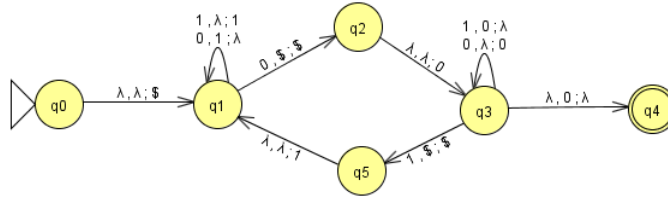


- Construct a pushdown automata that recognizes $\{w \mid w \text{ is an element of } \{0,1\}^* \text{ and } w \text{ has more 0's than 1's}\}$.



- Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$A \rightarrow BAB \mid B \mid \epsilon$$

$$B \rightarrow 00 \mid \epsilon$$

$$S \rightarrow BC \mid AB \mid BA \mid BB \mid DD \mid \epsilon$$

$$A \rightarrow BC \mid AB \mid BA \mid BB \mid DD$$

$$B \rightarrow DD$$

$$C \rightarrow AB$$

$$D \rightarrow 0$$

- Show that the class of context-free languages is closed under the union operation (construction and proof). The construction should be quite simple.

Proof. Define two context-free languages: $G_1 = (V_1, \Sigma, R_1, S_1)$ and $G_2 = (V_2, \Sigma, R_2, S_2)$ and also the language $G_U = (V_1 \cup V_2 \cup \{S\}, \Sigma, R_1 \cup R_2 \cup \{S \rightarrow S_1 \mid S_2\}, S)$ which is the union of G_1 and G_2 as the start variable of G_U points to both start variables of G_1 and G_2 . Additionally the rules and variables are shared (assuming the rules and variables are disjoint). After the start variable of G_U , subsequent steps use rules exclusively from G_1 or G_2 , not both. therefore all productions of G_U must be in the languages G_1 or G_2 . \square

- Show that the class of context-free languages is closed under the concatenation operation (construction and proof). The construction should be quite simple.

Proof. Define two context-free languages: $G_1 = (V_1, \Sigma, R_1, S_1)$ and $G_2 = (V_2, \Sigma, R_2, S_2)$ and also the language $G_C = (V_1 \cup V_2 \cup \{S\}, \Sigma, R_1 \cup R_2 \cup \{S \rightarrow S_1 S_2\}, S)$ which is the concatenation of G_1 and G_2 and the start variable of G_C concatenates both the start variables of G_1 and G_2 . So G_C produces words that start with G_1 and end with G_2 , thus all productions of G_C must be concatenations of G_1 and G_2 . \square

- Show that the class of context-free languages is closed under the star operation (construction and proof). The construction should be quite simple.

Proof. Define the context-free language $G_1 = (V, \Sigma, R, S)$. The star of this language would have to be able to generate Σ or a countably infinite amount of copies. So the start state would have to $S_0 \rightarrow \epsilon \mid S_0 S$. Therefore the language $G_S = (V, \Sigma, R \cup \{S_0 \rightarrow \epsilon \mid S_0 S\}, S_0)$ generates either ϵ or a sequence of many words in G_1 . \square

- Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA and define CFG $G = (V, \Sigma, R, S)$ as follows:

- $V = Q$;
- For each $q \in Q$ and $a \in \Sigma$, define rule $q \rightarrow aq'$ where $q' = \delta(q, a)$;
- For $q \in F$ define rule $q \rightarrow \epsilon$;
- $S = q_0$.

Prove $L(M) = L(G)$.

Proof. A language is regular if a DFA accepts it, so $L(M)$ is a regular language. By Corollary 2.32, the language must also be context-free. In order for $L(M) = L(G)$, the construction of G must be a direct translation of a DFA to GFA. G converts the states of a DFA to variables, defines rules that function similarly to the transition functions and uses a rule that moves to ϵ instead of using accept states. This construction successfully translates a DFA into a CFG. \square

7. Let $L = \{0^n 1^m 0^n 1^m \mid n, m \geq 0\}$. Show L is not context-free.

Proof. Assume L is context-free. Let p be the pumping length. Let $w = 0^p 1^p 0^p 1^p$, which means the options for vxy are 0^p , $0^p 1^p$, 1^p , $1^p 0^p$. Each option is really two options as the string is $0^p 1^p$ twice. If 0^p is pumped up, then there will be too many characters in one of the zero's. If $0^p 1^p$ is pumped up, then the left side of the word will be longer than the right side. If 1^p is pumped up then one of 1's will have too many characters than the other 1's. If $1^p 0^p$ is pumped up then the middle two 1's and 0's will be larger than the outer 1's and 0's when they need to be equal. Since every case of $uvxyz0$ is not in L , the language cannot be context-free. \square

8. Let $L = \{w \mid w \text{ is in } \{a, b, c, d\}^*, \text{ with the number of a's} = \text{number of b's and the number of c's} = \text{the number of d's}\}$. Show L is not context-free.
9. Let A and B be languages. We define $A \approx B = \{ab \mid a \text{ is an element of } A \text{ and } b \text{ is an element of } B \text{ and } |a| > |b|\}$. Show that if A and B are regular languages, then $A \approx B$ is a context free language.
10. Show $L = \{w \mid w \text{ is an element of } \{a, b, c, d, e, f\}^* \text{ such that the number of a's} + \text{number of b's} = \text{number of c's} + \text{number of d's} = \text{number of e's} + \text{number of f's}\}$ is not context-free.