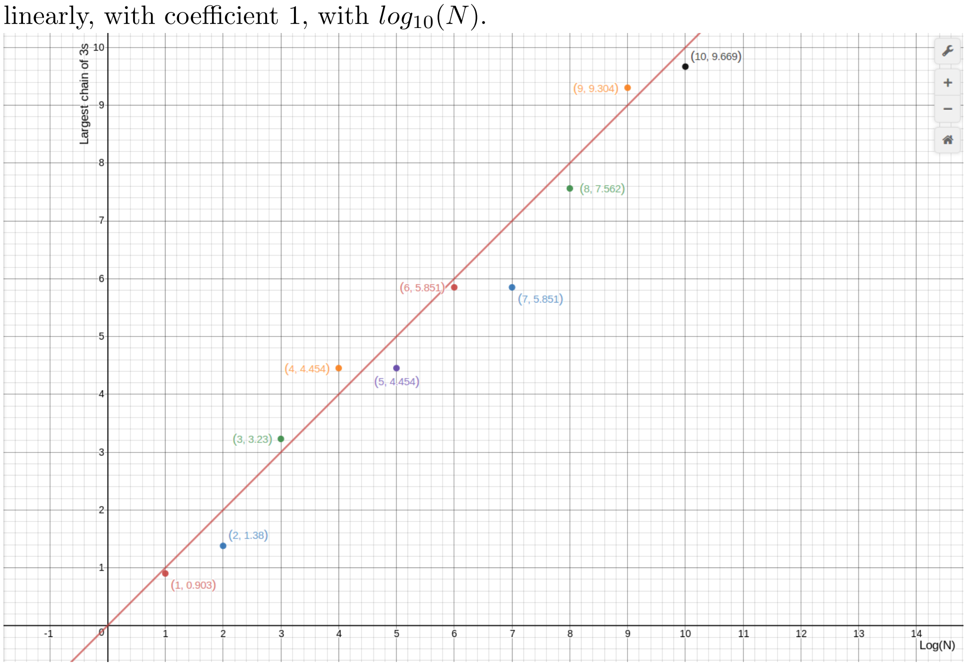
## 1. Consecutive 3s in the digits of $\pi$

We will investigate the number of consecutive 3s found in the first $N$ digits of $\pi$, then propose conjectures based on our results. To do so, we will use a python program which contains a text file of the first 10 billion digits of $\pi$.
First we will look at the longest chain of consecutive 3s in the first $N$ digits, $1 \leq N \geq 10^{10}$. Below is a table of the first $N$ for which the length of consecutive 3s increases by 1.

| Max length 3s | | $N$ |
|---|---|---|
| 1 | : | 8 |
| 2 | : | 24 |
| 3 | : | 1699 |
| 4 | : | 28469 |
| 5 | : | 28470 |
| 6 | : | 710104 |
| 7 | : | 710105 |
| 8 | : | 36488182 |
| 9 | : | 2011485314 |
| 10 | : | 4663739967 |

Graphing the lengths of consecutive 3s vs the log base 10 of the first $N$ for which this length appears, will yield the following graph.

From the tend line of $y = x$ we see that the maximum length scales roughly linearly, with coefficient 1, with $log_{10}(N)$.




**Conjecture 1.** *Let $l(N)$ be the size of the largest consecutive chain of 3s in the first $N$ digits of $\pi$. Then $l(N) \in \{n-1, n, n+1 : 10^n <= N <= 10^{n+1}\}$*

To motivate a bit more from our previous conjecture, we could think of a list of numbers, say $L$ with length $N$. Each number in this list, at index $i$, is modeled as a discrete uniform random variable taking on values between 0 and 9, call this random variable $R_i$. If we were to speculate about the largest consecutive chain of 3s in $L$, we would guess it to be around the integer closest to $log_{10}(N)$. The reason being, that any digit at index $i$ has a probability of $1/10$ to be a 3. Thus we would expect a chain of $n$ adjacent 3s to appear with probability $1/10^n$.

With idea consecutive 3s in a number of $N$ random digits, we can turn back to our observations of $\pi$. Our data indicates that we expect to find $n$ consecutive 3s every around $10^n$ digits into $\pi$. From this, we can speculate that each digit in $\pi$ has a $1/10$ chance to be a 3 and that each digit is independent of all other digits. To test this, we can again use our python program. We will test the frequency of the digits in the first 100 million digits of pi

| digit | frequency |
|---|---|
| 0 | 0.099944 |
| 1 | 0.0999333 |
| 2 | 0.1000306 |
| 3 | 0.0999964 |
| 4 | 0.1001093 |
| 5 | 0.1000466 |
| 6 | 0.0999337 |
| 7 | 0.1000207 |
| 8 | 0.0999814 |
| 9 | 0.100004 |

From this table, we see that the each digit shows up with approximate frequency 0.1, further strengthening the idea of uniformity in the digits of $\pi$.

**Conjecture 2.** *The digits of $\pi$ are randomly distributed, all with a probability 0.1.*

**Corollary 1.** *Let $l(N)$ be the size of the largest consecutive chain of 3s in the first $N$ digits of $\pi$. Suppose $I$ is an index such that $l(I-1) < l(I)$. Then $P(l(I) \ < \ l(I+1)) \ = \ 0.1$*

*Proof.* Suppose the digits of $\pi$ are random and uniform. Further suppose $l(I-1) < l(I)$, then the digit at index $I$, of $\pi$, is a 3 and this three is part the longest current chain of 3s in the first $I$ digits. Since each digit has probability 0.1 to any number 1, 2, ..., 9, the probability that the digit at index $I+1$ is a 3, is 0.1. $\square$