Introduction to Pattern Recognition Homework 3 Report: Decision Trees and Random Forests

1 Questions

1.1 Gini index or Entropy is often used for meaasuring the "best" splitting of the data. Please compute the entropy and Gini Index of the provided data by the formula below.

$$Gini = 1 - \sum_{j} p_{j}^{2}$$

$$Entropy = -\sum_{j} p_{j}log_{2}p_{j}$$

The **Gini index** and **Entropy** are two criteria we sometimes use to measure the purity or quality of a split on data. The Gini index tells us how mixed the classes are, and we would like to choose the split that results in the lowest amount of mixing. The entropy is a measure of how much information we gain by making a split on a certain feature.

For the provided dataset, we have the Gini index and entropy as:

Gini = 0.4628099173553719 Entropy = 0.9456603046006402

- 1.2 Implement the Decision Tree algorithm CART and train the model by the given arguments, and print the accuracy score on the test data.
- 1.2.1 Using criterion='gini', show the accuracy score of the test data by max_depth=3 and max_depth = 10, respectively.
- 1.2.2 Using max_depth=3, show the accuracy score of test data by criterion='gini' and criterion='entropy'.

2 Coding

- 2.1 Consider a dataset comprising 400 data points from Class C_1 and 400 data points from class C_2 . Supose that a tree model A splits these into (300, 100) at the first leaf node and (100,300) at the second leaf node, where (n,m) denotes that n points are assigned to C_1 and m points are assigned to C_2 . Similarly, suppose that a second tree model B splits them into (200, 400) and (200, 0). Evaluate the misclassification rate for the two trees and hence show that they are equal.
- 2.2 Similarly, evaluate the cross-entropy

$$Entropy = -\sum_{k=1}^{K} p_k log_2 p_k$$

and the Gini index

$$Gini = 1 - \sum_{k=1}^{K} p_k^2$$

for the two trees and show that they are both lower for tree B than for tree A. Define p_k to be the proportion of data points in region R assigned to class k, where k=1,...,K.