Unit 4: Simple Harmonic Motion

Period and Frequency

- frequency: number of oscillations completed each second [hertz, s^{-1}]
- period: time for 1 complete oscillation
- $T = \frac{1}{f}$
- $x(t) = x(t+T)$: must be true because displacement must return to initial value after 1 period T of motion

Periodic Motion (Harmonic Motion)

- definition: any motion that repeats itself at regular intervals
- in SHM, acceleration is proportional to displacement but opposite in sign
- in SHM, magnitude of velocity is 0 at greatest displacement and max at least displacement

Quantity	Equation	Amplitude
Displacement	$x(t) = x_m cos(\omega t + \phi)$	x_m
Velocity	$v(t) = -\omega x_m sin(\omega t + \phi)$	ωx_m
Acceleration	$a(t) = -\omega^2 x_m cos(\omega t + \phi) = -\omega^2 x(t)$	$\omega^2 x_m$

Term descriptions

- x_m: amplitude of motion, positive, amplitude is maximum displacement of particle in either direction
- $(\omega t + \phi)$: phase of motion; varies with time
- ϕ: phase constant (phase angle), value of ϕ depends on displacement and velocity of particle at time $t = 0$
- ω: angular frequency [radian per sec]; $\omega = \frac{2\pi}{T} = 2\pi f$

Force Law for Simple Harmonic Motion

- linear simple harmonic oscillator: linear indicates that F is proportional to x rather than to some power of x
- hookes law: $F = ma = -(m\omega^2)x = -kx$, therefore $k = m\omega^2$
- angular frequency: $w = \sqrt{\frac{k}{m}}$
- period: $T = 2\pi\sqrt{\frac{m}{k}}$

Energy in Simple Harmonic Motion

- $U(t) = \frac{1}{2}kx^2 = \frac{1}{2}x_m^2 cos^2(\omega t + \phi)$
- $K(t) = \frac{1}{2}mv^2 = \frac{1}{2}kx_m^2 sin^2(\omega t + \phi)$
- $E = U + K = \frac{1}{2}kx_m^2$
- mechanical energy of linear oscilattor is constant and independent of time