Math 327 Homework 6 Anchu A. Lee
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Question 8.2: The lengths of time, in minutes, that 10 patients waited in a doctors office before receiving treatment were recorded as follows: 5, 11, 9, 5, 10, 15, 6, 10, 5, and 10. Treating the data as a random sample, find

(a) the mean;
8.6 minutes

(b) the median;
9.5 minues

(c) the mode.
5 and 10 minutes

Question 8.6: Find the mean, median, and mode for the sample whose observations, 15, 7, 8, 95, 19, 12, 8, 22, and 14, represent the number of sick days claimed on 9 fed- eral income tax returns. Which value appears to be the best measure of the center of these data? State reasons for your preference.

Mean: 22.222...; Median: 14; Mode: 8
The median is the best mesure of the center for this data. Mean is skewed because of one outlier and mode doesn't reflect the center.

Question 8.12: The tar contents of 8 brands of cigarettes se- lected at random from the latest list released by the Federal Trade Commission are as follows: 7.3, 8.6, 10.4, 16.1, 12.2, 15.1, 14.5, and 9.3 milligrams. Calculate 7.3 8.6 9.3 10.4 12.2 14.5 15.1 16.1

(a) the mean;
11.69 milligrams

(b) the variance.
10.77 milligrams

Question 8.18: If the standard deviation of the mean for the sampling distribution of random samples of size 36 from a large or infinite population is 2, how large must the sample size become if the standard deviation is to be reduced to 1.2?

$4 = variance/(36)$, **so variance is 144. Then** $1.44 = 144/x$; **so the sample size must be at least 100 to have a standard deviation of 1.2.**

Question 8.22: The heights of 1000 students are approximately normally distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Suppose 200 random samples of size 25 are drawn from this pop- ulation and the means recorded to the nearest tenth of a centimeter. Determine

(a) the mean and standard deviation of the sampling distribution of $\bar{X}$;
Mean: 174.5cm; Standard deviation: $6.9/\sqrt{25} = 1.38$.

(b) the number of sample means that fall between 172.5 and 175.8 centimeters inclusive;
$P(172.45 < \bar{X} < 175.85)$, **as standard deviation is 1.38;** $z_1 = (172.45 - 174.5)/1.38 = -1.49$, $z_2 = (175.85 - 174.5)/1.38 = 0.98$. **then** $P(-1.49 < Z < 0.98)$ **which according to table A.3 is** $0.8365 - 0.0681$ **and the probability is 0.7684 and the number of samples is 154.**

(c) the number of sample means falling below 172.0 centimeters.
$P(\bar{X} < 171.95)$, **then** $z = (171.95 - 174.5)/1.38 = -1.85$ **then** $P(Z < -1.85)$ **and by table A.3 equals** 0.0322 **and the number of samples is 6.**

Question 8.24: If a certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms, what is the probability that a random sample of 36 of these resistors will have a com- bined resistance of more than 1458 ohms?

$P(\sum_{i=1}^{36} X_i > 1458) = P(\bar{X} > 1458/36)$ **now find for Z:** $\sigma = 2/\sqrt{36} = 1/3$ **and** $z = (40.5 - 40)/1.5 = 1.5$ **so now** $P(Z > 1.5)$ **which according to table A.3 is** $1 - 0.9332$ **and the answer is 0.0668**

Question 8.28: A random sample of size 25 is taken from a normal population having a mean of 80 and a standard deviation of 5. A second random sample of size 36 is taken from a different normal population having a mean of 75 and a standard deviation of 3. Find the probability that the sample mean computed from the 25 measurements will exceed the sample mean computed from the 36 measurements by at least 3.4 but less than 5.9. Assume the difference of the means to be measured to the nearest tenth.

Need to find $P(3.35 < \bar{X}_1 - \bar{X}_2 < 5.85)$**. The mean difference would be** $\mu_1 - \mu_2 = 80 - 75 = 5$**. and the standard deviation difference would be** $\sqrt{1 + 1/4} = 1.12$**. Then** $z_1 = (3.35 - 5)/1.12 = -1.47$ **and** $z_2 = (5.85 - 5)/1.12 = 0.76$**. Now** $P(-1.47 < Z < 0.76) = 0.7764 - 0.0708$ **so the answer is 0.7056**

Question 8.32: Two different box-filling machines are used to fill cereal boxes on an assembly line. The critical measurement influenced by these machines is the weight of the product in the boxes. Engineers are quite certain that the variance of the weight of product is $\sigma^2 = 1$ ounce. Experiments are conducted using both machines with sample sizes of 36 each. The sample averages for ma- chines A and B are $\bar{x}_A = 4.5$ ounces and $\bar{x}_B = 4.7$ ounces. Engineers are surprised that the two sample averages for the filling machines are so different.

(a) Use the Central Limit Theorem to determine $P(\bar{X}_B - \bar{X}_A \geq 0.2)$ under the condition that $\mu_A = \mu_B$.
$P(\bar{X}_B - \bar{X}_A \geq 0.2)$**.** $z = 0.2/\sqrt{\frac{1}{36} + \frac{1}{36}} = 0.85$ **Then** $P(Z \geq 0.85) = 1 - 0.8023 =$ **0.1977**

(b) Do the aforementioned experiments seem to, in any way, strongly support a conjecture that the popu- lation means for the two machines are different? Explain using your answer in (a).
No, it does not support the conjecture. If the two population means were different, then the results from (a) would be smaller.

Question 8.34: Two alloys A and B are being used to manufac- ture a certain steel product. An experiment needs to be designed to compare the two in terms of maximum load capacity in tons (the maximum weight that can be tolerated without breaking). It is known that the two standard deviations in load capacity are equal at 5 tons each. An experiment is conducted in which 30 specimens of each alloy (A and B) are tested and the results recorded as follows:
: $\bar{x}_A = 49.5,$ $\bar{x}_B = 45.5;$ $\bar{x}_A - \bar{x}_B = 4$.
The manufacturers of alloy A are convinced that this evidence shows conclusively that $\mu_A > \mu_B$ and strongly supports the claim that their alloy is superior. Man- ufacturers of alloy B claim that the experiment could easily have given $\bar{x}_A - \bar{x}_B = 4$ even if the two popula- tion means are equal. In other words, the results are inconclusive!

(a) Make an argument that manufacturers of alloy B are wrong. Do it by computing $P(\bar{X}_A - \bar{X}_B > 4 \mid \mu_A = \mu_B)$.
$\sqrt{5^2/30 + 5^2/30} = 1.29$ **so then** $(4 - 0)/1.29 = 3.19$**.** $P(\bar{X}_A - \bar{X}_B > 4 \mid \mu_A = \mu_B) = P(Z > 3.10) =$ **0.0010**

(b) Do you think these data strongly support alloy A?
As the answer from (a) is very miniscule, it supports alloy A's statement.

Question 8.38: For a chi-squared distribution, find

(a) $x^2_{0.005}$ when $v = 5$;
27.49

(b) $x^2_{0.05}$ when $v = 19$;
18.48

(c) $x^2_{0.01}$ when $v = 12$.
36.42

Question 8.40: For a chi-squared distribution, find x^2_α such that

(a) $P(X^2 > x^2_\alpha) = 0.01$ when $v = 21$;
$X^2_{0.01} =$ **38.93**

(b) $P(X^2 < x^2_\alpha) = 0.95$ when $v = 6$;
$X^2_{0.05} =$ **12.59**

(c) $P(x^2_\alpha < X^2 < 23.209) = 0.015$ when v = 10.
$X^2_{0.01} =$ **23.209 and** $0.01 + 0.015 = 0.025$ **so** $X^2_{0.025} =$ **20.483**

Question 8.42: The scores on a placement test given to college freshmen for the past five years are approximately normally distributed with a mean $\mu = 74$ and a variance $\sigma^2 = 8$. Would you still consider $\sigma^2 = 8$ to be a valid value of the variance if a random sample of 20 students who take the placement test this year obtain a value of $s^2 = 20$?

$X^2 = \frac{19 \cdot 20}{8} = 47.5$**. Meanwhile** $X^2_{0.01} = 36.191$ **so the value of variance is not valid.**

Question 8.44:

(a) Find $t_{0.023}$ when $v = 14$.
2.145

(b) Find $-t_{0.10}$ when $v = 10$.
-1.372

(c) find $t_0.995$ when $v = 7$.
-3.499

Question 8.46:

(a) Find $P(-t_{0.005} < T < t_{0.01})$ for $v = 20$.
$1 - 0.005 - 0.01 =$ **0.985**

(b) Find $P(T > -t_{0.025})$.
$1 - 0.025 =$ **0.975**

Question 8.48: A manufacturing firm claims that the batteries used in their electronic games will last an average of 30 hours. To maintain this average, 16 batteries are tested each month. If the computed t-value falls be- tween $-t_{0.025}$ and $t_{0.025}$, the firm is satisfied with its claim. What conclusion should the firm draw from a sample that has a mean of $\bar{x} = 27.5$ hours and a stan- dard deviation of $s = 5$ hours? Assume the distribution of battery lives to be approximately normal.

$t = \frac{27.5-30}{5/4} = -2$. **Meanwhile** $t_{0.025} = 2.131$ **and** $t_{-0.025} = -2.131$. **-2 lies between the two so the firm's claim is correct.**

Question 8.50: A maker of a certain brand of low-fat cereal bars claims that the average saturated fat content is 0.5 gram. In a random sample of 8 cereal bars of this brand, the saturated fat content was 0.6, 0.7, 0.7, 0.3, 0.4, 0.5, 0.4, and 0.2. Would you agree with the claim? Assume a normal distribution.

Mean = 0.457, Variance = 0.0336 so then $t = (0.475 - 0.5)/0.0645 = -0.39$. $P(\bar{X} < 0.475) = P(T < -0.39) =$ **0.34. I would not agree with the claim.**

Question 8.51: For an F-distribution, find

(a) $f_{0.05}$ with $v_1 = 7$ and $v_2 = 15$;
2.71

(b) $f_{0.05}$ with $v_1 = 15$ and $v_2 = 7$:
3.51

(c) $f_{0.01}$ with $v_1 = 24$ and $v_2 = 19$;
2.92

(d) $f_{0.95}$ with $v_1 = 19$ and $v_2 = 24$;
0.47

(e) $f_{0.99}$ with $v_1 = 28$ and $v_2 = 12$.
0.34

Question 8.52: Pull-strength tests on 10 soldered leads for a semiconductor device yield the following results, in pounds of force required to rupture the bond:

19.8	12.7	13.2	16.9	10.6
18.8	11.1	14.3	17.0	12.5

Another set of 8 leads was tested after encapsulation to determine whether the pull strength had been in- creased by encapsulation of the device, with the fol- lowing results:

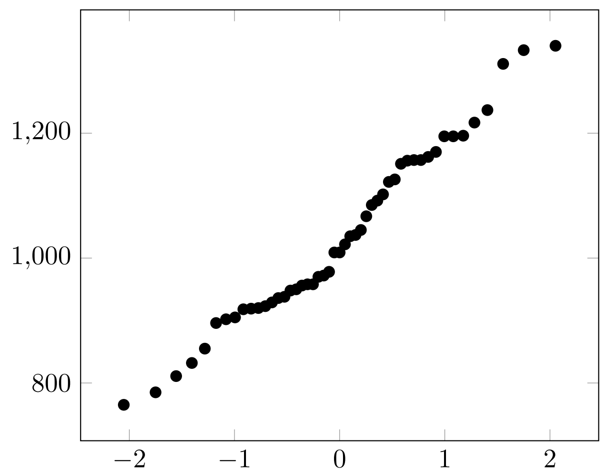
24.9	22.8	23.6	22.1	20.4	21.6	21.8	22.5

Comment on the evidence available concerning equal- ity of the two population variances.

Variance of the first group is 10.441 and the variance of the second group is 1.846. f **is then 5.66. This value is between** $f_{0.05}$ **and** $f_{0.01}$**, so** $P(F > 5.66)$ **is small and therefore the variances of the first group and the second group are not equal.**

Question 8.54: Construct a quantile plot of these data, which represent the lifetimes, in hours, of fifty 40-watt, 110-volt internally frosted incandescent lamps taken from forced life tests:

919	1196	785	1126	936	918
1156	920	948	1067	1092	1162
1170	929	950	905	972	1035
1045	855	1195	1195	1340	1122
938	970	1237	956	1102	1157
978	832	1009	1157	1151	1009
765	958	902	1022	1333	811
1217	1085	896	958	1311	1037
702	923				



Question 8.64: If S_1^2 and S_2^2 represent the variances of indepen- dent random samples of size $n_1 = 25$ and $n_2 = 31$, taken from normal populations with variances $\sigma_1^2 = 10$ and $\sigma_2^2 = 15$, respectively, find $P(\sigma_1^2/\sigma_2^2 > 1.26)$.

$P(s_1^2/s_2^2 > 1.26) = P((s_1^2/\sigma_1^2)/(s_2^2/\sigma_2^2) > (15 \cdot 1.26)/10) = P(F > 1.89)$ **Then** F **values** v_1 **is 24 and** v_2 **is 30 and the probability is 0.05.**