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1. Show that EQ_{CFG} is undecidable.

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2. Show that EQ_{CFG} is co-Turing-recognizable.

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3. Find a match in the following instance of the Post Correspondence Problem.

$$\left\{ \left[\frac{ab}{abab} \right], \left[\frac{b}{a} \right], \left[\frac{aba}{b} \right], \left[\frac{aa}{a} \right] \right\}$$

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4. Show that the Post Correspondence Problem is decidable over the unary alphabet $\Sigma = \{1\}$.

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5. In the silly Post Correspondence Problem, SPCP, the top string in each pair has the same length as the bottom string. Show that the SPCP is decidable.

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6. Show that A is Turing-recognizable iff $A \leq_m A_{TM}$

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7. **Rices theorem**. Let P be any nontrivial property of the language of a Turing machine. Prove that the problem of determining whether a given Turing machines language has property P is undecidable.

In more formal terms, let P be a language consisting of Turing machine descriptions where P fulfills two conditions. First, P is nontrivial - it contains some, but not all, TM descriptions. Second, P is a property of the TMs language-whenever $L(M_1) = L(M_2)$, we have $\langle M_1 \rangle \in P$ iff $\langle M_2 \rangle \in P$. Here, M_1 and M_2 are any TMs. Prove that P is an undecidable language.

Assume for the sake of contradiction that P is a decidable language satisfying the properties and let R_P be a TM that decides P. We show how to decide A_{TM} using R_P by constructing TM S. First, let T_\emptyset be a TM that always rejects, so $L(T_\emptyset) = \emptyset$. You may assume that $\langle T_\emptyset \rangle \notin P$ without loss of generality because you could proceed with P instead of P if $\langle T_\emptyset \rangle \in P$. Because P is not trivial, there exists a TM T with $\langle T \rangle \in P$. Design S to decide A_{TM} using R_P s ability to distinguish between T_\emptyset and T.

S = "On input $\langle M, w \rangle$:

1. Use M and w to construct the following TM M_w .

 $M_w =$ "On input x:

- 1. Simulate M on w. If it halts and rejects, reject. If it accepts, proceed to stage 2.
- 2. Simulate T on x. If it accepts, accept."
- 2. Use TM R_P to determine whether $\langle M_w \rangle \in P$. If YES, accept. if NO, reject."

TM M_w simulates T if M accept w. Hence $L(M_w)$ equals L(T) if M accepts w and \emptyset otherwise. Therefore $\langle M_w \rangle \in P$ iff M accepts w

8. Let

$$f(x) = \begin{cases} 3x + 1 & \text{for odd x} \\ x/2 & \text{for even x} \end{cases}$$

for any natural number x. If you start with an integer x and iterate f, you obtain a sequence, $x, f(x), f(f(x)), \dots$ Stop if you ever hit 1. For example, if x = 17, you get the sequence 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. Extensive computer tests have shown that every starting point between 1 and a large positive integer gives a sequence that ends in 1. But the question of whether all positive starting points end up at 1 is unsolved; it is called the 3x + 1 problem. Suppose that A_{TM} were decidable by a TM H. Use H to describe a TM that is guaranteed to state the answer to the 3x + 1 problem.

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9. Let $T = \{(i, j, k) \mid i, j, k \in N\}$. Show that T is countable.

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10. Review the way that we define sets to be the same size in Definition 4.12 (page 203). Show that is the same size is an equivalence relation.

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