Math 327 Homework 10 Anchu A. Lee
December 7, 2017

Question 10.2: A sociologist is concerned about the effectiveness of a training course designed to get more drivers to use seat belts in automobiles.

(a) What hypothesis is she testing if she commits a type I error by erroneously concluding that the training course is ineffective?

The training course is effective.

(b) What hypothesis is she testing if she commits a type II error by erroneously concluding that the training course is effective?

The training course is effective.

Question 10.4: A fabric manufacturer believes that the proportion of orders for raw material arriving late is $p = 0.6$. If a random sample of 10 orders shows that 3 or fewer arrived late, the hypothesis that $p = 0.6$ should be rejected in favor of the alternative $p < 0.6$. Use the binomial distribution.

(a) Find the probability of committing a type I error if the true proportion is $p = 0.6$

$\alpha = P(X \leq 24 \mid p = 0.6) = 0.0548$

(b) Find the probability of committing a type II error for the alternatives $p = 0.3$, $p = 0.4$, and $p = 0.5$.

For $p = 0.3 : \beta = P(X > 3 \mid p = 0.3) = 1 - 0.6496 = 0.3504$.
For $p = 0.4 : \beta = P(X > 3 \mid p = 0.4) = 1 - 0.3823 = 0.6177$.
For $p = 0.5 : \beta = P(X > 3 \mid p = 0.5) = 1 - 0.1719 = 0.8281$.

Question 10.6: The proportion of adults living in a small town who are college graduates is estimated to be $p = 0.6$. To test this hypothesis, a random sample of 15 adults is selected. If the number of college graduates in the sample is anywhere from 6 to 12, we shall not reject the null hypothesis that $p = 0.6$; otherwise, we shall conclude that $p \neq 0.6$.

(a) Evaluate α assuming that $p = 0.6$. Use the binomial distribution.

$\alpha = P(X \leq 5 \mid p = 0.6) + P(X \geq 13 \mid p = 0.6)$
$0.0338 + (1 - 0.9729)$
$0.0338 + 0.0271 = 0.0609$

(b) Evaluate β for the alternatives $p = 0.5$ and $p = 0.7$.

For $p = 0.5 : \beta = P(6 \leq X \leq 12 \mid p = 0.5)$
$0.9963 - 0.1509 = 0.8454$.

For $p = 0.7 : \beta = P(6 \leq X \leq 12 \mid p = 0.7)$
$0.8732 - 0.0037 = 0.8695$.

(c) Is this a good test procedure?

No, it is not a good test procedure for finding differences in p of size 0.1.

Question 10.8: In *Relief from Arthritis* published by Thorsons Publishers, Ltd., John E. Croft claims that over 40% of those who suffer from osteoarthritis receive measurable relief from an ingredient produced by a particular species of mussel found off the coast of New Zealand. To test this claim, the mussel extract is to be given to a group of 7 osteoarthritic patients. If 3 or more of the patients receive relief, we shall not reject the null hypothesis that $p = 0.4$; otherwise, we conclude that $p < 0.4$.

(a) Evaluate α, assuming that $p = 0.4$.

$n = 7, p = 0.4, \alpha = P(X \leq 2)$
$= 0.4199$.

(b) Evaluate β for the alternative $p = 0.3$.

$n = 5, p = 0.3, \beta = P(X \geq 3)$
$= 1 - P(X \leq 2)$
$= 1 - 0.6471 = 0.3529$

Question 10.14: A manufacturer has developed a new fishing line, which the company claims has a mean breaking strength of 15 kilograms with a standard deviation of 0.5 kilogram. To test the hypothesis that $\mu = 15$ kilograms against the alternative that $\mu < 15$ kilograms, a random sample of 50 lines will be tested. The critical region is defined to be $\bar{x} < 14.9$.

(a) Find the probability of committing a type I error when H_0 is true.

$$n = 50, \mu = 15, \sigma = 0.5, \sigma_{\bar{X}} = 0.5/\sqrt{50} = 0.071, z = 14.9 - 15/0.071 = -1.41$$
$$\alpha = P(Z < -1.41) = 0.0793$$

(b) Evaluate β for the alternatives $\mu = 14.8$ and $\mu = 14.9$ kilograms.

For $\mu = 14.8$, $z = 14.9 - 14.8/0.071 = 1.41$
$$\beta = P(Z > 1.41) = 0.0793$$

For $\mu = 14.9$, $z = 0$ and $\beta = P(Z > 0) = 0.5$.

Question 10.20: A random sample of 64 bags of white cheddar popcorn weighed, on average, 5.23 ounces with a standard deviation of 0.24 ounce. Test the hypothesis that $\mu = 5.5$ ounces against the alternative hypothesis, $\mu < 5.5$ ounces, at the 0.05 level of significance.

$z = \frac{5.23-5.5}{0.24/\sqrt{64}} = -9$, P-value $= P(Z < -9) \approx 0$

Popcorn, on average, weigh less than 5.5 ounces.

Question 10.22: In the American Heart Association journal *Hypertension*, researchers report that individuals who practice Transcendental Meditation (TM) lower their blood pressure significantly. If a random sample of 225 male TM practitioners meditate for 8.5 hours per week with a standard deviation of 2.25 hours, does that suggest that, on average, men who use TM meditate more than 8 hours per week? Quote a P-value in your conclusion.

$z = \frac{8.5-8}{2.25/\sqrt{225}} = 3.33$. P-value $= P(Z > 3.33) = 0.0004$.

Reject hypothesis $\mu = 8$, conclude that men who use TM meditate more than 8 hours a week.

Question 10.24: The average height of females in the freshman class of a certain college has historically been 162.5 centimeters with a standard deviation of 6.9 centimeters. Is there reason to believe that there has been a change in the average height if a random sample of 50 females in the present freshman class has an average height of 165.2 centimeters? Use a P-value in your conclusion. Assume the standard deviation remains the same.

$z = \frac{165.2-162.5}{6.9/\sqrt{50}} = 2.77$. P-value $= 2P(Z > 2.77) = 2 \cdot 0.0028 = 0.0056$

Reject $\mu = 162.5$ and conclude $\mu \neq 162.5$.

Question 10.26: According to a dietary study, high sodium intake may be related to ulcers, stomach cancer, and migraine headaches. The human requirement for salt is only 220 milligrams per day, which is surpassed in most single servings of ready-to-eat cereals. If a random sample of 20 similar servings of a certain cereal has a mean sodium content of 244 milligrams and a standard deviation of 24.5 milligrams, does this suggest at the 0.05 level of significance that the average sodium content for a single serving of such cereal is greater than 220 milligrams? Assume the distribution of sodium contents to be normal.

$\alpha = 0.05$, $df = 19$, Crit: $t > 1.729$.

$t = \frac{244-220}{24.5/\sqrt{20}} = 4.38$

Reject $\mu = 220$ milligrams and conclude $\mu > 220$ milligrams.

Question 10.30: A random sample of size $n_1 = 25$, taken from a normal population with a standard deviation $\sigma_1 = 5.2$, has a mean $\bar{x}_1 = 81$. A second random sample of size $n_2 = 36$, taken from a different normal population with a standard deviation $\sigma_2 = 3.4$, has a mean $\bar{x}_2 = 76$. Test the hypothesis that $\mu_1 = \mu_2$ against the alternative, $\mu_1 \neq \mu_2$. Quote a P-value in your conclusion.

$z = \frac{81-76}{\sqrt{\frac{5.2^2}{25} + \frac{3.4^2}{36}}} = 4.22$

P-value≈ 0

Conclude $\mu_1 \neq \mu_2$.

Question 10.32: *Amstat News* (December 2004) lists median salaries for associate professors of statistics at research institutions and at liberal arts and other institutions in the United States. Assume that a sample of 200 associate professors from research institutions has an average salary of \$70,750 per year with a standard deviation of \$6000. Assume also that a sample of 200 associate professors from other types of institutions has an average salary of \$65,200 with a standard deviation of \$5000. Test the hypothesis that the mean salary for associate professors in research institutions is \$2000 higher than for those in other institutions. Use a 0.01 level of significance.

$\alpha = 0.01$, Crit: $z > 2.33$

$z = \frac{(70750-65200)-2000}{\sqrt{\frac{6000^2}{200}+\frac{5000^2}{200}}} = 6.43$

P-value $= P(Z > 6.43) \approx 0$.

Reject $\mu_1 - \mu_2 = \$2000$ and conclude mean salary for associate professors is higher than in other institutions by \$2000.

Question 10.36: Engineers at a large automobile manufacturing company are trying to decide whether to purchase brand A or brand B tires for the companys new models. To help them arrive at a decision, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are as follows:

$$\begin{aligned} \text{Brand } A\text{: } \bar{x}_1 &= 37,900 \text{ kilometers,} \\ s_1 &= 5100 \text{ kilometers.} \\ \text{Brand } B\text{: } \bar{x}_1 &= 39,800 \text{ kilometers,} \\ s2 &= 5900 \text{ kilometers.} \end{aligned}$$

Test the hypothesis that there is no difference in the average wear of the two brands of tires. Assume the populations to be approximately normally distributed with equal variances. Use a P-value.

$s_p = \sqrt{\frac{5100^2+5900^2}{2}} = 5515$, $t = \frac{37900-39800}{5515\sqrt{\frac{1}{12}+\frac{1}{12}}} = -0.84$

With 22 degrees of freedome and $0.20 < P(T < -0.84) < 0.3$ so then $0.4 < P$-value < 0.6.

There is no difference in the average wear of the two brands of tires.

Question 10.40:

Calculate degrees of freedom: $v = \frac{(\frac{0.391478^2}{8}+\frac{0.214414^2}{24})^2}{\frac{(\frac{0.391478^2}{8})^2}{7}+\frac{(\frac{0.214414^2}{24})^2}{23}} = 8.44 \approx 8$.

$t = \frac{0.97625-0.91583}{\sqrt{\frac{0.391478^2}{8}+\frac{0.214414^2}{24}}} = -0.42$.

$0.3 < P(T < -0.42) < 0.4$

$0.6 < P$-value < 0.8.

Fail to reject hypothesis that $\mu_S = \mu_N$.

Question 10.42:

Criti: $t > 2.776$ with 4 degrees of freedom.

$\bar{d} = -0.1$, $s_d = 0.1414$,

$t = \frac{-0.1}{(\frac{0.1414}{\sqrt{5}})} = -1.58$

Conclude that two methods are not significantly different, $\mu_1 = \mu_2$.

Question 10.44:

Crit: $t > 2.365$ with 7 degrees of freedom.

$\bar{d} = 198.625$, $s_d = 210.165$

$t = \frac{198.625}{(\frac{210.165}{\sqrt{8}})} = 2.67$

Conclude that length of storage affects concentrations.

Question 10.52: For testing

$$H_0 : \mu = 14,$$
$$H_1 : \mu \neq 14,$$

an $\alpha = 0.05$ level t-test is being considered. What sample size is necessary in order for the probability to be 0.1 of falsely failing to reject H_0 when the true population mean differs from 14 by 0.5? From a preliminary sample we estimate to be 1.25.

$\sigma = 1.25$, $\beta = 0.1$, $\delta = 0.5$

$\Delta = \frac{0.5}{1.25} = 0.4$

From Table A.8: $n = 68$.

Question 10.56: Suppose that, in the past, 40% of all adults favored capital punishment. Do we have reason to believe that the proportion of adults favoring capital punishment has increased if, in a random sample of 15 adults, 8 favor capital punishment? Use a 0.05 level of significance.

Binomial: $p = 0.4$, $n = 15$, $x = 8$, $np_0 = 15 \cdot 0.4 = 6$

From Table A.1: P-value $= P(X \geq 8 \mid p = 0.4) = 1 - P(X \leq 7 \mid p = 0.4)$

$= 0.2131$, which is larger than $\alpha = 0.05$, so we do not have a reason to believe that the portion of adults favoring capital punishment has increased.

Question 10.58: It is believed that at least 60% of the residents in a certain area favor an annexation suit by a neighboring city. What conclusion would you draw if only 110 in a sample of 200 voters favored the suit? Use a 0.05 level of significance.

P-value $\approx P(Z < \frac{110-200 \cdot 0.6}{\sqrt{200 \cdot 0.6 \cdot 0.4}})$

$= P(Z < -1.44) = 0.0749$ which is greater than $\alpha = 0.05$, so at least 60% of the residents are in favor of annexation.

Question 10.64: In a study on the fertility of married women conducted by Martin OConnell and Carolyn C. Rogers for the Census Bureau in 1979, two groups of childless wives aged 25 to 29 were selected at random, and each was asked if she eventually planned to have a child. One group was selected from among wives married less than two years and the other from among wives married five years. Suppose that 240 of the 300 wives married less than two years planned to have children some day compared to 288 of the 400 wives married five years. Can we conclude that the proportion of wives married less than two years who planned to have children is significantly higher than the proportion of wives married five years? Make use of a P-value.

$\hat{p} = \frac{240+288}{300+400} = 0.7543$

$z = \frac{\frac{240}{300} - \frac{288}{400}}{\sqrt{0.7543 \cdot 0.2457 \cdot (\frac{1}{300} + \frac{1}{400})}} = 2.44$

P-value $= P(Z > 2.44) = 0.0073$

Conclude that the proportion of wives married less than two years who planned to have children is significantly higher than the proportion of wives married five years.

Question 10.68: Past experience indicates that the time required for high school seniors to complete a standardized test is a normal random variable with a standard deviation of 6 minutes. Test the hypothesis that $\sigma = 6$ against the alternative that $\sigma < 6$ if a random sample of the test times of 20 high school seniors has a standard deviation $s = 4.51$. Use a 0.05 level of significance.

Question 10.70: Past data indicate that the amount of money contributed by the working residents of a large city to a volunteer rescue squad is a normal random variable with a standard deviation of \$1.40. It has been suggested that the contributions to the rescue squad from just the employees of the sanitation department are much more variable. If the contributions of a random sample of 12 employees from the sanitation department have a standard deviation of \$1.75, can we conclude at the 0.01 level of significance that the standard deviation of the contributions of all sanitation workers is greater than that of all workers living in the city?

Question 10.76:

Question 10.78: