CS 373 Homework 7 Anchu A. Lee
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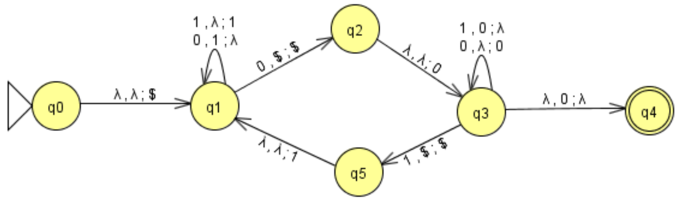
1. Construct a pushdown automata that recognizes $\{w \mid w \text{ is an element of } \{0,1\}^* \text{ and } w \text{ has more 0's than 1's }\}$.



2. Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$A \to BAB \mid B \mid \epsilon$$
$$B \to 00 \mid \epsilon$$

3. Show that the class of context-free languages is closed under the union operation (construction and proof). The construction should be quite simple.

4. Show that the class of context-free languages is closed under the concatenation operation (construction and proof). The construction should be quite simple.

5. Show that the class of context-free languages is closed under the star operation (construction and proof). The construction should be quite simple.

6. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA and define CFG $G = (V, \Sigma, R, S$ as follows:

   - $V = Q$;
   - For each $q \in Q$ and $a \in \Sigma$, define rule $q \to aq'$ where $q' = \delta(q, a)$;
   - For $q \in F$ define rule $q \to \epsilon$;
   - $S = q_0$.

   Prove $L(M) = L(G)$.

7. Let $L = \{0^n 1^m 0^n 1^m \mid n, m \geq 0\}$. Show L is not context-free.

8. Let $L = \{w \mid w \text{ is in } \{a, b, c, d\}^*$, with the number of a's = number of b's and the number of c's = the number of d's $\}$. Show L is not context-free.

9. Let A and B be languages. We define $A \approx B = \{ab \mid a \text{ is an element of } A \text{ and } b \text{ is an element of } B \text{ and } |a| > |b| \ \}$. Show that if A and B are regular languages, then $A \approx B$ is a context free language.

10. Show $L = \{w \mid w$is an element of $\{a, b, c, d, e, f\}^*$ such that the number of a's + number of b's = number of c's + number of d's = number of e's + number of f's$\}$ is not context-free.