NATIONAL CHENG KUNG UNIVERSITY

Optimization Design Homework 1

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1 Find the stationary points for the following functions. Also identify(for each stationary point), the local maximum, minimum, or neither (by using second order derivative or Hessian matrix.)

1.1 $f(x) = x^3 \exp(-x^2)$, for $-2 < x < 2$.

The stationary points are those where $f'(x) = 0$, so we first find the first derivative. By using the product rule,

$$f'(x) = 3x^2 \exp(-x^2) - 2x^4 \exp(-x^2) = -x^2 \exp(-x^2)(2x^4 - 3x^2) \tag{1.1}$$

which is zero at $x = 0$, and by solving $2x^4 - 3x^2$ we get the other roots $x = \pm\frac{\sqrt{3}}{\sqrt{2}}$. Theorem 2.2 gives the sufficient condition for a minimum or maximum point for single variables. We must find a point where $f^m(x^*) \neq 0$, so we find $f''(x)$. We again use the product rule on Equation 1.1 and factor the result to obtain

$$f''(x) = -x \exp(-x^2)(4x^4 + 2x^2 - 6) \tag{1.2}$$

where $f''(\frac{\sqrt{3}}{\sqrt{2}}) < 0$ and $f''(-\frac{\sqrt{3}}{\sqrt{2}}) > 0$ However, $f''(0) = 0$, so we find the third derivative by using the same procedure

$$f'''(x) = -x \exp(-x^2)(20x^3 + 6x + 8x^4 + 4x^2 - 12) - 6 \exp(-x^2) \tag{1.3}$$

where $f'''(0) = 6$, however since $n = 3$, 0 does not correspond to either a maximum nor minimum point. In the end, $-\frac{\sqrt{3}}{\sqrt{2}}$ is a minimum, 0 is neither, and $\frac{\sqrt{3}}{\sqrt{2}}$ is a maximum.

1.2 $f(x, y) = -x^2 - 3y^2 + 12xy$