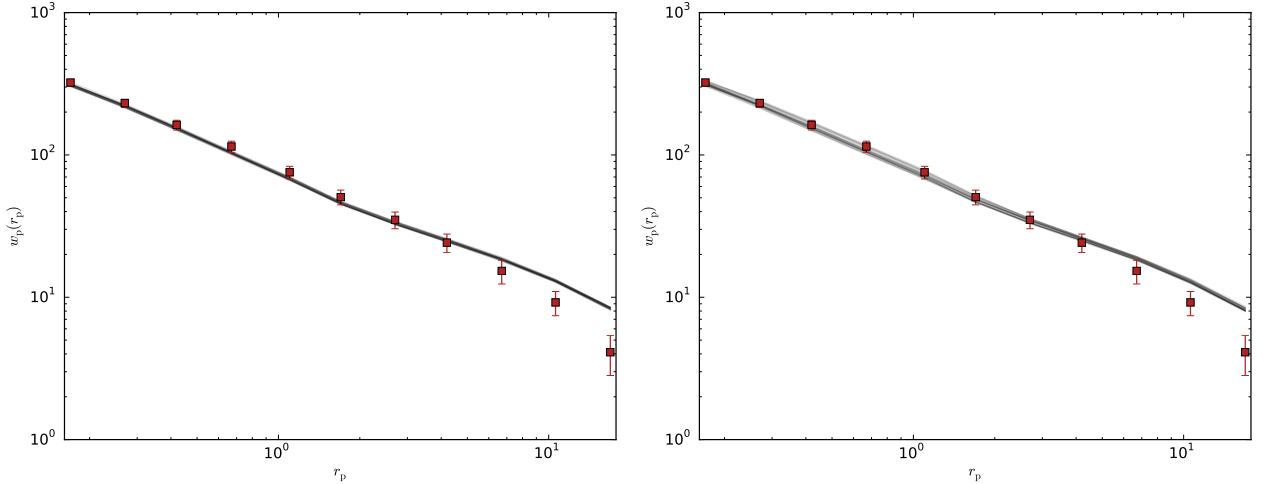
The inferred parameters from our standard analyses differ in several ways from the Zehavi et al. (2011) analysis. Firstly, in our re-analysis of the projected clustering data, we generally find all mass scales to be slightly higher than in the work of Zehavi et al. (2011). This difference is largely due to the slightly different cosmologies adopted in this work. The most important difference are in the values of  $\Omega_M$ , and  $\sigma_8$ . Zehavi et al. (2011) assumed  $\Omega_M = 0.25$  and  $\sigma_8 = 0.8$ , whereas in the present work, we use the BolshoiP simulation in which  $\Omega_M = 0.307$  and  $\sigma_8 = 0.82$ . Slightly larger mass scales are necessary in an analysis with higher  $\Omega_M$  and  $\sigma_8$  in order to maintain galaxy number densities with larger halo number densities.

A second noteworthy difference between the present

work and that of Zehavi et al. (2011) is that we find many parameters to be notably more poorly constrained. At the lower luminosity thresholds, for example, we constrain  $\log(M_{\min})$  and  $\sigma_{\log M}$  with several times lower precision than Zehavi et al. (2011). We do not show our constraints on  $\log(M_0)$  as they are very poor, with 1-sigma constraints  $\gtrsim 1$  dex for all samples. In several cases, the constraint on  $\log(M_0)$  is determined by the prior given in Table 3.2. This is in stark contrast to several of the results of Zehavi et al. (2011). For example, for the threshold sample with  $M_r < -19.5$  ( $M_r < -20.5$ ), Zehavi et al. (2011) quote  $\log(M_0) = 12.23 \pm 0.17$  ( $12.35 \pm 0.24$ ), whereas we infer  $\log(M_0) = 11.38^{+0.95}_{-1.57}$  ( $11.19^{+0.89}_{-1.39}$ ). Examining the form of Eq. (2), it is sensible that the parameter  $\log(M_0)$  should be unconstrained at the lower end, because the

Sample $M_r$	Authors	$\log(M_{\min})$	$\sigma_{\log M}$	$\log(M_1)$	$\alpha$	$A_{\text{cen}}$	$A_{\text{sat}}$	$\chi^2/\text{DoF}$
−21	Zehavi+11	$12.78 \pm 0.10$	$0.68 \pm 0.15$	$13.80 \pm 0.03$	$1.15 \pm 0.06$	—	—	3.1
−21	Zentner+16	$12.92^{+0.07}_{-0.11}$	$0.74^{+0.09}_{-0.16}$	$13.93^{+0.04}_{-0.05}$	$1.23^{+0.10}_{-0.12}$	—	—	1.59
−21	Zentner+16	$12.83^{+0.11}_{-0.09}$	$0.60^{+0.15}_{-0.17}$	$13.93^{+0.05}_{-0.08}$	$1.16^{+0.12}_{-0.14}$	$0.29^{+0.44}_{-0.35}$	$0.08^{+0.49}_{-0.36}$	1.34
−20.5	Zehavi+11	$12.14 \pm 0.03$	$0.17 \pm 0.15$	$13.44 \pm 0.03$	$1.15 \pm 0.03$	—	—	2.7
−20.5	Zentner+16	$12.25^{+0.07}_{-0.03}$	$0.23^{+0.17}_{-0.15}$	$13.59^{+0.02}_{-0.02}$	$1.20^{+0.04}_{-0.04}$	—	—	1.90
−20.5	Zentner+16	$12.32^{+0.13}_{-0.08}$	$0.45^{+0.21}_{-0.25}$	$13.59^{+0.04}_{-0.04}$	$1.14^{+0.05}_{-0.06}$	$> 0.08(90\%)$	$0.22^{+0.40}_{-0.31}$	1.40
−20	Zehavi+11	$11.83 \pm 0.03$	$0.25 \pm 0.11$	$13.08 \pm 0.03$	$1.00 \pm 0.05$	—	—	2.1
−20	Zentner+16	$11.95^{+0.11}_{-0.6}$	$0.37^{+0.23}_{-0.21}$	$13.28^{+0.03}_{-0.04}$	$1.16^{+0.04}_{-0.04}$	—	—	2.19
−20	Zentner+16	$12.23^{+0.33}_{-0.21}$	$0.84^{+0.37}_{-0.31}$	$13.20^{+0.06}_{-0.08}$	$1.05^{+0.06}_{-0.08}$	$> 0.28(99\%)$	$0.01^{+0.32}_{-0.26}$	1.16
−19.5	Zehavi+11	$11.57 \pm 0.04$	$0.17 \pm 0.13$	$12.87 \pm 0.03$	$0.99 \pm 0.04$	—	—	1.00
−19.5	Zentner+16	$11.76^{+0.33}_{-0.11}$	$0.51^{+0.51}_{-0.29}$	$13.05^{+0.04}_{-0.08}$	$1.12^{+0.04}_{-0.07}$	—	—	1.24
−19.5	Zentner+16	$11.80^{+0.36}_{-0.16}$	$0.63^{+0.53}_{-0.37}$	$13.04^{+0.09}_{-0.12}$	$1.06^{+0.07}_{-0.10}$	$> -0.01(84\%)$	$> -0.16(84\%)$	0.69
−19	Zehavi+11	$11.45 \pm 0.04$	$0.19 \pm 0.13$	$12.64 \pm 0.04$	$1.02 \pm 0.02$	—	—	1.8
−19	Zentner+16	$11.72^{+0.33}_{-0.19}$	$0.69^{+0.52}_{-0.46}$	$12.78^{+0.04}_{-0.04}$	$1.03^{+0.04}_{-0.04}$	—	—	2.77
−19	Zentner+16	$11.62^{+0.33}_{-0.13}$	$0.53^{+0.57}_{-0.35}$	$12.83^{+0.06}_{-0.07}$	$1.02^{+0.04}_{-0.04}$	$0.35^{+0.45}_{-0.66}$	$> 0.02(84\%)$	2.01

**Table 2.** Results of standard HOD fits to SDSS DR7  $w_p(r_p)$  as well as fits using a parameterized model of assembly bias. Assembly bias is quantified by the parameters  $A_{\text{cen}}$  ( $A_{\text{sat}}$ ) for central (satellite) galaxies. The secondary property that we assume to determine the galaxy HOD is halo concentration.  $A_{\text{cen,sat}} = 0$  means that there is no assembly bias while  $A_{\text{cen,sat}} = 1$  ( $A_{\text{cen,sat}} = -1$ ) means that galaxy abundance is maximally correlated (anticorrelated) with halo concentration at fixed  $M_{\text{vir}}$ . Thus the  $A_{\text{cen,sat}}$  parameters span the range  $[-1, 1]$ . If the constraints on  $A_{\text{cen}}$  and  $A_{\text{sat}}$  are unspecified in the table, then the model used to interpret the data does not include assembly bias. In our analyses, quoted parameter values with errors correspond to the median value of the parameter and the 16<sup>th</sup> and 84<sup>th</sup> percentiles. In cases for which the posterior is monotonic within the physical parameter space, we show one-sided percentiles.

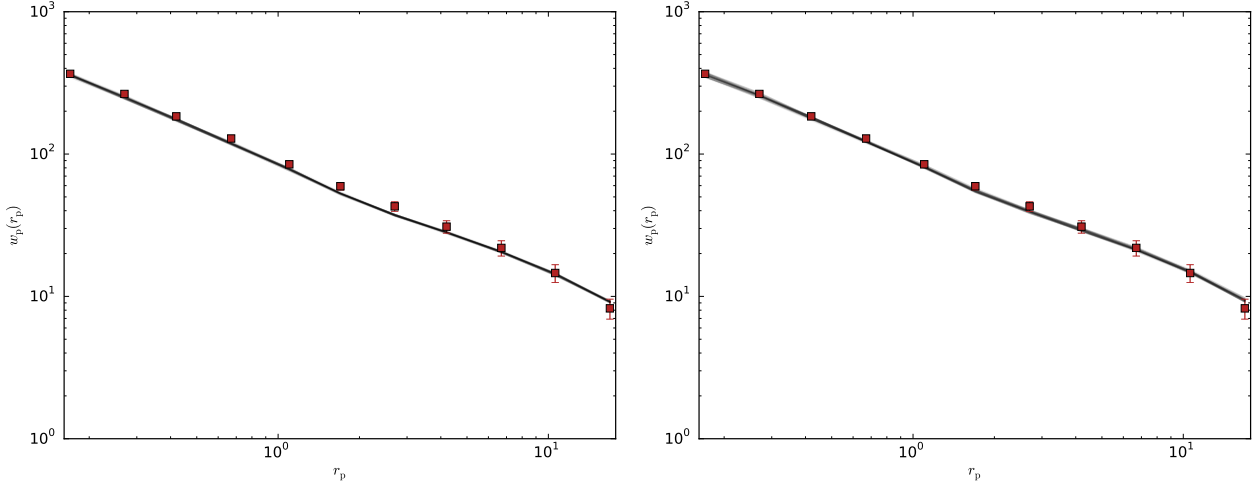


**Figure 2.** **Left:** The  $M_r < -19$  threshold sample projected correlation function with diagonal elements of covariance (points with errorbars). The grey lines are 50 randomly-selected HOD models that yield  $\Delta\chi^2 < 1$  compared to the best-fitting model. **Right:** Same as the left panel but using a fit to a Decorated HOD model that contain parameters to describe the strength of assembly bias.

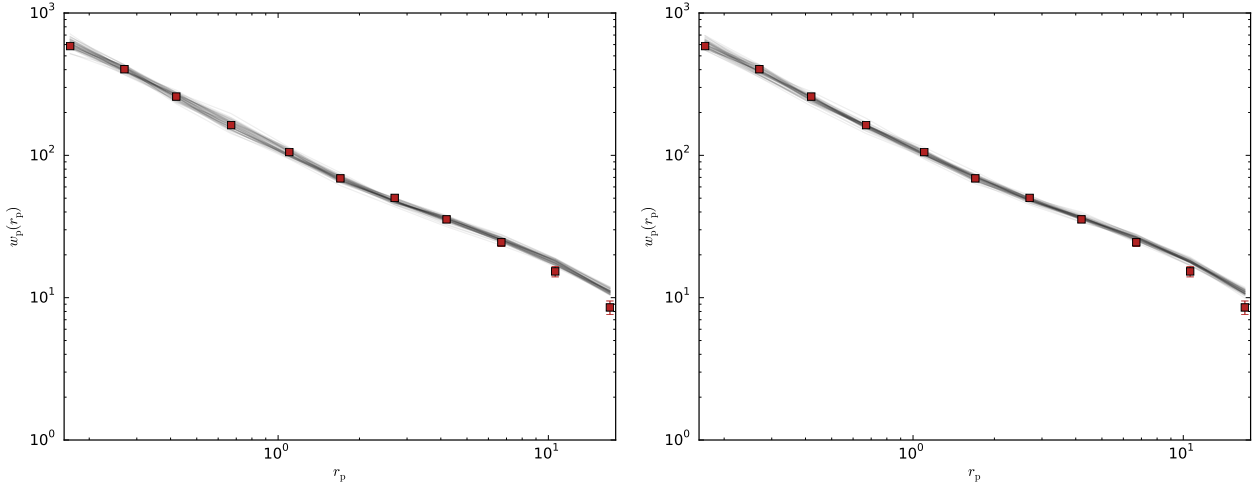
value of  $M_0$  does not alter the predicted satellite number once  $M_0 \ll M_1$ . Therefore, it seems likely that the tighter constraints quoted by Zehavi et al. (2011) must be an error.

Additionally, we have confirmed with a subset of the

authors of Zehavi et al. (2011) that the number of MCMC samples they included in their analysis was insufficient in a number of cases and that this can lead to a significant underestimation of the uncertainties on the inferred parameters, especially  $\log(M_{\min})$  and  $\sigma_{\log M}$  (Z. Zheng & I.



**Figure 3.** The same as Figure 2, but for the  $M_r < -20$  threshold sample.



**Figure 4.** The same as Figure 2, but for the  $M_r < -21$  threshold sample.

Zehavi, private communication). The analysis of Zehavi et al. (2011) used  $10^4$  samples, whereas we find several  $\times 10^6$  samples are often necessary for convergence. Additionally, we have recreated qualitatively similar behavior by considering only small subsets of our full MCMC chains. Consequently, insufficient sampling of the posterior seems to be the likely resolution of the discrepancies between our work and that of Zehavi et al. (2011).

Two degeneracies are manifest in Fig. 1 that are common to all of our analyses. The parameters  $\log(M_1)$  and  $\alpha$  are degenerate with each other and positively correlated. The parameter  $M_1$  is the mass scale at which a halo has one satellite on average, and  $\alpha$  is the power-law index describing the dependence of average satellite number on halo mass. Increasing  $M_1$  *decreases* the number of satellites in massive halos by increasing the mass scale where the power law abundance becomes operative. An increase in  $\alpha$  can partly compensate for an increase in  $M_1$  by increasing the rate at which average satellite number grows with halo mass.

As is evident in Figure 1,  $\log(M_{\min})$  and  $\sigma_{\log M}$  share a relatively narrow degeneracy as well. This degeneracy is largely induced by the measured number density of the sample. Increasing  $\log(M_{\min})$  decreases galaxy number density, but this can be compensated by an increase in  $\sigma_{\log M}$ , which places galaxies in a fraction of the considerably more numerous halos with masses less than  $M_{\min}$ . The consequence is that  $\log(M_{\min})$  and  $\sigma_{\log M}$  are degenerate with each other such that most of the posterior probability lies in a narrow band along which  $\log(M_{\min})$  and  $\sigma_{\log M}$  are positively correlated, as shown in Fig. 1. In the following plots, we suppress the parameter  $\sigma_{\log M}$ , in order to increase the clarity of the plots, because the viable range of  $\sigma_{\log M}$  is determined by this simple degeneracy with  $\log(M_{\min})$ .

The results of this subsection demonstrate that we achieve reasonable fits to projected galaxy clustering data using direct HOD population of a high-resolution numerical simulation of structure formation. These results also update and supersede existing constraints in the litera-

ture in at least three respects. First, we work within the best-fit Planck cosmology. Second, we perform our parameter inference analysis using direct population of halos identified in a numerical simulation of cosmological structure formation (BolshioP). This greatly mitigates modeling uncertainties associated with nonlinear density field evolution, scale-dependent halo bias, halo exclusion, or other effects that have been difficult to incorporate into analytical halo models with high precision. Third, we have explored the posteriors of the parameters with significantly more samples (roughly two orders of magnitude), thereby mitigating errors on inferred parameters and their errors induced by insufficient sampling of the posterior.

### 3.2 Analysis with Decorated HOD

We turn now to a discussion of our parameter inference analysis of projected galaxy clustering in Decorated HOD models that include a treatment of galaxy assembly bias. In this work, we consider only the simplest model of galaxy assembly bias, introducing only two new parameters,  $A_{\text{cen}}$  and  $A_{\text{sat}}$ , that describe the strength of central galaxy and satellite galaxy assembly bias respectively. These parameters are limited to values of  $-1 \leq A_{\text{cen,sat}} \leq 1$ , and  $A_{\text{cen,sat}} = 0$  when there is no galaxy assembly bias. In this work, we use halo concentration as our secondary halo property, so  $A_{\text{cen,sat}} = 1$  ( $A_{\text{cen,sat}} = -1$ ) means that the mean number of galaxies per halo is maximally correlated (anti-correlated) with halo concentration. The model and its implementation in `halotools` is discussed further in Section 2.2 above and in Hearin et al. (2016).

Examples of our fits are given in the right-hand panels of Figure 2, Figure 3, and Figure 4. The general trend that can be gleaned from these figures is that introducing assembly bias improves the ability of the predicted two-point functions to match the measured two-point functions across the transition from the one-halo (highly nonlinear) to two-halo (nearly linear) regimes near  $r_p \sim 2h^{-1}\text{Mpc}$ . This is most apparent for the  $M_r < -20$  threshold sample shown in Fig. 3. Visually, these differences appear to be small; however, Table 3.1 shows that they are statistically important.

The one-dimensional marginalized constraints on all parameters from these analyses are given in the lowest row of each luminosity threshold grouping in Table 3.1. In cases where the posterior on a parameter is monotonic within the physical parameter range, we quote an upper or lower limit on the parameter. One- and two-dimensional visualizations of the posteriors from our analysis are shown in Figure 5, Figure 6, and Figure 7.

Table 3.1 and Figures 5-?? all make several simple, generic points. Introducing additional parameter freedom associated with galaxy assembly bias generally increases the viable parameter space, even for the subset of standard HOD parameters. Constraints on the standard HOD parameters are generally less restrictive. This is exactly what is expected from the introduction of additional parameter freedom.

Focusing attention on the parameters describing

$M_r$ Threshold	$\Delta\text{BIC}$
-21	-0.54
-20.5	1.33
-20	4.56
-19.5	0.26
-19	4.37

**Table 3.** Change to the Bayesian Information Criterion,  $\Delta\text{BIC}$ , after introducing additional parametric freedom to accommodate galaxy assembly bias. Sign convention is such that positive values favor models including assembly bias, negative values favor standard HOD models with no assembly bias parameters. Changes in the Bayesian Information Criterion  $|\Delta\text{BIC}| \geq 5$  strongly favor one model over another.

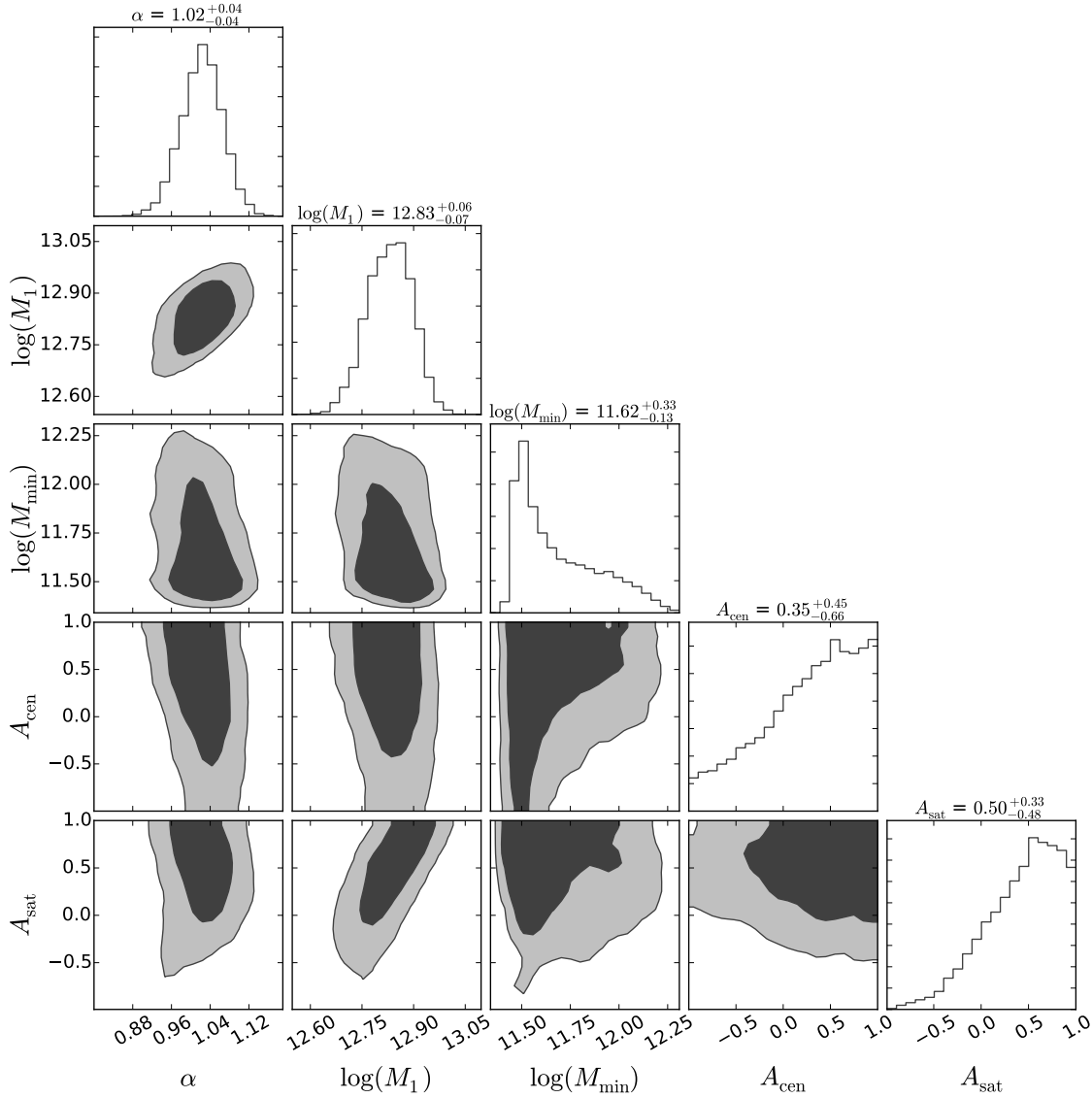
galaxy assembly bias, it is evident that these parameters are often quite poorly constrained by galaxy clustering data. This is important as it implies that galaxy clustering of the precision of SDSS DR7 measurements cannot rule out, or strongly restrict galaxy assembly bias in many cases. Nonetheless, it is apparent that the presence of assembly bias can alter the inferred HOD, or more generally, the inferred relationship between galaxies and halo mass. This is most evident for the  $M_r < -20$  threshold sample, for which there are significant differences in the inferred values of all baseline HOD parameters between models with and without galaxy assembly bias. Other threshold samples exhibit significant changes particularly for  $\alpha$ , and to a lesser degree for  $\log(M_{\text{min}})$  and  $\sigma_{\log M}$ .

Beyond those generic conclusions, a few specific cases are worthy of further examination. Consider the  $M_r < -20$  sample. The inferred value of  $A_{\text{cen}} > 0.28$  at 99% confidence. In this case the data strongly prefer  $A_{\text{cen}} > 0$  and thus strongly prefer galaxies to reside in halos of larger concentration at fixed halo mass. This particular threshold sample is the most significant outlier in this regard. Nevertheless, there are hints of assembly bias in other samples. Satellite galaxies show a marginal preference for occupying halos of higher concentration in the  $M_r < -19$  threshold sample. The  $M_r < -19$  sample exhibits weak preference for a positive correlation of galaxy occupation with halo concentration at fixed mass for both satellite galaxies and central galaxies. Continuing upward with luminosity, the  $M_r < -20.5$  sample exhibits a significant preference for central galaxy assembly bias. Lastly, there is no preference for either central galaxy or satellite galaxy assembly bias for the  $M_r < -21$  threshold sample, for which both  $A_{\text{cen}}$  and  $A_{\text{sat}}$  are consistent with zero within  $1\sigma$ . These data suggest that assembly bias may be present in the real universe.

## 4 CONCLUSIONS

We have re-analyzed the SDSS DR7 measurements of projected galaxy clustering,  $w_p$ , and number density,  $n_g$ , originally published in Zehavi et al. (2011). Our work is





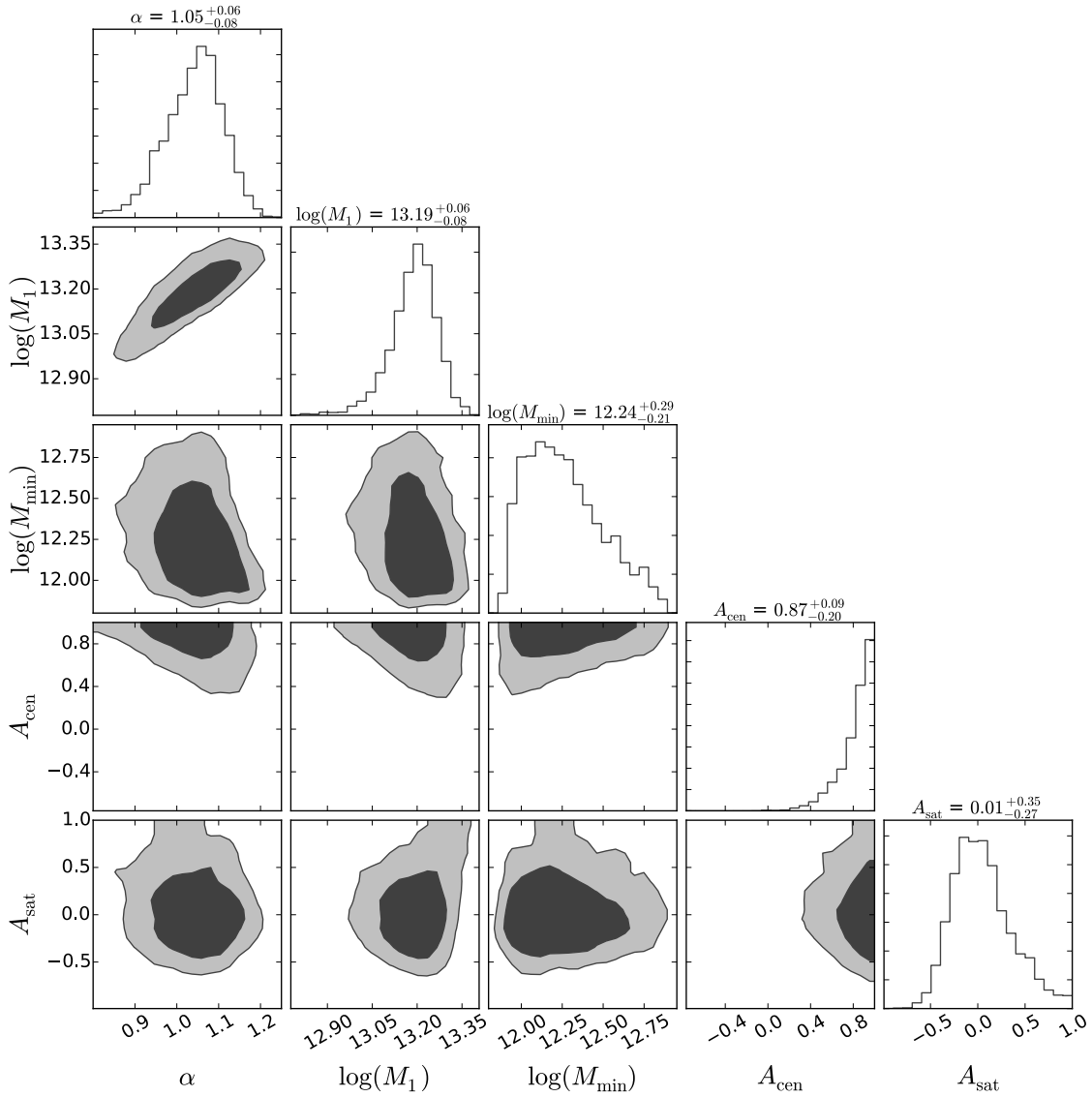
**Figure 5.** Two-dimensional marginalized constraints on decorated HOD parameters inferred from fits to  $w_p(r_p)$  data for the  $M_r < -19$  sample. The contours and histograms along the diagonal panels are as in Fig. 1. The decorated HOD models include a two-parameter model for assembly bias. The HOD parameter  $\log(M_0)$  is extremely poorly constrained by the data and has been suppressed for clarity. Likewise, as in Fig. 1,  $\sigma_{\log M}$  and  $\log(M_{\min})$  share a narrow degeneracy, so we have suppressed  $\sigma_{\log M}$  in order to make constraints on other parameters more easily visible.

especially novel in that we provide the first quantitative constraints on assembly bias derived from the Decorated HOD, an extension to the traditional HOD introduced in Hearin et al. (2016) developed for exactly this purpose. We enumerate our most important conclusions below.

- (i) It is not possible to rule out galaxy assembly bias using SDSS DR7 measurements of  $w_p$  and  $n_g$ .
- (ii) Decorated HOD fits to  $w_p$  and  $n_g$  favor significant levels of assembly bias, particularly in the lower luminosity thresholds we study. Both the  $M_r < -20, -20.5$  samples prefer relatively strong central galaxy assembly bias, while at lower luminosities, the  $M_r < -19$  sample favors satellite assembly bias.
- (iii) Galaxy assembly bias generally weakens for

brighter galaxy samples: Decorated HOD fits to the  $M_r < -21$  sample are consistent with both  $\mathcal{A}_{\text{bias}}^{\text{cens}} = 0$  and  $\mathcal{A}_{\text{bias}}^{\text{sats}} = 0$ . This consistent with the well-established result that *halo assembly bias* weakens with increasing halo mass over the dynamic range relevant to these galaxy samples (see, e.g., Figure 8 of Hearin et al. 2016, and references therein).

- (iv) Our posteriors and best-fit parameters summarized in Table 3.1 supersede the values published in Zehavi et al. (2011), as direct-mock-population together with the Decorated HOD allows us to account for highly significant systematics that have heretofore been neglected from all HOD fits to SDSS data. We note that our findings update the original Zehavi et al. (2011) results *even for our fits in which assembly bias has been*



**Figure 6.** The same as Figure 5, but for the  $M_r < -20$  sample.

fixed to zero, since the original results derive from unconverged MCMC chains that do not sufficiently sample the HOD model posteriors.

We conclude by noting that since  $w_p$  and  $n_g$  are already very well-measured in DR7, it is likely that further improvements on assembly bias constraints at low-redshift will require additional observational measurements, e.g., galaxy–galaxy lensing, group statistics, etc.

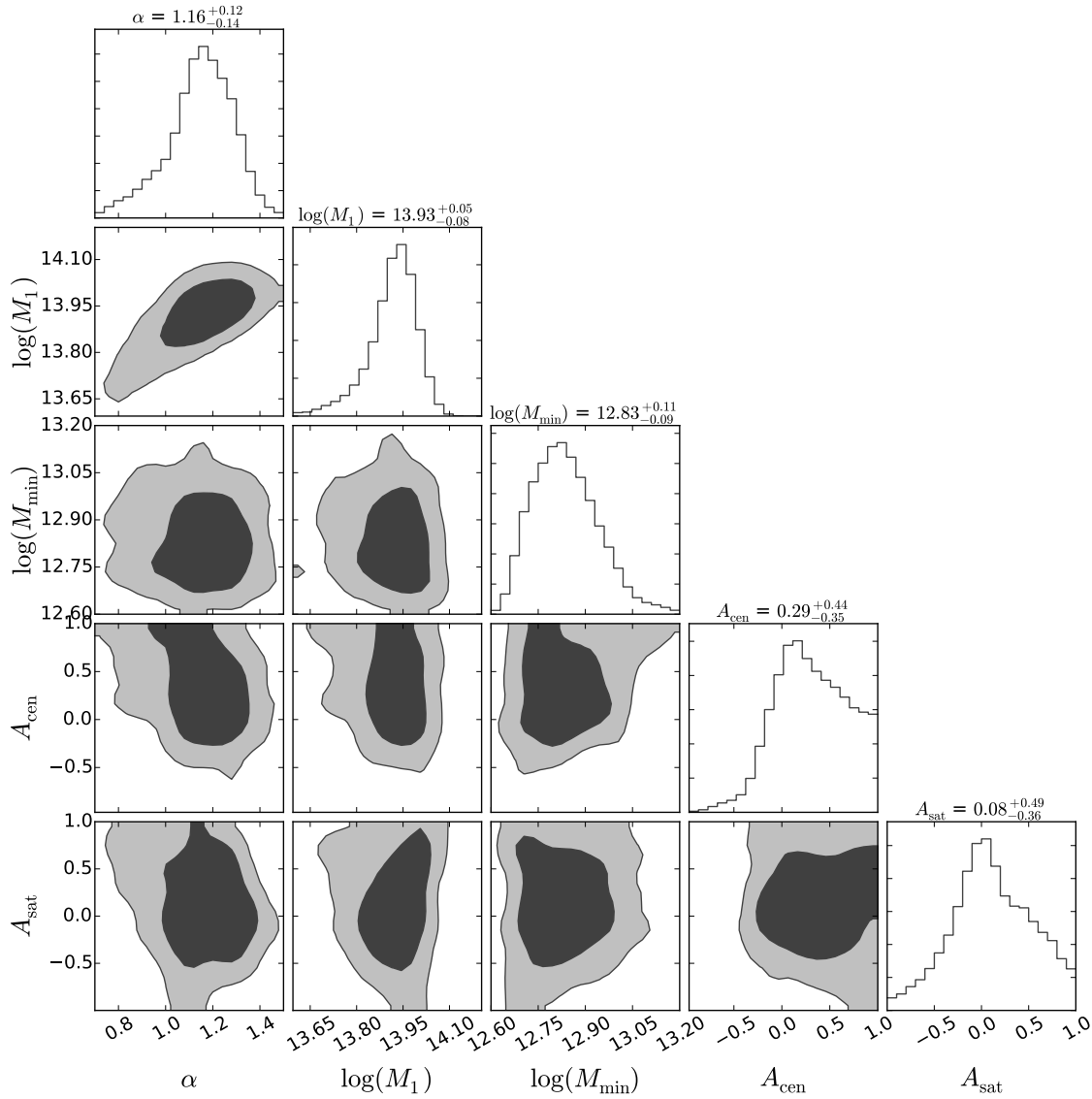
## 5 ACKNOWLEDGEMENTS

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puting Centre (LRZ, [www.lrz.de](http://www.lrz.de)). The Bolshoi simulations have been performed within the Bolshoi project of the University of California High-Performance Astro-Computing Center (UC-HiPACC) and were run at the NASA Ames Research Center. FvdB is supported by the US National Science Foundation through grant AST 1516962. **AZ; add yours as well.**

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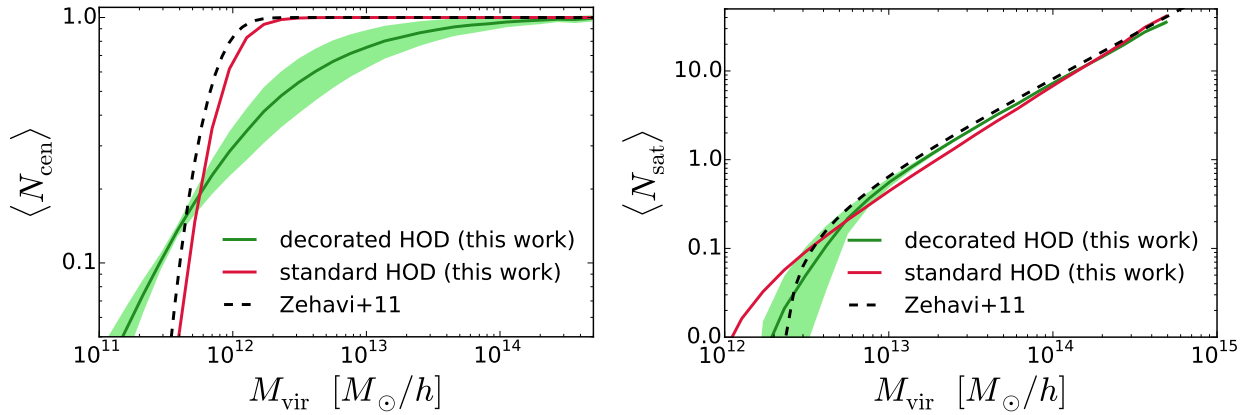
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**Figure 7.** The same as Figure 5, but for the  $M_r < -21$  sample.

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**Figure 8.** We show the first moments of the central and satellite occupation distributions from the best-fit models to the  $M_r < -20$  sample in the left and right panels, respectively. The green curve shows the HOD of our best-fit model that includes the effect of assembly bias; the red curve shows the HOD of the best-fit model in which assembly bias is assumed to be zero; the dashed black curve shows the best-fit HODs from Zehavi et al. (2011). The green error band around the decorated HOD results shows the variance in the HOD from randomly sampling one hundred points in parameter space with  $\chi^2_{\text{model}} - \chi^2_{\text{min}} < 1$ .

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