CS 373 Homework 7 Anchu A. Lee
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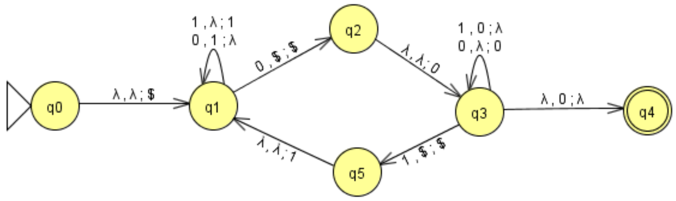
1. Construct a pushdown automata that recognizes $\{w \mid w$ is an element of $\{0,1\}^*$ and w has more 0's than 1's $\}$.



2. Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$A \to BAB \mid B \mid \epsilon$$
$$B \to 00 \mid \epsilon$$

$S \to BC \mid AB \mid BA \mid BB \mid DD \mid \epsilon$
$A \to BC \mid AB \mid BA \mid BB \mid DD$
$B \to DD$
$C \to AB$
$D \to 0$

3. Show that the class of context-free languages is closed under the union operation (construction and proof). The construction should be quite simple.

Proof. Define two context-free languages: $G_1 = (V_1, \Sigma, R_1, S_1)$ and $G_2 = (V_2, \Sigma, R_2, S_2)$ and also the language $G_U = (V_1 \cup V_2 \cup \{S\}, \Sigma, R_1 \cup R_2 \cup \{S \to S_1 \mid S_2\}, S)$ which is the union of G_1 and G_2 as the start variable of G_U points to both start variables of G_1 and G_2. Additionally the rules and variables are shared (assuming the rules and variables are disjoint). After the start variable of G_U, subsequent steps use rules exclusively from G_1 or G_2, not both. therefore all productions of G_U must be in the languages G_1 or G_2. □

4. Show that the class of context-free languages is closed under the concatenation operation (construction and proof). The construction should be quite simple.

Proof. Define two context-free languages: $G_1 = (V_1, \Sigma, R_1, S_1)$ and $G_2 = (V_2, \Sigma, R_2, S_2)$ and also the language $G_C = (V_1 \cup V_2 \cup \{S\}, \Sigma, R_1 \cup R_2 \cup \{S \to S_1 S_2\}, S)$ which is the concatenation of G_1 and G_2 ad the start variable of G_C concatenates both the start variables of G_1 and G_2. So G_C produces words that start with G_1 and end with G_2, thus all productions of G_C must be concatenations of G_1 and G_2. □

5. Show that the class of context-free languages is closed under the star operation (construction and proof). The construction should be quite simple.

Proof. Define the context-free language $G_1 = (V, \Sigma, R, S)$. The star of this language would have to be able to generate Σ or a countably infinite amount of copies. So the start state would have to $S_0 \to \epsilon \mid S_0 S$. Therefore the language $G_S = (V, \Sigma, R \cup \{S_0 \to \epsilon \mid S_0 S\}, S_0)$ generates either ϵ or a sequence of many words in G_1. □

6. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA and define CFG $G = (V, \Sigma, R, S)$ as follows:

- $V = Q$;
- For each $q \in Q$ and $a \in \Sigma$, define rule $q \to aq'$ where $q' = \delta(q, a)$;
- For $q \in F$ define rule $q \to \epsilon$;
- $S = q_0$.

Prove $L(M) = L(G)$.

Proof. A language is regular if a DFA accepts it, so $L(M)$ is a regular language. By Corollary 2.32, the language must also be context-free. In order for $L(M) = L(G)$, the construction of G must be a direct translation of a DFA to GFA. G converts the states of a DFA to variables, defines rules that function similarly to the transition functions and uses a rule that moves to ϵ instead of using accept states. This construction successfully translates a DFA into a CFG. □

7. Let $L = \{0^n 1^m 0^n 1^m \mid n, m \geq 0\}$. Show L is not context-free.

Proof. Assume L is context-free. Let p be the pumping length. Let $w = 0^p 1^p 0^p 1^p$, which means the options for vxy are 0^p, $0^p 1^p$, 1^p, $1^p 0^p$. Each option is really two options as the string is $0^p 1^p$ twice. If 0^p is pumped up, then there will be too many characters in one of the zero's. If $0^p 1^p$ is pumped up, then the left side of the word will be longer than the right side. If 1^p is pumped up then one of 1's will have too many characters than the other 1's. If $1^p 0^p$ is pumped up then the middle two 1's and 0's will be larger than the outer 1's and 0's when they need to be equal. Since every case of $uvxyz0$ is not in L, the language cannot be context-free. □

8. Let $L = \{w \mid w$ is in $\{a, b, c, d\}^*$, with the number of a's = number of b's and the number of c's = the number of d's $\}$. Show L is not context-free.

Proof. Assume L is a context-free language, let p be the pumping length. Let $w = a^p b^p c^p d^p$, which means the options for vxy are a^p, $a^p b^p$, b^p, $b^p c^p$, c^p, $c^p d^p$, d^p. If a^p is pumped up, then the number of a's do not equal to the number of b's. If b^p is pumped up, then the number of b's do not equal the number of a's. If $b^p c^p$ is pumped up, then the number of b's do not equal the number of a's and the number of c's do not equal the number of d's. If c^p is pumped up then the number of c's do not equal the number of d's. If d^p is pumped up then the number of d's do not equal the number of c's. That leaves only $a^p b^p$ and $c^p d^p$ that still remain in the language if pumped up. However, $|a^p b^p| \leq p$ which means p must be equal to 0. However $|xy|$ must be greater than zero and contradicts the only value of p that would make the first case of the pumping lemma true. The same is true for $c^p d^p$, so L must not be context-free. □

9. Let A and B be languages. We define $A \approx B = \{ab \mid a$ is an element of A and b is an element of B and $|a| > |b|\ \}$. Show that if A and B are regular languages, then $A \approx B$ is a context-free language.

Proof. $A \approx B$ must be accepted by a PDA if it is context-free. So let C be a PDA that recognizes A and D be a PDA that recognizes B. Then let G be a PDA that recognizes $A \approx B$. The start state of G would simply push a \$ to the stack and transition to the start state of C. Every transition in C also pushes a 0 onto the stack. Each accept state in C ϵ-transitions to the start state of D. Every transition in D also pops a 0 off the stack. If D reaches it's accept state and there are still 0's on the stack, then it transitions into the accept state of G. G then will only accept words a concatenate with b so long as $a > b$. □

10. Show $L = \{w \mid w$is an element of $\{a, b, c, d, e, f\}^*$ such that the number of a's + number of b's = number of c's + number of d's = number of e's + number of f's$\}$ is not context-free.