Unit 3: Rotation:

Angular Quantities

- counterclockwise positive, clockwise negative
- use units of radians and seconds; 1 rev= $360° = 2\pi$ rad
- Angular position: $\theta = \frac{s}{r}$, s is arc length; don't reset θ to 0 after each rotation because position, not displacement; $rads$
- Angular Velocity: $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$; $\omega = \frac{d\theta}{dt}$; $rads/sec$
- Angular Acceleration: $\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$; $\alpha = \frac{d\omega}{dt}$; $rads/sec^2$

Right-hand rule for Angular Quantities

- fingers point in direction of rotation, thumb will point in direction of angular quantity
- rigid body rotating around the direction of angular vector
- angular displacements can't be treated as vectors because fails commutative property of vector addition

Equations of motion for constant linear and angular acceleration

Linear Equation	Angular Equation
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Relationships between Linear and Angular Variables (all in radians)

- in uniform circular motion with rigid body, all particles make 1 revolution in same amount of time (all have same angular speed ω); linear speed v will increase the farther the particle is from the axis of rotation
- position: $s = \theta r$ (θ must be in radians)
- speed: $v = \omega r$ (linear velocity always tangent to circular path of point)
- T is the period (time for 1 revolution); $T = \frac{1}{f} = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$
- tangential acceleration (present when $\alpha \neq 0$): $a_t = \alpha r$
- radial acceleration (present when $\omega \neq 0$): $a_r = \frac{v^2}{r} = \omega^2 r$
- can use $a = \sqrt{a_t^2 + a_r^2}$ because 2 components are perpendicular

Kinetic Energy of Rotation

- can't apply $K = \frac{1}{2}mv^2$ because would only give KE of center of mass, which is 0
- can't apply $K = \sum \frac{1}{2}m_i v_i^2$ because v_i not same for all particles, replace using $v = \omega r$
- $K = \frac{1}{2}I\omega^2$ (in radians)

Rotational Inertia

- SI unit is $kg * m^2$
- general idea is that inertia is angular equivalent of mass; the farther mass is distributed from the rotation axis, the more the inertia
- rotational inertia (moment of inertia): constant for a particular rigid body and rotation axis
- Point masses (particles): $I = \sum m_i r_i^2$
- Continuous rigid body (look at table of 9 common body shapes): $I = \int r^2 dm$
- Parallel Axis theorem: $I = I_{com} + mh^2$; h is perpendicular axis between given axis and **parallel** axis through center of mass

Torque

- 1st expression: $\tau = (r)(F sin\phi) = rF_t$; F_t is tangential force, radial component F_r doesn't cause rotation because parallel to rotation axis
- 2nd expression: $\tau = (r sin\phi)(F) = r_\perp F$; moment arm: $r_\perp$; line of action is extended line; ϕ is angle between $\vec{r}$ and $\vec{F}$
- net torque (τ_{net}) is sum of individual torques (superposition principle)
- Units is newton-meter ($N * m$) **but never joules**

Newton's second law for rotation

- $\tau = I\alpha$; α must be in radians; angular equivalent of $F = ma$

Work and Rotational Kinetic Energy

- Angular Work-kinetic energy theorem: $W = \Delta KE = K_f - K_i = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$
- calculate work with rotation equivalent of force: $W = \int_{\theta_i}^{\theta_f} \tau d\theta$
- rotational equivalent for power: $P = \frac{dW}{dt} = \tau\omega$

Similarities between Translational and Rotational Motion

Pure Translation (Fixed Direction)	Pure Rotation (Fixed Axis)
x	θ
$v = dx/dy$	$\omega = d\theta/dt$
$a = dv/dt$	$\alpha = d\omega/dt$
mass (m)	rotational inertia (I)
$F_{net} = ma$	$\tau_{net} = I\alpha$
Work ($W = \int F dx$)	Work ($W = \int \tau d\theta$)
KE ($K = \frac{1}{2}mv^2$)	KE ($K = \frac{1}{2}I\omega^2$)
Power (constant force) $P = Fv$	constant torque $P = \tau\omega$
Work-KE theorem $W = \Delta K$	$W = \Delta K$

Rolling as Translation and Rotation Combined