CS 373 Homework 5 Anchu A. Lee
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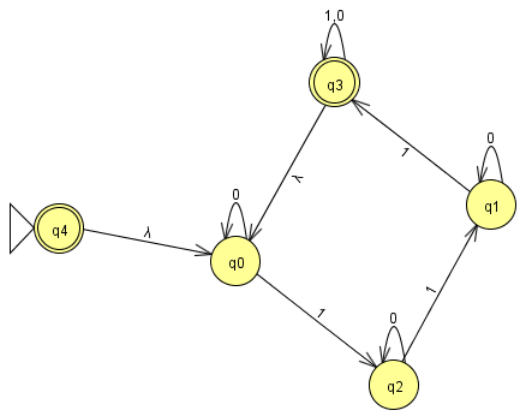
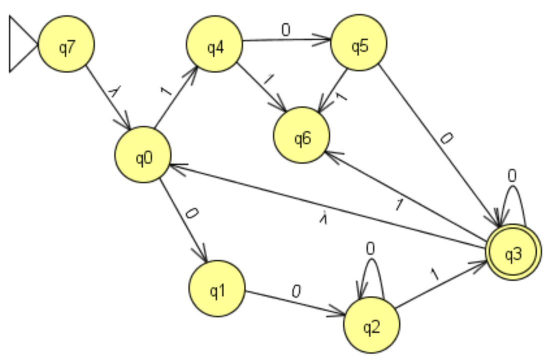
Question 1.10a Construct NFA that recognizes the star of the language in Exercise 1.6b

$$\{w \mid w \text{ contains at least three 1s}\}$$



Question 1.10b Same as before, but Exercise 1.6j

$$\{w \mid w \text{ contains at least two 0s and at most one 1}\}$$



Question 1.29b Use the pumping lemma to show that the language is not regular

$$A_2 = \{www \mid w \in \{a, b\}^*\}$$

Proof. Assume A_2 is regular, then there must exist a number n that is the pumping length. Test with the word $k = b^{n/2}b^{n/2}b^{n/2}$. $|k| > n$. Due to the nature of the language, there is only one way to split the word to satisfy the language. $x = b^{n/2}$; $y = b^{n/2}$; $z = b^{n/2}$. $|xy| \leq n$ and $|y| \geq 1$. Now consider pumping it up with xy^iz for $i = 2$. xyz is not in L because it is $b^{n/2}b^nb^{n/2}$ which does not match the definition of the language. Therefore our assumption was incorrect and A_2 is not regular. □

Question 1.46a Prove the following language is not regular using pumping lemma or closure of the class of regular languages under union, intersection, and compliment.

$$\{0^n1^m0^n \mid m, n \geq 0\}$$

Proof. Assume this language (A) is regular, so then there must exist a variable p, the pumping length. Choose $w = 0^p10^p$ as the test word. $|w| > p$ and $w \in A$. As $|xy| \leq p$, x and y must be composed of only zeros. Additionally, as $|y| > 0$, y would then have to equal 0^k for some $k > 0$. For xy^iz, choose $i = 0$ and the resulting word should still be in A. However $xy^0z = xz = 0^{p-k}10^p$. This resulting word is not in A therefore our assumption was incorrect. □

Question 1.46c Same as before

$$\{w \mid w \in \{0,1\}^* \text{ is not a palindrome}\}$$

Proof. Remember that w is a palindrome if $w = w^R$. Assume that the language L is regular. Then the compliment of L should also be regular. $L' = \{w \mid w \in \{0,1\}^* \text{ is a palindrome}\}$ is also regular. Now we can use the pumping lemma on L'.
There exists a p by the pumping lemma. Choose the word 0^p10^p. $|w| \geq p$. Because $|xy| \leq p$, x, y must be composed of only zeros. Additionally as $|y| > 0$, y would have to equal 0^k for some $k > 0$. Finally take xy^iz for $i = 0$. Then the resulting word would equal $0^{p-k}10^p$ which cannot be a palindrome since $p - k < p$. This contradicts the assumptiong that L is regular. □

Question 1.46d Same as before

$$\{wtw \mid w, t \in \{0.1\}^+\}$$

Proof. Assume the language (L) is regular. Then by the pumping lemma, there exists a p. Choose the word $d = 0^p110^p1$. $|d| \geq p$. To comply with the conditions for pumping lemma, x and y must both consist of only zeros as $|xy| \leq p$. That means $y = 0^k$ for some $k > 0$. Next, for some i, $xy^iz \in L$. Set $i = 2$ and the resulting word is $0^{p+k}110^p1$. As $p + k > p$ this word cannot be in the language L and therefore the assumption that L is regular is false. □

Question 1.47 Let $\sum = \{1, \#\}$ and let

$$Y = \{w \mid w = x_1\#x_2\#...\#x_k \text{ for } k \geq 0, \text{ each } x_i \in 1^*, \text{ and } x_i \neq x_j \text{ for } i \neq j\}$$

Prove Y is not regular.

Proof. Assume the language Y is regular. Then let p be the pumping length from the pumping lemma. Consider the word $w = 1^p\#1^{p+1}\#...\#1^{2p}$. $|xy| \leq p$ so x and y must compose of only 1s. Then $y = 1^k$ for some $k > 0$. Now consider $t = xy^iz$ for $i = 2$. t can also be written out as $t = t_0\#t_1\#...\#t_u$ where $u = p$, $t_0 = 1^{p+|y|}$ and for $1 \leq j \leq p$, $t_j = 1^{p+j}$. Since $1 \leq |y| \leq p$, we can find that $p + 1 \leq (p + |y|) \leq 2p$ and then $t_0 = t_{p+|y|}$. Two series of 1s are equal to each other and therefore t cannot be in the language Y. This contradicts our assumption that Y was regular. □

Question 1.49

- Let $B = \{1^ky \mid y \in \{0,1\}^* \text{ and } y \text{ contains at least } k \text{ 1s, for } k \geq 1\}$.
  Show that B is a regular language.

  Proof. If B is a regular language, then it can be expressed by a regular expression. Set $k = 2$ $110^*10^*1(0 \cup 1)^*$. This language is regular. □

- Let $C = \{1^ky \mid y \in \{0,1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for } k \geq 1\}$.
  Show that C isn't a regular language.

  Proof. Assume that C is a regular language, then let p be the pumping length from the pumping lemma. Consider the word $s = 1^p0^p1^p$. As $|xy| \leq p$, x and y must both only consist of 1s. Then $y = 1^t$ for some $t > 0$. Now consider xy^iz for $i = 0$ which would look like $1^{p-t}0^p1^p$. As $p - t < p$ the word is not in C and violates the assumption that C is a regular language. □

Show that $\{0^n 1^m 2^k \mid k$ divides $n+m\}$ is not regular.
Convert the following NFA to a DFA: