APMA 1655 Chapter 2 Notes: Basics of Probability

Basic Terminology

- Sample space: collection of all possible outcomes denoted by Ω or S
- Event: any subset of sample space
- notions on events: $A \subseteq B$, $A \cap B$=AB; $A \cup B$ and $\bar{A}$; events A and B are mutually exclusive or disjoint if $A \cap B = \emptyset$
- distributive laws: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$; $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Probability Axioms

- Let sample space be Ω; let P(A) denote the probability of event A
  - $0 \leq P(A) \leq 1$ for every event $A \subseteq \Omega$
  - $P(\Omega) = 1$
  - if $A_1, A_2, ...$ is a sequence of mutually exclusive events, then:

$$P(A_1 \cup A_2 \cup ...) = \Sigma_{i=1}^{\inf} P(A_i)$$

Method of Counting in Discrete Probability

- let sample space Ω have finitely many equally likely sample points
- $P(A)$ =number of sample points in A/total number of sample points in Ω

Combinatorial Numbers

- factorial: $n! = n \times (n-1) \times ... \times 1$ with $0! = 1$
- binomial coefficients:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

  for $0 \leq k \leq n$

- binomial expansion:

$$(x+y)^n = \Sigma_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Conditional Probability

- given that B happens, what is the probability if A happening?
- $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
- $P(A \cap B) = P(B)P(A|B) = P(A)P(B|A)$

Independence

- $\{A_1, ..., A_n\}$ are independent if any subcollection $\{A_{j1}, ..., A_{jk}\}$ satisfies:

$$P(A_{j1} \cap A_{j2} ... \cap A_{jk}) = P(A_{j1})P(A_{j2})...P(A_{jk})$$

- A and B are independent if $P(A \cap B) = P(A)P(B)$

Law of Total Probability and Bayes' Rule

- tree image on Chapter 2 slide 17
- Let $\{B_1, ..., B_2, ..., B_n\}$ be a partition of sample space Ω, that is the B_i's are disjoint and $\cap_{i=1}^{n} B_i = \Omega$
- Law of Total Probability: $P(A) = \Sigma_{i=1}^{n} P(A|B_i)P(B_i)$
- Bayes' Rule: $P(B_k|A) = \frac{P(A|B_k)P(B_k)}{\Sigma_{i=1}^{n} P(A|B_i)P(B_i)}$

Random Variables

- a function X: $\Omega \to \mathbb{R}$
- random vector: collection of random variables