1 Reducible/Irreducible polynomial

1.1 Reducible

f(x) can be written as a product of two polynomials in $\mathbb{F}[x]$ both of postive degree.

1.2 Irreducible

otherwise

1.3 Example in $\mathbb{R}$[x]

$x^2 - a$ is reducible. but $x^2 + 1$ is irreducible but reducible over $\mathbb{C}$[x].

2 Conjugate Roots Theorem

Let $f(x)\epsilon\mathbb{C}[x]$ have all coefficients real. If $c\epsilon\mathbb{C}$ is a root of f(x) then so is $\bar{c}$(complex conjugate.

3 Example

LEt $f(x) = x^4 + 3x^2 + 5x + 4$ in $\mathbb{Z}_{\ntriangleright}$ Write table to find all roots.

x	0	1	2	3	4	5	6
f(x)	4	6	0	1	6	1	3

so x=2 is the only root of f(x). Thus (x-2) is a factor.

Apply DAP over $\mathbb{Z}_{\ntriangleright}$ divide f(x) by (x-2)
Answer $h(x) = x^3 + 2x^2 + 5$
Key Point If α is a root of h(x) then by FT and TD α is a root of f(x).
2 is a root fo h(x) so h(2)=0
divide by (x-2) to get $k(x) = x^2 + 4x + 1$
Check root of k(x) to be root of f(x). $k(2) = 2 \neq 0$
Thus k(x) has no roots over $\mathbb{Z}_{\ntriangleright}$
hence k(x) is irreducible over $\mathbb{Z}_{\ntriangleright}$ **Conclusion** $f(x) = (x-2)^2(x^2 + 4x + 1$.

4 Proof of CPN

Restate CPN: Let $f(x)\epsilon\mathbb{C}[x]$ of degree n. Then $\exists n$ complex roots not necessarily distinct.