CS 373 Homework 8 Anchu A. Lee
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1. This exercise concerns TM M_2 , whose description and state diagram appear in Example 3.7. In each of the parts, give the sequence of configurations that M_2 enters when started on the indicated input string.

   (a) 0.
   $q_1 0 \sqcup \quad \sqcup q_2 \sqcup \quad \sqcup\sqcup q_{accept}$

   (b) 000.
   $q_1 000 \sqcup \quad \sqcup q_2 00 \sqcup \quad \sqcup\sqcup q_3 0 \sqcup \quad \sqcup\sqcup 0 q_4 \sqcup \quad \sqcup\sqcup 0 \sqcup q_{reject}$

2. This exercise concerns TM M_1 , whose description and state diagram appear in Example 3.9. In each of the parts, give the sequence of configurations that M_1 enters when started on the indicated input string.

   (a) 1#1.
   $q_1 1\#1\sqcup \quad \mathrm{x} q_3 \#1\sqcup \quad \mathrm{x}\# q_5 1\sqcup \quad \mathrm{x} q_6 \#\mathrm{x}\sqcup \quad q_7 \mathrm{x}\#\mathrm{x}\sqcup \quad \mathrm{x} q_1 \#\mathrm{x}\sqcup \quad \mathrm{x}\# q_8 \mathrm{x}\sqcup \quad \mathrm{x}\#\mathrm{x} q_8 \sqcup \quad \mathrm{x}\#\mathrm{x}\sqcup q_{accept}$

   (b) 1##1.
   $q_1 1\#\#1\sqcup \quad \mathrm{x} q_3 \#\#1\sqcup \quad \mathrm{x}\# q_5 \#1\sqcup \quad \mathrm{x}\#\# q_{reject} 1\sqcup$

3. Describe a Turing machine, sequence of steps, that recognizes $\{w \mid w$ is an element of $\{a, b, c\}^*$ such that the number of a's in $w <$ the number of b's in w and the number of a's in $w =$ the number of c's in $w\}$

   (1) Place symbol at the left side of tape
   (2) Scan right for a, if found: mark it, else: go to step 6
   (3) Rewind
   (4) Scan right for b, if found: mark it, else: Halt and Reject (a must be $< b$)
   (5) Rewind and go to step 2.
   (6) Rewind
   (7) Scan right for a', if found: mark it, else: go to step 11
   (8) Rewind
   (9) Scan right for c, if found: mark it, else: Halt and Reject (a must be $= c$)
   (10) go to step 6.
   (11) Scan right for c, if found: Halt and Reject, else: Halt and Accept.

4. Show the equivalent transitions for a 2-PDA for the Turing machine transitions $(q_i, X) \to (q_j, A, L)$ and $(q_i, X) \to (q_j, A, R)$ (in state q_i read X, write A, and move left or right and transition to state q_j). The transitions for a 2-PDA are of the form $(q_i, X, S_1, S_2) \to (q_j, T_1, T_2)$ (in state q_i, read X, pop S_1 from stack 1, pop S_2 from stack 2, transition to state q_j, push T_1 onto stack 1 and push T_2 onto stack 2). You don't have to prove the transitions are equivalent, just tell me what they are.

5. Give implementation-level descriptions of Turing machines that decide the following languages over the alphabet $\{0, 1\}$. $\{w \mid w$ does not contain twice as many 0's as 1's$\}$

   (1) Place symbol at left side of tape
   (2) Rewind
   (3) Scan right for 1, if found: mark it, else: go to step 9
   (4) Rewind
   (5) Scan right for 0, if found: mark it, else: Halt and Accept
   (6) Rewind
   (7) Scan right for 0, if found: mark it, else: Halt and Accept
   (8) Go to step 2.
   (9) Rewind
   (10) Scan right for $0'$, if found: Halt and Reject, else: Halt and Accept

6. Prove the class of Turing recognizable languages is closed under the union operation (construction and proof)

Proof. Let M_1 and M_2 be two Turing machines that recognize languages L_1 and L_2 respectfully. Then let M_3 be a machine that will run input w alternately between machines M_1 and M_2. If a machine accepts, M_3 accepts. If both machines reject then M_3 rejects. As $w \in L_1 \cup L_2$, the string can be $w \in L_1$, then the M_1 portions of M_3 will accept it. If the string is $w \in L_2$, then the M_2 portions of M_3 will accept it. If the string is $w \notin L_1 \cup L_2$ then $w \notin L_1$ and $w \notin L_2$ and so M_3 will not accept w. Therefore M_3 recognizes $L_1 \cup L_2$. □

7. Prove the class of decidable languages is closed under concatenation (construction and proof)
8. Prove the class of decidable languages is closed under intersection (construction and proof)
9. Prove the class of Turing recognizable languages is closed under the star operation (construction and proof)
10. Show that a language is decidable iff some enumerator enumerates the language in the standard string order.