Diophantine Equations to the Power of n

MATC15 - Project - Draft 1

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Conjecture: Let x be an arbitrary integer.

$x^n = \sum_{i=1}^{n} y_i^n$ has an integer solution such that $y_i \neq x, \forall i$.

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1 Introduction

The objective of this project is to investigate the conjecture above: whether or not we can always find at least one integer solution to equations of the form $x^n = y_0^n + \cdots + y_n^n$ given any x, excluding trivial solutions involving y_i's$= 0$ or x.

Some of this investigation and research will involve:

- Finding parameters and conditions for possible valid solutions
- Computational analysis on random integers raised to the power of n and finding an integer solution to the sum.
- Noting differences between even and odd n.
- Identifying different families of solutions that take on a similar form.

Though this conjecture may be false, we hope to investigate as much as we can on the matter and provide some deeper research to the subject.

2 References

- Drago Bajc, **Power solutions of some Diophantine equations**, *The Mathematical Gazette*, 97:538, 107-110 (2013). https://www.jstor.org/stable/24496765

  Mentions form of above conjecture and states that solutions have been found in some cases but not in other cases, such as $n = 6$. Considers above

conjecture with x^k instead of x^n, where $(k, n) = 1$ and provides a general form for these solutions.

- **Computing Minimal Equal Sums Of Like Powers**,
http://euler.free.fr/index.htm

    Website dedicated to finding and compiling examples and counterexamples of Euler's sums of powers conjecture, which states that if a sum of n positive kth powers equals one kth power, then $n >= k$. Includes many resources we can look into.

- **BEST KNOWN SOLUTIONS**,
http://euler.free.fr/records.htm

    Extensive list of aforementioned examples and counterexamples to Euler's sums of powers conjecture.

- L. Jacobi, D. Madden, **On** $a^4 + b^4 + c^4 + d^4 = (a + b + c + d)^4$,
The American Mathematical Monthly, 115:3, 230-236 (2008).
https://doi.org/10.1080/00029890.2008.11920519

    Discusses specific case of the conjecture with $n = 4$. Also discusses relation of Euler's conjecture and related Diophantine equations to the topic of elliptic curves.

- T. Roy and F. J. Sonia, **A Direct Method To Generate Pythagorean Triples And Its Generalization To Pythagorean Quadruples And n-tuples**,
https://arxiv.org/ftp/arxiv/papers/1201/1201.2145.pdf

    Gives methods for finding Pythagorean n-tuples, sums of n squares that result in a square. Might be able to reduce some cases into one of these cases.

- D. R. Heath-Brown, W. M. Lioen and H. J. J. Te Riele, **On Solving the Diophantine Equation** $x^3 + y^3 + z^3 = k$ **on a Vector Computer**,
Mathematics of Computation, 61:203, 235-244 (1993)

    Presents detailed algorithm for the $n = 3$ case, might be able to apply similar principles with higher n values.

- L. J. Lander, T. R. Parkin and J. L. Selfridge **A survey of equal sums of like powers**,
Mathematics of Computation, 21, 446-459 (1967).
https://www.ams.org/journals/mcom/1967-21-099/S0025-5718-1967-0222008-0/S0025-5718-1967-0222008-0.pdf

    Presents various solutions to powers of Diophantine equations, including the $n = 4$ and $n = 5$ cases of the conjecture.

- J. Leech, **On** $A^4 + B^4 + C^4 + D^4 = E^4$,
  Mathematical Proceedings of the Cambridge Philosophical Society, 54(4), 554-555, (1958).
  doi.org/10.1017/S0305004100003091

  Brief paper outlining found solutions for the $n = 4$ case and considerations that reduce the number of possible solutions that need to be checked.