Example of Higher-order Uncertainty Analysis

Determine the uncertainty of the mass flow rate of air through a chocked orifice (see governing equation on the laboratory 1 assignment) and the uncertainty in the Reynolds number of the same flow through a tube with a known diameter (0.5 $\pm$ 0.01 inches). Hypothetical values are supplied for the instrument uncertainty as well as from repeated measurements. Be sure to apply an uncertainty tree to help with the analysis.

In class on Thursday I will show the solution and answer questions, but we will not have time for me to work through the entire solution.

$\dot{m}$ ***

$$C \frac{P_0 A_t}{\sqrt{R T_0}} \sqrt{\frac{2k}{(k+1)} \left(\frac{2}{k+1}\right)^{2/(k-1)}}$$

- C
  - bias
  - ~~prec~~
- P_0
  - prec
  - bias
- $A_t = \frac{\pi d_t^2}{4}$
  - d_t
    - ~~prec~~
    - bias
      - Re
        - $\frac{\rho 4 Q}{\pi D_d \mu} = \frac{4 \dot{m}}{\pi D_d \mu}$
          - D (tube)
            - prec.
            - bias
          - μ (~~~ constant~~)
          - $\dot{m}$
- R
  - const.
- T_0
  - prec.
  - bias
- k (~ap constant)

$$A_t = \frac{\pi d^2}{4}$$

- d
  - prec
  - bias

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$$\dot{m} = \frac{C P_0 A_t}{\sqrt{R T_0}} \underbrace{\sqrt{\frac{2k}{(k+1)}\left(\frac{2}{k+1}\right)^{2/(k-1)}}}_{F}$$

$$\frac{\partial \dot{m}}{\partial C} = \frac{P_0 A_t}{\sqrt{R T_0}} F \qquad u_{\dot{m},C} = F\frac{P_0 A_t}{\sqrt{R T_0}} u_C$$

$$\frac{\partial \dot{m}}{\partial P_0} = \frac{C A_t}{\sqrt{R T_0}} F \qquad u_{\dot{m},P_0} = \frac{F C A_t}{\sqrt{R T_0}} u_{P_0}$$

$$\frac{\partial \dot{m}}{\partial A_t} = \frac{C P_0 F}{\sqrt{R T_0}} \qquad u_{\dot{m},A_t} = \frac{F C P_0}{\sqrt{R T_0}} u_{A_t}$$

$$\frac{\partial \dot{m}}{\partial T_0} = \frac{-C P_0 A_t F}{2 R^{1/2} T_0^{3/2}} \qquad u_{\dot{m},T_0} = \frac{-C P_0 A_t F}{2 R^{1/2} T_0^{3/2}} u_{T_0}$$

B.4 $$u_{\dot{m}} = \sqrt{u_{\dot{m},C}^2 + u_{\dot{m},P_0}^2 + u_{\dot{m},A_t}^2 + u_{\dot{m},T_0}^2}$$

$$Re = \frac{4\dot{m}}{\pi D \mu}$$

$$\frac{\partial Re}{\partial \dot{m}} = \frac{4}{\pi D \mu} \qquad u_{Re,\dot{m}} = \frac{4}{\pi D \mu} u_{\dot{m}}$$

$$\frac{\partial Re}{\partial D} = \frac{-4\dot{m}}{\pi D^2 \mu} \qquad u_{Re,D} = \frac{4\dot{m}}{\pi D^2 \mu} u_D$$

$$u_{Re} = \sqrt{u_{Re,\dot{m}}^2 + u_{Re,D}^2}$$

$$A_t = \pi d^2/4$$

$$\frac{\partial A_t}{\partial d} = \frac{2\pi d}{4} \qquad u_{A_t} = \frac{2\pi d}{4} u_{d_t}$$

Set	Po (PSIG), i.e., pressure tank (i.e, stagnation)	To (oC), i.e., temperature in tank (i.e., stagnation)	Stagnation Pressure in kPa absolute	Stagnation Temp. in Kelvin
1	25	23.2	273.4137931	296.2
2	22	23.1	252.7241379	296.1
3	32.7	23.5	326.5172414	296.5
4	21.2	23.1	247.2068966	296.1
5	26	23.7	280.3103448	296.7
6	29.1	23.3	301.6896552	296.3
7	25.2	23.7	274.7931034	296.7

** Note that the values are hypothetical

	Stagnation Pressure in kPa absolute	Stagnation Temp. in Kelvin
Mean	279.5221675	296.3714286
Stdev	27.45494221	0.262769136
Precision Uncertainty	25.3925013	0.243029673

student-t distribution (95% confidence)

2.447

Note that this value is for 6 degrees of freedom (i.e., 7 samples)

Set	Po (PSIG), i.e., pressure tank	To (oC), i.e., temperature in tank	C	D (in)		Stagnation Pressure in kPa absolute	Stagnation Temp. in Kelvin	C (discharge coefficient)	D (m)
	2.5 (linearity uncertainty)	0.4 (manufacturer)	0.99 $\pm$ 0.01	0.3 $\pm$ 0.01		17.24137931	0.4		0.00762
	1% of full-scale (up to 30 psi)					2.068965517			
					Bias Uncertainty	17.36507353	0.4	0.01	0.000254

** Note that the values are hypothetical

	Pressure (kPa)	Temp (K)	C	D (m)	area of throat (m^2)	Ratio of specific heats	Viscosity (kg-m/s)	Ideal gas constant (kJ/kg-K)
Avererage value	279.5221675	296.3714286	0.99	0.00762	4.56037E-05	1.4	0.00001846	0.287
Uncertainty	30.76239427	0.468042116	0.01	0.000254	3.04024E-06	n/A	n/A	n/A

Evaluation of uncertainty for mass flow rate (kg/s)

	Constant for m_dot		0.684731456
	Sensitivity to independent variable		Uncertainty for that indepdent variable
C		0.000946405	9.46405E-06
stag pressure		3.35194E-06	0.000103114
Area of throat		20.54530602	6.24628E-05
stag temp		-1.58069E-06	-7.39828E-07

Uncertainty in mass flow rate	0.00012093
Average mass flow rate	0.000936941
Percent uncertainty	12.9069254

Evaluation of uncertainty in Re

	Sensitivity to independent variable		Uncertainty for that indepdent variable
m_dot		9051560.334	1094.608179
D		-1112963.49	-282.6927264

Uncertainty in mass flow rate	1130.522995
Average Re	8480.781792
Percent uncertainty	13.33041013