Unit 2: Work, Energy Momentum:

Kinetic Energy:

- motion energy
- Equation: $KE = \frac{1}{2}mv^2$
- Units: 1 Joule = 1 J = $\frac{1kg*m^2}{s^2}$

Work:

- Work is energy transferred to or from an object via a force acting on the object, energy transferred to object is positive work, energy transferred from the object is negative work
- Work-kinetic energy theorem:

$$W = \Delta KE = K_f - K_i = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

- For a **constant** force and particle-like object:

$$W = F_x * d = F * d * cos\theta = \vec{F} \cdot \vec{d}$$

- Units: 1 Joule = 1 J = $\frac{1kg*m^2}{s^2} = 1N * m$

Types of Work

- Gravitational Force: constant force (7-6 pg 147)
- Spring Force: variable force because force is function of x,

$$\vec{F_s} = -k\vec{d}$$

- Work done by spring force (**note** that initial term is in front, W_s is positive if the block ends up closer to relaxed position when x = 0 then it was initially and vice versa):

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Work Done by General Variable Force

- 1d: $W = \int_{x_i}^{x_f} F(x)d_x$
- 3d: force in 3d is $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$, displacement in 3d is $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$, instantaneous work in 3d is $dW = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz$, Work is:

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

- Work-Kinetic Energy Theorem works with a Variable force

Power

- rate at which Work is done by a force
- average power: $P_{avg} = \frac{W}{\Delta t}$
- instantaneous power: $P = \frac{dW}{dt} = F * v$
- Units: $1watt = 1W = 1\frac{J}{s}$
- more generally, power is rate at which energy is transferred by a force from one type to another, then average power is $P_{avg} = \frac{\Delta E}{\Delta t}$, and instantaneous power is $P = \frac{dE}{dt}$

Conservative Forces

- conservative force: gravitational force and spring force, $W_1 = -W_2$
- non-conservative force: kinetic frictional force and drag force
- net work done by a conservative force on a particle moving on a closed path is zero (same start and end)
- work done by a conservative force on a particle moving between two points doesn't depend on the path taken by the particle

Potential Energy

- General relationship between conservative force and potential energy: $\Delta U = -W = -\int_{x_i}^{x_f} F(x)dx$
- Elastic Potential Energy: plugging in $F_x = -kx$ into the previous equation returns $\Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$, setting $x_i = 0$ we get $U(x) = \frac{1}{2}kx^2$

Conservation of Mechanical Energy

- mechanical energy defined as: $E_{mec} = K + U$
- Conservation of mechanical energy: in an isolated system with only conservative forces, the kinetic energy and potential energy can change but there sum E_{mec} can't change
- Isolated system is when there is no external force from an object outside the system that causes energy changes inside the system
- Equation for isolated system with only conservative forces inside of system:

$$\Delta E_{mec} = (K_f - K_i) + (U_f - U_i) = \Delta K + \Delta U = 0$$

- mechanical energy involves KE and U

Work Done on a System by an External Force

- Work is energy transferred to or from a system by means of an external force acting on that system, positive work is transfer of energy into system and vice versa
- Friction creates thermal energy: $\Delta E_{th} = f_k * d$
- No friction: $W = \Delta E_{mec}$
- With friction: $W = \Delta E_{mec} + \Delta E_{th}$

Conservation of Energy

- Total energy E of a system can change only by amounts of energy that are transferred to or from the system
- Isolated system: total energy E of an isolated system cannot change: $\Delta E = \Delta K + \Delta U = 0$

- When external forces/internal energy transfers are involved ΔE equals the net force done on the system

Potential Energy Curves

- Because $\Delta U(x) = -W = -F(x)\Delta x$, the force in 1d motion can be expressed as $F(x) = \frac{-dU(x)}{dx}$
- Turning points: When $K = 0$ because $U = E$; particle changes direction because force is positive
- Neutral Equilibrium: 1st derivative is not zero, 2nd derivative is zero; stationary marble placed on a horizontal tabletop
- Unstable Equilibrium: 1st derivative is not zero; at a peak and concave down; pushing particle left or right will cause particle to not return to original position
- Stable Equilibrium: 1st derivative is zero; particle will return to its original position at some point; concave up

Center of Mass

- center of mass of a system of particles is as if all the system's mass were concentrated there and all external forces were applied there
- Location, so units in distance (i.e. meters)
- Equation for system of particles: $\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r_i}$ where $\vec{r_i} = x_i\hat{i} + y_i\hat{j} + z_i\hat{k}$
- Equation for solid body with uniform density: $x_{com} = \frac{1}{M}\int xdm$
- Can also use planes of symmetry
- Motion of the center of mass for a system of particles: $\vec{F}_{net} = M\vec{a}_{com}$, $\vec{F}_{net}$ is the net force of all external forces acting on the system
- Uniform objects have uniform density (mass per unit of volume)
- Density (3d): $\rho = \frac{dm}{dv} = \frac{M}{V}$
- Area (2d): $\sigma = \frac{dm}{da} = \frac{N}{A}$
- Length (1d): $\lambda = \frac{dm}{dl} = \frac{M}{L}$

Linear Momentum

- Definition: $\vec{p} = m\vec{v}_{com}$
- Newton's 2nd law: $\vec{F}_{net} = \frac{d\vec{P}}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$; if mass doesn't change with respect to time: $\vec{F}_{net} = m\frac{dv}{dt} = ma$
- Units: $\frac{kg*m}{s} = N*s$
- momentum is a vector, has multiple directions
- linear momentum can only be changed by an external net force
- Conservation of linear momentum (total momentum at time t_i equals total momentum at later time t_f): $\vec{P}_i = \vec{P}_f$; occurs if next external force is 0
- External forces can be accounted for with impulse: $p_i + J_{ext} = p_f$

Impulse

- Impulse-linear momentum theorem: $\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$
- Impulse definition: $\vec{J} = \int_{t_i}^{t_f}\vec{F}(t)dt = F_{avg}\Delta t$
- Impulse cause changes in momentum, impulse is a vector

Inelastic Collisions

- Inelastic collisions: objects stick together, largest amount of mechanical energy is lost, KE is not conserved; if system is closed and isolated, linear momentum of system *must* be conserved
- Inelastic collision momentum equation (i and f are right before and after collision): $\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$

Elastic Collisions

- Elastic collisions: mechanical energy is conserved, if system is closed and isolated, momentum is also conserved
- 2 equations for velocity of each particle