

Introduction to Pattern Recognition Homework 1

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1. **Suppose that we have three colored boxes R (red), B (blue), and G (green). Box R contains 3 apples, 4 oranges, and 3 guavas, box B contains 2 apples, 0 oranges, and 2 guavas, and box G contains 12 apples, 4 oranges, and 4 guavas. If a box is chosen at random with probabilities $p(R) = 0.2, p(B) = 0.4, p(G) = .4$, and a piece of fruit is removed from the box (with equal probability of selecting any of the items in the box), then what is the probability of selecting guava? If we observe that the selected fruit is in fact an apple, what is the probability that it came from the blue box?**

For the first question, let b_i refer to choosing box i , and g_i refer to choosing a guava from box i . If we want the probabilities in general of choosing a guava from each of the boxes, we need to add the products $p(b_i)p(g_i)$. Thus our formula becomes

$$\sum_{i=0}^N p(b_i)p(g_i)$$

which evaluates to

$$(.3)(.2) + (.5)(.4) + (.4)(.2) = 0.34$$

For the second question, we want to calculate the probability that the blue box was chosen given that we have chosen an apple. Let a be the event of choosing an apple, and B be the event of choosing the blue box. Then, we would like to find $p(B|a)$. By using **Baye's Theorem**, which states

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)}$$

we can find each of the required probabilities. We are given that $p(B) = .4$. To find $p(a|B)$, we find the probability of choosing an apple from box B , which is $\frac{2}{2+2} = 0.5$. Finally, to find $p(a)$, we repeat the process from the first part, and add the probabilities of choosing box i and picking an apple from that box, which in the end comes out to .5. Thus, our final calculation is

$$p(B|a) = \frac{(.5)(.4)}{(.5)} = .4$$

2. **Using the definition**

$$\text{var}[f] = E[(f(x) - E[f(x)])^2]$$

show that $\text{var}[f(x)]$ satisfies

$$\text{var}[f(x)] = E[f(x)^2] - E[f(x)]^2$$