CS 373 Homework 9 Anchu A. Lee
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1. Let $\text{Some}_{DFA} = \{ \langle A \rangle \mid A$ is a DFA and $L(A)$ is not empty and $L(A)$ is not equal to Σ^* $\}$. Show that Some_{DFA} is decidable.

   Construct a Turing machine M to decide the Some_{DFA} problem.

   On input d:

   1. Check if d is a DFA, if not reject.
   2. Run $\langle d \rangle$ on a Turing machine T that decides E_{DFA}, if T rejects then continue, if T accepts, then reject.
   3. Construct a DFA d^C that is the compliment of d.
   4. Run $\langle \text{d}^C \rangle$ on a turing machine T, if T rejects then accept d, if T accepts then reject d.

   If d is not a DFA, then M rejects. If $L(d)$ is empty, then M rejects. If the compliment of d is empty, meaning $L(d) = \Sigma^*$, then M rejects. Otherwise M accepts. All conditions are handled in M so $M = \text{Some}_{DFA}$.

2. Let $Alot_{RE} = \{ \langle A \rangle \mid A$ is a regular expression and $L(A)$ is infinite $\}$. Show that $Alot_{RE}$ is decidable.

3. Let $Complimentary_{RE,DFA} = \{ \langle A, B \rangle \mid A$ is a regular expression and B is a DFA such that $L(A) \cup L(B) = \Sigma^*$ and $L(A) \cap L(B) = \emptyset \}$. Show that $Complimentary_{RE,DFA}$ is decidable.

4. Let $ALL_{DFA} = \{ \langle A \rangle \mid A$ is a DFA and $L(A) = \Sigma^* \}$. Show that ALL_{DFA} is decidable.

5. Let $N_{\varepsilon CFG} = \{ \langle G \rangle \mid G$ is a CFG and G does not generate the empty string $\}$. Show that $N_{\varepsilon CFG}$ is decidable.

6. Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. We describe the functions $f : X \to Y$ and $g : X \to Y$ in the following tables. Answer each part and give a reason for each negative answer.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

n	$g(n)$
1	10
2	9
3	8
4	7
5	6

   (a) Is f onto?
   (b) Is f a correspondence?
   (c) Is g onto?
   (d) Is g a correspondence?

7. Let $U = \{ \langle A, B, C \rangle \mid A, B, C$ are DFA's and $|L(A)| = |L(B)| + |L(C)|$ $\}$. Show that U is decidable.

8. Let $A = \{ \langle R \rangle \mid R$ is a regular expression describing a language containing at least one string w that has 111 as a substring (i.e., $w = x111y$ for some x and y)$\}$. Show that A is decidable.

9. Let $E_{PDA} = \{ \langle P \rangle \mid P$ is a pushdown automata and $L(P)$ is empty $\}$. Show E_{PDA} is decidable.

10. A ***useless state*** in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.