1. Exercise 2.1

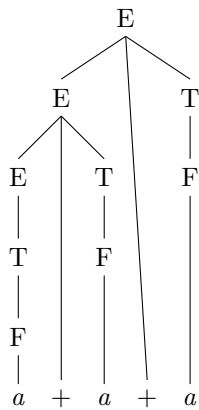   a. $E \Rightarrow T \Rightarrow F \Rightarrow a$

   ```
   E
   |
   T
   |
   F
   |
   a
   ```

   b. $E \Rightarrow E + T \Rightarrow T + T \Rightarrow F + T \Rightarrow a + T \Rightarrow a + F \Rightarrow a + a$

   ```
           E
          /|\
        E     T
        |     |
        T     F
        |
        F
        |
        a  +  a
   ```

   c. $E \Rightarrow E + T \Rightarrow E + T + T \Rightarrow T + T + T \Rightarrow T + T + F \Rightarrow T + F + F \Rightarrow F + F + F \Rightarrow F + F + a \Rightarrow$
      $F + a + a \Rightarrow a + a + a$

   ```
              E
             /|\
           E     T
          /|\    |
        E   T    F
        |   |
        T   F
        |
        F
        |
        a + a + a
   ```

   d. $E \Rightarrow T \Rightarrow F \Rightarrow (E) \Rightarrow (T) \Rightarrow (F) \Rightarrow ((E)) \Rightarrow ((T)) \Rightarrow ((F)) \Rightarrow ((a))$

```
         E
         |
         T
         |
         F
        /|\
       / E \
      /  |  \
     /   T   \
    /    |    \
   /     F     \
  /    / | \    \
 /    / E \ \    \
 |   /  |  \ |   |
 |  /   T   \|   |
 |  |   |   ||   |
 |  |   F   ||   |
 |  |   |   ||   |
 (  (   a   )    )
```

2. Construct a pushdown automata that recognizes

$$\{w \mid w \in \{0,1\}^* \text{ s.t. the number of 0's in } w \text{ is equal to the number of 1's in } w\}$$

3. Exercise 2.2

    a. Use the languages $A = \{a^m b^n c^n \mid m, n \geq 0\}$ and $B = \{a^n b^n c^m \mid m, n \geq 0\}$ together with Example 2.36 to show that the class of context-free languages is not closed under intersection.

    The language A is context-free as there exists a CFG G_1:

    $$S \to XY$$
    $$X \to aX \mid \epsilon$$
    $$Y \to bYc \mid \epsilon$$

    The language B is context-free as there exists a CFG G_2:

    $$S \to XY$$
    $$X \to aXb \mid \epsilon$$
    $$Y \to cY \mid \epsilon$$

    However $A \cap B$ is the language $\{a^n b^n c^n\}$ and Example 2.36 says that language is not context-free. Therefore by proof by contradiction, context-free languages are not closed under intersection.

b. Use part (a) and DeMorgan's law (Theorem 0.20) to show that the class of context-free languages is not closed under complementation.

Suppose that context-free languages are closed under complementation. Then the complement of A and B, A', B' should also be context-free. Since context-free languages are closed under union, then $A' \cup B'$ should also be a context-free language. By DeMorgan's law, $A' \cup B' = A \cap B$, however that is not the case as proved in part (b). By proof of contradiction, context-free languages are not closed under complementation.

4. Exercise 2.4b

5. Give a CFG for

$$\{0^a 1^b 2^c 3^d 4^e 5^f \mid \text{such that } a, b, c, d, e, f \geq 0 \text{ and } a + b = d + e\}$$

6. Exercise 2.4e: Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is $\{0, 1\}$.

$$\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$$

7. Put the rules following in Chomsky normal form (assume that S is the new start variable)

$$S \rightarrow aAA \mid aBC \mid abc$$
$$A \rightarrow AA \mid Aa \mid ab$$
$$B \rightarrow aaBC \mid BC$$
$$C \rightarrow a \mid bc$$

8. Exercise 2.15: Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generated by CFG $G = (V, \Sigma, R, S)$. Add the new rule $S \rightarrow SS$ and call the resulting grammar G'. This grammar is supposed to generate $A*$.

9. Show the following is context free using a CFG

$$\{xy \mid x, y \in \{0, 1\}^*, \ |x| = |y|, \ y \neq x^R\}$$

10. Construct a pushdown automata that recognizes

$$\{w \mid w \text{ is an element of } \{a, b, c, d\}^* \text{ such that the number of a's in } w \text{ plus the number of b's in } w \text{ is} $$
$$\text{equal to the number of c's in } w \text{ plus the number of d's in } w\}$$