1. Let $Some_{DFA} =\{ \langle A \rangle \mid A$ is a DFA and $L(A)$ is not empty and $L(A)$ is not equal to Σ^* $\}$. Show that $Some_{DFA}$ is decidable.

2. Let $Alot_{RE} =\{ \langle A \rangle \mid A$ is a regular expression and $L(A)$ is infinite $\}$. Show that $Alot_{RE}$ is decidable.

3. Let $Complimentary_{RE,DFA} =\{ \langle A, B \rangle \mid A$ is a regular expression and B is a DFA such that $L(A) \cup L(B) = \Sigma^*$ and $L(A) \cap L(B) =\emptyset\}$. Show that $Complimentary_{RE,DFA}$ is decidable.

4. Let $ALL_{DFA} =\{ \langle A \rangle \mid A$ is a DFA and $L(A) = \Sigma^*\}$. Show that ALL_{DFA} is decidable.

5. Let $N_{\varepsilon CFG} =\{ \langle G \rangle \mid G$ is a CFG and G does not generate the empty string $\}$. Show that $N_{\varepsilon CFG}$ is decidable.

6. Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. We describe the functions $f : X \to Y$ and $g : X \to Y$ in the following tables. Answer each part and give a reason for each negative answer.

n	$f(n)$		n	$g(n)$
1	6		1	10
2	7		2	9
3	6		3	8
4	7		4	7
5	6		5	6

   (a) Is f onto?

   (b) Is f a correspondence?

   (c) Is g onto?

   (d) Is g a correspondence?

7. Let $U =\{ \langle A, B, C \rangle \mid A, B, C$ are DFA's and $|L(A)| = |L(B)| + |L(C)|$ $\}$. Show that U is decidable.

8. Let $A =\{ \langle R \rangle \mid R$ is a regular expression describing a language containing at least one string w that has 111 as a substring (i.e., $w = x111y$ for some x and y)$\}$. Show that A is decidable.

9. Let $E_{PDA} =\{ \langle P \rangle \mid P$ is a pushdown automata and $L(P)$ is empty $\}$. Show E_{PDA} is decidable.

10. A **useless state** in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.