

As a part of this course, you are asked to work on a project. The project is worth 15% of your total mark. You can choose to work on a project individually or in a team. There are three types of projects you can choose from. Regardless of the project type or the number of team members, you should follow the following timeline. All submissions are done through Quercus.

Table 1: Project Time-line

	Deadline	weight
Draft one	March 11th	2%
Draft two	March 25th	6%
Final draft and the cover report	April 8th	7%

If you work individually, you don't need to submit a cover report. If you work in a team, you should submit a cover report: a paragraph in which you clearly indicate the role of each of the group members in the project. All member should sign this paragraph.

## **Type I**

In Type I project, you choose a topic that you like to learn more about. It could be a topic we covered in class and you like to expand your knowledge on, or a topic that interests you. There is a list of suggested topics you can choose from. You are not limited to that list. Any topic that is related to the material we learned in this course is worth exploring. When you choose a topic you should think about following questions:

- Do I have enough background to learn this topic in about a month mostly on my own or with my peers?
- I am asked to present this topic (in written form or a video). what would my presentation look like? what would be my outline?

Ask me on piazza if you are not sure whether your topic is suitable for this project.

The goal of type I project is to give you an opportunity to read and learn a topic you enjoy, think about how to present it to someone at the same math level as you are but who may be new to your topic and to give you a platform to present it. Your final draft can be a 15-20 minute video or a typed report (5-10 pages) on your topic aimed at your number theory classmates.

You can work with at most 2 other student for Type A project.

### **Draft one**

This should contain:

1. Name of everyone in your team
2. Your topic
3. List of references you are planing to look at. A reference can be a book chapter, online source, course notes, etc.

Criteria to get full mark on the this draft:

- Meeting the deadline
- Containing all the requesting information

## Draft Two

Draft two is a hand written or typed outline of your final draft (regardless of whether your final draft is a video or a written report). Draft two should include

1. Title
2. Any amendment to your choice of topic, and references. For instance, you may list a book in draft one, and after looking more closely realize that you won't be able to read it. Instead you find a lecture note that summarizes the book. You should remove the book and add the note to your list of references.
3. An outline for your report/presentation: This draft should contain the mathematical meat of your project. It is not marked for being well written, good use of notation or style. You have the chance to improve these in the final version. It is marked for **coherence** and **mathematical correctness**. By the time you finish this draft, you should have a complete understanding of your project. This draft should be 1 to 5 pages. You will be marked on:
  - Meeting the deadline
  - A logical path, with some details, that guides a reader to the goal of the project.
  - Structure
  - Mathematical correctness.

## Final Draft

As final draft, you can choose to submit a typed write-up or a 15-20 minutes video presentation aimed at your number theory peers. Depending on the quality of your work, we can submit your report or video to the UTSC Center for Teaching and Learning Math in Action Journal. If accepted, your work will be available, and citable through their website. This is particularly good for you if you wish to apply for graduate school and you want to show you have been proactive in learning and presenting mathematics.

Your final report should be mathematically correct, complete and coherent regardless of the format. You can assume the knowledge we learned in this course. Your final draft will be marked based on

- Meeting the deadline.

- Applying any feedback from the first draft if you got any.
- Citation: regardless if the format of your final draft you should give citation to any source you use to learn from and to where to find a proof to any result that you use without proving
- Alignment with the topic and structure in your second draft.
- Mathematical completeness and correctness

## **Type II**

In type II, you can choose a concept to numerically or visually explore, make conjectures based on your numerical evidence and make an attempt to proof your conjecture. This type suits you best if you are (or some one else in your team is) comfortable programming and already have some number theoretic question in mind that has been bugging you and you want to numerically explore it or if you have some ideas on visualizing a number theoretic concept. Some examples are

- Visualizing gap between primes (see an example in page 55 Weissman for inspiration)
- Numerical exploitation of behavior of multiplicative functions such as divisor sum functions
- Moats on Gaussian primes
- Approximating number of Gaussian integers or Gaussian primes within a given geometric shape.

You can work with at most 3 other students in this project.

### **Draft one**

This should contain:

1. Name of everyone in your team
2. Your topic
3. List of references you are planing to look at. A reference can be a book chapter, online source, course notes, etc.
4. A brief description fo your plan/idea

Criteria to get full mark on the this draft:

- Meeting the deadline
- Containing all the requesting information

## **Draft Two**

Draft two is a hand written or typed outline of your final draft (regardless of whether your final draft is a video or a written report). Draft two should include

1. Title
2. Any amendment to your choice of topic, and references.
3. A summary of what you tried and the results you got. Include any image or numerical table. Any conjecture that is backed up by your evidence. Any ideas and suggestions on how to proceed based on evidence.

You will be graded based on

- Meeting the deadline
- Structure
- Meaningful computational or visualization attempts
- Ideas and suggestions on how to proceed based on evidence.

## **Final Draft**

As final draft, you can choose to submit a typed write-up or a 15-20 minutes video presentation aimed at your number theory peers. Depending on the quality of your work, we can submit your report or video to the UTSC Center for Teaching and Learning Math in Action Journal or at CMS Studc to be presented in their next conference. If accepted, your work will be available, and citable through their website. This is particularly good for you if you wish to apply for graduate school and you want to show you have been interested in research and presenting mathematics during your undergrad.

Your final report should be mathematically correct, complete and coherent regardless of the format. You can assume the knowledge we learned in this course. Your final draft will be marked based on

- Meeting the deadline.
- Applying any feedback from the first draft if you got any.
- Citation: regardless of the format of your final draft you should give citation to any source you use to learn form and to where to find a proof to any result that you use without proving.

- Alignment with the topic and structure in your second draft.
- Mathematical completeness and correctness

## Type III

One interesting feature of number theory is that most of interesting questions in number theory can be posed and described only using elementary mathematics. Although in most cases, we need modern mathematics to solve them, there are topics that can be followed to certain level only using only elementary mathematics.

In Type III project, you choose a topic that can be described using only highschool mathematics and present it aimed at grade 9-12 students. You can choose to present the topic in a write-up or in a 15-20 minutes video. I provide a list of topics that you can choose from, but you are not limited to these topics. If you chose to make a video, we can share that with DDSB to make it available to schools to use.

You can work with at most 3 other students in this project.

### Draft one

This should contain:

1. Name of everyone in your team
2. Your topic
3. List of references you are planning to look at. A reference can be a book chapter, online source, course notes, etc.

Criteria to get full mark on this draft:

- Meeting the deadline
- Containing all the requested information

### Draft Two

Draft two is a hand written or typed outline of your final draft (regardless of whether your final draft is a video or a written report). Draft two should include

1. Title
2. Any amendment to your choice of topic, and references. For instance, you may list a book in draft one, and after looking more closely realize that you won't be able to read it. Instead you find a lecture note that summarizes the book. You should remove the book and add the note to your list of references.



3. A lesson plan: how are you planning to present the topic? how do you motivate the topic? what are the examples you will give? how much time you are planing to spend on each? what mathematical background are you assuming? how much theory are you planning to include

You will be graded on

- Meeting the deadline
- Structure of your plan
- Choice of examples to present

### **Final Draft**

As final draft, you can choose to submit a typed write-up or a 15-20 minutes video presentation aimed at grade 9-12 students. If you chose to make a video, we can share that with DDSB to make it available to schools to use. Your final report should be mathematically correct, complete and coherent regardless of the format and aimed the the correct audience. You will be graded on

- Meeting the deadline.
- Applying any feedback from the first draft if you got any.
- Citation: regardless if the format of your final draft you should give citation to any source you use to learn form and to where to find a proof to any result that you use without proving
- Alignment with the topic and structure in your second draft.
- Mathematical completeness and correctness
- How engaging and inviting your work is

### **Suggested Topics for Type I**

1. p-adic numbers
2. Constructable numbers (chapter 3 of [2])
3. Riemann Zeta function
4. Dirichlet convolution

5. Hardy–Littlewood circle method
6. Euler product
7. Elliptic curves

### **Suggested Topics for Type III**

1. Number series (there is a beautiful argument in Chapter 0 page 3-10 of [2] on counting and adding visually using partnering and triplication techniques.)
2. Perfect numbers (look at problems in either of our textbooks)
3. Linear Diophantine Equations (the coin problems make a great hands on activity, you can look at this worksheet for ideas as well [7])
4. Modular arithmetic (chapter 6 of [2] gives a nice visual way of thinking about modular arithmetic that doesn't require any background, you can also look at [5])
5. Diffie–Hellman key exchange and Discrete logarithm problem (You shouldn't assume students know what logarithm is and have to build up from scratch.)
6. Divisibility tests (both of our textbooks have material on this)
7. Calendar Problem (this is more of a problem solving/algorithm problem. You can read more about it in [4] page 13.)
8. Quadratic residue (we will cover this in class a la chapter 8 of [2] or Sec. 5.3 of [1], there is also a lesson plan you can look at [6])
9. Magic Triangles (this is at the crossroad of number theory and discrete math, you can read more about it here [8])
10. You can get more ideas by leafing through CMS Crux [9].

## **References**

- [1] *Elementary Number Theory*, C. Vanden Eynden
- [2] *An Illustrated Theory of Numbers*, M. Wiessman
- [3] *The Book of Numbers*

- [4] Calander Problem
- [5] Modular arithmetic
- [6] gcd and Quadratic Residue
- [7] Diophantine
- [8] Magic Triangles
- [9] CMS Crux