

Unit 6: Magnetism

How Magnetic Fields are Produced

- 1st way: elementary particles such as electrons have intrinsic magnetic field around them
- magnetic fields of electrons in certain materials add together to give a net magnetic field around the material
- when a charged particle moves through a magnetic field, a magnetic force \vec{F}_B acts on the particle
- 2nd way: use moving electrically charged particles (such as current in a wire) to make an electromagnet; current produces a magnetic field

Magnetic Field Definition and Magnetic Force on Particle

- $\vec{F}_B = q\vec{v} \times \vec{B}$
- magnitude of \vec{F}_B is $F_B = |q|vB\sin\phi$; ϕ is the angle between \vec{v} and \vec{B}
- direction of \vec{F}_B always perpendicular to \vec{v} and \vec{B}
- magnitude of \vec{F}_B max when \vec{v} and \vec{B} are perpendicular
- RHR: thumb points in \vec{v} , 4 fingers point in \vec{B} , palm points in \vec{F}_B dir. if positive charge (other dir. if negative)
- SI Units of Magnetic Field: $1 \text{ tesla} = 1 \frac{\text{newton}}{(\text{coulomb/second})(\text{meter})} = 1 \frac{\text{newton}}{(\text{ampere})(\text{meter})}$

Magnetic Field Lines

- field lines emerge from north pole of magnet and enter at south pole
- opposite magnetic poles attract each other, like magnetic poles repel each other
- direction of tangent to magnetic field line at any point gives the direction of \vec{B} at that point
- magnetic field is stronger where the lines are closer together
- On Earth, field lines travel from geographic south pole (Antarctic) to geographic north pole (Arctic)

Circulating Charged Particles

- if particle moves in circle at constant speed, net force acting on particle is constant in magnitude of points toward center of circle, perpendicular to the velocity vector
- $|q|vB = \frac{mv^2}{r}$

Magnetic Force on a Current-Carrying Wire

- $\vec{F}_B = i\vec{L} \times \vec{B}$
- magnitude: $F_B = iLB\sin\phi$; ϕ is angle between \vec{L} and \vec{B}
- differential equation: $d\vec{F}_B = i d\vec{L} \times \vec{B}$; break up wire into little segments

- RHR (field): thumb in dir. of current, flat hand points to region of interest, pads of fingers point in dir. of \vec{B}
- current travels opposite of electron flow
- RHR (loop of charge with current flowing): curl fingers in dir. if i, thumb points dir. of \vec{B} in center of the loop
- current-carrying wire generates a magnetic field that forms a circle around the wire

Calculating the Magnetic Field Due to a Current

- moving charged particle produces magnetic field around itself
- Biot-Savart Law: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{s} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{id\vec{s} \cdot r \sin\theta}{r^3}$

Magnetic Field Due to a Current in a Long Straight Wire

- magnitude of magnetic field at a perpendicular distance R from a long (infinite) straight wire carrying a current i is: $B = \frac{\mu_0 i}{2\pi R}$
- magnetic field lines produced by a current in a long straight wire form concentric circles around the wire; greater magnitude closer to wire

Magnetic Field Due to a Current in a Circular Arc of Wire

- $B = \int d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{s} \times \vec{r}}{r^3} = \int \frac{\mu_0}{4\pi} \frac{id\vec{s} \cdot r \sin(90)}{r^3} = \frac{\mu_0 i \phi}{4\pi r}$
- can replace ds with $r d\phi$
- this equation gives the magnetic field only at the center of the curvature of a circular arc of current
- must use radians for ϕ , not degrees

Force Between Two Parallel Currents

- to find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field
- parallel currents attract each other, and antiparallel currents repel each other

Ampere's Law

- can find the net magnetic field due to any distribution of currents by first writing the differential magnetic field $d\vec{B}$ due to a current length element and then summing the contributions of $d\vec{B}$ from all the elements
- Ampere's Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$
- loop on integral sign means that the dot product $\vec{B} \cdot d\vec{s}$ is to be integrated around a closed (Amperian) Loop; current i_{enc} is the net current encircled by that closed loop
- RHR: curl right hand around the Amperian loop, with the four fingers pointing in the direction of integration; a current in the loop in same direction as outstretched thumb is positive, opposite direction is negative

- currents outside of Amperian loop don't contribute to the magnetic field because they cancel out

Magnetic Field Outside a Long Straight Wire with Current

- $\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r) = \mu_0 i$

Magnetic Field Inside a Long Straight Wire with Current

- if current is uniformly distributed, current i_{enc} encircled by the loop is proportional to the area encircled by the loop: $i_{enc} = i \frac{\pi r^2}{\pi R^2}$
- $\oint \vec{B} \cdot d\vec{s} = B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}$

Magnetic Field of a Solenoid

- solenoid: long, tightly wound helical coil of wire; assume that the length of the solenoid is much greater than the diameter
- magnetic field outside ideal solenoid is zero

Magnetic Field of a Toroid

- toroid: hollow solenoid that has been curved until its two ends meet, forming a sort of hollow bracelet

Electromagnetic Induction

- motor: electrical energy to mechanical energy; moving charge in properly aligned magnetic field will produce force on a wire
- generator: mechanical energy to electrical energy; moving wire, in properly aligned magnetic field, will produce a force on a charge
- electromagnetic induction: creation of a potential difference by moving a conductor through a magnetic field
- induced current: current produced in a loop
- induced emf (\mathcal{E}): work done per unit charge to produce the induced current (to move the conduction electrons that constitute the current)
- induction: process of producing the current and emf

Faraday's Law of Induction

- gives the magnitude of induced voltage
- emf and current can be induced in a loop by changing the amount of magnetic field passing through the loop; actual number of field lines passing through the loop does not matter, only rate matters
- Magnetic flux through area A: $\Phi_B = \int \vec{B} \cdot d\vec{A}$; $d\vec{A}$ is a vector of magnitude dA that is perpendicular to a differential area dA
- special case when $\vec{B} \perp$ area A, \vec{B} uniform: $\Phi_B = BA$
- SI Units of magnetic flux: 1 weber = 1Wb = 1 T · m²

- magnitude of the emf \mathcal{E} induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time

- Faraday's Law: $\mathcal{E} = -\frac{d\Phi_B}{dt}$

Faraday's Law Requirements

- Faraday's Law requires change in magnetic flux
- change the magnitude B of the magnetic field within the coil
- change either the total area of the coil or the portion of that area that lies within the magnetic field (for example by expanding the coil or sliding it into or out of the field)
- change the angle between the direction of the magnetic field \vec{B} and the plane of the coil (for example, by rotating the coil so that field \vec{B} is first perpendicular to the plane of the coil and then is along that plane)

Lenz's Law

- determines direction of an induced current in a loop: an induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current (conservation of energy)
- RHR: curled straight right-hand rule: rotate fingers in direction of current, straight thumb represents direction of B
- \vec{B}_i always opposes the change in the flux of \vec{B} but does not necessarily mean that \vec{B}_i is opposite \vec{B}
- 1: dir of B_0 , 2: is B_0 increasing or decreasing, 3: oppose change, 4: RHR loop for I of $B_{induced}$

Induced Electric Fields

- a changing magnetic field produces an electric field
- an electric field is needed to do the work of moving the conduction electrons in the current
- both induced electric field and electric field produced by static charges exert force $q_0 \vec{E}$ on particle of charge q_0
- electric field lines produced by changing magnetic field is a set of concentric circles
- $\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$
- induced electric fields not produced by static charges but by a changing magnetic flux
- electric potential has meaning only for electric fields that are produced by static charges; it has no meaning for electric fields that are produced by induction