Unit 7: Circuits

Electric Current

- how current is created: battery has potential, which creates an electric field, which generates a force on the electrons, causing them to move
- SI Unit of current: 1 ampere = 1A = 1 coulomb per second = 1 C/s
- current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction

Current Density

- current density: $\vec{J}$; magnitude of J equal to current per unit area through conductor element
- $i = \int \vec{J} \cdot d\vec{A}$
- if current is uniform across surface and parallel to $d\vec{A}$: $i = \int JdA = JA$

Drift Speed

- drift speed: v_d; speed electrons move with direction opposite of applied electric field that causes the current
- positive charge carriers drift at speed v_d in the direction of the applied electric field $\vec{E}$, same direction as J
- $\vec{J} = (ne)\vec{v}_d$: n is the number of charge carriers per unit volume, e is charge of an electron

Resistance and Resistivity

- resistance is a property of an object (resistor); resistivity is a property of a material (copper)
- $R = \frac{V}{i}$ (definition of R)
- SI units of resistance: 1 ohm = 1 Ω = 1 volt per ampere = 1 V/A
- resistivity $\rho = \frac{E}{J}$
- SI units of resistivity: units of E/units of J = $\frac{V/m}{A/m^2} = Vm/A = \Omega * m$
- above equations only hold for isotropic materials, whose electrical properties are the same in all directions
- relationship between resistance and resistivity: $R = \rho \frac{L}{A}$

Ohm's Law

- given device (conductor, resistor etc.) obeys Ohm's Law (R = V/i) if resistance is independent of applied potential difference V

Power

- power P (rate of electrical energy transfer) in an electrical device across which a potential difference V is maintained is: $P = iV$

Work, Energy, and Emf

- emf device: maintains a potential difference between a pair of terminals (i.e. battery)
- emf points from negative terminal toward the positive terminal
- when emf device not connected, no net flow of charge carriers, when connected, net flow of positive charge carriers from negative terminal to positive terminal
- emf vs electric field: in emf device, positive charge carriers move form lower electric potential region (and thus low electric potential energy) to high electric potential region and higher electric potential energy (opposite of electric field the terminals does, which is directed from positive terminal to negative)
- $\mathcal{E} = \frac{dW}{dq}$: emf of an emf device is the work per unit charge that the device does in moving charge frow its low-potential terminal to its high-potential terminal
- SI Unit of emf: joule per coulomb, aka volt
- ideal emf device: no internal resistance between terminals; potential difference between terminals is equal to emf of device
- real emf device: has internal resistance to internal movement of charge; when not in circuit (no current), potential difference between terminals is equal to emf; when device has current through it, potential difference between terminals differs from emf
- batteries connected so that currents are in opposite directions situation: actual direction of the current in the circuit is determine by the battery with the larger emf

Current in Single-Loop Circuits

- energy method for finding current: emf $\mathcal{E}$ is the energy per unit charge transferred to the moving charges by the battery: $i = \frac{\mathcal{E}}{R}$
- Kirchhoff's loop rule: the algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero; can apply loop rule in either direction (based on conservation of energy)
- Resistance Rule: for a move through a resistance in the direction of the current, the change in potential is -iR; in the opposite direction it is +iR
- EMF Rule: for a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathcal{E}$; in the opposite direction it is $-\mathcal{E}$

Resistance in Single-Loop Circuits

- internal resistance: treat as if resistor is right next to battery
- resistances in series: when a potential difference V is applied across resistances connected in series, the resistances have identical currents i. The sum of the potential differences across the resistances is equal to the applied potential difference V

- resistances connected in series can be replaced with an equivalent resistance R_{eq} that has the same current i and the same total potential difference V as the actual resistances; $R_{eq} = \sum_{j=1}^{n} R_j$ (n resistances in series)

Potential Differences

- to find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the changes in potential you encounter
- grounding a circuit: potential defined to be 0 at grounding point in the circuit
- $P_{emf} = i\mathcal{E} = i^2 r$

Multiloop Circuits

- Kirchhoff's junction rule: the sum of the currents entering any junction must be equal to the sum of the currents leaving that junction (based on conservation of charge)
- resistances in parallel: when a potential difference V is applied across resistances connected in parallel, the resistances all have that same potential difference V
- resistances connected in parallel can be replaced with an equivalent resistance R_{eq} that has the same potential difference V and the same total current i as the actual resistances; $\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$ (n resistances in parallel)

Ammeters and Voltmeters

- ammeter: used to measure currents; to measure current, insert ammeter so that current passes through the meter; resistance of ammeter must be very small compared to other resistances in the circuit, otherwise the ammeter will change the current being measured
- voltmeter: used to measure potential differences; to measure potential difference between 2 points, connect voltmeter at two points without breaking/cutting the wire; resistance of voltmeter must be much larger compared to other resistors in the circuit, otherwise creates a new path for current to travel

RC (resistor-capacitor) Circuits

- loop rule for 1 capacitor, 1 resistor, 1 battery circuit: $\Delta V = \mathcal{E} - iR - \frac{q}{C} = 0$
- $i = \frac{dq}{dt} = (\frac{\mathcal{E}}{R})e^{-t/RC}$
- $R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E}$; $q = C\mathcal{E}(1 - e^{-t/RC})$
- q/C is negative because capacitor's top plate which is connected to the battery's positive terminal, is at a higher potential than the lower plate; thus there is a drop in potential as we move down through the capacitor
- as soon as circuit is complete, charge (current) begins to flow between a capacitor plate and a battery terminal on each side of the capacitor
- current increases charge q on the plates and the potential difference V across the capacitor
- when potential difference of capacitor equals potential difference of battery, current is 0
- a capacitor that is being charged initially acts like ordinary connecting wire relative to the charging circuit. A long time later, it acts like a broken wire
- Time constant: $\tau = RC$; the greater the time constant, the greater the charging time

Time Constant

- $\tau = RC$
- bigger tie constant means longer charging time

Capacitor Discharge

- $R\frac{dq}{dt} + \frac{q}{C} = 0$ which is $q = q_0 e^{-t/RC}$
- $q_0 = CV_0$ is the initial charge on the capacitor

Definition of Capacitance

- can come in any shape/size; consists of 2 isolated conductors (called plates)
- when a capacitor is charged, its plates have charges of equal magnitudes but opposite signs: +q and -q; charge of capacitor is q (net charge of capacitor is 0)
- plates are conductors, therefore they have equipotential surfaces (all points on a plate are at the same electric potential)
- $q = CV$: charge q and potential difference V are proportional, C, the proportionality constant, is the capacitance of the capacitor
- capacitance depends only on the geometry of the plates and not their charge or potential difference; capacitance is a measure of how much charge must be put on the plates to produce a certian potential difference between them
- SI Units: 1 farad = 1F = 1 coulomb per volt = 1 C/V

Charging a Capacitor

- when a capacitor and a battery are put in a completed circuit, battery's electric field drives electrons onto one side of the plate, charging the other with an equal magnitude but opposite in sign charge
- initially, when plates are uncharged, potential difference between them is zero
- as plates become oppositely charged, potential difference increases until it equals the potential difference V between the terminals of the battery; capacitor is then fully charged

Calculating the Capacitance

- parallel plate capacitor: $C = \frac{\epsilon_0 A}{d}$; A is plate area, d is plate separation; $epsilon_0$ is the permittivity constant
- completely discharged capacitor acts like a wire

- completely charged capacitor acts like a broken wire, no current

Capacitors in Parallel

- capacitors connected in parallel can be replaced with an equivalent capacitor that has the same total charge q and the same potential difference V as the actual capacitors
- $C_{eq} = \sum_{j=1}^{n} C_j$: n capacitors in parallel

Capacitors in Series

- capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same total potential difference V as the actual series capacitors
- $\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$: n capacitors in series

Energy Stored in an Electric Field

- work must be done by an external agent like a battery to charge a capacitor
- as charge accumulates on the capacitor plates, increasing amounts of work must be done to transfer additional electrons (more electrons, more opposing force)
- work required to charge a capacitor is stored in the form of electric potential energy U in the electric field between the plates
- $U = \frac{q^2}{2C} = \frac{1}{2}CV^2$
- potential energy of charged capacitor may be viewed as being stored in the electric field between its plates
- Energy density (u): potential energy per unit volume between the plates
- parallel plate capacitor energy density: $u = \frac{U}{Ad} = \frac{CV^2}{2Ad} = \frac{1}{2}Q\Delta V$

Inductors and Inductance

- definition of inductance: $L = \frac{N\phi_B}{i}$
- SI units of inductance: 1 henry = 1 H = 1 $T * m^2/A$
- inductance depends only on geometry of device

Self-Induction

- an induced emf $\mathcal{E}_L$ appears in any coil in which the current is changing with time
- self induced emf: $\mathcal{E}_L = -L\frac{di}{dt}$
- magnitude of current has no influence on magnitude of the induced emf; only the rate of change of the current counts
- direction of self-induced emf is given from Lenz's Law: self induced emf $\mathcal{E}_L$ opposes change in current (reason for negative sign in equation)

RL (Resistor-Inductor) Circuits

- initially an inductor acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wire
- application of loop rule in circuit: $\mathcal{E} - iR - L\frac{di}{dt} = 0$ goes to $i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau_L})$
- inductive time constant: $\tau_L = \frac{L}{R}$

Current Decay in Inductor

- $L\frac{di}{dt} + iR = 0$
- $i = \frac{\mathcal{E}}{R}e^{-t/\tau_l} = i_0 e^{-t/\tau_l}$

Energy Stored in a Magnetic Field

- magnetic energy: $U_B = \frac{1}{2}Li^2$
- U_B represents total energy stored by an inductor L carrying a current i