Project 3

Robert Ottolia

Luke Brewbaker

Andrew Brown

**Problem 1:** Transshipment Model

Part A

i.

min(15\*P1W1R1 + 16\*P1W1R2 + 17\*P1W1R3 + 20\*P1W1R4 + 27\*P1W2R3 + 23\*P1W2R4 + 25\*P1W2R5 + 29\*P1W2R6 + 16\*P2W1R1 + 17\*P2W1R2 + 18\*P2W1R3 + 21\*P2W1R4 + 20\*P2W2R3 + 16\*P2W2R4 + 18\*P2W2R5 + 22\*P2W2R6 + 18\*P3W1R1 + 19\*P3W1R2 + 20\*P3W1R3 + 23\*P3W1R4 + 20\*P3W2R3 + 16\*P3W2R4 + 18\*P3W2R5 + 22\*P3W2R6 + 23\*P3W3R4 + 21\*P3W3R5 + 21\*P3W3R6 + 15\*P3W3R7 + 26\*P4W2R3 + 22\*P4W2R4 + 24\*P4W2R5 + 28\*P4W2R6 + 22\*P4W3R4 + 20\*P4W3R5 + 20\*P4W3R6 + 14\*P4W3R7)

( Supply Constraints)

P1W1R1 + P1W1R2 + P1W1R3 + P1W1R4 + P1W2R3 + P1W2R4 + P1W2R5 + P1W2R6 <= 150

P2W1R1 + P2W1R2 + P2W1R3 + P2W1R4 + P2W2R3 + P2W2R4 + P2W2R5 + P2W2R6 <= 450

P3W1R1 + P3W1R2 + P3W1R3 + P3W1R4 + P3W2R3 + P3W2R4 + P3W2R5 + P3W2R6 + P3W3R4 + P3W3R5 + P3W3R6 + P3W3R7 <= 250

P4W2R3 + P4W2R4 + P4W2R5 + P4W2R6 + P4W3R4 + P4W3R5 + P4W3R6 + P4W3R7 <= 150

( Demand Constraints)

P1W1R1 + P2W1R1 + P3W1R1 >= 100

P1W1R2 + P2W1R2 + P3W1R2 >= 150

P1W1R3 + P1W2R3 + P2W1R3 + P2W2R3 + P3W1R3 + P3W2R3 + P4W2R3 >= 100

P1W1R4 + P1W2R4 + P2W1R4 + P2W2R4 + P3W1R4 + P3W2R4 + P3W3R4 + P4W2R4 + P4W3R4 >= 200

P1W2R5 + P2W2R5 + P3W2R5 + P3W3R5 + P4W2R5 + P4W3R5 >= 200

P1W2R6 + P2W2R6 + P3W2R6 + P3W3R6 + P4W2R6 + P4W3R6 >= 150

P3W3R7 + P4W3R7 >= 100

ii.

For this part we used Lindo. The input was as follows:

MIN 15 P1W1R1 + 16 P1W1R2 + 17 P1W1R3 + 20 P1W1R4 + 27 P1W2R3 + 23 P1W2R4 + 25 P1W2R5 + 29 P1W2R6 + 16 P2W1R1 + 17 P2W1R2 + 18 P2W1R3 + 21 P2W1R4 + 20 P2W2R3 + 16 P2W2R4 + 18 P2W2R5 + 22 P2W2R6 + 18 P3W1R1 + 19 P3W1R2 + 20 P3W1R3 + 23 P3W1R4 + 20 P3W2R3 + 16 P3W2R4 + 18 P3W2R5 + 22 P3W2R6 + 23 P3W3R4 + 21 P3W3R5 + 21 P3W3R6 + 15 P3W3R7 + 26 P4W2R3 + 22 P4W2R4 + 24 P4W2R5 + 28 P4W2R6 + 22 P4W3R4 + 20 P4W3R5 + 20 P4W3R6 + 14 P4W3R7

ST

! Supply Constraints

P1W1R1 + P1W1R2 + P1W1R3 + P1W1R4 + P1W2R3 + P1W2R4 + P1W2R5 + P1W2R6 <= 150

P2W1R1 + P2W1R2 + P2W1R3 + P2W1R4 + P2W2R3 + P2W2R4 + P2W2R5 + P2W2R6 <= 450

P3W1R1 + P3W1R2 + P3W1R3 + P3W1R4 + P3W2R3 + P3W2R4 + P3W2R5 + P3W2R6 + P3W3R4 + P3W3R5 + P3W3R6 + P3W3R7 <= 250

P4W2R3 + P4W2R4 + P4W2R5 + P4W2R6 + P4W3R4 + P4W3R5 + P4W3R6 + P4W3R7 <= 150

! Demand Constraints

P1W1R1 + P2W1R1 + P3W1R1 >= 100

P1W1R2 + P2W1R2 + P3W1R2 >= 150

P1W1R3 + P1W2R3 + P2W1R3 + P2W2R3 + P3W1R3 + P3W2R3 + P4W2R3 >= 100

P1W1R4 + P1W2R4 + P2W1R4 + P2W2R4 + P3W1R4 + P3W2R4 + P3W3R4 + P4W2R4 + P4W3R4 >= 200

P1W2R5 + P2W2R5 + P3W2R5 + P3W3R5 + P4W2R5 + P4W3R5 >= 200

P1W2R6 + P2W2R6 + P3W2R6 + P3W3R6 + P4W2R6 + P4W3R6 >= 150

P3W3R7 + P4W3R7 >= 100

END

And the results were:

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 17100.00

VARIABLE VALUE REDUCED COST

P1W1R1 0.000000 210.000000

P1W1R2 50.000000 0.000000

P1W1R3 100.000000 0.000000

P1W1R4 0.000000 5.000000

P1W2R3 0.000000 10.000000

P1W2R4 0.000000 8.000000

P1W2R5 0.000000 8.000000

P1W2R6 0.000000 9.000000

P2W1R1 100.000000 0.000000

P2W1R2 100.000000 0.000000

P2W1R3 0.000000 0.000000

P2W1R4 0.000000 5.000000

P2W2R3 0.000000 2.000000

P2W2R4 50.000000 0.000000

P2W2R5 200.000000 0.000000

P2W2R6 0.000000 1.000000

P3W1R1 0.000000 2.000000

P3W1R2 0.000000 2.000000

P3W1R3 0.000000 2.000000

P3W1R4 0.000000 7.000000

P3W2R3 0.000000 2.000000

P3W2R4 150.000000 0.000000

P3W2R5 0.000000 0.000000

P3W2R6 0.000000 1.000000

P3W3R4 0.000000 7.000000

P3W3R5 0.000000 3.000000

P3W3R6 100.000000 0.000000

P3W3R7 0.000000 0.000000

P4W2R3 0.000000 9.000000

P4W2R4 0.000000 7.000000

P4W2R5 0.000000 7.000000

P4W2R6 0.000000 8.000000

P4W3R4 0.000000 7.000000

P4W3R5 0.000000 3.000000

P4W3R6 50.000000 0.000000

P4W3R7 100.000000 0.000000

iii.

According to Lindo, you would want to do the following:

P1W1R2 50.000000 (ship 50 refrigerators from P1 to R1 through W1)

P1W1R3 100.000000 (ship 100 refrigerators from P1 to R3 through W1)

P2W1R1 100.000000 (ship 100 refrigerators from P2 to R1 through W1)

P2W1R2 100.000000 (ship 100 refrigerators from P2 to R2 through W1)

P2W2R4 50.000000 (ship 50 refrigerators from P2 to R4 through W2)

P2W2R5 200.000000 (ship 200 refrigerators from P2 to R5 through W2)

P3W2R4 150.000000 (ship 150 refrigerators from P3 to R4 through W2)

P3W3R6 100.000000 (ship 100 refrigerators from P3 to R6 through W3)

P4W3R6 50.000000 (ship 50 refrigerators from P4 to R6 through W3)

P4W3R7 100.000000 (ship 100 refrigerators from P4 to R7 through W3)

Part B

We can see from the cost table that plants P1 and P2 cannot ship to warehouse 3. We can also see that warehouse W1 cannot ship to retailers R5, R6 or R7. This means that the demand from retailers R5, R6 and R7 must be met by supply strictly coming from warehouse W3 (with the elimination of W2). So, since only ports P3 and P4 can ship to W3, we can add up their supply and see if it is enough to offset the demand for ports R5, R6 and R7.

The results are:

Supply from P3 + P4: 400

Demand from R5, R6, R7: 450

Since the demand is greater than supply without W2, there is no feasible model to ship all refrigerators using only warehouse 1 and 3.

Part C

Yes it is possible, since we have seen the demand above is 450, and the supply is 400 (without W2). So if you add back in W2, even capping it at 100, it is feasible to meet demand. To solve this we added a line to the Lindo input supply constraints, as follows:

P1W2R3 + P1W2R4 + P1W2R5 + P1W2R6 + P2W2R3 + P2W2R4 + P2W2R5 + P2W2R6 + P3W2R3 + P3W2R4 + P3W2R5 + P3W2R6 + P4W2R3 + P4W2R4 + P4W2R5 + P4W2R6 <=100

Basically just adding up all possible routes through W2, and capping them at 100 refrigerators. The optimal was found at:

VARIABLE VALUE REDUCED COST

P1W1R1 50.000000 0.000000 (Ship 50 refrigerators from P1 to R1 through W1)

P1W1R3 100.000000 0.000000 (Ship 100 refrigerators from P1 to R3 through W1)

P2W1R1 50.000000 0.000000 (Ship 50 refrigerators from P2 to R1 through W1)

P2W1R2 150.000000 0.000000 (Ship 150 refrigerators from P2 to R2 through W1)

P2W1R4 150.000000 0.000000 (Ship 150 refrigerators from P2 to R4 through W1)

P2W2R4 50.000000 0.000000 (Ship 50 refrigerators from P2 to R4 through W2)

P2W2R5 50.000000 0.000000 (Ship 50 refrigerators from P2 to R5 through W2)

P3W3R5 100.000000 0.000000 (Ship 100 refrigerators from P3 to R5 through W3)

P3W3R6 150.000000 0.000000 (Ship 150 refrigerators from P3 to R6 through W3)

P4W3R5 50.000000 0.000000 (Ship 50 refrigerators from P4 to R5 through W3)

P4W3R7 100.000000 0.000000 (Ship 100 refrigerators from P4 to R7 through W3)

And the total cost comes to $18,300.00.

Part D

Cijk = Unit transportation cost from source i, through warehouse j, to destination k

ai = amount of supply at source i

bk = amount of demand at retailer k

Xik = number of refrigerators distributed from origin i to retailer k

Minimize z =

Subject to:

Xik >= 0

**Problem 2: A mixture problem**

Part A

i)

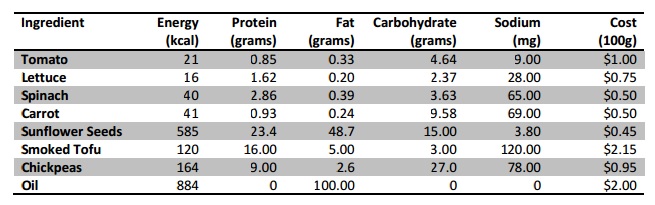
Each ingredient of the 8 ingredients in the salad will be assigned as if it is an array. The optimal solution will be a combination of the ingredients. Variables are:

* T = Tomato
* L = Lettuce
* S = Spinach
* C = Carrot
* SS = Sunflower Seeds
* ST = Smoked Tofu
* CP = Chickpeas
* O = Oil

Additionally, each ingredient will have an assigned variable for nutritional value as follows:

* P = Protein
* F = Fat
* C = Carbohydrates
* S = Sodium
* LG = Leafy Greens

Given nutritional content of the ingredients:



Our goal is to minimize the amount of calories but still meet a specific nutritional requirement. Each salad must contain:

* At least 2 but no more than 8 ingredients.
* At least 15 grams of protein
* At least 2 and at most 8 grams of fat
* At least 4 grams of carbohydrates
* At most 200mg of sodium
* At least 40% leafy greens by mass

Given the provided requirements for a salad we can derive some constraints for our linear program:

* P >= 15
* 2 <= F <=8
* C >= 4
* S <= .2
* LG >= .4(mass)

ii)

Using LINDO, we can determine the optimal solution for this program with the following code:

MIN 21T + 16L + 40S + 41C + 585SS + 120ST + 164CP + 884O

ST

0.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP >= 15

0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O >= 2

0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O <= 8

4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4

9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP <= 200

0.4L + 0.4S 0.6T

0.6C

0.6SS

0.6ST

0.6CP

0.6O

>= 0

T >= 0

L >= 0

S >= 0

C >= 0

SS >= 0

ST >= 0

CP >= 0

O >= 0

END

Results of the code will be attached to this report titled Problem 2-A Lindo Results.txt.

LP OPTIMUM FOUND AT STEP 3

OBJECTIVE FUNCTION VALUE

1) 114.7126

VARIABLE VALUE REDUCED COST

T 0.000000 6.276346

L 0.574713 0.000000

S 0.000000 12.785481

C 0.000000 16.499697

SS 0.000000 389.725952

ST 0.879310 0.000000

CP 0.000000 49.335754

O 0.000000 884.000000

iii)

The lowest calorie salad solution:

|  |  |  |  |
| --- | --- | --- | --- |
| Ingredient | Calories | Cost ($) | Nutrition |
| Lettuce (1.220836 units) | 20 | $0.92 | 1.98g Protein  .24g Fat  2.89g Carbohydrates  34mg Sodium |
| Smoked Tofu (.813890 units) | 98 | $1.75 | 13.02g Protein  4.07g Fat  2.44g Carbohydrates  98mg Sodium |
| Totals: |  |  |  |
| Lettuce and Smoked Tofu | 118 | $2.67 | 15g Protein  4.31g Fat  5.33g Carbohydates  132mg Sodum |

As you can see, this combination meets all of our requirements outlined above.

Part B

i)

The goal for this problem is to minimize cost associated with the salad, while still meeting the minimum nutritional requirements. We will reuse the same variables and nutritional table as in Part A for this problem, so the constraints will be:

* P >= 15
* 2 <= F <=8
* C >= 4
* S <= .2
* LG >= .4(mass)

ii)

Again we will be using LINDO for this solution:

MIN 1T + 0.75L + 0.5S + 0.5C + 0.45SS + 2.15ST + .95CP + 2O

ST

0.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP >= 15

0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O >= 2

0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O <= 8

4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4

9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP <= 200

0.4L + 0.4S - 0.6T - 0.6C - 0.6SS - 0.6ST - 0.6CP - 0.6O >= 0

T >= 0

L >= 0

S >= 0

C >= 0

SS >= 0

ST >= 0

CP >= 0

O >= 0

END

Results will be attached to this report as Problem 2-B Lindo Results.txt.

LP OPTIMUM FOUND AT STEP 10

OBJECTIVE FUNCTION VALUE

1) 1.676833

VARIABLE VALUE REDUCED COST

T 0.000000 1.075116

L 0.000000 0.414501

S 1.525128 0.000000

C 0.000000 0.558887

SS 0.103289 0.000000

ST 0.000000 0.344260

CP 0.913462 0.000000

O 0.000000 7.668215

iii)

Given the results, the following is the solution for a low cost salad:

|  |  |  |  |
| --- | --- | --- | --- |
| Ingredient | Calories | Cost ($) | Nutrition |
| Spinach (1.525128 units) | 61 | $0.76 | 4.36g Protein  0.60g Fat  5.54g Carbohydrates  99mg Sodium |
| Sunflower Seeds (0.103289 units) | 60 | $0.05 | 2.42g Protein  5.02g Fat  1.55g Carbohydrates  < 1mg Sodium |
| Chickpeas (0.913462 units) | 150 | $0.87 | 8.22g Protein  2.38g Fat  24.66g Carbohydrates  71mg Sodium |
| Totals: |  |  |  |
| Spinach, Sunflower Seeds, and Chickpeas | 270 | $1.68 | 15g Protein  8g Fat  31.75g Carbohydrates  171mg Sodium (rounded up) |

As we can see, these results meet our requirements for being low cost and also meeting the specified nutritional values.

Part C

i)

Suppose you want to sell a salad that is both low calorie and low cost. In this case, we want to sell a salad for $5.00, and make a profit of at least $3.00. However, you can advertise the salad as low calorie and potentially sell more if it is under 250 calories.

The objective for this problem is to optimize both cost and minimize calories. We believe advertising a salad as low calorie will sell more and therefore generate more revenue. We will use Lindo again for this:

MIN 1T + 0.75L + 0.5S + 0.5C + 0.45SS + 2.15ST + .95CP + 2O

ST

21T + 16L + 40S + 41C + 585SS + 120ST + 164CP + 884O <= 250

0.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP >= 15

0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O >= 2

0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O <= 8

4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4

9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP <= 200

0.4L + 0.4S - 0.6T - 0.6C - 0.6SS - 0.6ST - 0.6CP - 0.6O >= 0

T >= 0

L >= 0

S >= 0

C >= 0

SS >= 0

ST >= 0

CP >= 0

O >= 0

END

Results will be attached to this report as Problem 2-C Lindo Results.txt.

LP OPTIMUM FOUND AT STEP 10

OBJECTIVE FUNCTION VALUE

1) 1.721142

VARIABLE VALUE REDUCED COST

T 0.000000 1.066125

L 0.000000 0.407098

S 1.426983 0.000000

C 0.000000 0.589763

SS 0.101109 0.000000

ST 0.128708 0.000000

CP 0.721505 0.000000

O 0.000000 8.618567

ii)

The low cost, 250 calorie salad solution:

|  |  |  |  |
| --- | --- | --- | --- |
| Ingredient | Calories | Cost ($) | Nutrition |
| Spinach (1.426983 units) | 57 | $0.71 | 4.08g Protein  0.56g Fat  5.18g Carbohydrates  93mg Sodium |
| Sunflower Seeds (0.101109 units) | 59 | $0.05 | 2.37g Protein  4.92g Fat  1.52g Carbohydrates  <1mg Sodium |
| Smoked Tofu (0.128708 units) | 15 | $0.28 | 2.06g Protein  0.64g Fat  0.39g Carbohydrates  15mg Sodium |
| Chickpeas (0.721505 units) | 118 | $0.69 | 6.49g Protein  1.88g Fat  19.48g Carbohydrates  56mg Sodium |
| Totals |  |  |  |
| Spinach, Sunflower Seeds, Smoked Tofu, and Chickpeas | 249 | $1.73 | 15g Protein  8g Fat  26.57g Carbohydrates  165mg Sodium |

iii)

For this solution we emphasized the 250 calorie cap so that the salad can be marketed as low calorie to achieve more sales. However, you could also emphasize the $3.00 profit and get a different answer, but the salad would likely not be able to be sold as "low calorie". However, if you were to sell the salad in our solution, you could still market it as a low calorie salad and make greater than then $3.00 profit so we believe this is the best solution.

**Problem 3:** Solving shortest path problems using linear programming.

Part A)

|  |  |
| --- | --- |
| VARIABLE | VALUE |
| B | 2 |
| C | 3 |
| D | 8 |
| E | 9 |
| F | 6 |
| G | 8 |
| H | 9 |
| I | 8 |
| J | 10 |
| K | 12 |
| L | 15 |
| M | 17 |
| A | 0 |

The Lindo input was as follows:

max b + c + d + e + f + g + h + i + j + k + l + m

ST

a = 0

b - a <= 2

c - a <= 3

d - a <= 8

h - a <= 9

a - b <= 4

c - b <= 5

e - b <= 7

f - b <= 4

d - c <= 10

b - c <= 5

g - c <= 9

i - c <= 11

f - c <= 4

a - d <= 8

g - d <= 2

j - d <= 5

f - d <= 5

f - d <= 1

h - e <= 5

c - e <= 4

i - e <= 10

i - f <= 2

g - f <= 2

d - g <= 2

j - g <= 8

k - g <= 12

i - h <= 5

k - h <= 10

a - i <= 20

k - i <= 6

j - i <= 2

m - i <= 12

i - j <= 2

k - i <= 4

l - j <= 5

h - k <= 10

m - k <= 10

m - l <= 2

END

Part B

Adding in vertex Z with no path to vertex a resulted in Lindo throwing an error. It said unbounded solution.

Here is the input used:

NBOUNDED VARIABLES ARE:

SLK 41

Z

OBJECTIVE FUNCTION VALUE

1) 0.9999990E+08

And here is the result:

max b + c + d + e + f + g + h + i + j + k + l + m

ST

a = 0

b - a <= 2

c - a <= 3

d - a <= 8

h - a <= 9

a - b <= 4

c - b <= 5

e - b <= 7

f - b <= 4

d - c <= 10

b - c <= 5

g - c <= 9

i - c <= 11

f - c <= 4

a - d <= 8

g - d <= 2

j - d <= 5

f - d <= 5

f - d <= 1

h - e <= 5

c - e <= 4

i - e <= 10

i - f <= 2

g - f <= 2

d - g <= 2

j - g <= 8

k - g <= 12

i - h <= 5

k - h <= 10

a - i <= 20

k - i <= 6

j - i <= 2

m - i <= 12

i - j <= 2

k - i <= 4

l - j <= 5

h - k <= 10

m - k <= 10

m - l <= 2

m - z <= 5

END

Part C

|  |  |
| --- | --- |
| Vertex to M | Shortest Path |
| A | 17 |
| B | 15 |
| C | 15 |
| D | 12 |
| E | 19 |
| F | 11 |
| G | 14 |
| H | 14 |
| I | 9 |
| J | 7 |
| K | 10 |
| L | 2 |
| M | 0 |

Lindo input:

MAX a + b + c + d + e + f + g + h + i + j + k + l

ST

m = 0

a - b <= 2

a - c <= 3

a - d <= 8

a - h <= 9

b - a <= 4

b - c <= 5

b - e <= 7

b - f <= 4

c - d <= 10

c - b <= 5

c - g <= 9

c - i <= 11

c - f <= 4

d - a <= 8

d - g <=2

d - j <= 5

d - f <= 1

e - h <= 5

e - c <= 4

e - i <= 10

f - i <= 2

f - g <= 2

g - d <= 2

g - j <= 8

g - k <= 12

h - i <= 5

h - k <= 10

i - a <= 20

i - k <= 6

i - j <= 2

i - m <= 12

j - i <= 2

j - k <= 4

j - l <= 5

k - h <= 10

k - m <= 10

l - m <= 2

END

Part D

For this problem, we used methods from part C and part A. We ran lindo to get all vertices to i, and then i to all vertices. Then these two amounts were added together for all spots in the matrix below.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F | G | H | I | J | K | L | M |
| A | 28 | 30 | 31 | 36 | 37 | 34 | 36 | 24 | 8 | 10 | 14 | 15 | 17 |
| B | 26 | 28 | 29 | 34 | 35 | 32 | 34 | 22 | 6 | 8 | 12 | 13 | 15 |
| C | 26 | 28 | 29 | 34 | 35 | 32 | 34 | 22 | 6 | 8 | 12 | 13 | 15 |
| D | 23 | 25 | 26 | 31 | 32 | 29 | 31 | 19 | 3 | 5 | 9 | 10 | 12 |
| E | 30 | 32 | 33 | 38 | 39 | 36 | 38 | 26 | 10 | 12 | 16 | 17 | 19 |
| F | 22 | 24 | 25 | 30 | 31 | 28 | 30 | 18 | 2 | 4 | 8 | 9 | 11 |
| G | 25 | 27 | 28 | 33 | 34 | 31 | 33 | 21 | 5 | 7 | 11 | 12 | 14 |
| H | 25 | 27 | 28 | 33 | 34 | 31 | 33 | 21 | 5 | 7 | 11 | 12 | 14 |
| I | 20 | 22 | 23 | 28 | 29 | 26 | 28 | 16 | 0 | 2 | 6 | 7 | 9 |
| J | 22 | 24 | 25 | 30 | 31 | 28 | 30 | 18 | 2 | 4 | 8 | 9 | 11 |
| K | 35 | 37 | 38 | 43 | 44 | 41 | 43 | 31 | 15 | 17 | 21 | 22 | 24 |
| L | NP | NP | NP | NP | NP | NP | NP | NP | NP | NP | NP | NP | NP |
| M | NP | NP | NP | NP | NP | NP | NP | NP | NP | NP | NP | NP | NP |

Here is the lindo input used:

For ALL VERTICES TO i:

max a+b+c+d+e+f+g+h+i+j+k

st

i = 0

a - b <= 2

a - c <= 3

a - d <= 8

a - h <= 9

b - a <= 4

b - c <= 5

b - e <= 7

b - f <= 4

c - d <= 10

c - b <= 5

c - g <= 9

c - i <= 11

c - f <= 4

d - a <= 8

d - g <= 2

d - j <= 5

d - f <= 1

e - h <= 5

e - c <= 4

e - i <= 10

f - i <= 2

f - g <= 2

g - d <= 2

g - j <= 8

g - k <= 12

h - i <= 5

h - k <= 10

i - a <= 20

i - k <= 6

i - j <= 2

i - m <= 12

j - i <= 2

j - k <= 4

j - l <= 5

k - h <= 10

k - m <= 10

l - m <= 2

end

And here is the Lindo input used for i to all other vertices:

max a+b+c+d+e+f+g+h+i+j+k+l+m

st

i = 0

b - a <= 2

c - a <= 3

d - a <= 8

h - a <= 9

a - b <= 4

c - b <= 5

e - b <= 7

f - b <= 4

d - c <= 10

b - c <= 5

g - c <= 9

i - c <= 11

f - c <= 4

a - d <= 8

g - d <= 2

j - d <= 5

f - d <= 1

h - e <= 5

c - e <= 4

i - e <= 10

i - f <= 2

g - f <= 2

d - g <= 2

j - g <= 8

k - g <= 12

i - h <= 5

k - h <= 10

a - i <= 20

k - i <= 6

j - i <= 2

m - i <= 12

i - j <= 2

k - j <= 4

l - j <= 5

h - k <= 10

m - k <= 10

m - l <= 2

end