Math 327 Homework 3 Anchu A. Lee
September 20, 2017

Question 3.4
$S = \{HHH, THHH, HTHHH, TTHHH, TTTHHH, HTTHHH, THTHHH, HHTHHH\}$
S is discrete because you cannot flip a fraction of a heads or tails.

Question 3.10
The probability of rolling any side of a fair six sided die is $\frac{1}{6}$, so the formula for probability distribution is $f(x) = \frac{1}{6}$ for $x = 1, 2, 3, 4, 5, 6$ Equal chance of getting any side.

Question 3.12

- $P(T = 5) = F(5) - F(4) = \frac{1}{4}$
- $P(T > 3) = 1 - F(3) = \frac{1}{2}$
- $P(1.4 < T < 6) = F(6) - F(1.4) = \frac{1}{2}$
- $P(T \leq 5 \mid T \geq 2) = \frac{P(2 \leq T \leq 5)}{P(T \geq 2)} = \frac{F(5)-F(2)}{1-F(2)} = \frac{2}{3}$

Question 3.14
$x = \frac{12}{60}$

- $F(x) = F(0.2) = 1 - e^{-8(0.2)} = 0.79810...$
- $f(x) = \frac{dF}{dx} = 8e^{-8x}$ when $x > 0$
  $\int_0^{0.2} f(x)dx = \int_0^{0.2} 8e^{-8x}dx = 8\int_0^{0.2} e^{-8x}dx = -e^{-8x}\Big|_0^{0.2} = 0.79810...$

Question 3.18

- $P(X < 4) = \int_2^4 \frac{2(1+x)}{27}dx = 0.59259...$
- $P(3 \leq X < 4) = \int_3^4 \frac{2(1+x)}{27}dx = 0.33333...$

Question 3.20
$F(x) = \int_2^x \frac{2(1+t)}{27}dt = \frac{2}{27} \cdot \int_2^x 1 + tdt = \frac{2}{27}\left(t + \frac{t^2}{2}\right)\Big|_2^x = \frac{(x+4)(x-2)}{27}$
$P(3 \leq X < 4) = F(4) - F(3) = \frac{(4+4)(4-2)}{27} - \frac{(3+4)(3-2)}{27} = 0.33333...$

Question 3.24
$\binom{10}{4}$ ways of selecting 4 CDs from 10. We want x number of jazz CDs from 5 $\binom{5}{x}\binom{5}{4-x}$
$f(x) = \frac{\binom{5}{x}\binom{5}{4-x}}{\binom{10}{4}}$
$x = 0, 1, 2, 3, 4$

Question 3.30

- $1 = k\int_{-1}^{1}(3 - x^2) = \frac{16}{3}k$, $k = \frac{3}{16}$
- $P(X < 0.5) = \int_{-1}^{0.5} \frac{3}{16}(3 - x^2)dx = 0.7734375$
- $F(x) = \int_{-1}^{x} \frac{3}{16}(3 - t^2)dt = (3t - \frac{1}{3}t^3)\big|_{-1}^{x} = \frac{1}{2} + \frac{9}{16}x - \frac{x^3}{16}$
  $P(|X| < 0.7) = P(X < -0.8) + P(X > 0.8) = F(-0.8) + 1 - F(0.8) = 0.164$

Question 3.32

- $\int_0^1 5(1 - y)^4 dy = 1$

- $P(Y < 0.1) = \int_0^{0.1} 5(1-y)^4 dy = 0.40951$
- $P(Y > 0.5) = (1-0.5)^5 = 0.03125$

Question 3.38

- $P(X \leq 2, Y = 1) = f(0,1) + f(1,1) + f(2,1) = \frac{1}{30} + \frac{2}{30} + \frac{3}{30} = \frac{1}{5}$
- $P(X > 2, Y \leq 1) = f(3,0) + f(3,1) = \frac{3}{30} + \frac{4}{30} = \frac{7}{30}$
- $P(X > Y) = f(1,0) + f(2,0) + f(3,0) + f(2,1) + f(3,1) + f(3,2) = \frac{1}{30} + \frac{2}{30} + \frac{3}{30} + \frac{3}{30} + \frac{4}{30} + \frac{5}{30} = \frac{18}{30}$
- $P(X + Y = 4) = f(1,3) + f(2,2) = \frac{4}{30} + \frac{4}{30} = \frac{8}{30}$

Question 3.40

- $g(x) = \int_0^1 \frac{2}{3}(x+2y)dy = \frac{2}{3}\int_0^1 (x+2y)dy = \frac{2}{3}(x+1)$, for $0 \leq x \leq 1$
- $h(y) = \int_0^1 \frac{2}{3}(x+2y)dx = \frac{1}{3}(1+4y)$, for $0 \leq y \leq 1$
- $P(X < 0.5) = \int_0^{0.5} \frac{2}{3}(x+1)dx = 0.41666...$

Question 3.44

- $1 = \int_{30}^{50}\int_{30}^{50} k(x^2+y^2)dxdy = k\int_{30}^{50} 20y^2 + \frac{98000}{3}dy = k\frac{3920000}{3}$, so $k = \frac{3}{3920000}$
- $P(30 \leq X \leq 40, 40 \leq Y < 50) = \frac{3}{3920000}\int_{40}^{50}\int_{30}^{40}(x^2+y^2)dxdy = \frac{3}{3920000}\int_{40}^{50} 10y^2 + \frac{37000}{3}dy = \frac{3}{3920000}\frac{980000}{3} = 0.25$
- $P(30 \leq X \leq 40, 30 \leq Y < 40) = \frac{3}{3920000}\int_{30}^{40}\int_{30}^{40}(x^2+y^2)dxdy = \frac{3}{3920000}\int_{30}^{40} 10y^2 + \frac{37000}{3}dy = \frac{3}{3920000}\frac{740000}{3} = 0.18877...$

Question 3.46

- $g(0) = \frac{1}{30} + \frac{2}{30} = \frac{3}{30}$
  $g(1) = \frac{1}{30} + \frac{2}{30} + \frac{3}{30} = \frac{6}{30}$
  $g(2) = \frac{2}{30} + \frac{3}{30} + \frac{4}{30} = \frac{9}{30}$
  $g(3) = \frac{3}{30} + \frac{4}{30} + \frac{5}{30} = \frac{12}{30}$
- $h(0) = \frac{1}{30} + \frac{2}{30} + \frac{3}{30} = \frac{6}{30}$
  $h(1) = \frac{1}{30} + \frac{2}{30} + \frac{3}{30} + \frac{4}{30} = \frac{10}{30}$
  $h(2) = \frac{2}{30} + \frac{3}{30} + \frac{4}{30} + \frac{5}{30} = \frac{14}{30}$

Question 3.50

- $g(2) = 0.10 + 0.20 + 0.10 = 0.40$
  $g(4) = 0.15 + 0.30 + 0.15 = 0.60$
- $h(1) = 0.10 + 0.15 = 0.25$
  $h(3) = 0.20 + 0.30 = 0.50$
  $h(5) = 0.10 + 0.15 = 0.25$

Question 3.68

- $g(x) = \int_1^2 \frac{3x-y}{9}dy = \frac{1}{6}(2x-1)$ for $1 < x < 3$
  $h(y) = \int_1^3 \frac{3x-y}{9}dx = -\frac{2}{9}(y-6)$ for $1 < y < 2$
- No, because $g(x)g(y) \neq f(x,y)$
- $P(X > 2) = \int_2^3 \frac{1}{6}(2x-1)dx = \frac{2}{3}$

Question 3.80

Let X be the number of components that work. $P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$
$P(X = 3) = \binom{5}{3}(0.92)^3(1 - 0.92)^2 = 0.49836...$
$P(X = 4) = \binom{5}{4}(0.92)^4(1 - 0.92) = 0.28656...$
$P(X = 5) = \binom{5}{5}(0.92)^5 = 0.65908...$
$P(X \geq 3) = 0.99547...$