CS 373 Homework 8 Anchu A. Lee
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1. Show that EQ_{CFG} is undecidable.

   was in class

2. Show that EQ_{CFG} is co-Turing-recognizable.

   was in class

3. Find a match in the following instance of the Post Correspondence Problem.

$$\{[\frac{ab}{abab}], [\frac{b}{a}], [\frac{aba}{b}], [\frac{aa}{a}]\}$$

   was in class

4. Show that the Post Correspondence Problem is decidable over the unary alphabet $\Sigma = \{1\}$.

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5. In the *silly Post Correspondence Problem, SPCP*, the top string in each pair has the same length as the bottom string. Show that the *SPCP* is decidable.

   was in class

6. Show that A is Turing-recognizable iff $A \leq_m A_{TM}$

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7. **Rices theorem**. Let P be any nontrivial property of the language of a Turing machine. Prove that the problem of determining whether a given Turing machines language has property P is undecidable.
   In more formal terms, let P be a language consisting of Turing machine descriptions where P fulfills two conditions. First, P is nontrivial - it contains some, but not all, TM descriptions. Second, P is a property of the TMs language- whenever $L(M_1) = L(M_2)$, we have $\langle M_1 \rangle \in P$ iff $\langle M_2 \rangle \in P$. Here, $M_1 and M_2$ are any TMs. Prove that P is an undecidable language.

   Assume for the sake of contradiction that P is a decidable language satisfying the properties and let R_P be a TM that decides P. We show how to decide A_{TM} using R_P by constructing TM S. First, let $T_\emptyset$ be a TM that always rejects, so $L(T_\emptyset) = \emptyset$. You may assume that $\langle T_\emptyset \rangle \notin P$ without loss of generality because you could proceed with P instead of P if $\langle T_\emptyset \rangle \in P$. Because P is not trivial, there exists a TM T with $\langle T \rangle \in P$. Design S to decide A_{TM} using R_Ps ability to distinguish between $T_\emptyset$ and T.

   $S =$"On input $\langle M, w \rangle$:
   1. Use M and w to construct the following TM M_w.
      $M_w =$ "On input x:
      1. Simulate M on w. If it halts and rejects, reject. If it accepts, proceed to stage 2.
      2. Simulate T on x. If it accepts, accept."
   2. Use TM R_P to determine whether $\langle M_w \rangle \in P$. If YES, accept. if NO, reject."

   TM M_w simulates T if M accept w. Hence $L(M_w)$ equals $L(T)$ if M accepts w and $\emptyset$ otherwise. Therefore $\langle M_w \rangle \in P$ iff M accepts w

8. Let

$$f(x) = \begin{cases} 3x + 1 & \text{for odd x} \\ x/2 & \text{for even x} \end{cases}$$

for any natural number x. If you start with an integer x and iterate f, you obtain a sequence, $x, f(x), f(f(x)), \ldots$. Stop if you ever hit 1. For example, if $x = 17$, you get the sequence 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. Extensive computer tests have shown that every starting point between 1 and a large positive integer gives a sequence that ends in 1. But the question of whether all positive starting points end up at 1 is unsolved; it is called the $3x + 1$ problem. Suppose that A_{TM} were decidable by a TM H. Use H to describe a TM that is guaranteed to state the answer to the $3x + 1$ problem.

was in class

9. Let $T = \{(i, j, k) \mid i, j, k \in N\}$. Show that T is countable.

   was in class

10. Review the way that we define sets to be the same size in Definition 4.12 (page 203). Show that is the same size is an equivalence relation.

    was in class