Graph Theory Homework 1

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1 (True or False) There exists a simple graph with 6 vertices, whose degrees are $2, 3, 3, 4, 4, 5$.

This statement is **false**. According to the Degree-Sum Theorem, given a vertex v and its degree $d(v)$, the sum the degrees is twice the number of edges in our graph.

$$\sum_{v \in V(G)} d(v) = 2e(G) \tag{1}$$

When counting the degrees, each edge gets counted exactly twice, since one edge connects one vertex u to another vertex v. The total number of degrees then has to be an even number. A corollary of this rule is that a graph can have only an even number of vertices with odd degrees, since an even number of odd degrees would result in an even degree count. In the list above, we have three vertices with odd degree, which violates the degree sum rule. Such a graph can not exist.

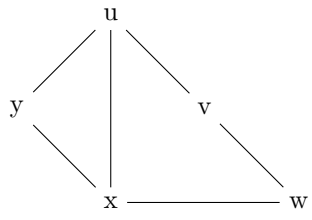
2 Determine the values of m, n such that a complete bipartite graph $K_{m,n}$ is a complete graph.

A **complete graph** is a graph for which there exists an edge between any pair of vertices $u, v, u \neq v$. In a bipartite graph, the vertices are split into two sets such that their union is $V(G)$. All the vertices from one partition have an edge to all the vertices in the other partition.

The values $m = 1, n = 1$ satisfy the constraints. For the graph $K_{1,1}$, there is an edge connecting every pair of distinct vertices, which satisfies the definition of a complete graph, and also every node in one partition is connected to all the nodes in the other partition, satisfying the definition of a bipartite graph.

3 Briefly describe the definition of a simple graph.

4 Draw the complement of the following graph



(2)

5 **Describe the definition of self-complementary.**

6 **What is a walk? What is a trail? Is a walk a trail?**

7 **Describe the degree-sum formula.**

8 **Prove or disprove: If u and v are the only vertices of odd degree in a graph G, then G contains a $u - v$ path.**

9 **(True or False) Every disconnected graph has an isolated vertex.**

10 **Prove or disprove. If every vertex of a simple graph G has degree 2, then G is a cycle.**