MTH 452/552

Systems of ODEs.

$u''(t) = -u(t) + e^t$

Transform into a first order system.

$w_1(t) = u(t)$

$w_2(t) = u'(t)$

$w_1'(t) = u'(t) = w_2(t)$

$w_2'(t) = u''(t) = -u(t) + e^t = -w_1(t) + e^t$

$$\underbrace{\begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}'}_{w'(t)} = \underbrace{\begin{bmatrix} w_2(t) \\ -w_1(t) + e^t \end{bmatrix}}_{f(t,w)} = $$

$$w'(t) = f(t, w) = \begin{bmatrix} w_2 \\ -w_1 + e^t \end{bmatrix}$$

$u_1''' + 4t^2 u_2' = e^t$

$u_3' = -4u_1 + u_2$

$u_2' = -u_3 + u_1$

Convert to 1st order system

$w_1 = u_1$, $w_2 = u_2$, $w_3 = u_3$

$w_4 = u_1'$, " "

$w_5 = u_1''$

$w_1' = u_1' = w_4$

$w_2' = u_2'' = -w_3 + w_1$

$w_3' = u_3' = -4u_1 + u_2 = -4w_1 + w_2$

$w_4' = w_5$

$w_5' = u_1''' = -4t^2 u_2' + e^t = -4t^2(-u_3 + u_1) + e^t$

$= -4t^2(-w_3 + w_1) + e^t$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix}' = \begin{bmatrix} w_4 \\ -w_3 + w_1 \\ -4w_1 + w_2 \\ w_5 \\ -4t^2(w_1 - w_3) + e^t \end{bmatrix}$$

See Example 5.1 in text.

Initial value problem:

$u' = f(t, u)$ ODE

$u(t_0) = u_0$ Initial condition.

Most simple numerical method,

$$u'(t) \approx \frac{u(t + \Delta t) - u(t)}{\Delta t}$$

$= f(t, u(t))$. Now solve for $u(t + \Delta t)$

$u(t + \Delta t) = u(t) + \Delta t\, f(t, u(t))$

"Euler method".