Homework 2; Thursday 23 February
To be handed in no later than 9.15 on Thursday 2 March

1. Suppose
$$\pi(a,b) \propto a^4\, b^6\, e^{-a-b-3ab}$$
for $a,b>0$ is required to be sampled. Find
$$\pi(a|b) \quad \text{and} \quad \pi(b|a)$$
and hence find a transition density $p(a_{n+1},b_{n+1}|a_n,b_n)$ which has π as the stationary density.
Implement the chain and use the output to evaluate the integral
$$I = \int\int a\, b\, \pi(a,b)\, da\, db.$$

2. Suppose we wish to sample from
$$\pi(x) \propto \frac{\exp(-\frac{1}{2}x^2)}{1+x^2}$$
using a Metropolis–Hastings algorithm with density $q(x'|x) = N(x'|x,\sigma^2)$ for some σ as the proposal density.

Describe what you think might be the problems encountered if (i) σ is too small and (ii) σ is too big. Run the algorithm with such σ to verify your conclusions.

Without using any theory, find what you think is a suitable σ and run the algorithm with this σ to evaluate
$$I = \int x\, \pi(x)\, dx.$$
What happens if instead you use $q(x'|x) = N(x'|-x,\sigma^2)$.

3. Suppose a posterior density is given by
$$f(\theta|\text{data}) \propto e^{\theta a}\, e^{-ne^{\theta}}\, e^{-\frac{1}{2}\theta^2}$$
for some $a>0$ and n integer, and $-\infty<\theta<+\infty$. In fact this is a Poisson model with mean e^{θ} and standard normal prior for θ.
Find a Markov chain for sampling from f and implement it.