Math 327 Homework 3 Anchu A. Lee
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Question 4.2

x	0	1	2	3
$f(x)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

$E(X) = \sum_{x=0}^{3} x \cdot f(x) = 0 \cdot \frac{27}{64} + 1 \cdot \frac{27}{64} + 2 \cdot \frac{9}{64} + 3 \cdot \frac{1}{64} = \frac{3}{4}$

Question 4.4
$P(H) = \frac{3}{4}$, $P(T) = \frac{1}{4}$

x	0	1	2
$f(x)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

$E(X) = 0 \cdot \frac{9}{16} + 1 \cdot \frac{6}{16} + 2 \cdot \frac{1}{16} = \frac{1}{2}$

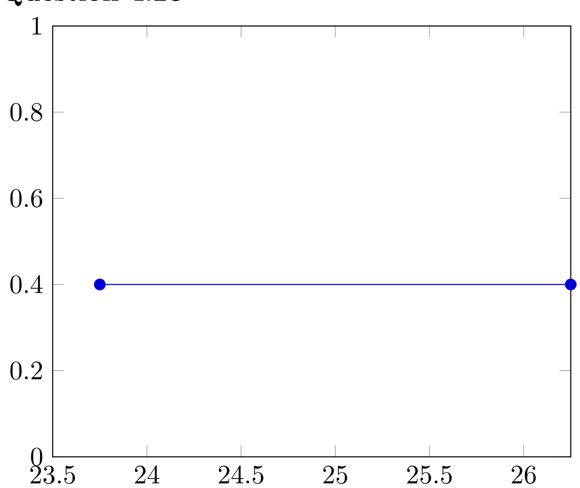
Question 4.10
$\mu_X = 1 \cdot 0.17 + 2 \cdot 0.50 + 3 \cdot 0.33 = 2.16$
$\mu_Y = 1 \cdot 0.23 + 2 \cdot 0.50 + 3 \cdot 0.27 = 2.04$

Question 4.14
$E(X) = \int_0^1 \frac{x \cdot 2(x+2)}{5} dx = \frac{8}{15}$

Question 4.18
$E(X^2) = \sum_{x=0}^{3} x^2 \cdot f(x) dx = 0 \cdot \frac{27}{64} + 1 \cdot \frac{27}{64} + 4 \cdot \frac{9}{64} + 9 \cdot \frac{1}{64} = \frac{9}{8}$

Question 4.20
$E(e^{2X/3}) = \int_0^\infty e^{2x/3} \cdot e^{-x} dx = 3$

Question 4.28



$E(X) = \int_{23.75}^{26.25} \frac{2}{5} x dx = 25$
Not suprised, this is the expected value as it is exactly in the middle of the interval.

Question 4.34
$\mu_X = (-2)(0.3) + (3)(0.2) + (5)(0.5) = 2.5$
$\sigma_X^2 = \sum_x x^2 f(x) - \mu_X^2$
$\sum_x x^2 f(x) = (4)(0.3) + (9)(0.2) + (25)(0.5) = 15.5$
$\sigma_X^2 = 15.5 - 2.5^2 = 9.25$
$\sigma = 3.04138...$

Question 4.38
$\mu_X = \frac{8}{15}$
$\sigma_X^2 = \int_0^1 x^2 \frac{2(x+2)}{5} dx - \mu_X^2$
$\int_0^1 x^2 \frac{2(x+2)}{5} dx = \frac{11}{30}$
$\sigma_X^2 = \frac{11}{30} - (\frac{8}{15})^2 = \frac{37}{450}$

Question 4.40
$g(X) = 3X^2 + 4$
$\mu_{g(X)} = \int_0^1 (3x^2 + 4) \frac{2(x+2)}{5} dx = 5.1$
$\sigma_{g(X)}^2 = \int_0^1 (3x^2 + 4)^2 \frac{2(x+2)}{5} dx - \mu_X^2$
$\int_0^1 (3x^2 + 4)^2 \frac{2(x+2)}{5} dx = \frac{671}{25}$
$\sigma_{g(X)}^2 = \frac{671}{25} - (5.1)^2 = 0.83$

Question 4.46
$k = (\frac{3}{392})10^{-4}$, $g(x) = k(20x^2 + \frac{98000}{3})$
$u_X = \int_{30}^{50} x \cdot (\frac{3}{392})10^{-4}(20x^2 + \frac{98000}{3})dx = 40.81632...$
$u_Y = \int_{30}^{50} y \cdot (\frac{3}{392})10^{-4}(20y^2 + \frac{98000}{3})dy = 40.81632...$
$E(XY) = \int_{30}^{50} \int_{30}^{50} kxy(x^2 + y^2)dydx = k \int_{30}^{50} 800x^3 + 1360000x dx$
$E(XY) = k \int_{30}^{50} 800x^3 + 1360000x dx = 1665.30612...$
$\sigma_{XY} = E(XY) - u_X u_Y = -0.6642...$

Question 4.58
$E(Y) = 60E(X^2) + 39E(X)$
$E(X) = \int_0^1 x^2 + \int_1^2 x(2-x) = 1$
$E(X^2) = \int_0^1 x^3 + \int_1^2 x^2(2-x) = \frac{7}{6}$
$E(Y) = (60)(\frac{7}{6}) + (39)(1) = 109$ kwh

Question 4.60
$E(2X - 3Y) = E(2X) - E(3Y)$
$E(X) = (2)(0.40) + (5)(0.60) = 3.20$
$E(Y) = (1)(0.25) + (3)(0.50) + (5)(0.25) = 3$

- $E(2X - 3Y) = 2E(X) - 3E(Y) = (2)(3.20) - (3)(3) = -2.60$
- $E(XY) = E(X)E(Y) = (3.20)(3) = 9.60$

Question 4.62
$\sigma_Z^2 = \sigma_{-2X+4Y-3}^2 = (-2)^2\sigma_X^2 + 4^2\sigma_Y^2 = 4\sigma_X^2 + 16\sigma_Y^2$
$4\sigma_X^2 + 16\sigma_Y^2 = (4)(5) + (16)(3) = 68$

Question 4.64
$E(X) = \int_2^\infty \frac{8}{x^3}$
$E(Y) = \int_0^1 2y$
$E(Z) = E(XY) = E(X)E(Y) = \int_2^\infty x\frac{8}{x^3} \cdot \int_0^1 2y^2 = \frac{8}{3}$

Question 4.76
$\mu = 60$ and $\sigma = 6$
By Chebyshev's theorem: $1 - \frac{1}{k^2} \leq P(\mu - k\sigma < X < \mu + k\sigma)$
$84 = \mu + k\sigma$, $k = 4$
$\mu - k\sigma = 60 - (4)(6) = 36$
$1 - \frac{1}{16} \leq P(36 < X < 84) \leq P(X < 84)$

$1 - \frac{1}{16} = 0.9375$
$P(X \geq 4) \leq 1 - 0.9375 = 0.0625$

Question 4.92
No, X and Y are not independent because $g(0)h(0) = (0.17)(0.23)$ while $f(0,0) = 0.10$

Question 4.98