This homework focused on calculating the most probable macrostate of a system based on the thermodynamic probability. The total energy of the system, or internal energy is defined as

$$U = \sum_{i=1}^{\#\,levels} N_i \epsilon_i$$

Where N_i is the number of particles in energy state ϵ_i. Similarly, the total number of particles is as follows:

$$N = \sum_{i=1}^{\#\,levels} N_i$$

The thermodynamic probability function is a measure of the most likely macrostate present in a system, where a macrostate is a specification of the number of particles in each energy level that satisfies the above two equations. The macrostate, or arrangement of N_i, that can be made up of the greatest number of microstates (greatest Ω) is the most likely macrostate. The value of Ω was calculated as follows:

$$\ln(\Omega) = \sum_{i=1}^{\#\,levels} N_i \ln\left(\frac{g_i}{N_i}\right) + N$$

With an internal energy of 1000 units, number of particles N=2000, degeneracy $g_i = 10{,}000$, and available energy levels of 0,1,2 the most probable macrostate consisted of $N_0 = 1232,\ N_1 = 536,\ N_2 = 232$ with a thermodynamic probability $\Omega = \exp(7021)$. This was found by iteration over all possible combinations of N_i, and each combination was found using the first two equations of this summary. To calculate the entropy of the most likely macrostate Boltzmann's Law was used:

$$S = k \ln(\Omega)$$

The most likely macrostate is coincident with the macrostate with the highest entropy and was found to be $S = 9.694 * 10^{-23} kJ/K$

If the total energy of the system is increased then we would expect the distribution of particles in each energy level to shift toward the higher energy levels; the percentage increase of N_2 would increase significantly, N_1 would increase to a lesser extent, and N_0 would decrease. At the same time, the number of possible microstates Ω that correspond to the most likely macrostate would increase, as seen in Figure 1.

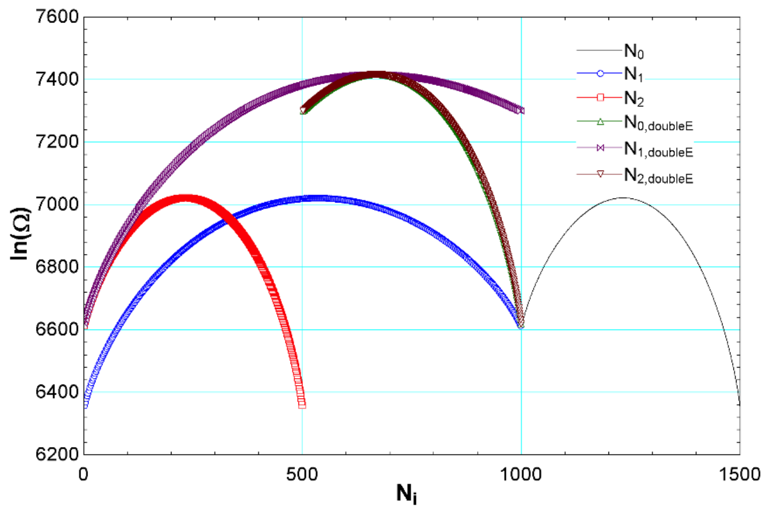

Figure 1: thermodynamic probability as a function of all three particle distributions. Curves are shown for N_1, N_2, and N_3 with internal energies at both 1000 and 2000 units to demonstrate trends of increasing energy. Note that each curve for a given energy level has the same maximum value of Ω, which corresponds to the macrostate with the greatest number of possible microstates which is therefore the most probable macrostate.