Math 327 Homework 9 Anchu A. Lee
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Question 9.54: A manufacturer of MP3 players conducts a set of comprehensive tests on the electrical functions of its product. All MP3 players must pass all tests prior to being sold. Of a random sample of 500 MP3 players, 15 failed one or more tests. Find a 90% confidence interval for the proportion of MP3 players from the population that pass all tests.

Answer: $n = 500$, $\hat{p} = 485/500 = 0.97$, $\hat{q} = 0.03$, $z_{0.05} = 1.645$
$1.645 \cdot \sqrt{\frac{0.97 \cdot 0.03}{500}} = 0.013$, $0.97 - 0.013 < p < 0.97 + 0.013$
$\boldsymbol{0.957 < p < 0.983}$

Question 9.56: A geneticist is interested in the proportion of African males who have a certain minor blood disorder. In a random sample of 100 African males, 24 are found to be afflicted.

(a) Compute a 99% confidence interval for the proportion of African males who have this blood disorder.

(b) What can we assert with 99% confidence about the possible size of our error if we estimate the proportion of African males with this blood disorder to be 0.24?

Answer: $n = 100$, $\hat{p} = 24/100 = 0.24$, $\hat{q} = 0.76$, $z_0.005 = 2.575$
$2.575 \cdot \sqrt{\frac{0.24 \cdot 0.76}{100}} = 0.110$
(a) $\boldsymbol{0.130 < p < 0.350}$
(b) **0.110**

Question 9.64: A study is to be made to estimate the proportion of residents of a certain city and its suburbs who favor the construction of a nuclear power plant near the city. How large a sample is needed if one wishes to be at least 95% confident that the estimate is within 0.04 of the true proportion of residents who favor the construction of the nuclear power plant?

Answer: $e = 0.04$, $z_{0.025} = 1.96$
$n = \frac{(1.96)^2}{(4 \cdot 0.04)^2} \approx \mathbf{601}$

Question 9.66: Ten engineering schools in the United States were surveyed. The sample contained 250 electrical engineers, 80 being women; 175 chemical engineers, 40 being women. Compute a 90% confidence interval for the difference between the proportions of women in these two fields of engineering. Is there a significant difference between the two proportions?

Answer: $n_1 = 250$, $\hat{p}_1 = 80/250 = 0.32$, $n_2 = 175$, $\hat{p}_2 = 40/175 = 0.2286$, $z_{0.05} = 1.645$
$(0.32 - 0.2286) \pm (1.645)\sqrt{\frac{0.32 \cdot 0.68}{250} + \frac{0.2286 \cdot 0.7714}{175}}$
$0.0914 \pm 0.713,$
$\boldsymbol{0.0201 < p_1 - p_2 < 0.1627}$. Yes, there are significantly more women in electrical engineering than in chemical engineering.

Question 9.68: In the study *Germination and Emergence of Broccoli*, conducted by the Department of Horticulture at Virginia Tech, a researcher found that at 5°C, 10 broccoli seeds out of 20 germinated, while at 15°C, 15 out of 20 germinated. Compute a 95% confidence interval for the difference between the proportions of germination at the two different temperatures and decide if there is a significant difference.

Answer: $n_1 = 20$, $\hat{p}_1 = 0.50$, $n_2 = 20$, $\hat{p}_2 = 0.75$, $z_{0.025} = 1.96$
$(0.5 - 0.75) \pm (1.96)\sqrt{\frac{0.5 \cdot 0.5}{20} + \frac{0.75 \cdot 0.25}{20}}$
-0.25 ± 0.2899
$\boldsymbol{-0.5399 < p_1 - p_2 < 0.0399}$, The interval includes 0 so significance cannot be shown.

Question 9.70: According to *USA Today* (March 17, 1997), women made up 33.7% of the editorial staff at local TV stations in the United States in 1990 and 36.2% in 1994. Assume 20 new employees were hired as editorial staff.

(a) Estimate the number that would have been women in 1990 and 1994, respectively.

(b) Compute a 95% confidence interval to see if there is evidence that the proportion of women hired as editorial staff was higher in 1994 than in 1990.

Answer: $n_1 = 20$, $\hat{p}_1 = 0.337$, $n_2 = 20$, $\hat{p}_2 = 0.361$
(a) $n_1\hat{p}_1 = 20 \cdot 0.337 \approx 7$, $n_2\hat{p}_2 = 20 \cdot 0.362 \approx 7$.
(b) $z_{0.025} = 1.96$
$(0.337 - 0.362) \pm (1.96)\sqrt{\frac{0.337 \cdot 0.663}{20} + \frac{0.362 \cdot 0.638}{20}}$
-0.025 ± 0.295
$\boldsymbol{-0.320 < p_1 - p_2 < 0.270}$. No evidence that the proportion of women hired was higher in 1994. Includes 0.

Question 9.72: A random sample of 20 students yielded a mean of $\bar{x} = 72$ and a variance of $s^2 = 16$ for scores on a college placement test in mathematics. Assuming the scores to be normally distributed, construct a 98% confidence interval for σ^2.

Answer: $s^2 = 16$, $v = 19$, $\chi^2_{0.01} = 36.191$, $\chi^2_{0.99} = 7.633$
$\frac{19 \cdot 16}{36.191} < \sigma^2 < \frac{19 \cdot 16}{7.633}$
$\boldsymbol{8.4 < \sigma^2 < 39.827}$

Question 9.78: Construct a 90% confidence interval for σ_1^2/σ_2^2 in Exercise 9.43 on page 295. Were we justified in assuming that $\sigma_1^2 \neq \sigma_2^2$ when we constructed the confidence interval for $\mu_1 - \mu_2$?

Answer: $s_1^2 = 5000^2$, $s_2^2 = 6100^2$, $f_{0.05}(11, 11) = 2.82$.
$(\frac{5000}{6100})^2 \cdot \frac{1}{2.82} < \frac{\sigma_1^2}{\sigma_2^2} < (\frac{5000}{6100})^2 \cdot (2.82)$
$\boldsymbol{0.238 < \frac{\sigma_1^2}{\sigma_2^2} < 1.895}$. The interval contains 1 so it was not reasonable to assume $\boldsymbol{\sigma_1^2 \neq \sigma_2^2}$

Question 9.90: According to the *Roanoke Times*, McDonalds sold 42.1% of the market share of hamburgers. A random sample of 75 burgers sold resulted in 28 of them being from McDonalds. Use material in Section 9.10 to determine if this information supports the claim in the *Roanoke Times*.

Answer: $n = 75$, $x = 28$, $\hat{p} = 28/75 = 0.3733$, $z_{0.025} = 1.96$
$0.3733 \pm (1.96)\sqrt{\frac{0.3733 \cdot 0.6267}{75}} = 0.3733 \pm 0.1095$
$\boldsymbol{0.2638 < p < 0.4828}$. Interval contains 0.421 so claim is reasonable.

Question 9.96: An anthropologist is interested in the proportion of individuals in two Indian tribes with double occipital hair whorls. Suppose that independent samples are taken from each of the two tribes, and it is found that 24 of 100 Indians from tribe A and 36 of 120 Indians from tribe B possess this characteristic. Construct a 95% confidence interval for the difference $p_B p_A$ between the proportions of these two tribes with occipital hair whorls.

Answer: $n_A = 100$, $\hat{p}_A = 24/100 = 0.24$, $n_B = 120$, $\hat{p}_B = 36/120 = 0.30$, $z_{0.025} = 1.96$
$(0.30 - 0.24) \pm (1.96)\sqrt{\frac{0.24 \cdot 0.76}{100} + \frac{0.3 \cdot 0.7}{120}} = 0.06 \pm 0.117$
$\boldsymbol{-0.057 < p_B - p_A < 0.177}$.

Question 9.106: A random sample of 30 firms dealing in wireless products was selected to determine the proportion of such firms that have implemented new software to improve productivity. It turned out that 8 of the 30 had implemented such software. Find a 95% confidence interval on p, the true proportion of such firms that have implemented new software.

Answer: $n = 30$, $x = 8$, $z_{0.025} = 1.96$
$\frac{8}{30} \pm (1.96)\sqrt{\frac{(4/15) \cdot (11/15)}{30}} = \frac{8}{30} \pm 0.158$
$\boldsymbol{0.108 < p < 0.425}$.

Question 9.108: A manufacturer turns out a product item that is labeled either "defective" or "not defective." In order to estimate the proportion defective, a random sample of 100 items is taken from production, and 10 are found to be defective. Following implementation of a quality improvement program, the experiment is conducted again. A new sample of 100 is taken, and this time only 6 are found to be defective.

(a) Give a 95% confidence interval on $p_1 p_2$, where p_1 is the population proportion defective before improvement and p_2 is the proportion defective after improvement.

(b) Is there information in the confidence interval found in (a) that would suggest that $p_1 > p_2$? Explain.

Answer: $n_1 = 100$, $\hat{p}_1 = 0.1$, $n_2 = 100$, $\hat{p}_2 = 0.06$.
(a) $z_{0.025} = 1.96$
$(0.1 - 0.06) \pm (1.96)\sqrt{\frac{0.1 \cdot 0.9}{100} + \frac{0.06 \cdot 0.94}{100}} = 0.04 \pm 0.075$
$\boldsymbol{-0.035 < p_1 - p_2 < 0.115}$.
(b) Interval contains 0, not enough evidence to suggest $\boldsymbol{p_1 > p_2}$.