Andrew Codispoti
20554012
105

1

$\int_0^1 sin(3t^2)\, dt$
Let $g(x) = sin(u), u = 3t^2$

$$\begin{aligned} g(0) &= 0 \\ g'(0) &= cos(u) = 1 \\ g''(0) &= -sin(u) = 0 \\ g^3(0) &= -cos(u) = -1 \\ g^4(0) &= sin(u) = 0 \\ g^5(0) &= cos(u) = 1 \end{aligned}$$

$$P_{5,0}(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \tag{1}$$

Substituting $x = 3t^2$

$$P_{10,0}(t) = 3t^2 - \frac{27t^6}{6} + \frac{243t^{10}}{120} \tag{2}$$

$$P_{11,0}(x) = \int_0^x 3t^2 - \frac{27t^6}{6} + \frac{243t^{10}}{120} dt \tag{3}$$

$$\int_0^1 f(t) = \int_0^1 P_{10,0}(t) + \int_0^1 R_{5,0}(u) \tag{4}$$

$$t\ \epsilon[0,1], \quad u\ \epsilon[0,3]$$

$$|R_{5,0}(u)| \leq |f^{(6)}(3)| \leq -sin(3) \leq .15 \tag{5}$$

$$|R_{5,0}(u)| \leq \frac{.15(3)^6}{6!} \tag{6}$$

$$\leq .15187 \tag{7}$$

Check:

$$\int_0^1 sin(3t^2)dt - \int_0^1 P_{10,0}(t) = \tag{8}$$

2

3

4