CS 373 Homework 6 Anchu A. Lee
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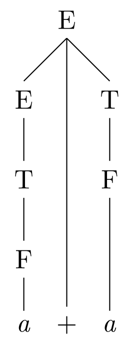
1. Exercise 2.1

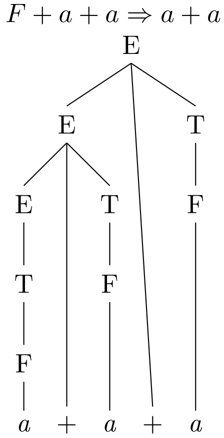
   a. $E \Rightarrow T \Rightarrow F \Rightarrow a$

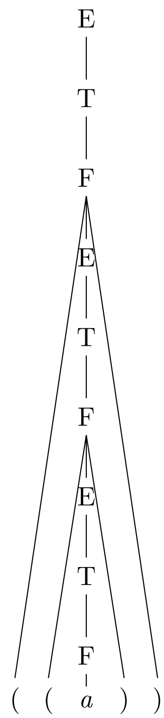
   b. $E \Rightarrow E + T \Rightarrow T + T \Rightarrow F + T \Rightarrow a + T \Rightarrow a + F \Rightarrow a + a$

   c. $E \Rightarrow E + T \Rightarrow E + T + T \Rightarrow T + T + T \Rightarrow T + T + F \Rightarrow T + F + F \Rightarrow F + F + F \Rightarrow F + F + a \Rightarrow F + a + a \Rightarrow a + a + a$

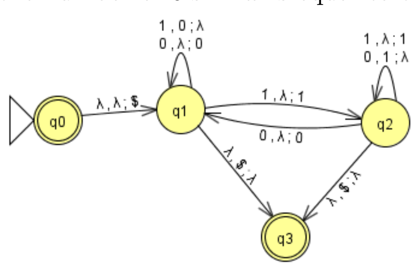
   

   d. $E \Rightarrow T \Rightarrow F \Rightarrow (E) \Rightarrow (T) \Rightarrow (F) \Rightarrow ((E)) \Rightarrow ((T)) \Rightarrow ((F)) \Rightarrow ((a))$



2. Construct a pushdown automata that recognizes

$$\{w \mid w \in \{0,1\}^* \text{ s.t. the number of 0's in } w \text{ is equal to the number of 1's in } w\}$$



3. Exercise 2.2

   a. Use the languages $A = \{a^m b^n c^n \mid m, n \geq 0\}$ and $B = \{a^n b^n c^m \mid m, n \geq 0\}$ together with Example 2.36 to show that the class of context-free languages is not closed under intersection.

   The language A is context-free as there exists a CFG G_1:

$$S \to XY$$
$$X \to aX \mid \epsilon$$
$$Y \to bYc \mid \epsilon$$

   The language B is context-free as there exists a CFG G_2:

$$S \to XY$$
$$X \to aXb \mid \epsilon$$
$$Y \to cY \mid \epsilon$$

   However $A \cap B$ is the language $\{a^n b^n c^n\}$ and Example 2.36 says that language is not context-free. Therefore by proof by contradiction, context-free languages are not closed under intersection.

b. Use part (a) and DeMorgan's law (Theorem 0.20) to show that the class of context-free languages is not closed under complementation.

   Suppose that context-free languages are closed under complementation. Then the complement of A and B, A', B' should also be context-free. Since context-free languages are closed under union, then $A' \cup B'$ should also be a context-free language. By DeMorgan's law, $A' \cup B' = A \cap B$, however that is not the case as proved in part (b). By proof of contradition, context-free languages are not closed under complementation.

4. Exercise 2.4b: Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is $\{0, 1\}$.

$$\{w \mid w \text{ starts and ends with the same symbol}\}$$

$$\begin{aligned} S &\to 0X0 \mid 1X1 \\ X &\to 0X \mid 1X \mid \epsilon \end{aligned}$$

5. Give a CFG for

$$\{0^a 1^b 2^c 3^d 4^e 5^f \mid \text{such that } a, b, c, d, e, f \geq 0 \text{ and } a + b = d + e\}$$

$$\begin{aligned} S &\to TW \\ T &\to 0T4 \mid U \\ U &\to 1U3 \mid V \\ V &\to 2V \mid \epsilon \\ W &\to 5W \mid \epsilon \end{aligned}$$

6. Exercise 2.4e: Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is $\{0, 1\}$.

$$\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$$

$$S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon$$

7. Put the rules following in Chomsky normal form (assume that S is the new start variable)

$$\begin{aligned} S &\to aAA \mid aBC \mid abc \\ A &\to AA \mid Aa \mid ab \\ B &\to aaBC \mid BC \\ C &\to a \mid bc \end{aligned}$$

$$\begin{aligned} S &\to DI \mid DH \mid DJ \\ A &\to AA \mid AD \mid DE \\ B &\to GH \mid BC \\ C &\to a \mid EF \\ D &\to a \\ E &\to b \\ F &\to c \\ G &\to DD \\ H &\to BC \\ I &\to AA \\ J &\to EF \end{aligned}$$

8. Exercise 2.15: Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generated by CFG $G = (V, \Sigma, R, S)$. Add the new rule $S \rightarrow SS$ and call the resulting grammar G'. This grammar is supposed to generate $A*$.

   Suppose a context-free language B with the corresponding grammar $G = \{\{S\}, \{(,)\}, \{S \rightarrow (S), S \rightarrow \epsilon\}, S\}$. Following the construction, we add $S \rightarrow SS$ to the new grammar G'. However, $(()())$ is in G' but is not in B^*. So the new grammar G' does not generate A^*.

9. Show the following is context-free using a CFG

$$\{xy \mid x, y \in \{0,1\}^*,\ |x| = |y|,\ y \neq x^R\}$$

10. Construct a pushdown automata that recognizes

$\{w \mid w$ is an element of $\{a, b, c, d\}^*$ such that the number of a's in w plus the number of b's in w is equal to the number of c's in w plus the number of d's in $w\}$