Midterm 1: Chapters 1 to 4

1	2	3	4	5	6	7	8	9	10
22.43	10.25	23.71	21.77	22.11	18.71	19.77	20.33	20.17	21.12

(a) $\bar{x} = \frac{\Sigma_{i=1}^{10} x_i}{10} = 20.037$. $\tilde{x} = \frac{20.33+21.12}{2} = 20.725$
(b) $s_x^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = 13.935$. Split into 4 sections, numbers seperating are quartiles. Last minus first is IQR $= 22.11 - 19.77 = 2.34$
(c) Trimmed mean of 10%: remove 10% from highest and lowest. $= 20.801$. Close to median but more than mean; data is slightly skewed to the left.
(d) Set decimal point to |.
(e) Away from Q1 and Q3 by 1.5·IQR are outliers. Left dot is minimum, start of box is Q1, middle line is median, end of box is Q3, last dot is maximum.

A: polluted, B: test detects pollution, $P(A) = 0.2$, $P(B \mid A) = 0.60$, $P(B \mid A') = 0.3$
(a) $P(A \cap B) = P(A)P(B \mid A) = 0.2 \cdot 0.6 = 0.12$
(b) $P(B) = P(B \cap A) + P(B \cap A') = 0.12 + P(A')P(B \mid A') = 0.12 + 0.8 \cdot 0.3 = 0.36$
(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.36 - 0.12 = 0.44$
(d) $P(A' \mid B') = \frac{P(A' \cap B')}{P(B')} = \frac{1-0.44}{0.64} = 0.875$ (e) $P(P(A \cap B) \neq 0)$ so are not mutually exclusive. $P(A \cap B) \neq P(A)P(B)$ so are not independent.

X number of cash registers being used for location 1, Y the number used at the same time for location 2.

$f(x,y)$		$y=0$	$y=1$	$y=2$
x	0	0.10	0.05	0.05
	1	0.10	0.20	0.05
	2	0.05	0.10	0.30
$h(y)$		0.25	0.35	0.40

(a) Marginal probability mass functions, add the rows for X; columns for Y. $h(y)$ example.
(b) Cumulative distribution function of X: 0 if $x < 0$, 0.2 if $0 \leq x < 1$. 0.55 if $1 \leq x < 2$. 1 if $x \geq 2$. So $F(1.5) = 0.55$.
(c) Conditional distribution of Y given $X = 2$, $f(y \mid X = 2)$: $\frac{0.05}{0.45}$ when $y = 0$, $\frac{0.10}{0.45}$ when $y = 1$, $\frac{0.30}{0.45}$ when $y = 2$.
(d) Mean of X: $\mu_X = E(x) = \Sigma_{x=0}^{2} x g(x) = 0.35 + 2 \cdot 0.45 = 1.25$ Variance of X: $\sigma_X^2 = E(X^2) - \mu_X^2 = \Sigma_{x=0}^{2} x^2 g(x) - \mu_X^2 = 0.35 + 4 \cdot 0.45 - 1.25^2 = 0.5875$
(e) $\sigma_{XY} = E(XY) - \mu_X \mu_Y = (0.20 \cdot 1 \cdot 1 + 0.10 \cdot 2 \cdot 1 + 0.05 \cdot 1 \cdot 2 + 0.30 \cdot 2 \cdot 2) - 1.25 \cdot 1.15 = 0.2625 \neq 0$ so X and Y are not independent.

X be a continuous random variable with probability density function $f(x) = Cx^2$ if $-2 < x < 1$ and zero otherwise.
(a) Find C, it must make the function equal to 1 for the interval. $1 = \int_{-2}^{1} Cx^2 dx$, $C = \frac{1}{3}$
(b) $\mu = E[X] = \int_{-2}^{1} x(\frac{1}{3}x^2)dx = -\frac{5}{4}$. $\sigma^2 = E[X^2] - \mu^2 = \int_{-2}^{1} x^2(\frac{1}{3}x^2)dx - (-\frac{5}{4})^2 = 0.6375$
(c) $P[X < -1] = P(-2 < x < -1) = \int_{-2}^{-1} \frac{1}{3}x^2 dx = \frac{7}{9}$
(d) $P[-1 < X \leq 3] = P(-1 < X < 1) + P(1 < X < 3) = \int_{-1}^{1} \frac{1}{3}x^2 + \int_{1}^{3} \frac{1}{3}x^2 = \frac{2}{9} + 0 = \frac{2}{9}$
(e) $g(X) = 4X - 3$. $\mu_{g(X)} = \int_{-2}^{1} g(x)f(x)dx = \int_{-2}^{1}(4X-3)(\frac{1}{3}x^2) = -8$. $\sigma_{g(X)}^2 = E(g(X)^2) - \mu_{g(X)}^2 = \int_{-2}^{1}(4X-3)^2(\frac{1}{3}x^2) - 64 = 10.2$

Combinations Different order is still same set. $\binom{n}{r} = \frac{n!}{r!(n-r)!}$
Permutations Different order is different set. $({}_nP_r) = \frac{n!}{(n-r)!}$
Chebyshev's Theorem The probability that a random variable X will be within k standard deviations of the mean is $P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2 y}$.

Midterm 2: Chapters 5 to 9

There are on average, 3 potholes in a section of 1 mile. Over time, so use Poisson Distribution $\frac{e^{-\lambda t}(\lambda t)^x}{x!}$, $\lambda = 3$
(a) Probability at least 2 potholes appear in a section of 1 mile. $P(X < 2) = P(X = 0) + P(X = 1) = \frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} = 0.20$
(b) Probability at least 2 but less than 4 appear in a section of 1 mile. $P(2 \leq X < 4) = P(X = 2) + P(X = 3) = \frac{e^{-3}3^2}{2!} + \frac{e^{-3}3^3}{3!} = 0.45$
(c) Exactly 5 potholes occur in a section of 2 miles. $P(X = 5) = \frac{e^{-(3\cdot 2)} \cdot (3 \cdot 2)^5}{5!} = 0.161$
(d) Probability of having less than 2 potholes in exactly 2 of the next 5 miles. Binomial, either succeed or fail. $b(2; 5, 0.2) = \binom{5}{2} \cdot 0.2^2 \cdot 0.8^3 = 0.20$
(e) Probability the first section of 1 mile with less than 2 potholes is the 4th mile. Geometric, xth trial is the first success. $g(4, 0.20) = (0.20)(0.8)^{4-1} = 0.1024$

Life of a electrical switch has an exponential distribution with a mean of 1.45 ($\beta = 1.45$) and parameter β given by $f(x; \beta) = \frac{1}{\beta} e^{-x/\beta}$ when $x > 0$ otherwise 0. β is the mean time between failures.
(a) Probability a randomly selected switch fails within a year. $P(X < 1) = 1 - e^{(-1/1.45)\cdot 1} = 0.50$
(b) Switch functions more than a year but less than 2.9 years. $P(1 < X < 2.9) = \int_{1}^{2.9} \frac{1}{1.45} e^{-x/1.45} = 0.37$
(c) Switch was functioning for 2 years, probability that it will function for another 3.48 years. $P(x > 3.48 + 2 \mid X > 2) = P(X > 3.48)$ memoryless $= e^{-3.48/1.45} = 0.09$
(d) If 10 switches are installed what is the probability exactly 3 fail during the first year? $Y \sim b(10, 0.5)$, $P(Y = 3) = \binom{10}{3} 0.5^3 \cdot 0.5^7 = 0.117$
(e) If 100 switches are installed what is the probability that at least 40 but less than 55 fail within the first year? $Y \sim b(100, 0.5)$, $\mu_Y = n \cdot p = 50$, $\sigma_Y = n \cdot p \cdot q = 25$. Because $np = nq = 50 > \sqrt{npq} = 5$, use Normal distribution to approximate. $Y \approx Normal(50, 5)$. $P(40 \leq Y < 55) = P(39.5 \leq Y < 54.5) = P(\frac{39.5-50}{5} \leq \frac{Y-50}{5} < \frac{54.5-50}{5}) = P(-2.1 \leq Z < 0.9)$ Check table values. $= 0.8159 - 0.0179 = 0.798$.
