Problem 1

In this problem, we have to minimize the function $f(x) = x^3 \exp(-x^2)$ within the interval $[-2, 2]$. We compare the number of iterations used by the golden section method and quadratic interpolation to achieve a good result.

In the golden section method we repeatedly bracket the minimum in a smaller and smaller interval based on the function value evaluated at two points x_1 and x_2. We also have two endpoints a, b inside of which we conduct our search. The points x_1, x_2 are selected using the golden ratio such that the distances between the endpoints and x_1, x_2 are equal to the golden ratio. Once the difference between the endpoints is lower than some tolerance ϵ, we stop the algorithm.

We compare the golden section method with the quadratic interpolation method, which instead tries to use a parabola to find the minimum point. We calculate a point x^*, given by the function

$$x^* = \frac{1}{2} \times \frac{f(x_l)(x_m^2 - x_u^2) + f(x_m)(x_u^2 - x_l^2) + f(x_u)(x_l^2 - x_m^2)}{f(x_l)(x_m - x_u) + f(x_m)(x_u - x_l) + f(x_u)(x_l - x_m)} \tag{1}$$

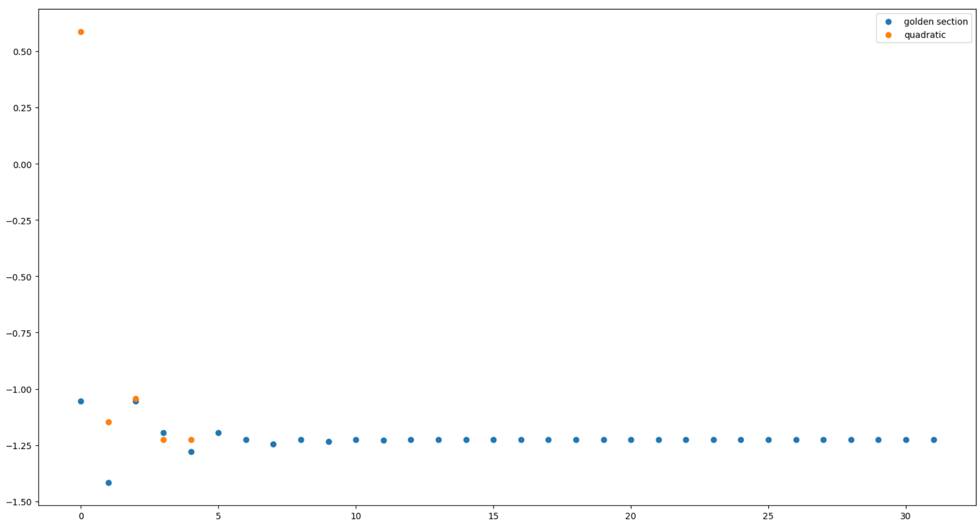

Figure 1: Number of iterations necessary for golden section and quadratic interpolation methods to finish. The tolerance for both is 1×10^{-7}. The y-value is the function value of $\min(x_1, x_2)$ for golden section and x^* for the quadratic interpolation.

Problem 2

For the second and third problem, we want to minimize the function

$$f(x_1, x_2) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$

called Himmelblau's function.

For the second problem we use the Hooke-Jeeves method to find the function minimum. This algorithm consists of three parts. First, for a dimension i and using our current minimum u, we evaluate the function at $f(u + \nabla u)$ and $f(u - \nabla u)$ to see which direction along i we should follow. Essentially we modify ∇u along i to see how we should move along i.

Second, we see if the terminating conditions are met, or whether the norm of the direction vector is zero. This means we cannot further minimize the function value by moving u.

Finally, if we can still continue the algorithm, we check how far along ∇u we can move. We evaluate $f(u + \nabla u)$ until we stop getting lower function values.

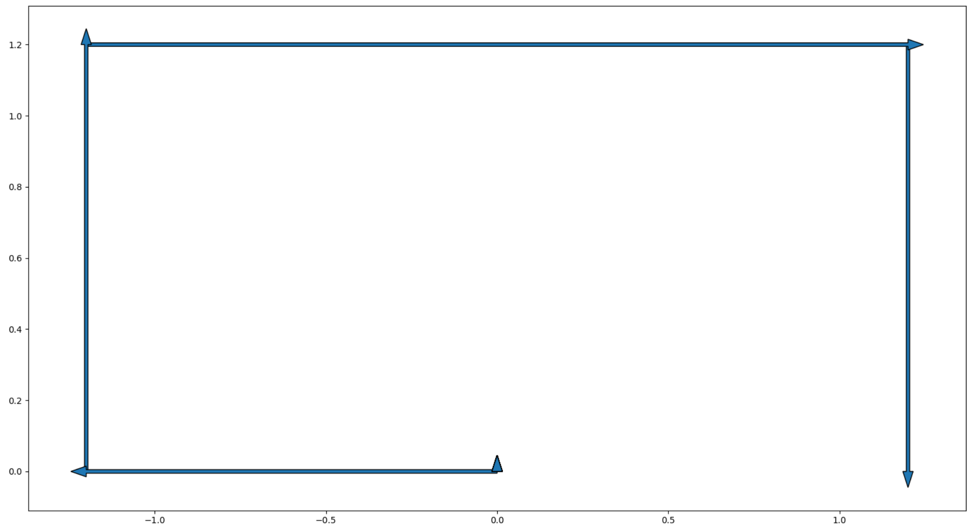

Figure 2: The direction of our search vector ∇u when the step size is 0.6. ∇u changes along the x and y dimension for the length of our algorithm or until its norm is 0.

Problem 3

For the third problem we still minimize Himmelblau' s function, however here we use the downhill simplex method. With this method, we first define an $n+1$ dimensional shape as our simplex, and ensure that by the end, the minimum is contained within that simple.

For $n = 2$ our simplex is a triangle. We can perform three operations on our simplex: reflection, expansion and contraction.

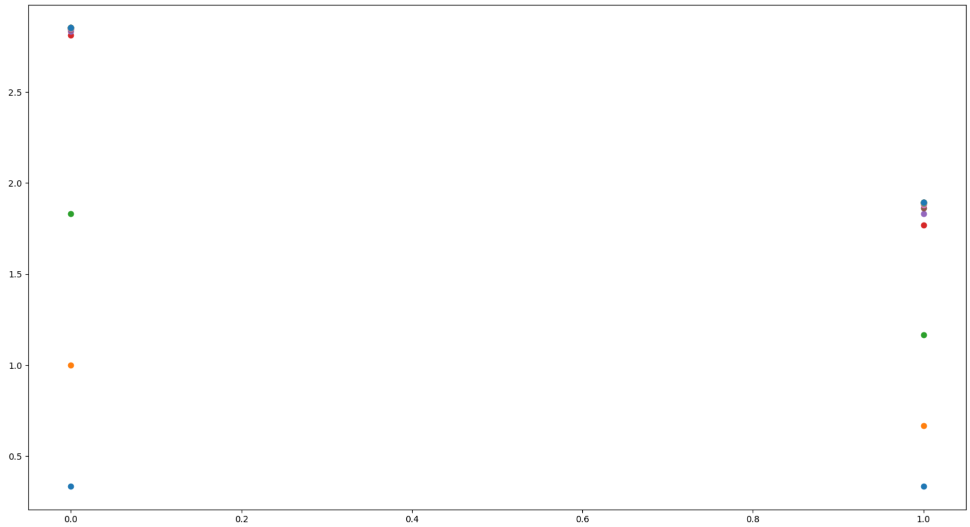

Figure 3: Simplex centroids. Towards the end of the iterations the centroids cluster around $(2.8, 1.9)$