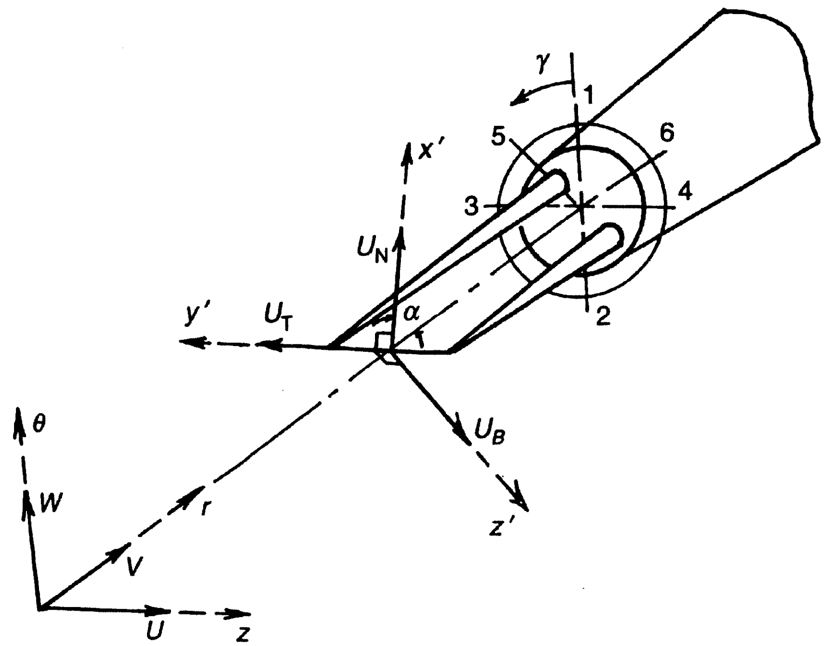

Fig. 6.2. The wire-fixed and space-fixed coordinate systems for the single-element multiposition method. The probe-stem is aligned with the radial direction, *r*.

velocity component, $\overline{W}$, is the largest component, then the series expansion must be carried out in terms of $\overline{W}$. The resultant modified equations are described by Nabhani (1989), Nabhani and Bruun (1990), and Bruun *et al.* (1993).

Using eqns (6.4) and (6.6), solutions may be obtained in either of the space-fixed coordinate systems shown in Figs 6.1 and 6.2. Alternatively, the mean-flow direction may first be determined, and the hot-wire probe then aligned with this direction. Such a technique is described by Bissonnette and Mellor (1974).

6.1.2 V_e-solution procedures

Equations (6.4) and (6.6) for $\bar{V}_e$ and $\overline{v_e^2}$ are coupled with the mean-velocity components and the Reynolds stresses occurring in both equations. Different solution procedures have been proposed depending on whether the mean-velocity vector has one, two, or three components in the coordinate system selected for the signal analysis.

6.1.2.1 One-component mean velocity

A simple solution procedure exists when the probe-stem is aligned with the mean-flow direction, $\bar{U}$. In this coordinate system, $\bar{V}$ and $\overline{W}$ are zero, and eqn (6.6) for $\overline{v_e^2}$ will not contain any mean-velocity terms. The solution of the Reynolds stresses is therefore decoupled from the mean-velocity evaluation. De Grande and Kool (1981) have presented an SY-probe method, in which the probe was placed at twenty one different positions at each point to increase the accuracy of the measured Reynolds stresses. However, to simplify the signal analysis, most procedures have involved both an SN and an SY probe. For an SN-probe ($\alpha = 0°$) placed in the (x, y)-plane ($\gamma = 0°$), the equations for $\bar{V}_e$ and $\overline{v_e^2}$ are to second order (Section 4.12.2.3)

$$\bar{V}_{e0} = \bar{U}\left(1 + \frac{1}{2}h^2\frac{\overline{w^2}}{\bar{U}^2}\right) \tag{6.8}$$

and

$$\overline{v_{e0}^2} = \overline{u^2}. \tag{6.9}$$

Equation (6.6) for an SY-probe contains contributions from all six Reynolds stresses for most values of γ. However, equations with only two of the velocity fluctuations can be obtained if the SY-probe is located in either the (x, y)- or (x, z)-plane. On placing the SY-probe at positions 1 ($\gamma = 0°$), 2 ($\gamma = 180°$) in the (x, y)-plane, and 3 ($\gamma = 90°$) and 4 ($\gamma = 270°$) in the (x, z)-plane (see Fig. 6.1), the corresponding equations for $\overline{v_e^2}$ will be

$$\overline{v_{e1}^2} = D_1\overline{u^2} + D_4\overline{v^2} + D_3\overline{uv}, \tag{6.10a}$$

$$\overline{v_{e2}^2} = D_1\overline{u^2} + D_4\overline{v^2} - D_3\overline{uv}, \tag{6.10b}$$

$$\overline{v_{e3}^2} = D_1\overline{u^2} + D_4\overline{w^2} - D_3\overline{uw}, \tag{6.10c}$$

$$\overline{v_{e4}^2} = D_1\overline{u^2} + D_4\overline{w^2} + D_3\overline{uw}, \tag{6.10d}$$

where $D_1 = A_1 = \cos^2\alpha + k^2\sin^2\alpha$, $D_3 = (1 - k^2)\sin 2\alpha$, and $D_4 = D_3^2/(4D_1)$, as listed in Table 6.2.

Taking the difference between eqns (6.10a–b) and similarly for eqns (6.10c–d) the shear stresses $\overline{uv}$ and $\overline{uw}$ can be evaluated from

$$\overline{uv} = \frac{1}{2D_3}(\overline{v_{e1}^2} - \overline{v_{e2}^2}), \tag{6.11a}$$

$$\overline{uw} = \frac{1}{2D_3}(\overline{v_{e4}^2} - \overline{v_{e3}^2}), \tag{6.11b}$$

TABLE 6.2 *The coefficients D_1–D_6*

$D_1 = A_1 = \cos^2\alpha + k^2\sin^2\alpha$
$D_2 = \sin^2\alpha + k^2\cos^2\alpha$
$D_3 = (1 - k^2)\sin 2\alpha$
$D_4 = D_3^2/(4D_1)$
$D_5 = (D_2 - D_4)/(4D_1)$
$D_6 = h^2/(4D_1)$

and a summation of eqns (6.10a–b) and (6.10c–d) gives two linear equations in $\overline{u^2}$, $\overline{v^2}$, and $\overline{w^2}$.

$$\overline{v_{e1}^2} + \overline{v_{e2}^2} = 2D_1\overline{u^2} + 2D_4\overline{v^2}, \tag{6.12a}$$

$$\overline{v_{e3}^2} + \overline{v_{e4}^2} = 2D_1\overline{u^2} + 2D_4\overline{w^2}. \tag{6.12b}$$

By measuring $\overline{u^2}$ with an *SN*-probe (eqn (6.9)), the corresponding values of $\overline{v^2}$ and $\overline{w^2}$ can be calculated from eqns (6.12a–b). Knowing the value of $\overline{w^2}$ the value of $\bar{U}$ can be determined from eqn (6.8).

The value of the shear stress $\overline{vw}$ can be calculated from two additional measurements at, for example, position 5 ($\gamma = 45°$) and 6 ($\gamma = -45°$); see Fig. 6.1:

$$\overline{v_{e5}^2} = D_1\overline{u^2} + \frac{1}{2}D_4\overline{v^2} + \frac{1}{2}D_4\overline{w^2} + \frac{\sqrt{2}}{2}D_3\overline{uv} - \frac{\sqrt{2}}{2}D_3\overline{uw} - D_4\overline{vw}, \tag{6.13a}$$

$$\overline{v_{e6}^2} = D_1\overline{u^2} + \frac{1}{2}D_4\overline{v^2} + \frac{1}{2}D_4\overline{w^2} + \frac{\sqrt{2}}{2}D_3\overline{uv} + \frac{\sqrt{2}}{2}D_3\overline{uw} + D_4\overline{vw}, \tag{6.13b}$$

By taking the difference between eqns (6.13a–b), $\overline{vw}$ can be calculated from

$$\overline{vw} = \frac{1}{2D_4}\left[\left(\overline{v_{e6}^2} - \overline{v_{e5}^2}\right) - \sqrt{2}D_3\overline{uw}\right], \tag{6.14}$$

with $\overline{uw}$ being determined from eqn (6.11b). Vagt and Fernholz (1979) have described a special *SY*-probe and a related signal-analysis technique for evaluating $\overline{vw}$. It should be noted that the six equations in eqns (6.10a–d) and (6.13a–b) for the six Reynolds stresses are not independent, due to the symmetry of the coefficients. An additional, independent equation is needed; this is usually eqn (6.9) for the *SN*-probe. To overcome the two-probe problem, Hooper (1980) and Sampath *et al.* (1982) modified an *X*-probe into a probe containing an *SN* and an *SY* element, which reduced the experimental time significantly.

6.1.2.2 *Two-component mean velocity*

In many flow situations the mean-velocity vector will have two significant components in the space-fixed coordinate system selected. Bissonnette and Mellor (1974) investigated a swirling flow, which had significant axial, $\bar{U}$, and tangential, $\bar{W}$, mean-velocity components (see the notation in Fig. 6.2). They used both an *SN* and an *SY* hot-wire probe, which were inserted radially, that is perpendicularly to the mean flow field. At each point the magnitude and direction of the mean-velocity vector was first identified by means of the *SN*-probe. Rotating the probe around its stem, they recorded the voltage output as a function of the roll angle, γ. As described in Section 4.11.1, this type of measurement can determine the