Math 327 Homework 5 Anchu A. Lee
October 12, 2017

Question 5.2: Twelve people are given two identical speakers, which they are asked to listen to for differences, if any. Suppose that these people answer simply by guessing. Find the probability that three people claim to have heard a difference between the two speakers.

$\binom{\text{\# of people}}{\text{\# of success}} \cdot \text{probability of success}^{(\text{\# of success})} \cdot \text{probability of failure}^{(\text{\# of fail})}$
$\binom{12}{3}(\frac{1}{2})^3(\frac{1}{2})^9 = \frac{55}{1024} \approx \mathbf{0.0537}$

Question 5.4: In a certain city district, the need for money to buy drugs is stated as the reason for 75% of all thefts. Find the probability that among the next 5 theft cases reported in this district,

(a) exactly 2 resulted from the need for money to buy drugs;

$\binom{5}{2}(\frac{3}{4})^2(\frac{1}{4})^3 = \frac{45}{512} \approx \mathbf{0.08789}$

(b) at most 3 resulted from the need for money to buy drugs.

$\binom{5}{3}(\frac{3}{4})^3(\frac{1}{4})^2 = \frac{135}{512}$
$\binom{5}{2}(\frac{3}{4})^2(\frac{1}{4})^3 = \frac{45}{512}$
$\binom{5}{1}(\frac{3}{4})^1(\frac{1}{4})^4 = \frac{15}{1024}$
$\binom{5}{0}(\frac{3}{4})^0(\frac{1}{4})^5 = \frac{1}{1024}$
$\frac{135}{512} + \frac{45}{512} + \frac{15}{1024} + \frac{1}{1024} = \frac{47}{128} \approx \mathbf{0.367}$

Question 5.6: According to a survey by the Administrative Management Society, one-half of U.S. companies give employees 4 weeks of vacation after they have been with the company for 15 years. Find the probability that among 6 companies surveyed at random, the number that give employees 4 weeks of vacation after 15 years of employment is

(a) anywhere from 2 to 5;

$\binom{6}{2}(\frac{1}{2})^2(\frac{1}{2})^4 + \binom{6}{3}(\frac{1}{2})^3(\frac{1}{2})^3 + \binom{6}{4}(\frac{1}{2})^4(\frac{1}{2})^2 + \binom{6}{5}(\frac{1}{2})^5(\frac{1}{2})^1$
$\frac{15}{64} + \frac{20}{64} + \frac{15}{64} + \frac{6}{64} = \frac{56}{64} \approx \mathbf{0.875}$

(b) fewer than 3.

$\binom{6}{2}(\frac{1}{2})^2(\frac{1}{2})^4 + \binom{6}{1}(\frac{1}{2})^1(\frac{1}{2})^5 + \binom{6}{0}(\frac{1}{2})^0(\frac{1}{2})^6$
$\frac{15}{64} + \frac{6}{64} + \frac{1}{64} = \frac{22}{64} \approx \mathbf{0.34375}$

Question 5.14: The percentage of wins for the Chicago Bulls basketball team going into the playoffs for the 199697 season was 87.7. Round the 87.7 to 90 in order to use Table A.1.

(a) What is the probability that the Bulls sweep (4-0) the initial best-of-7 playoff series?

$\binom{4}{4}(0.90)^4(0.10)^0 \approx \mathbf{0.65}$

(b) What is the probability that the Bulls win the initial best-of-7 playoff series?

$\binom{7}{4}(0.90)^4(0.10)^3 + \binom{7}{5}(0.90)^5(0.10)^2 + \binom{7}{6}(0.90)^6(0.10)^1 + \binom{7}{7}(0.90)^7(0.10)^0 \approx \mathbf{0.997}$

(c) What very important assumption is made in answering parts (a) and (b)?

The probability that the Bulls win any game is always 90%.

Question 5.16: Suppose that airplane engines operate independently and fail with probability equal to 0.4. Assuming that a plane makes a safe flight if at least one-half of its engines run, determine whether a 4-engine plane or a 2-engine plane has the higher probability for a successful flight.

4-engine plane: $\binom{4}{2}(0.60)^2(0.40)^2 + \binom{4}{3}(0.60)^3(0.40)^1 + \binom{4}{4}(0.60)^4(0.40)^0 \approx 0.8208$
2-engine plane: $\binom{2}{1}(0.60)^1(0.40)^1 + \binom{2}{2}(0.60)^2(0.40)^0 \approx 0.84$
2-engine plane has higher chance for a successful flight.

Question 5.20: According to USA Today (March 18, 1997), of 4 million workers in the general workforce, 5.8% tested positive for drugs. Of those testing positive, 22.5% were cocaine users and 54.4% marijuana users.

(a) What is the probability that of 10 workers testing positive, 2 are cocaine users, 5 are marijuana users, and 3 are users of other drugs?

$\binom{10}{2,5,3}(0.225)^2(0.544)^5(0.231)^3 = \frac{10!}{2!5!3!}(0.225)^2(0.544)^5(0.231)^3 \approx \mathbf{0.0749}$

(b) What is the probability that of 10 workers testing positive, all are marijuana users?

$\binom{10}{0,10,0}(0.225)^0(0.544)^{10}(0.231)^0 = \frac{10!}{0!10!0!}(0.225)^0(0.544)^{10}(0.231)^0 \approx \mathbf{0.00227}$

(c) What is the probability that of 10 workers testing positive, none is a cocaine user?

$\binom{10}{0,10}(0.225)^0(0.775)^{10} = \frac{10!}{0!10!}(0.225)^0(0.775)^{10} \approx \mathbf{0.0782}$

Question 5.28: A manufacturer knows that on average 20% of the electric toasters produced require repairs within 1 year after they are sold. When 20 toasters are randomly selected, find appropriate numbers x and y such that

(a) the probability that at least x of them will require repairs is less than 0.5;

$n = 20$, $p = 0.20$, $P(X \geq x) \leq 0.5$ and $P(X < x) > 0.5$ gives $x = 3$

(b) the probability that at least y of them will not require repairs is greater than 0.8.

$p = 0.80$, $P(Y \geq y) \geq 0.8$ and $P(Y < y) < 0.2)$ gives $y = 15$

Question 5.30: To avoid detection at customs, a traveler places 6 narcotic tablets in a bottle containing 9 vitamin tablets that are similar in appearance. If the customs official selects 3 of the tablets at random for analysis, what is the probability that the traveler will be arrested for illegal possession of narcotics?

$n = 3$, $N = 15$, $k = 6$, $x = 0$
$1 - \frac{\binom{6}{0}\cdot\binom{9}{3}}{\binom{15}{3}} = \frac{53}{65}$

Question 5.32: From a lot of 10 missiles, 4 are selected at random and fired. If the lot contains 3 defective missiles that will not fire, what is the probability that

(a) all 4 will fire?
$x = 4$, $N = 10$, $n = 4$, $k = 7$
$\frac{\binom{7}{4}\binom{3}{0}}{\binom{10}{4}} = \frac{1}{6}$

(b) at most 2 will not fire?
$k = 3$
$\sum_{x=0}^{2} \frac{\binom{3}{x}\binom{7}{4-x}}{\binom{10}{4}} = \frac{29}{30}$

Question 5.38: Among 150 IRS employees in a large city, only 30 are women. If 10 of the employees are chosen at random to provide free tax assistance for the residents of this city, use the binomial approximation to the hypergeometric distribution to find the probability that at least 3 women are selected.

$p = \frac{30}{150} = 0.2$
$1 - \sum_{x=0}^{2}\binom{10}{x}(0.2)^x(0.8)^{10-x} \approx \mathbf{0.322}$

Question 5.44: An urn contains 3 green balls, 2 blue balls, and 4 red balls. In a random sample of 5 balls, find the probability that both blue balls and at least 1 red ball are selected.

$$\frac{\binom{2}{2}\binom{4}{1}\binom{3}{2}}{\binom{9}{5}} + \frac{\binom{2}{2}\binom{4}{2}\binom{3}{1}}{\binom{9}{5}} + \frac{\binom{2}{2}\binom{4}{3}\binom{3}{0}}{\binom{9}{5}} = \frac{17}{63}$$

Question 5.50: Find the probability that a person flipping a coin gets

(a) the third head on the seventh flip;
$\binom{6}{2}(\frac{1}{2})^7 \approx \mathbf{0.117}$

(b) the first head on the fourth flip.
$(\frac{1}{2})(\frac{1}{2})^3 = \frac{1}{16}$

Question 5.54: According to a study published by a group of University of Massachusetts sociologists, about two-thirds of the 20 million persons in this country who take Valium are women. Assuming this figure to be a valid estimate, find the probability that on a given day the fifth prescription written by a doctor for Valium is

(a) the first prescribing Valium for a woman;
Geometric $= g(5; 2/3) = (\frac{2}{3})(\frac{1}{3})^4 = \frac{2}{243}$

(b) the third prescribing Valium for a woman.
Negative Binomial $= b^*(5; 3, 2/3) = \binom{4}{2}(\frac{2}{3})^3(\frac{1}{3})^2 = \frac{16}{81}$

Question 5.56: On average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection

(a) exactly 5 accidents will occur?
$p(5; 3) = \sum_{x=0}^{5} p(x; 3) - \sum_{x=0}^{4} p(x; 3) \approx \mathbf{0.1008}$

(b) fewer than 3 accidents will occur?
$\sum_{x=0}^{2} p(x; 3) \approx \mathbf{0.4232}$

(c) at least 2 accidents will occur?
$1 - \sum_{x=0}^{1} p(x; 2) \approx 1 - 0.1991 = \mathbf{0.8009}$

Question 5.60: The average number of field mice per acre in a 5-acre wheat field is estimated to be 12. Find the probability that fewer than 7 field mice are found

(a) on a given acre;
$\mu = 12$, $r = 6$, $\sum_{x=0}^{6} p(x; 12) \approx \mathbf{0.0458}$

(b) on 2 of the next 3 acres inspected.
$x = 2$, $n = 3$, $p = 0.0458$
$\binom{3}{2}(0.0458)^2(0.9542)^1 \approx \mathbf{0.0060}$

Question 5.62: The probability that a student at a local high school fails the screening test for scoliosis (curvature of the spine) is known to be 0.004. Of the next 1875 students at the school who are screened for scoliosis, find the probability that

(a) fewer than 5 fail the test;
$\mu = (1875)(0.004) = 7.5$, $r = 4$, $\approx \mathbf{0.1321}$

(b) 8, 9 , or 10 fail the test.
$\mu = 7.5$, $r = 10$ minus $r = 7 \approx 0.8622 - 0.5246 = \mathbf{0.3376}$

Question 5.70: A company purchases large lots of a certain kind of electronic device. A method is used that rejects a lot if 2 or more defective units are found in a random sample of 100 units.

(a) What is the mean number of defective units found in a sample of 100 units if the lot is 1% defective?
$\mu = \mathbf{1}$

(b) What is the variance?
$\sigma^2 = \mathbf{0.99}$

Question 5.72: Potholes on a highway can be a serious problem, and are in constant need of repair. With a particular type of terrain and make of concrete, past experience suggests that there are, on the average, 2 potholes per mile after a certain amount of usage. It is assumed that the Poisson process applies to the random variable number of potholes.

(a) What is the probability that no more than one pothole will appear in a section of 1 mile?
$\sum_{x=0}^{1} \frac{e^{-2} \cdot 2^x}{x!} \approx \mathbf{0.406}$

(b) What is the probability that no more than 4 potholes will occur in a given section of 5 miles?
$\mu = 2 \cdot 5 = 10$ so $\sum_{x=0}^{4} \frac{e^{-10} \cdot 10^x}{x!} \approx \mathbf{0.02925}$

Question 5.76: The refusal rate for telephone polls is known to be approximately 20%. A newspaper report indicates that 50 people were interviewed before the first refusal.

(a) Comment on the validity of the report. Use a probability in your argument.
$\sum_{x=51}^{\infty}(0.2)(1 - 0.2)^x \approx 0.0000114$. an extremely rare event, but not impossible.

(b) What is the expected number of people interviewed before a refusal?
$\mu = \frac{1}{0.20} - 1 = \mathbf{4}$

Question 5.78: An automatic welding machine is being considered for use in a production process. It will be considered for purchase if it is successful on 99% of its welds. Otherwise, it will not be considered efficient. A test is to be conducted with a prototype that is to perform 100 welds. The machine will be accepted for manufacture if it misses no more than 3 welds.

(a) What is the probability that a good machine will be rejected?
$1 - \sum_{x=0}^{3} \binom{100}{x} (0.01)^x (1-0.01)^{100-x} \approx \mathbf{0.01837}$

(b) What is the probability that an inefficient machine with 95% welding success will be accepted?
$\sum_{x=0}^{3} \binom{100}{x} (0.05)^x (1-0.05)^{100-x} \approx \mathbf{0.257839}$

Question 5.94: A production process produces electronic component parts. It is presumed that the probability of a defective part is 0.01. During a test of this presumption, 500 parts are sampled randomly and 15 defectives are observed.

(a) What is your response to the presumption that the process is 1% defective? Be sure that a computed probability accompanies your comment.
$1 - \sum_{x=0}^{14} \binom{500}{x} (0.01)^x (0.99)^{500-x} \approx \mathbf{0.000206}$
Extraordinarily rare event, so the presumption that the process defective rate is 0.01 is wrong.

(b) Under the presumption of a 1% defective process, what is the probability that only 3 parts will be found defective?
$\binom{500}{3} (0.01)^3 (0.99)^4 97 \approx \mathbf{0.14023}$

(c) Do parts (a) and (b) again using the Poisson approximation.
(a) $\mu = 5$, $p = 0.01$, $r = 14$. $1 - 0.9998 = \mathbf{0.0002}$
(b) $r = 3$ minus $r = 2$; $= 0.2650 - 0.1247 = \mathbf{0.1403}$