Midterm 1: Chapters 1 to 4

1	2	3	4	5	6	7	8	9	10
22.43	10.25	23.71	21.77	22.11	18.71	19.77	20.33	20.17	21.12

(a) $\bar{x} = \frac{\Sigma_{i=1}^{10} x_i}{10} = 20.037$. $\tilde{x} = \frac{20.33+21.12}{2} = 20.725$
(b) $s_x^2 = \frac{\Sigma(x_i-\bar{x})^2}{n-1} = 13.935$. Split into 4 sections, numbers seperating are quartiles. Last minus first is IQR $= 22.11 - 19.77 = 2.34$
(c) Trimmed mean of 10%: remove 10% from highest and lowest. $= 20.801$. Close to median but more than mean; data is slightly skewed to the left.
(d) Set decimal point to |.
(e) Away from Q1 and Q3 by 1.5·IQR are outliers. Left dot is minimum, start of box is Q1, middle line is median, end of box is Q3, last dot is maximum.

A: polluted, B: test detects pollution, $P(A) = 0.2$, $P(B \mid A) = 0.60$, $P(B \mid A') = 0.3$
(a) $P(A \cap B) = P(A)P(B \mid A) = 0.2 \cdot 0.6 = 0.12$
(b) $P(B) = P(B \cap A) + P(B \cap A') = 0.12 + P(A')P(B \mid A') = 0.12 + 0.8 \cdot 0.3 = 0.36$
(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.36 - 0.12 = 0.44$
(d) $P(A' \mid B') = \frac{P(A' \cap B')}{P(B')} = \frac{1-0.44}{0.64} = 0.875$ (e) $P(P(A \cap B) \neq 0)$ so are not mutually exclusive. $P(A \cap B) \neq P(A)P(B)$ so are not independent.

X number of cash registers being used for location 1, Y the number used at the same time for location 2.

$f(x,y)$		$y=0$	$y=1$	$y=2$
x	0	0.10	0.05	0.05
	1	0.10	0.20	0.05
	2	0.05	0.10	0.30
$h(y)$		0.25	0.35	0.40

(a) Marginal probability mass functions, add the rows for X; columns for Y.
(b) Cumulative distribution function of X: 0 if $x < 0$, 0.2 if $0 \leq x < 1$. 0.55 if $1 \leq x < 2$. 1 if $x \geq 2$. So $F(1.5) = 0.55$.
(c) Conditional distribution of Y given $X = 2$, $f(y \mid X = 2)$: $\frac{0.05}{0.45}$ when $y = 0$, $\frac{0.10}{0.45}$ when $y = 1$, $\frac{0.30}{0.45}$ when $y = 2$.
(d) Mean of X: $\mu_X = E(x) = \Sigma_{x=0}^{2} x g(x) = 0.35 + 2 \cdot 0.45 = 1.25$ Variance of X: $\sigma_X^2 = E(X^2) - \mu_X^2 = \Sigma_{x=0}^{2} x^2 g(x) - \mu_X^2 = 0.35 + 4 \cdot 0.45 - 1.25^2 = 0.5875$
(e) $\sigma_{XY} = E(XY) - \mu_X \mu_Y = (0.20 \cdot 1 \cdot 1 + 0.10 \cdot 2 \cdot 1 + 0.05 \cdot 1 \cdot 2 + 0.30 \cdot 2 \cdot 2) - 1.25 \cdot 1.15 = 0.2625 \neq 0$ so X and Y are not independent.

X be a continuous random variable with probability density function $f(x) = Cx^2$ if $-2 < x < 1$ and zero otherwise.
(a) Find C, it must make the function equal to 1 for the interval. $1 = \int_{-2}^{1} Cx^2 dx$, $C = \frac{1}{3}$
(b) $\mu = E[X] = \int_{-2}^{1} x(\frac{1}{3}x^2)dx = -\frac{5}{4}$. $\sigma^2 = E[X^2] - \mu^2 = \int_{-2}^{1} x^2(\frac{1}{3}x^2)dx - (-\frac{5}{4})^2 = 0.6375$
(c) $P[X < -1] = P(-2 < x < -1) = \int_{-2}^{-1} \frac{1}{3}x^2 dx = \frac{7}{9}$
(d) $P[-1 < X \leq 3] = P(-1 < X < 1) + P(1 < X < 3) = \int_{-1}^{1} \frac{1}{3}x^2 + \int_{1}^{3} \frac{1}{3}x^2 = \frac{2}{9} + 0 = \frac{2}{9}$
(e) $g(X) = 4X - 3$. $\mu_{g(X)} = \int_{-2}^{1} g(x)f(x)dx = \int_{-2}^{1} (4X-3)(\frac{1}{3}x^2) = -8$. $\sigma_{g(X)}^2 = E(g(X)^2) - \mu_{g(X)}^2 = \int_{-2}^{1} (4X-3)^2(\frac{1}{3}x^2) - 64 = 10.2$