

October 24, 2017

Question 6.2: Suppose X follows a continuous uniform distribution from 1 to 5. Determine the conditional probability $P(X > 2.5 | X \leq 4)$

$$\frac{P(2.5 < X \leq 4)}{P(X \leq 4)} = \frac{4-2.5}{4-1} = \frac{1.5}{3}$$

Question 6.4: A bus arrives every 10 minutes at a bus stop. It is assumed that the waiting time for a particular individual is a random variable with a continuous uniform distribution.

- (a) What is the probability that the individual waits more than 7 minutes?

$$P(X > 7) = \frac{10-7}{10} = \frac{3}{10}$$

- (b) What is the probability that the individual waits between 2 and 7 minutes?

$$P(2 < X < 7) = \frac{7-2}{10} = \frac{5}{10}$$

Question 6.6: Find the value of z if the area under a standard normal curve

- (a) to the right of z is 0.3622;

The left of z is then $1 - 0.3622 = 0.6378$. 0.6378 can be approximated as 0.6368 so $z = 0.35$ by Table A.3.

- (b) to the left of z is 0.1131;

The right of z is then $0.1131 - 1 = -0.8869$. So by table A.3, $z = -1.21$

- (c) between 0 and z , with $z > 0$, is 0.4838;

Area left of z is $0.5 + 0.4838 = 0.9838$. So $z = 2.14$.

- (d) between z and z , with $z > 0$, is 0.9500.

Area left of z is $0.25 + 0.95 = 0.975$, so $z = 1.96$.

Question 6.10: According to Chebyshev's theorem, the probability that any random variable assumes a value within 3 standard deviations of the mean is at least $8/9$. If it is known that the probability distribution of a random variable X is normal with mean μ and variance σ^2 , what is the exact value of $P(\mu - 3\sigma < X < \mu + 3\sigma)$?

Question 6.12: The loaves of rye bread distributed to local stores by a certain bakery have an average length of 30 centimeters and a standard deviation of 2 centimeters. Assuming that the lengths are normally distributed, what percentage of the loaves are

- (a) longer than 31.7 centimeters?
 (b) between 29.3 and 33.5 centimeters in length?
 (c) shorter than 25.5 centimeters?

Question 6.22: If a set of observations is normally distributed, what percent of these differ from the mean by

- (a) more than 1.3σ ?
 (b) less than 0.52σ ?

Question 6.26: A process yields 10% defective items. If 100 items are randomly selected from the process, what is the probability that the number of defectives

- (a) exceeds 13?
 (b) is less than 8?

Question 6.30: A drug manufacturer claims that a certain drug cures a blood disease, on the average, 80% of the time. To check the claim, government testers use the drug on a sample of 100 individuals and decide to accept the claim if 75 or more are cured.

- (a) What is the probability that the claim will be rejected when the cure probability is, in fact, 0.8?
- (b) What is the probability that the claim will be accepted by the government when the cure probability is as low as 0.7?

Question 6.34: A pair of dice is rolled 180 times. What is the probability that a total of 7 occurs

- (a) at least 25 times?
- (b) between 33 and 41 times inclusive?
- (c) exactly 30 times?

Question 6.40: In a certain city, the daily consumption of water (in millions of liters) follows approximately a gamma distribution with $\alpha = 2$ and $\beta = 3$. If the daily capacity of that city is 9 million liters of water, what is the probability that on any given day the water supply is inadequate?

Question 6.42: Suppose that the time, in hours, required to repair a heat pump is a random variable X having a gamma distribution with parameters $\alpha = 2$ and $\beta = 1/2$. What is the probability that on the next service call

- (a) at most 1 hour will be required to repair the heat pump?
- (b) at least 2 hours will be required to repair the heat pump?

Question 6.46: The life, in years, of a certain type of electrical switch has an exponential distribution with an average life $\beta = 2$. If 100 of these switches are installed in different systems, what is the probability that at most 30 fail during the first year?

Question 6.50: If the proportion of a brand of television set requiring service during the first year of operation is a random variable having a beta distribution with $\alpha = 3$ and $\beta = 2$, what is the probability that at least 80% of the new models of this brand sold this year will require service during their first year of operation?

Question 6.54: The lifetime, in weeks, of a certain type of transistor is known to follow a gamma distribution with mean 10 weeks and standard deviation $\sqrt{50}$ weeks.

- (a) What is the probability that a transistor of this type will last at most 50 weeks?
- (b) What is the probability that a transistor of this type will not survive the first 10 weeks?

Question 6.56: Rate data often follow a lognormal distribution. Average power usage (dB per hour) for a particular company is studied and is known to have a lognormal distribution with parameters $\mu = 4$ and $\sigma = 2$. What is the probability that the company uses more than 270 dB during any particular hour?

Question 6.58: The number of automobiles that arrive at a certain intersection per minute has a Poisson distribution with a mean of 5. Interest centers around the time that elapses before 10 automobiles appear at the intersection.

- (a) What is the probability that more than 10 automobiles appear at the intersection during any given minute of time?
- (b) What is the probability that more than 2 minutes elapse before 10 cars arrive?