Chapter 3 Notes: Discrete Rand Var

Definitions and Terminologies

- discrete random variable: can only take finitely many possible values
- distribution: the probabilistic/statistical behavior of a random variable
- probability mass function: $p_i = p(x_i) = P(X = x_i)$, then $p_i \geq 0$ and $\Sigma_i p_i = 1$
- cumulative distribution function: for every $x \in \mathbb{R}$, define: $F(x) = P(X \leq x) = \Sigma_{x_i \leq x} p(x_i)$

CDF properties

- always increasing
- $F(-\infty) = 0$, $F(\infty) = 1$
- step function that jumps at x_i with jump size p_i
- continuous from the right

Expected Value

- $E[X] = \Sigma_i x_i P(X = x_i) = \Sigma_i x_i p_i$

Functions of a Random Variable

- let h: $\mathbb{R} \to \mathbb{R}$ be a function, then h(X) is a random variable
- expectation of h(X): $E[h(X)] = \Sigma_i h(x_i) P(X = x_i)$
- let a, b be real numbers, then $E[aX + b] = aE[X] + b$
- for an functions $h_1, ..., h_k : \mathbb{R} \to \mathbb{R}$, $E[h_1(X) + ... + h_k(X)] = E[h_1(X)] + ... + E[h_k(X)]$

Variance and Standard Deviation

- Variance describes variation of a random variable (average of square of standard deviations from the mean)
- $\text{Var}[X] = E[(X - E[X])^2]$
- standard deviation: Std[X]=$\sqrt{Var[X]}$
- Properties:
  - $\text{Var}[X] = 0$ if and only if P(X=c)=1 for some constant c
  - $\text{Var}[X] = E[X^2] - E^2[X]$
  - let a, b be real numbers, then $\text{Var}[aX + b] = a^2 \text{Var}[X]$

Bernoulli Random Variable

- type of binomial random variable
- X take values in $\{0, 1\}$ and $P(X = 1) = p$, $P(X = 0) = 1 - p$
- bernoulli denoted by B(1;p)

Binomial Random Variable

- X takes values in $\{0, 1, ..., n\}$ and $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$
- binomial distribution denoted by B(n;p)
- expectation: $E[X] = np$
- variance: $\text{Var}[X] = np(1 - p)$

Maximum Likelihood Estimate (MLE)

- X is B(n;p), probability is: $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$
- MLE: find the p that maximizes the likelihood given X=k: $\hat{p} = \frac{k}{n} = \frac{X}{n}$

Geometric Distribution

- geometric random variable: X takes values in $\{1, 2, ...\}$ and $P(X = k) = p(1 - p)^{k-1}$
- expectation: $E[X] = \frac{1}{p}$
- variance: $\text{Var}[X] = \frac{1-p}{p^2}$

Poisson Distribution

- Poisson random variable: X takes values in $\{0, 1, 2, ...\}$ and $P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}$
- $e^x = \Sigma_{k=0}^{\infty} \frac{x^k}{k!}$
- expectation: $E[X] = \lambda$
- variance: $\text{Var}[X] = \lambda$
- binomial approximation of poisson: let $\lambda > 0$ be any fixed number, when n is large: $B(n; \lambda/n) = Poisson(\lambda)$