Unit 5: Electric Charge, Field, and Potential:
Electric Charge

- object is charged when there is a charge imbalance
- opposite charges attract, same repel
- conductors: materials through which charge can move rather freely (some metals)
- insulators: materials through which charge cannot move through freely (rubber)
- grounding: setting up pathway of conductors between object and earth, discharges object by neutralizing the object (eliminating charge)
- atoms: consists of protons and neutrons packed tightly into nucleus at center; some outermost electrons move around (conduction electrons)
- only conduction electrons can move, positive ions are fixed in place; object can only become positively charged through removal of negative charges

Coulomb's Law for Electrostatic Force

- $F = k\frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$
- $k = \frac{1}{4\pi\varepsilon_0} = 8.99 * 10^9 \frac{N*m^2}{C^2}$
- $\varepsilon_0 = 8.85 * 10^{-12} \frac{C^2}{N*m^2}$
- electrostatic forces may be either attractive or repulsive
- SI unit: coulomb; $1C = (1amperes)(1second)$;
- follows principle of superposition: $\vec{F}_{1,net} = \vec{F}_{1,2} + \vec{F}_{1,3} + ... + \vec{F}_{1,n}$

Shell theorems

- shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at its center (assuming shell's charge is much greater than particle's charge)
- if a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell
- excess charge on spherical shell spreads uniformly over external surface

Charge and Conservation

- elementary charge, $e = 1.602 * 10^{-19}C$
- object can only have full parts of e (charge of +10e, -6e, not 3.57e)
- conservation of charge: net charge of any isolated system cannot change, charges are transferred to and from objects

Electric Fields

- vector field
- $\vec{E} = \frac{\vec{F}}{q_0}$; $\vec{E}$ is magnitude of electric field; q_0 is a test charge at the point in question; $\vec{F}$ is the force that acts on the test charge
- SI unit for $\vec{E}$ is $\frac{N}{C}$; newtons per coulomb; can also be $\frac{V}{m}$, volts per meter
- field lines and electric field vectors: at any point, direction of straight field line or direction of tangent to curved field line is $\vec{E}$; number of lines per unit area measured in plane perpendicular to lines is proportional to magnitude of $\vec{E}$ ($\vec{E}$ is large when field lines are dense and vice versa)
- electric field lines extend away from positive charge (originate) and toward negative charge (terminate)
- electric dipole: two charges of equal magnitude but opposite sign

Electric Field Due to Point Charge

- $\vec{E} = \frac{\vec{F}}{q_0} = \frac{k|q|}{r^2}$; $\vec{E}$ is electric field due to point charge q; dir. of $\vec{F}$ is directly away from q if positive and directly toward q if negative
- superposition principle: $\vec{E} = \vec{E_1} + \vec{E_2} ... + \vec{E_n}$

A Point Charge in an Electric Field

- electrostatic force acting on the particle q: $\vec{F} = q\vec{E}$
- electrostatic force $\vec{F}$ acting on q in field $\vec{E}$ has the direction of $\vec{E}$ is q is positive and negative direction of $\vec{E}$ if q is negative

Electric Field due to Line of Charge

- when dealing with continuous charge distributions, express charge on an object with charge density rather than total charge
- general strategy out dq of charge, find dE due to that element, then integrate dE over entire line of charge
- Ring: use trig to get E direction (angle is a constant); replace r^2 with $z^2 + R^2$; integrate over s (circumference of ring)
- Circular Arc: use trig to get E direction (angle is a constant); can use ds = $rd\theta$
- Straight line with point P on an extension of the line: in E expression, replace r with x and integrate over x from end to end of line
- Straight line with point P at perpendicular distance y from line of charge: replace r in E expression with expression involving x and y;

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

Electric Field Due to a Charged Disk

- $\rho = \frac{dq}{dA} = \frac{dq}{2\pi r dr}$
- use equation E from ring of charge and use the above substitution

Flux of an Electric Field

- $\Delta\vec{A}$: vector coming out of small squares of area ΔA; each vector $\Delta\vec{A}$ is perpendicular to the Gaussian surface and directed way from the interior of the surface
- electric flux through a Gaussian surface: $\Phi = \oint \vec{E} \cdot d\vec{A}$

Gauss's Law

- relates the electric fields at points on a closed Gaussian surface to the net charge enclosed by that surface
- $\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$
- if q_{enc} is positive, net flux is outward; if q_{enc} negative, net flux is inward
- any charge outside of a Gaussian surface doesn't affect net flux (same amount of field lines entering surface as leaving)
- if an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor; none of the excess charge will be found within the body of the conductor (electric field within conductor is zero)
- removing chunk from within Gaussian surface doesn't charge distribution because all charge remains on outside surface
- electric field is set up by charges, not conductor; conductor simply provides an initial pathway for the charges to take up their positions
- cylindrical: $\Phi = EAcos\theta = E2\pi rh$; $E2\pi rh\epsilon_0 = q_{enc} = \lambda h$; $E = \frac{\lambda h}{2\pi r\epsilon_0}$
- plane: $\Phi = \frac{q_{enc}}{\epsilon_0} = 2EA = \frac{\sigma A}{\epsilon_0}$; $E = \frac{\sigma}{2\epsilon_0}$

Gauss's Law to Coulomb's Law

- can derive Coulomb's Law from Gauss' Law (use a spherical Gaussian surface, surface areas is $4\pi r^2$)
- $\Phi = \frac{q_{enc}}{\epsilon_0} = \oint \vec{E} \cdot d\vec{A}$
- $q_{enc} = \epsilon_0 \oint \vec{E} \cdot d\vec{A} = 4\pi r^2 \epsilon_0 E$
- $E = \frac{q_{enc}}{4\pi\epsilon_0 r^2} = \frac{kq}{r^2}$

Electric Potential Energy

- energy stored by a charge due to its position in an electric field (energy of a charged object in an external electric field)
- Units: joules (J)
- positive charge loses EPE when it moves in the direction of the electric field lines
- like GPE, EPE = 0 line needs to be set; usually at location of lowest EPE
- $\Delta U = U_f - U_i = -W$
- work W done by electrostatic force on particle is path independent (work same for all paths from i to f)

Electric Potential

- amount of electric potential energy per charge (analogous to electric field which is forces per charge, scalar value); property of an electric field, regardless of whether a charged object has been placed in that field;
- potential energy per unit charge at a point in an electric field
- $V = \frac{U}{q} = \frac{EPE}{q}$; V is electric potential, EPE is electric potential energy; q is charge placed at that location
- units: volt (V) = 1 joule/coulomb
- Electric Potential difference: $\Delta V = V_{final} - V_{initial} = \frac{\Delta U}{q} = \frac{-W}{q}$;

Work Done by an Applied Force

- $\Delta K = K_f - K_i = W_{app} + W$
- if starts and stops at test: $W_{app} = -W$
- work W_{app} done by our applied force during the move is equal to the negative of the work W done by the electric field (if no change in KE)

Equipotential Surfaces

- places which have the same electric potential
- because of path independence of work, $W = 0$ for any path connecting i and f on a given equipotential surface regardless of whether that path lies entirely on that surface
- equipotential surfaces are always perpendicular to electric field lines

Potential from Electric Field

- $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$

Potential Due to Point charge

- $V = \frac{kq}{r}$
- sign of V is same as sign of q; positively charged particle produces a positive electric potential, negatively charged particle produces negative electric potential
- equation gives potential outside or on the external surface of a spherically symmetric charge distribution

Potential Due to a Group of Point Charges

- $V = k\sum_{i=1}^{n} \frac{q_i}{r_i}$
- sum is algebraic sum, electric potential is scalar

Potential Due to a Continuous Charge Distribution

- $V = \int \frac{kdq}{r}$

Calculating Electric Field from the Potential

- $E_s = -\frac{\partial V}{\partial s}$
- component of $\vec{E}$ in any direction is the negative of the rate at which the electric potential changes with distance in that direction

Electric Potential Energy of a System of Point Charges

- electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance
- $U = W_{app} = q_2 V = k\frac{q_1 q_2}{r}$
- if charges have the same sign, W_{app} needs to do positive work to push them together against their mutual repulsion; if charges have opposite signs, we have to do negatie work against their mutual attraction to bring them together if they are to be stationary

Potential of a Charged Isolated Conductor

- $\vec{E} = 0$ for all points inside an isolated conductor; excess charge placed on an isolated conductor lies entirely on its surface (even if conductor has an empty internal cavity)
- an excess charge placed on a conductor will, in equilibrium state, be located entirely on the outer surface of the conductor; the charge will distribute itself so that the entire conductor, including interior points, is at a uniform potential