

1. Exercise 2.1

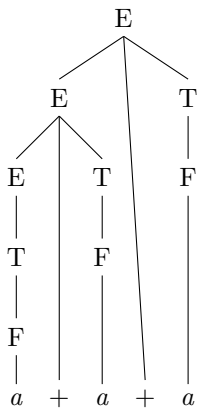
a. $E \Rightarrow T \Rightarrow F \Rightarrow a$



b. $E \Rightarrow E + T \Rightarrow T + T \Rightarrow F + T \Rightarrow a + T \Rightarrow a + F \Rightarrow a + a$



c. $E \Rightarrow E + T \Rightarrow E + T + T \Rightarrow T + T + T \Rightarrow T + T + F \Rightarrow T + F + F \Rightarrow F + F + F \Rightarrow F + F + a \Rightarrow F + a + a \Rightarrow a + a + a$



d. $E \Rightarrow T \Rightarrow F \Rightarrow (E) \Rightarrow (T) \Rightarrow (F) \Rightarrow ((E)) \Rightarrow ((T)) \Rightarrow ((F)) \Rightarrow ((a))$

- b. Use part (a) and DeMorgan's law (Theorem 0.20) to show that the class of context-free languages is not closed under complementation.

Suppose that context-free languages are closed under complementation. Then the complement of A and B , A' , B' should also be context-free. Since context-free languages are closed under union, then $A' \cup B'$ should also be a context-free language. By DeMorgan's law, $A' \cup B' = A \cap B$, however that is not the case as proved in part (b). By proof of contradiction, context-free languages are not closed under complementation.

4. Exercise 2.4b: Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is $\{0, 1\}$.

$$\{w \mid w \text{ starts and ends with the same symbol}\}$$

$$\begin{aligned} S &\rightarrow 0X0 \mid 1X1 \\ X &\rightarrow 0X \mid 1X \mid \epsilon \end{aligned}$$

5. Give a CFG for

$$\{0^a 1^b 2^c 3^d 4^e 5^f \mid \text{such that } a, b, c, d, e, f \geq 0 \text{ and } a + b = d + e\}$$

$$\begin{aligned} S &\rightarrow TW \\ T &\rightarrow 0T4 \mid U \\ U &\rightarrow 1U3 \mid V \\ V &\rightarrow 2V \mid \epsilon \\ W &\rightarrow 5W \mid \epsilon \end{aligned}$$

6. Exercise 2.4e: Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is $\{0, 1\}$.

$$\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$$

$$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon$$

7. Put the rules following in Chomsky normal form (assume that S is the new start variable)

$$\begin{aligned} S &\rightarrow aAA \mid aBC \mid abc \\ A &\rightarrow AA \mid Aa \mid ab \\ B &\rightarrow aaBC \mid BC \\ C &\rightarrow a \mid bc \end{aligned}$$

$$\begin{aligned} S &\rightarrow DI \mid DH \mid DJ \\ A &\rightarrow AA \mid AD \mid DE \\ B &\rightarrow GH \mid BC \\ C &\rightarrow a \mid EF \\ D &\rightarrow a \\ E &\rightarrow b \\ F &\rightarrow c \\ G &\rightarrow DD \\ H &\rightarrow BC \\ I &\rightarrow AA \\ J &\rightarrow EF \end{aligned}$$

8. Exercise 2.15: Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generated by CFG $G = (V, \Sigma, R, S)$. Add the new rule $S \rightarrow SS$ and call the resulting grammar G' . This grammar is supposed to generate A^* .

Suppose a context-free language B with the corresponding grammar $G = \{\{S\}, \{(\cdot, \cdot)\}, \{S \rightarrow (S), S \rightarrow \epsilon\}, S\}$. Following the construction, we add $S \rightarrow SS$ to the new grammar G' . However, $((\cdot))$ is in G' but is not in B^* . So the new grammar G' does not generate A^* .

9. Show the following is context-free using a CFG

$$\{xy \mid x, y \in \{0, 1\}^*, |x| = |y|, y \neq x^R\}$$

$$S \rightarrow T$$

$$T \rightarrow 0T1 \mid 1T0 \mid \epsilon$$

10. Construct a pushdown automata that recognizes

$\{w \mid w \text{ is an element of } \{a, b, c, d\}^* \text{ such that the number of a's in } w \text{ plus the number of b's in } w \text{ is equal to the number of c's in } w \text{ plus the number of d's in } w\}$

