CS 373 Homework 6 Anchu A. Lee

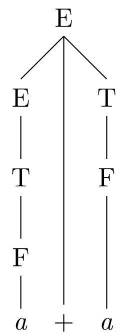
October 7, 2017

1. Exercise 2.1

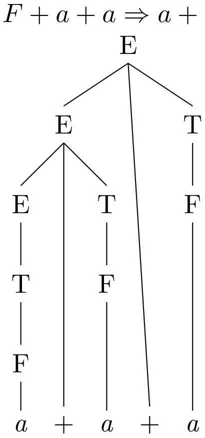
   a. $E \Rightarrow T \Rightarrow F \Rightarrow a$

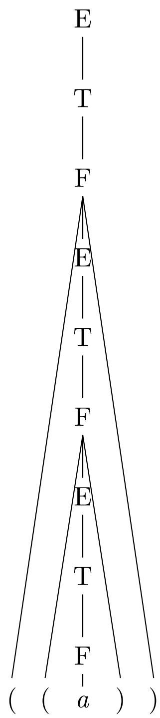
   b. $E \Rightarrow E + T \Rightarrow T + T \Rightarrow F + T \Rightarrow a + T \Rightarrow a + F \Rightarrow a + a$

   c. $E \Rightarrow E + T \Rightarrow E + T + T \Rightarrow T + T + T \Rightarrow T + T + F \Rightarrow T + F + F \Rightarrow F + F + F \Rightarrow F + F + a \Rightarrow F + a + a \Rightarrow a + a + a$

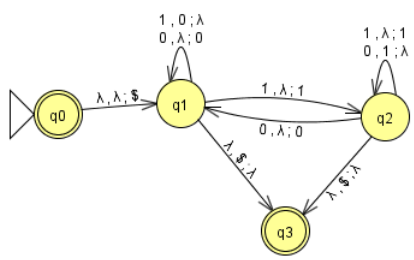
   

   d. $E \Rightarrow T \Rightarrow F \Rightarrow (E) \Rightarrow (T) \Rightarrow (F) \Rightarrow ((E)) \Rightarrow ((T)) \Rightarrow ((F)) \Rightarrow ((a))$



2. Construct a pushdown automata that recognizes

$$\{w \mid w \in \{0,1\}^* \text{ s.t. the number of 0's in } w \text{ is equal to the number of 1's in } w\}$$



3. Exercise 2.2

   a. Use the languages $A = \{a^m b^n c^n \mid m, n \geq 0\}$ and $B = \{a^n b^n c^m \mid m, n \geq 0\}$ together with Example 2.36 to show that the class of context-free languages is not closed under intersection.

   b. Use part (a) and DeMorgan's law (Theorem 0.20) to show that the class of context-free languages is not closed under complementation.

4. Exercise 2.4b

5. Give a CFG for

$$\{0^a 1^b 2^c 3^d 4^e 5^f \mid \text{such that } a, b, c, d, e, f \geq 0 \text{ and } a + b = d + e\}$$

6. Exercise 2.4e: Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is $\{0, 1\}$.

$$\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$$

7. Put the rules following in Chomsky normal form (assume that S is the new start variable)

$$\begin{aligned}S &\to aAA \mid aBC \mid abc \\ A &\to AA \mid Aa \mid ab \\ B &\to aaBC \mid BC \\ C &\to a \mid bc\end{aligned}$$

8. Exercise 2.15: Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generated by CFG $G = (V, \Sigma, R, S)$. Add the new rule $S \to SS$ and call the resulting grammar G'. This grammar is supposed to generate $A*$.

9. Show the following is context free using a CFG

$$\{xy \mid x, y \in \{0,1\}^*,\ |x| = |y|,\ y \neq x^R\}$$

10. Construct a pushdown automata that recognizes

$\{w \mid w$ is an element of $\{a, b, c, d\}^*$ such that the number of a's in w plus the number of b's in w is equal to the number of c's in w plus the number of d's in $w\}$