

Math 327 Homework 3
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Question 4.2

x	0	1	2	3
$f(x)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

$$E(X) = \sum_{x=0}^3 x \cdot f(x) = 0 \cdot \frac{27}{64} + 1 \cdot \frac{27}{64} + 2 \cdot \frac{9}{64} + 3 \cdot \frac{1}{64} = \frac{3}{4}$$

Question 4.4

$$P(H) = \frac{3}{4}, P(T) = \frac{1}{4}$$

x	0	1	2
$f(x)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

$$E(X) = 0 \cdot \frac{9}{16} + 1 \cdot \frac{6}{16} + 2 \cdot \frac{1}{16} = \frac{1}{2}$$

Question 4.10

$$\mu_X = 1 \cdot 0.17 + 2 \cdot 0.50 + 3 \cdot 0.33 = 2.16$$

$$\mu_Y = 1 \cdot 0.23 + 2 \cdot 0.50 + 3 \cdot 0.27 = 2.04$$

Question 4.14

$$E(X) = \int_0^1 \frac{x \cdot 2(x+2)}{5} dx = \frac{8}{15}$$

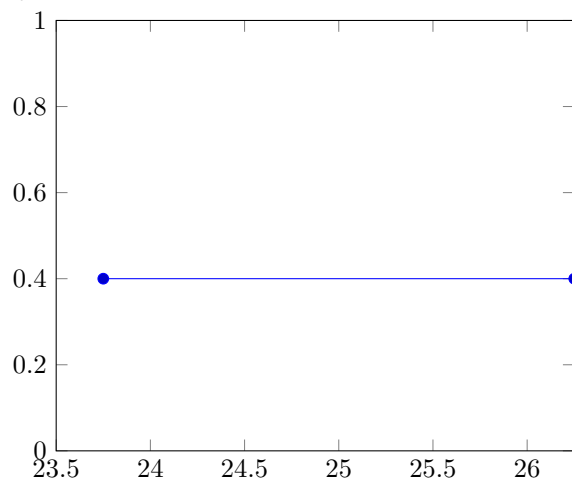
Question 4.18

$$E(X^2) = \sum_{x=0}^3 x^2 \cdot f(x) = 0 \cdot \frac{27}{64} + 1 \cdot \frac{27}{64} + 4 \cdot \frac{9}{64} + 9 \cdot \frac{1}{64} = \frac{9}{8}$$

Question 4.20

$$E(e^{2X/3}) = \int_0^\infty e^{2x/3} \cdot e^{-x} dx = 3$$

Question 4.28



$$E(X) = \int_{23.75}^{26.25} \frac{2}{5} x dx = 25$$

Not suprised, this is the expected value as it is exactly in the middle of the interval.

Question 4.34

$$\mu_X = (-2)(0.3) + (3)(0.2) + (5)(0.5) = 2.5$$

$$\sigma_X^2 = \sum x^2 f(x) - \mu_X^2$$

$$\sum x^2 f(x) = (4)(0.3) + (9)(0.2) + (25)(0.5) = 15.5$$

$$\sigma_X^2 = 15.5 - 2.5^2 = 9.25$$

$$\sigma = 3.04138...$$

Question 4.38

$$\begin{aligned}\mu_X &= \frac{8}{15} \\ \sigma_X^2 &= \int_0^1 x^2 \frac{2(x+2)}{5} dx - \mu_X^2 \\ \int_0^1 x^2 \frac{2(x+2)}{5} dx &= \frac{11}{30} \\ \sigma_X^2 &= \frac{11}{30} - \left(\frac{8}{15}\right)^2 = \frac{37}{450}\end{aligned}$$

Question 4.40

$$\begin{aligned}g(X) &= 3X^2 + 4 \\ \mu_{g(X)} &= \int_0^1 (3x^2 + 4) \frac{2(x+2)}{5} dx = 5.1 \\ \sigma_{g(X)}^2 &= \int_0^1 (3x^2 + 4)^2 \frac{2(x+2)}{5} dx - \mu_{g(X)}^2 \\ \int_0^1 (3x^2 + 4)^2 \frac{2(x+2)}{5} dx &= \frac{671}{25} \\ \sigma_{g(X)}^2 &= \frac{671}{25} - (5.1)^2 = 0.83\end{aligned}$$

Question 4.46

$$\begin{aligned}k &= \left(\frac{3}{392}\right)10^{-4}, g(x) = k(20x^2 + \frac{98000}{3}) \\ u_X &= \int_{30}^{50} x \cdot \left(\frac{3}{392}\right)10^{-4}(20x^2 + \frac{98000}{3})dx = 40.81632... \\ u_Y &= \int_{30}^{50} y \cdot \left(\frac{3}{392}\right)10^{-4}(20y^2 + \frac{98000}{3})dy = 40.81632... \\ E(XY) &= \int_{30}^{50} \int_{30}^{50} kxy(x^2 + y^2)dydx = k \int_{30}^{50} 800x^3 + 1360000xdx \\ E(XY) &= k \int_{30}^{50} 800x^3 + 1360000xdx = 1665.30612... \\ \sigma_{XY} &= E(XY) - u_X u_Y = -0.6642...\end{aligned}$$

Question 4.58

$$\begin{aligned}E(Y) &= 60E(X^2) + 39E(X) \\ E(X) &= \int_0^1 x^2 + \int_1^2 x(2-x) = 1 \\ E(X^2) &= \int_0^1 x^3 + \int_1^2 x^2(2-x) = \frac{7}{6} \\ E(Y) &= (60)\left(\frac{7}{6}\right) + (39)(1) = 109 \text{ kwh}\end{aligned}$$

Question 4.60

$$\begin{aligned}E(2X - 3Y) &= E(2X) - E(3Y) \\ E(X) &= (2)(0.40) + (5)(0.60) = 3.20 \\ E(Y) &= (1)(0.25) + (3)(0.50) + (5)(0.25) = 3\end{aligned}$$

- $E(2X - 3Y) = 2E(X) - 3E(Y) = (2)(3.20) - (3)(3) = -2.60$
- $E(XY) = E(X)E(Y) = (3.20)(3) = 9.60$

Question 4.62

$$\begin{aligned}\sigma_Z^2 &= \sigma_{-2X+4Y-3}^2 = (-2)^2\sigma_X^2 + 4^2\sigma_Y^2 = 4\sigma_X^2 + 16\sigma_Y^2 \\ 4\sigma_X^2 + 16\sigma_Y^2 &= (4)(5) + (16)(3) = 68\end{aligned}$$

Question 4.64

$$\begin{aligned}E(X) &= \int_2^\infty \frac{8}{x^3} \\ E(Y) &= \int_0^1 2y \\ E(Z) &= E(XY) = E(X)E(Y) = \int_2^\infty x \frac{8}{x^3} \cdot \int_0^1 2y^2 = \frac{8}{3}\end{aligned}$$

Question 4.76

$$\begin{aligned}\mu &= 60 \text{ and } \sigma = 6 \\ \text{By Chebyshev's theorem: } 1 - \frac{1}{k^2} &\leq P(\mu - k\sigma < X < \mu + k\sigma) \\ 84 &= \mu + k\sigma, k = 4 \\ \mu - k\sigma &= 60 - (4)(6) = 36 \\ 1 - \frac{1}{16} &\leq P(36 < X < 84) \leq P(X < 84)\end{aligned}$$

$$1 - \frac{1}{16} = 0.9375$$

$$P(X \geq 4) \leq 1 - 0.9375 = 0.0625$$

Question 4.92

No, X and Y are not independent because $g(0)h(0) = (0.17)(0.23)$ while $f(0, 0) = 0.10$

Question 4.98

$$\bullet \quad \begin{array}{c|ccc} x & 0 & 1 & 2 \\ \hline g(x) & 0.20 & 0.32 & 0.48 \end{array} \quad \begin{array}{c|ccc} y & 0 & 1 & 2 \\ \hline g(y) & 0.26 & 0.35 & 0.39 \end{array}$$

Given $Y = 2$

$$\begin{array}{c|ccc} y & 0 & 1 & 2 \\ \hline f(x|2) & \frac{4}{39} & \frac{5}{39} & \frac{30}{39} \end{array}$$

$$\bullet \quad E(X) = (0)(0.20) + (1)(0.32) + (2)(0.48) = 1.28$$

$$\sigma_x^2 = (0)(0.20) + (1)(0.32) + (4)(0.48) - \mu_x^2 = 2.24 - 1.28^2 = 0.6016$$

$$\bullet \quad E(X | Y = 2) = (0)\left(\frac{4}{39}\right) + (1)\left(\frac{5}{39}\right) + (2)\left(\frac{30}{39}\right) = \frac{5}{3}$$

$$\sigma_{X|Y=2}^2 = (0)\left(\frac{4}{39}\right) + (1)\left(\frac{5}{39}\right) + (4)\left(\frac{30}{39}\right) - \mu_{X|Y=2}^2 = \frac{125}{39} - \left(\frac{5}{3}\right)^2 = \frac{50}{117}$$