Math 327 Homework 8 Anchu A. Lee
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Question 9.2: An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

$96 = 100(1-\alpha)$, so $\alpha = 0.04$ and $z_{0.02} = 2.05$. $\sigma = 40$, $\bar{x} = 780$, $n = 30$.
$780 - (2.05)(\frac{40}{\sqrt{30}}) < \mu < 780 + (2.05)(\frac{40}{\sqrt{30}})$
$\mathbf{765.02 < \boldsymbol{\mu} < 794.97}$

Question 9.4: The heights of a random sample of 50 college students showed a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters.

(a) Construct a 98% confidence interval for the mean height of all college students.

$n = 50$, $\bar{x} = 174.5$, $\sigma = 6.9$, $98 = 100(1-\alpha)$, $\alpha = 0.02$ and $z_{0.01} = 2.33$
$174.5 - (2.33)(\frac{6.9}{\sqrt{50}}) < \mu < 174.5 + (2.33)(\frac{6.9}{sqrt50})$
$\mathbf{172.23 < \boldsymbol{\mu} < 176.77}$

(b) What can we assert with 98% confidence about the possible size of our error if we estimate the mean height of all college students to be 174.5 centimeters?

$\boldsymbol{error < (2.33)(\frac{6.9}{\sqrt{50}}) = 2.27}$

Question 9.6: How large a sample is needed in Exercise 9.2 if we wish to be 96% confident that our sample mean will be within 10 hours of the true mean?

$10 = (2.05)(\frac{40}{\sqrt{n}})$, $\boldsymbol{n = 67.24 \approx 68}$.

Question 9.10: A random sample of 12 graduates of a certain secretarial school typed an average of 79.3 words per minute with a standard deviation of 7.8 words per minute. Assuming a normal distribution for the number of words typed per minute, find a 95% confidence interval for the average number of words typed by all graduates of this school.

$n = 12$, $\bar{x} = 79.3$, $\sigma = 7.8$, $95 = 100(1-\alpha)$, $\alpha = 0.05$, $t_{0.025} = 2.201$
$79.3 - (2.201)(\frac{7.8}{\sqrt{12}}) < \mu < 79.3 + (2.201)(\frac{7.8}{\sqrt{12}})$
$\mathbf{74.34 < \boldsymbol{\mu} < 84.26}$

Question 9.12: A random sample of 10 chocolate energy bars of a certain brand has, on average, 230 calories per bar, with a standard deviation of 15 calories. Construct a 99% confidence interval for the true mean calorie content of this brand of energy bar. Assume that the distribution of the calorie content is approximately normal.

$n = 10$, $\bar{x} = 230$, $\sigma = 15$, $99 = 100(1-\alpha)$, $\alpha = 0.01$, $t_{0.005} = 3.250$
$230 - (3.250)(\frac{15}{\sqrt{10}}) < \mu < 230 + (3.250)(\frac{15}{\sqrt{10}})$
$\mathbf{214.58 < \boldsymbol{\mu} < 245.42}$

Question 9.36: Two kinds of thread are being compared for strength. Fifty pieces of each type of thread are tested under similar conditions. Brand A has an average tensile strength of 78.3 kilograms with a standard deviation of 5.6 kilograms, while brand B has an average tensile strength of 87.2 kilograms with a standard deviation of 6.3 kilograms. Construct a 95% confidence interval for the difference of the population means.

$n_A = 50$, $\bar{x}_A = 78.3$, $\sigma_A = 5.6$, $n_B = 50$, $\bar{x}_B = 87.2$, $\sigma_B = 6.3$, $95 = 100(1-\alpha)$, $\alpha = 0.05$, $z_{0.025} = 1.96$
$(87.2 - 78.3) - (1.96)(\sqrt{\frac{5.6^2}{50} + \frac{6.3^2}{50}}) < \mu_A - \mu_B < (87.2 - 78.3) + (1.96)(\sqrt{\frac{5.6^2}{50} + \frac{6.3^2}{50}})$
$\mathbf{6.56 < \boldsymbol{\mu_A - \mu_B} < 11.24}$

Question 9.38: Two catalysts in a batch chemical process, are being compared for their effect on the output of the process reaction. A sample of 12 batches was prepared using catalyst 1, and a sample of 10 batches was prepared using catalyst 2. The 12 batches for which catalyst 1 was used in the reaction gave an average yield of 85 with a sample standard deviation of 4, and the 10 batches for which catalyst 2 was used gave an average yield of 81 and a sample standard deviation of 5. Find a 90% confidence interval for the difference between the population means, assuming that the populations are approximately normally distributed with equal variances.

$n_1 = 12$, $\bar{x}_1 = 85$, $\sigma_1 = 4$, $n_2 = 10$, $\bar{x}_2 = 81$, $\sigma_2 = 5$, $s_p = \sqrt{\frac{(12-1)4^2+(10-1)5^2}{12+10-2}} = 4.47$, $90 = 100(1-\alpha)$, $\alpha = 0.1$, $t_{1,0.05} = 1.725$
$(85 - 81) - (1.725)(4.47)(\sqrt{\frac{1}{12} + \frac{1}{10}}) < \mu_1 - \mu_2 < (85 - 81) + (1.725)(4.47)(\sqrt{\frac{1}{12} + \frac{1}{10}})$
$\mathbf{0.7 < \boldsymbol{\mu_1 - \mu_2} < 7.3}$

Question 9.40: In a study conducted at Virginia Tech on the development of ectomycorrhizal, a symbiotic relationship between the roots of trees and a fungus, in which minerals are transferred from the fungus to the trees and sugars from the trees to the fungus, 20 northern red oak seedlings exposed to the fungus *Pisolithus tinctorus* were grown in a greenhouse. All seedlings were planted in the same type of soil and received the same amount of sunshine and water. Half received no nitrogen at planting time, to serve as a control, and the other half received 368 ppm of nitrogen in the form $NaNO_3$. The stem weights, in grams, at the end of 140 days were recorded as follows:

No Nitrogen	Nitrogen
0.32	0.26
0.53	0.43
0.28	0.47
0.37	0.49
0.47	0.52
0.43	0.75
0.36	0.79
0.42	0.86
0.38	0.62
0.43	0.46

Construct a 95% confidence interval for the difference in the mean stem weight between seedlings that receive no nitrogen and those that receive 368 ppm of nitrogen. Assume the populations to be normally distributed with equal variances.

$n_1 = 10$, $\bar{x}_1 = 0.399$, $\sigma_1 = 0.073$, $n_2 = 10$, $\bar{x}_2 = 0.565$, $\sigma_2 = 0.187$, $s_p = \sqrt{\frac{(10-1)0.073^2+(10-1)0.187^2}{10+10-2}} = 0.142$, $95 = 100(1-\alpha)$, $\alpha = 0.05$, $t_{0.025} = 2.101$

$(0.565 - 0.399) - (2.101)(0.142)(\sqrt{\frac{1}{10} + \frac{1}{10}}) < \mu_1 - \mu_2 < (0.565 - 0.399) + (2.101)(0.142)(\sqrt{\frac{1}{10} + \frac{1}{10}})$

$\mathbf{0.033 < \boldsymbol{\mu_1 - \mu_2} < 0.299}$

Qustion 9.42: An experiment reported in Popular Science compared fuel economies for two types of similarly equipped diesel mini-trucks. Let us suppose that 12 Volkswagen and 10 Toyota trucks were tested in 90 kilometer-per-hour steady-paced trials. If the 12 Volkswagen trucks averaged 16 kilometers per liter with a standard deviation of 1.0 kilometer per liter and the 10 Toyota trucks averaged 11 kilometers per liter with a standard deviation of 0.8 kilometer per liter, construct a 90% confidence interval for the difference between the average kilometers per liter for these two mini-trucks. Assume that the distances per liter for the truck models are approximately normally distributed with equal variances.

$n_V = 12$, $\bar{x}_V = 16$, $\sigma_V = 1$, $n_T = 10$, $\bar{x}_T = 11$, $\sigma_T = 0.8$, $s_p = \sqrt{\frac{(12-1)1^2+(10-1)0.8^2}{12+10-2}} = 0.92$, $90 = 100(1-\alpha)$, $\alpha = 0.1$, $t_{0.05} = 1.725$

$(16 - 11) - (1.725)(0.92)(\sqrt{\frac{1}{12} + \frac{1}{10}}) < \mu_V - \mu_T < (16 - 11) + (1.725)(0.92)(\sqrt{\frac{1}{12} + \frac{1}{10}})$

$\mathbf{4.3 < \boldsymbol{\mu_V - \mu_T} < 5.68}$

Question 9.44: Referring to Exercise 9.43, find a 99% confidence interval for $\mu_1 - \mu_2$ if tires of the two brands are assigned at random to the left and right rear wheels of 8 taxis and the following distances, in kilometers, are recorded

Taxi	Brand A	Brand B	d_i
1	34,400	36,700	-2300
2	45,500	46,800	-1300
3	36,700	37,700	-1000
4	32,000	31,100	900
5	48,400	47,800	600
6	32,800	36,400	-3600
7	38,100	38,900	-800
8	30,100	31,500	-1400

Assume that the differences of the distances are approximately normally distributed.

$n = 8$, $\bar{d} = -1112.5$, $\sigma = 1454.5$, $99 = 100(1-\alpha)$, $\alpha = 0.01$, $t_{0.005} = 3.499$

$(-1112.5) - (3.499)\frac{1454.5}{\sqrt{8}} < \mu_D < (-1112.5) + (3.499)\frac{1454.5}{\sqrt{8}}$

$\mathbf{-2911.84 < \boldsymbol{\mu_D} < 686.84}$

Question 9.46: The following data represent the running times of films produced by two motion-picture companies.

Company	Time (minutes)
I	103 94 110 87 98
II	97 82 123 92 175 88 118

Compute a 90% confidence interval for the difference between the average running times of films produced by the two companies. Assume that the running-time differences are approximately normally distributed with unequal variances.

$n_1 = 5$, $\bar{x}_1 = 98.4$, $\sigma_1 = 8.74$, $n_2 = 7$, $\bar{x}_2 = 110.71$, $\sigma_2 = 32.19$, $90 = 100(1-\alpha)$, $\alpha = 0.1$, $v = \frac{(8.74^2/5+32.19^2/7)^2}{(8.74^2/5)^2/(5-1)+(32.19^2/7)^2/(7-1)} \approx$ 7 so $t_{0.05} = 1.895$.

$$(110.71 - 98.4) - (1.895)\sqrt{\frac{8.74^2}{5} + \frac{32.19^2}{7}} < \mu_1 - \mu_2 < (110.71 - 98.4) + (1.895)\sqrt{\frac{8.74^2}{5} + \frac{32.19^2}{7}}$$

$$\boldsymbol{-11.9 < \mu_1 - \mu_2 < 36.53}$$

Question 9.48: An automotive company is considering two types of batteries for its automobile. Sample information on battery life is collected for 20 batteries of type A and 20 batteries of type B. The summary statistics are $\bar{x}_A = 32.91$, $\bar{x}_B = 30.47$, $s_A = 1.57$, and $s_B = 1.74$. Assume the data on each battery are normally distributed and assume $\sigma_A = \sigma_B$

$n_A = 20$, $\bar{x}_A = 32.91$, $\sigma_A = 1.57$, $n_B = 20$, $\bar{x}_B = 30.47$, $\sigma_B = 1.74$, $s_p = \sqrt{\frac{(20-1)1.57^2+(20-1)1.74^2}{20+20-2}} = 1.66$

(a) Find a 95% confidence interval on $\mu_A - \mu_B$.

$95 = 100(1 - \alpha)$, $\alpha = 0.05$, $t_{0.025} \approx 2.021$

$$(32.91 - 30.47) - (2.021)(1.66)\sqrt{\frac{1}{20} + \frac{1}{20}} < \mu_A - \mu_B < (32.91 - 30.47) + (2.021)(1.66)\sqrt{\frac{1}{20} + \frac{1}{20}}$$

$$1.38 < \mu_A - \mu_B < 3.5$$

(b) Draw a conclusion from (a) that provides insight into whether A or B should be adopted.

A should be adopted because it has a longer battery life.

Question 9.50: Two levels (low and high) of insulin doses are given to two groups of diabetic rats to check the insulinbinding capacity, yielding the following data:

Low dose:	$n_1 = 8$	$\bar{x}_1 = 1.98$	$s_1 = 0.51$
High dose:	$n_2 = 13$	$\bar{x}_2 = 1.30$	$s_2 = 0.35$

Assume that the variances are equal. Give a 95% confidence interval for the difference in the true average insulin-binding capacity between the two samples.

$s_p = \sqrt{\frac{(8-1)0.51^2+(13-1)0.35^2}{8+13-2}} = 0.416$, $95 = 100(1 - \alpha)$, $\alpha = 0.05$, $t_{0.025} = 2.093$

$$(1.98 - 1.30) - (2.093)(0.416)\sqrt{\frac{1}{8} + \frac{1}{13}} < \mu_1 - \mu_2 < (1.98 - 1.30) + (2.093)(0.416)\sqrt{\frac{1}{8} + \frac{1}{13}}$$

$$\boldsymbol{0.289 < \mu_1 - \mu_2 < 1.07}$$

Question 9.92: A study was undertaken at Virginia Tech to determine if fire can be used as a viable management tool to increase the amount of forage available to deer during the critical months in late winter and early spring. Calcium is a required element for plants and animals. The amount taken up and stored in plants is closely correlated to the amount present in the soil. It was hypothesized that a fire may change the calcium levels present in the soil and thus affect the amount available to deer. A large tract of land in the Fishburn Forest was selected for a prescribed burn. Soil samples were taken from 12 plots of equal area just prior to the burn and analyzed for calcium. Postburn calcium levels were analyzed from the same plots. These values, in kilograms per plot, are presented in the following table:

	Calcium Level (kg/plot)	
Plot	Preburn	Postburn
1	50	9
2	50	18
3	82	45
4	64	18
5	82	18
6	73	9
7	77	32
8	54	9
9	23	18
10	45	9
11	36	9
12	54	9

Construct a 95% confidence interval for the mean difference in calcium levels in the soil prior to and after the prescribed burn. Assume the distribution of differences in calcium levels to be approximately normal.