Introduction to Pattern Recognition Homework 2 Report

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April 24, 2020

1 Coding

1.1 Compute the mean vectors $m_i, (i = 1, 2)$ of each 2 classes on training data

1.2 Compute the within-class scatter matrix S_W on training data.

1.3 Compute the between-class scatter matrix S_B on training data.

1.4 Compute the Fisher's linear discriminant W on training data.

1.5 Project the testing data by linear discriminant to get the class prediction by nearest-neighbor rule and calculate your accuracy score on testing data.

1.6 Plot the 1) best projection line on the training data and slow the slope and intercept on the title 2)colorize the data with each class 3) project all data points on your projection line.

2 Questions

2.1 Show that maximization of the class separation criterion given by $L(\lambda, w) = w^T(m_2 - m_1) + \lambda(w^T w - 1)$ with respect to w, using a Lagrange multiplier to enforce the constraint $w^T w = 1$, leads to the result that $w \propto (m_2 - m_1)$.

Lagrange Multipliers allow us to maximize a function L given a constraint g. To do this, we have the initial equation $\nabla L(w, \lambda) = \alpha \nabla g(w, \lambda)$. Where α is the Lagrange Multiplier in this equation. Since we apply the multiplier to each variable individually, then we have to find the multiplier based on the component functions for L and g. Thus we have to solve the system of equations

$$L_\lambda = \alpha g_\lambda \qquad L_w = \alpha g_w$$

So, we have to differentiate with respect to λ and w, since we are dealing with the gradient vector. When we set up the equations, we get the system of equations

$$w^T w - 1 = 0 \tag{1a}$$

$$m_2 - m_1 = \alpha \tag{1b}$$

$$w^T w = 1 \tag{1c}$$

Then, multiplying (1c) by (1b), we get

$$(m_2 - m_1)(w^T w) = \alpha$$

which, when divided by $m_2 - m_1$ gives

$$w^T w = \frac{\alpha}{m_2 - m_1}$$

Since $w_T w$ is proportional to $m_2 - m_1$, then w will also be. Thus, using Lagrange Multipliers we have shown that $w \propto m_2 - m_1$

2.2 Using eq 2 and eq 3, derive the result eq 4 for the posterior class probability in the two-class generative model with Gaussian densities, and verify the results eq 5 and eq 6 for the parameters w and w_0.

$$\frac{1}{1 + exp(-\alpha) = \phi(\alpha)} \tag{2}$$

$$a = ln\frac{p(x|C_1)p(C_1)}{p(x|C_2)p(C_2)} \tag{3}$$

$$p(C_1|x) = \phi(w^T x + w_0) \tag{4}$$

$$w = \sum^{-1}(\mu_1 - \mu_2) \tag{5}$$

$$w_0 = -\frac{1}{2}\mu_1^T \sum{}^{-1}\mu_1 + \frac{1}{2}\mu_2^T \sum{}^{-1}\mu_2 + ln\frac{p(C_1)}{p(C_2)} \tag{6}$$