

In this assignment, your task is to find solutions of Minesweeper problems using Backtrack Search. Here we will consider a type of Minesweeper problems: In a board where part of the cells are already marked as hints, your task is to find an arrangement of mines in the other cells such that the hints are correct. An example is given below:

**Problem:**

			1	1	
	3				0
2	3		3	2	2
		2			
	2	2	3		3
	1				1

**One Solution:**

			1	1	
	3				0
2	3		3	3	2
		2			
	2	2	3		3
	1				1



The formulation of this task as a constraint satisfaction problem will be like:

- The cells are indexed starting from the top-left cell at (0,0). The hint (0~8) at cell (x,y) is  $h(x,y)$ .
- The value assigned to cell (x,y) is  $n(x,y)$ , which can only be zero or one, representing whether it contains a mine.
- Each non-hint cell represents a variable. The initial domains are  $\{0,1\}$ .
- Each hint gives a constraint in the form of an equation. For example:  

$$n(0,0) + n(1,0) + n(2,0) + n(0,1) + n(2,1) + n(2,2) = h(1,1) = 3.$$
- There is a global constraint that the summation of all the variables equals the total number of mines.

For problems given in this assignment, we will use 6x6 boards with 16 hints (17 constraints), 20 variables, and 10 mines in total. Note: Solutions may not be unique.

Notes about implementation:

- Implement backtrack search with a stack. Each node contains (1) the list of all the variable+value assignments made from the root to that node, and (2) the domains of all the unassigned variables at that node. Initialize the stack with a node containing an empty assignment list and initial domains of all the variables.
- When popping a node from the stack:
  - (Optional) Do the forward checking to update the domains of unassigned variables.
  - Generate the child nodes from all the variable+value combinations applicable after the current node. Insert them to the stack (in increasing order of preference, if any heuristic is used).

Consistency check for forward checking:

- For each constraint, we compute the lower and upper bounds of the sum of the variables given their domains, and compare the bounds with the hint.
- If the lower bound is larger than the hint, the constraint cannot be satisfied.
- If the upper bound is smaller than the hint, the constraint cannot be satisfied.
- If the lower bound equals the hint, the domains of all the unassigned variable in the constraint should be limited to their respective minimal values.
- If the upper bound equals the hint, the domains of all the unassigned variable in the constraint should be limited to their respective maximal values.
- For other cases, the domains are unchanged.

Heuristics

- MRV: This means that variables whose domains have become singletons are assigned first.

Figure 1 shows four 6x6 grids, labeled (a), (b), (c), and (d), illustrating different configurations of blue and orange cells. The grids are arranged in a row. Each grid has 6 rows and 6 columns. The cells are colored either blue or orange. The configurations are as follows:

- (a) Blue cells: (1,4), (1,5), (2,2), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,2), (4,3), (4,4), (4,5), (4,6), (5,2), (5,3), (5,4), (5,5), (5,6), (6,2), (6,3), (6,4), (6,5), (6,6). Orange cells: (1,1), (1,2), (1,3), (1,6), (2,1), (2,3), (2,4), (2,5), (3,5), (3,6), (4,1), (4,5), (5,1), (5,6), (6,1), (6,5), (6,6).
- (b) Blue cells: (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,2), (4,3), (4,4), (4,5), (4,6), (5,2), (5,3), (5,4), (5,5), (5,6), (6,2), (6,3), (6,4), (6,5), (6,6). Orange cells: (1,1), (1,2), (1,3), (1,6), (2,1), (2,5), (2,6), (3,5), (3,6), (4,1), (4,5), (5,1), (5,6), (6,1), (6,5), (6,6).
- (c) Blue cells: (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,2), (4,3), (4,4), (4,5), (4,6), (5,2), (5,3), (5,4), (5,5), (5,6), (6,2), (6,3), (6,4), (6,5), (6,6). Orange cells: (1,1), (1,6), (2,1), (2,6), (3,5), (3,6), (4,1), (4,5), (5,1), (5,6), (6,1), (6,5), (6,6).
- (d) Blue cells: (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,2), (4,3), (4,4), (4,5), (4,6), (5,2), (5,3), (5,4), (5,5), (5,6), (6,2), (6,3), (6,4), (6,5), (6,6). Orange cells: (1,1), (1,6), (2,1), (2,6), (3,5), (3,6), (4,1), (4,5), (5,1), (5,6), (6,1), (6,5), (6,6).