The purpose of this assignment was to test and demonstrate performance trends of GPUs when multiplying, multiplying and adding, and multiplying and reducing large arrays of numbers by using OpenCL.

Question 1

What machine did you run the experiment on?

The experiment was run on Rabbit at a time when the load average was approximately 0.75.

Question 2

Show tables and graphs of the array multiplication and array multiplication with addition timing experiments.

Graphs of the computation speed versus the global size and local work group size can be found in Figures 1 and 2. A table of the values used to generate these graphs can be found in Table 1 of Appendix 1.1.

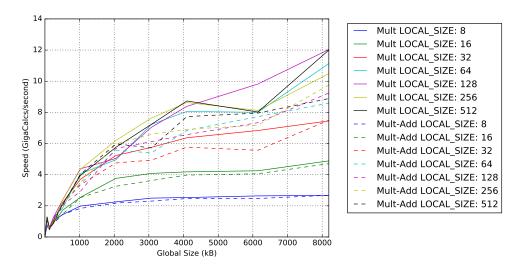


Figure 1: Speed of the multiplication and multiplication with addition experiments vs. global size. Note that the solid lines signify multiplication, while the dashed lines signify multiplication with addition.

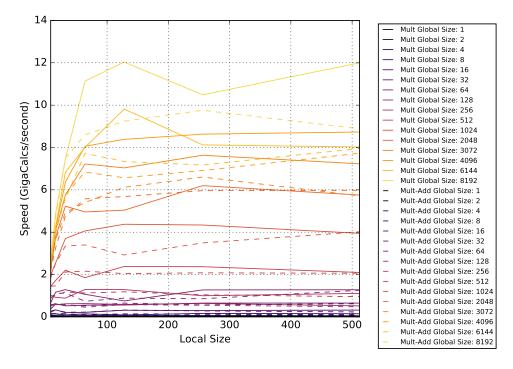


Figure 2: Speed of the multiplication and multiplication with addition experiments vs. local size. Note that the solid lines signify multiplication, while the dashed lines signify multiplication with addition.

Question 3

What patterns are you seeing in the performance curves of array multiplication and array multiplication-addition?

Question 4

Why do you think the patterns of array multiplication and array multiplication-addition look this way?

Question 5

 $What is the performance \ difference \ between \ doing \ a \ multiply \ and \ doing \ a \ multiply-add?$

Question 6

What does the difference in performance between doing a multiply and a multiply-add mean for the proper use of GPU parallel computing?

Question 7

Show a table and graph for the performance of the multiplication-reduction experiment.

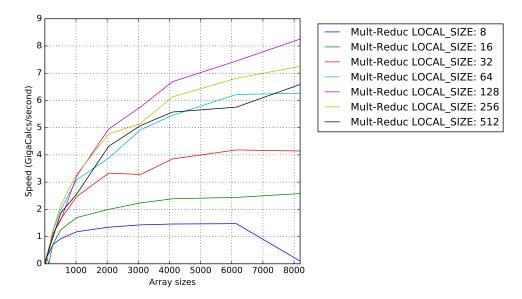


Figure 3: Speed of the multiplication with reduction experiment vs. global size.

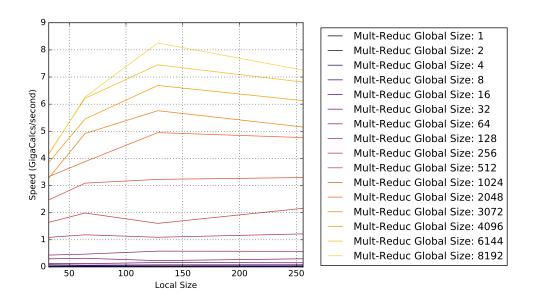


Figure 4: Speed of the multiplication with reduction experiment vs. local size.

Question 8

What patterns are you seeing in the performance curves of the multiplication-reduction experiment?

Question 9

Why do you think the patterns of the multiplication-reduction experiment look this way?

Question 10

What do the patterns in the multiplication-reduction experiment mean for the proper use of GPU parallel computing?

1 Appendices

1.1 Tables of Generated Data

Table 1: Raw Data Generated

Mode	NKB	LOCAL_SIZE	NUM_WORK_GROUPS	GigaCalcPerSecond
Multiply	1	8	128	0.012
Multiply	2	8	256	0.039
Multiply	4	8	512	0.055
Multiply	8	8	1024	0.162
Multiply	16	8	2048	0.221
Multiply	32	8	4096	0.371
Multiply	64	8	8192	0.663
Multiply	128	8	16384	0.575
Multiply	256	8	32768	1.028
Multiply	512	8	65536	1.439
Multiply	1024	8	131072	1.983
Multiply	2048	8	262144	2.254
Multiply	3072	8	393216	2.495
Multiply	4096	8	524288	2.532
Multiply	6144	8	786432	2.637
Multiply	8192	8	1048576	2.669
Multiply	1	16	64	0.020
Multiply	2	16	128	0.040
Multiply	4	16	256	0.040
	8	16	512	0.061
Multiply	I			0.131
Multiply	16	16	1024	
Multiply	32	16	2048	0.620
Multiply	64	16	4096	1.167
Multiply	128	16	8192	0.570
Multiply	256	16	16384	0.923
Multiply	512	16	32768	1.719
Multiply	1024	16	65536	2.543
Multiply	2048	16	131072	3.761
Multiply	3072	16	196608	4.080
Multiply	4096	16	262144	4.181
Multiply	6144	16	393216	4.251
Multiply	8192	16	524288	4.885
Multiply	1	32	32	0.019
Multiply	2	32	64	0.028
Multiply	4	32	128	0.075
Multiply	8	32	256	0.094
Multiply	16	32	512	0.199
Multiply	32	32	1024	0.522
Multiply	64	32	2048	1.286
Multiply	128	32	4096	0.619
Multiply	256	32	8192	0.886
Multiply	512	32	16384	2.207
Multiply	1024	32	32768	3.703
Multiply	2048	32	65536	5.230
Multiply	3072	32	98304	5.754
Multiply	4096	32	131072	6.370
	6144	32	196608	6.835
Multiply				
Multiply Multiply	8192	32 64	262144 16	7.453 0.015

Multiply	2	64	32	0.038
Multiply	4	64	64	0.074
Multiply	8	64	128	0.143
Multiply	16	64	256	0.217
Multiply	32	64	512	0.554
Multiply	64	64	1024	1.067
Multiply	128	64	2048	0.628
Multiply	256	64	4096	1.296
Multiply	512	64	8192	1.853
Multiply	1024	64	16384	4.060
Multiply	2048	64	32768	4.957
Multiply	3072	64	49152	7.220
	4096	64	65536	8.061
Multiply		64		
Multiply	6144		98304	8.002
Multiply	8192	64	131072	11.136
Multiply	1	128	8	0.017
Multiply	2	128	16	0.030
Multiply	4	128	32	0.066
Multiply	8	128	64	0.093
Multiply	16	128	128	0.314
Multiply	32	128	256	0.561
Multiply	64	128	512	0.754
Multiply	128	128	1024	0.590
Multiply	256	128	2048	1.289
Multiply	512	128	4096	2.381
Multiply	1024	128	8192	4.375
Multiply	2048	128	16384	5.040
Multiply	3072	128	24576	7.037
Multiply	4096	128	32768	8.384
Multiply	6144	128	49152	9.817
Multiply	8192	128	65536	12.035
Multiply	1	256	4	0.020
Multiply	2	256	8	0.040
Multiply	4	256	16	0.066
Multiply	8	256	32	0.165
Multiply	16	256	64	0.299
Multiply	32	256	128	0.651
Multiply	64	256	256	1.274
Multiply	128	256	512	0.634
Multiply	256	256	1024	0.999
Multiply	512	256	2048	2.371
Multiply	1024	256	4096	4.339
Multiply	2048	256	8192	6.198
Multiply	3072	256	12288	7.634
Multiply	4096	256	16384	8.630
Multiply	6144	256	24576	8.124
Multiply	8192	256	32768	10.489
Multiply	1	512	2	0.021
Multiply	2	512	4	0.040
Multiply	4	512	8	0.055
Multiply	8	512	16	0.149
Multiply	16	512	32	0.321
Multiply	32	512	64	0.652
Multiply	64	512	128	1.282
Multiply	128	512	256	0.521
Multiply	256	512	512	1.109
winipiy	230	J12	312	1.109

Multiply	512	512	1024	2.098
Multiply	1024	512	2048	3.946
Multiply	2048	512	4096	5.755
Multiply	3072	512	6144	7.230
Multiply	4096	512	8192	8.731
Multiply	6144	512	12288	8.024
Multiply	8192	512	16384	11.997
Multiply-Add	1	8	128	0.017
Multiply-Add	2	8	256	0.039
Multiply-Add	4	8	512	0.039
Multiply-Add	8	8	1024	0.125
Multiply-Add	16	8	2048	0.123
Multiply-Add	32	8	4096	0.561
Multiply-Add	64	8	8192	0.882
Multiply-Add	128	8	16384	0.882
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Multiply-Add	256	8	32768	0.892
Multiply-Add	512 1024	8	65536	1.446
Multiply-Add		8	131072	1.836
Multiply-Add	2048	8	262144	2.175
Multiply-Add	3072	8	393216	2.303
Multiply-Add	4096	8	524288	2.468
Multiply-Add	6144	8	786432	2.470
Multiply-Add	8192	8	1048576	2.677
Multiply-Add	1	16	64	0.016
Multiply-Add	2	16	128	0.033
Multiply-Add	4	16	256	0.061
Multiply-Add	8	16	512	0.156
Multiply-Add	16	16	1024	0.330
Multiply-Add	32	16	2048	0.627
Multiply-Add	64	16	4096	1.111
Multiply-Add	128	16	8192	0.624
Multiply-Add	256	16	16384	1.034
Multiply-Add	512	16	32768	1.466
Multiply-Add	1024	16	65536	2.525
Multiply-Add	2048	16	131072	3.253
Multiply-Add	3072	16	196608	3.624
Multiply-Add	4096	16	262144	3.975
Multiply-Add	6144	16	393216	4.058
Multiply-Add	8192	16	524288	4.707
Multiply-Add	1	32	32	0.021
Multiply-Add	2	32	64	0.024
Multiply-Add	4	32	128	0.080
Multiply-Add	8	32	256	0.150
Multiply-Add	16	32	512	0.223
Multiply-Add	32	32	1024	0.567
Multiply-Add	64	32	2048	1.123
Multiply-Add	128	32	4096	0.579
Multiply-Add	256	32	8192	1.175
Multiply-Add	512	32	16384	2.119
Multiply-Add	1024	32	32768	3.346
Multiply-Add	2048	32	65536	4.753
Multiply-Add	3072	32	98304	4.935
Multiply-Add	4096	32	131072	5.765
Multiply-Add	6144	32	196608	5.567
Multiply-Add	8192	32	262144	7.500
Multiply-Add	1	64	16	0.020
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Multiply-Add	2	64	32	0.024
Multiply-Add	4	64	64	0.075
Multiply-Add	8	64	128	0.098
Multiply-Add	16	64	256	0.222
Multiply-Add	32	64	512	0.520
Multiply-Add	64	64	1024	0.740
Multiply-Add	128	64	2048	0.458
Multiply-Add	256	64	4096	1.127
Multiply-Add	512	64	8192	2.159
Multiply-Add	1024	64	16384	3.397
Multiply-Add	2048	64	32768	5.574
Multiply-Add	3072	64	49152	5.398
Multiply-Add	4096	64	65536	6.841
Multiply-Add	6144	64	98304	7.723
Multiply-Add	8192	64	131072	8.594
Multiply-Add	1	128	8	0.020
Multiply-Add	2	128	16	0.027
Multiply-Add	4	128	32	0.074
Multiply-Add	8	128	64	0.142
Multiply-Add	16	128	128	0.321
Multiply-Add	32	128	256	0.652
Multiply-Add Multiply-Add	64	128	512	0.886
Multiply-Add Multiply-Add	128	128	1024	0.577
Multiply-Add	256	128	2048	1.185
Multiply-Add Multiply-Add	512	128	4096	2.046
Multiply-Add Multiply-Add	1024	128	8192	2.924
Multiply-Add Multiply-Add	2048	128	16384	5.684
Multiply-Add Multiply-Add	3072	128	24576	6.119
Multiply-Add Multiply-Add	4096	128	32768	6.571
Multiply-Add Multiply-Add	6144	128	49152	7.338
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Multiply-Add Multiply-Add	1	256	4	0.020
Multiply-Add Multiply-Add	2	256	8	0.040
Multiply-Add Multiply-Add	4	256	16	0.079
Multiply-Add Multiply-Add	8	256	32	0.079
Multiply-Add	16	256	64	0.282
Multiply-Add Multiply-Add	32	256	128	0.566
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Multiply-Add	128	256	512	0.550
Multiply-Add	256	256	1024 2048	1.042
Multiply-Add	512	256	4096	2.080
Multiply-Add	1024	256		3.494
Multiply-Add	2048	256	8192	5.963
Multiply-Add	3072	256	12288	6.604
Multiply-Add	4096	256	16384	6.912
Multiply-Add	6144	256	24576	7.172
Multiply-Add	8192	256	32768	9.757
Multiply-Add	1	512	2	0.019
Multiply-Add	2	512	4	0.029
Multiply-Add	4	512	8	0.082
Multiply-Add	8	512	16	0.104
Multiply-Add	16	512	32	0.235
Multiply-Add	32	512	64	0.621
Multiply-Add	64	512	128	1.256
Multiply-Add	128	512	256	0.477
Multiply-Add	256	512	512	0.955

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	Multiply+Reduction	1	64	16	0.004

Multiply+Reduction	2	64	32	0.009
Multiply+Reduction	4	64	64	0.021
Multiply+Reduction	8	64	128	0.041
Multiply+Reduction	16	64	256	0.065
Multiply+Reduction	32	64	512	0.122
Multiply+Reduction	64	64	1024	0.319
Multiply+Reduction	128	64	2048	0.472
Multiply+Reduction	256	64	4096	1.185
Multiply+Reduction	512	64	8192	1.988
Multiply+Reduction	1024	64	16384	3.089
Multiply+Reduction	2048	64	32768	3.883
Multiply+Reduction	3072	64	49152	4.920
Multiply+Reduction	4096	64	65536	5.454
Multiply+Reduction	6144	64	98304	6.219
Multiply+Reduction	8192	64	131072	6.265
Multiply+Reduction		128	8	0.005
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Multiply+Reduction			16	0.010
Multiply+Reduction	8	128 128	32	0.019
Multiply+Reduction Multiply+Reduction			128	
1 7	16 32	128	128 256	0.077
Multiply+Reduction	64	128	512	1
Multiply+Reduction	l	128		0.233
Multiply+Reduction	128	128	1024	0.582
Multiply+Reduction	256	128	2048	1.094
Multiply+Reduction	512	128	4096	1.606
Multiply+Reduction	1024	128	8192	3.227
Multiply+Reduction	2048	128	16384	4.950
Multiply+Reduction	3072	128	24576	5.759
Multiply+Reduction	4096	128	32768	6.688
Multiply+Reduction	6144	128	49152	7.448
Multiply+Reduction	8192	128	65536	8.254
Multiply+Reduction	1	256	4	0.004
Multiply+Reduction	2	256	8	0.009
Multiply+Reduction	4	256	16	0.018
Multiply+Reduction	8	256	32	0.041
Multiply+Reduction	16	256	64	0.080
Multiply+Reduction	32	256	128	0.154
Multiply+Reduction	64	256	256	0.300
Multiply+Reduction	128	256	512	0.575
Multiply+Reduction	256	256	1024	1.218
Multiply+Reduction	512	256	2048	2.161
Multiply+Reduction	1024	256	4096	3.295
Multiply+Reduction	2048	256	8192	4.768
Multiply+Reduction	3072	256	12288	5.157
Multiply+Reduction	4096	256	16384	6.130
Multiply+Reduction	6144	256	24576	6.813
Multiply+Reduction	8192	256	32768	7.253
Multiply+Reduction	1	512	2	0.004
Multiply+Reduction	2	512	4	0.010
Multiply+Reduction	4	512	8	0.017
Multiply+Reduction	8	512	16	0.036
Multiply+Reduction	16	512	32	0.080
Multiply+Reduction	32	512	64	0.167
Multiply+Reduction	64	512	128	0.245
Multiply+Reduction	128	512	256	0.527
Multiply+Reduction	256	512	512	0.956

Multiply+Reduction	512	512	1024	1.865
Multiply+Reduction	1024	512	2048	2.564
Multiply+Reduction	2048	512	4096	4.315
Multiply+Reduction	3072	512	6144	5.077
Multiply+Reduction	4096	512	8192	5.573
Multiply+Reduction	6144	512	12288	5.753
Multiply+Reduction	8192	512	16384	6.587

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