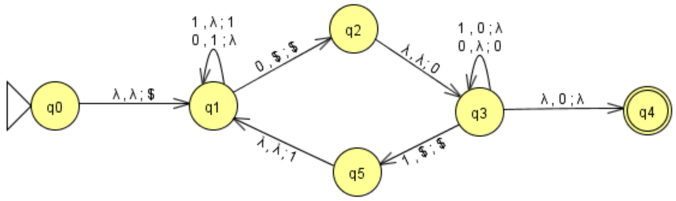
1. Construct a pushdown automata that recognizes $\{w \mid w \text{ is an element of } \{0,1\}^* \text{ and } w \text{ has more 0's than 1's }\}$.



2. Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$A \to BAB \mid B \mid \epsilon$$
$$B \to 00 \mid \epsilon$$

$S \to BC \mid AB \mid BA \mid BB \mid DD \mid \epsilon$
$A \to BC \mid AB \mid BA \mid BB \mid DD$
$B \to DD$
$C \to AB$
$D \to 0$

3. Show that the class of context-free languages is closed under the union operation (construction and proof). The construction should be quite simple.

Proof. Define two context-free languages: $G_1 = (V_1, \Sigma, R_1, S_1)$ and $G_2 = (V_2, \Sigma, R_2, S_2)$ and also the language $G_U = (V_1 \cup V_2 \cup \{S\}, \Sigma, R_1 \cup R_2 \cup \{S \to S_1 \mid S_2\}, S)$ which is the union of G_1 and G_2 as the start variable of G_U points to both start variables of G_1 and G_2. Additionally the rules and variables are shared (assuming the rules and variables are disjoint). After the start variable of G_U, subsequent steps use rules exclusively from G_1 or G_2, not both. therefore all productions of G_U must be in the languages G_1 or G_2. □

4. Show that the class of context-free languages is closed under the concatenation operation (construction and proof). The construction should be quite simple.

Proof. Define two context-free languages: $G_1 = (V_1, \Sigma, R_1, S_1)$ and $G_2 = (V_2, \Sigma, R_2, S_2)$ and also the language $G_C = (V_1 \cup V_2 \cup \{S\}, \Sigma, R_1 \cup R_2 \cup \{S \to S_1 S_2\}, S)$ which is the concatenation of G_1 and G_2 ad the start variable of G_C concatenates both the start variables of G_1 and G_2. So G_C produces words that start with G_1 and end with G_2, thus all productions of G_C must be concatenations of G_1 and G_2. □

5. Show that the class of context-free languages is closed under the star operation (construction and proof). The construction should be quite simple.

Proof. Define the context-free language $G_1 = (V, \Sigma, R, S)$. The star of this language would have to be able to generate Σ or a countably infinite amount of copies. So the start state would have to $S_0 \to \epsilon \mid S_0 S$. Therefore the language $G_S = (V, \Sigma, R \cup \{S_0 \to \epsilon \mid S_0 S\}, S_0)$ generates either ϵ or a sequence of many words in G_1. □

6. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA and define CFG $G = (V, \Sigma, R, S)$ as follows:

   - $V = Q$;
   - For each $q \in Q$ and $a \in \Sigma$, define rule $q \to aq'$ where $q' = \delta(q, a)$;
   - For $q \in F$ define rule $q \to \epsilon$;
   - $S = q_0$.

   Prove $L(M) = L(G)$.

7. Let $L = \{0^n 1^m 0^n 1^m \mid n, m \geq 0\}$. Show L is not context-free.

8. Let $L = \{w \mid w$ is in $\{a, b, c, d\}^*$, with the number of a's = number of b's and the number of c's = the number of d's $\}$. Show L is not context-free.

9. Let A and B be languages. We define $A \approx B = \{ab \mid a$ is an element of A and b is an element of B and $|a| > |b| \}$. Show that if A and B are regular languages, then $A \approx B$ is a context free language.

10. Show $L = \{w \mid w$is an element of $\{a, b, c, d, e, f\}^*$ such that the number of a's + number of b's = number of c's + number of d's = number of e's + number of f's$\}$ is not context-free.