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Homework 2 Executive Summary

Summary

A piston/cylinder/spring closed system was analyzed using principles derived from the first law of thermodynamics. The pressure forces on the piston are greater than the force on the piston from the spring, so the piston will move to the right until x is approximately 0.4587 m when the piston latch is released, assuming no heat interaction. The helium expands until it reaches 197.1 kPa and the air compresses until it reaches 131.4 kPa. The helium cools down to 218.5 K and the air heats up to 313.6 K from their initial values of 16.85°C in the adiabatic process. The equation for an adiabatic process is $PV^n = constant$, where *P* is pressure, *V* is volume, and *n* is the adiabatic index, which is equal to the ratio of the specific heats in this example.

Assuming that there is a heat interaction between the gases and the cylinder and piston walls, the entire assembly will reach a thermal equilibrium at the initial temperature of 290 K. The piston moves further to the right until x is approximately 0.5508 m because heat is transferred from the air to the helium. When the piston has reached equilibrium while the system has uniform temperature, the pressure in the helium will be 217.8 kPa, and the pressure of the air will be 138.9 kPa.

Friction was not fully considered in the analysis conducted. Friction works against the movement of the piston, which causes the piston to travel shorter distances in the direction of travel than depicted in the results given above. More friction is associated with traveling less distance, and smaller amounts of friction would be associated with traveling greater distance. If there were enough friction (i.e. the piston and the cylinder walls are rusted together), the piston would not move at all. Despite not accounting for friction when solving for the position of the piston, friction is needed to bring the piston to a stop, or otherwise the position will continually oscillate.

Assumptions

It was assumed that the piston came to a stop at the location where all forces balanced without the effects of friction factored in. The cylinder did not give off or take in any energy during the analysis (i.e. the outer surfaces of the cylinder are well insulated). Friction did not add heat to the gases, which could potentially change the equilibrium. Ideal gases were assumed.

Approach

The expansion and compression in part B are adiabatic because there is assumed to be no heat transfer between the fluids or the piston. The equations for adiabatic compression (for the air) and adiabatic expansion (for the helium) were entered into EES along with an equation for the force balance on the piston to obtain the result.

The entire cylinder assembly was considered in part C, and there was no net work into or out of the system. Based on this, it was observed that the internal energy of the system remained constant, which allowed us to assume that the temperature at thermal equilibrium after the latch was released is the same as the temperature of the system prior to releasing the latch (i.e. the process was isothermal). Equations for isothermal expansion (for the helium) and isothermal compression (for the air) were entered into EES along with a force balance equation to obtain the result.

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"Q 3.C-1"
"part b"


"Known"
P_He_1=400[kPa]                                              "Initial Pressure of Helium"
d=0.2[m]                                                     "Inside Diameter of cylinder"
T_1=290[K]                                                   "Initial Temperature of helium and air"
K=4.500[kN/m]
P_Air_1=100[kPa]                                             "Initial Pressure of Air"
L_1_He=0.3[m]                                                "Length of the cylinder that helium occupies"


"Calculation"
T_1_C=converttemp(K,C,290)                                   "Converting the initial temperatures to Kelvin"
L_1_Air=1.2-0.3-0.005                                        "Length of the cylinder that air occupies"
A=pi/4*d^2                                                   "Area of the piston"
c_p_he=cp(He,T=T_1_C)                                        "Specific heat at constant pressure of helium at inital state"
c_p_air=cp(Air,T=T_1_C)                                      "Specific heat at constant pressure of air at inital state"
c_v_he=cv(He,T=T_1_C)                                        "Specific heat at constant pressure of helium at inital state"
c_v_air=cv(Air,T=T_1_C)                                      "Specific heat at constant pressure of air at inital state"
gamma_he=c_p_he/c_v_he                                       "Adiabatic index of helium"
gamma_air=c_p_air/c_v_air                                    "Adiabatic index of air"
P_He_1*((A*L_1_He)^(gamma_he))=P_He_2*((A*x)^(gamma_he))     "Reversible adiabatic process"
(P_He_2-P_Air_2)*A=K*x                                       "Force balance at final state"
L_2_air=1.2-0.005                                            "Total length minus the thickness of piston"
P_Air_1*((A*(L_1_Air))^(gamma_air))=P_Air_2*((A*(L_2_air-x))^(gamma_air))   "Formula for Reversible adiabatic process relating pressure and  volume"
P_He_1^(1-gamma_he)*T_1^(gamma_he)=P_He_2^(1-gamma_he)*T_He_2^(gamma_he)    "Formula for Reversible adiabatic process relating pressure and temperature"
P_Air_1^(1-gamma_air)*T_1^(gamma_air)=P_Air_2^(1-gamma_air)*T_Air_2^(gamma_air)   "Formula for Reversible adiabatic process relating pressure and temperature"
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Q 3.C-1

part b

Known

$P_{He,1}$ = 400 [kPa] *Initial Pressure of Helium*

d = 0.2 [m] *Inside Diameter of cylinder*

$T_1 = 290$ [K] *Initial Temperature of helium and air*

$K = 4.5$ [kN/m]

$P_{Air,1} = 100$ [kPa] *Initial Pressure of Air*

$L_{1,He} = 0.3$ [m] *Length of the cylinder that helium occupies*

Calculation

$T_{1,C} = \mathbf{ConvertTemp}\left[K, C, 290\right]$ *Converting the initial temperatures to Kelvin*

$L_{1,Air} = 1.2 - 0.3 - 0.005$ *Length of the cylinder that air occupies*

$A = \frac{\pi}{4} \cdot d^2$ *Area of the piston*

$c_{p,he} = \mathbf{Cp}\left[He, T = T_{1,C}\right]$ *Specific heat at constant pressure of helium at inital state*

$c_{p,air} = \mathbf{Cp}\left[Air, T = T_{1,C}\right]$ *Specific heat at constant pressure of air at inital state*

$c_{v,he} = \mathbf{Cv}\left[He, T = T_{1,C}\right]$ *Specific heat at constant pressure of helium at inital state*

$c_{v,air} = \mathbf{Cv}\left[Air, T = T_{1,C}\right]$ *Specific heat at constant pressure of air at inital state*

$\gamma_{he} = \frac{c_{p,he}}{c_{v,he}}$ *Adiabatic index of helium*

$\gamma_{air} = \frac{c_{p,air}}{c_{v,air}}$ *Adiabatic index of air*

$P_{He,1} \cdot \left[A \cdot L_{1,He}\right]^{\gamma_{he}} = P_{He,2} \cdot \left[A \cdot x\right]^{\gamma_{he}}$ *Reversible adiabatic process*

$\left[P_{He,2} - P_{Air,2}\right] \cdot A = K \cdot x$ *Force balance at final state*

$L_{2,air} = 1.2 - 0.005$ *Total length minus the thickness of piston*

$P_{Air,1} \cdot \left[A \cdot L_{1,Air}\right]^{\gamma_{air}} = P_{Air,2} \cdot \left[A \cdot \left(L_{2,air} - x\right)\right]^{\gamma_{air}}$ *Formula for Reversible adiabatic process relating pressure and volume*

$P_{He,1}{}^{\left[1 - \gamma_{he}\right]} \cdot T_1{}^{\gamma_{he}} = P_{He,2}{}^{\left[1 - \gamma_{he}\right]} \cdot T_{He,2}{}^{\gamma_{he}}$ *Formula for Reversible adiabatic process relating pressure and temperature*

$P_{Air,1}{}^{\left[1 - \gamma_{air}\right]} \cdot T_1{}^{\gamma_{air}} = P_{Air,2}{}^{\left[1 - \gamma_{air}\right]} \cdot T_{Air,2}{}^{\gamma_{air}}$ *Formula for Reversible adiabatic process relating pressure and temperature*

SOLUTION

Unit Settings: SI C kPa kJ mass deg

$A = 0.03142$ [m^2]	$c_{p,air} = 1.004$ [kJ/kg-K]	$c_{p,he} = 5.193$ [kJ/kg-K]
$c_{v,air} = 0.7173$ [kJ/kg-K]	$c_{v,he} = 3.116$ [kJ/kg-K]	$d = 0.2$ [m]
$\gamma_{air} = 1.4$	$\gamma_{he} = 1.667$	$K = 4.5$ [kN/m]
$L_{1,Air} = 0.895$ [m]	$L_{1,He} = 0.3$ [m]	$L_{2,air} = 1.195$ [m]

$P_{Air,1}$ = 100 [kPa]	$P_{Air,2}$ = 131.4 [kPa]	$P_{He,1}$ = 400 [kPa]
$P_{He,2}$ = 197.1 [kPa]	T_1 = 290 [K]	$T_{1,C}$ = 16.85 [C]
$T_{Air,2}$ = 313.6 [K]	$T_{He,2}$ = 218.5 [K]	x = 0.4587 [m]

No unit problems were detected.

KEY VARIABLES

x = 0.4587 [m]	*part b*
$P_{Air,2}$ = 131.4 [kPa]	*part b*
$P_{He,2}$ = 197.1 [kPa]	*part b*
$T_{Air,2}$ = 313.6 [K]	*part b*
$T_{He,2}$ = 218.5 [K]	*part b*

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"Q3.C-1"
"part c"

"knowns"
P_He_1=400[kPa]                 "Initial Pressure of Helium"
P_Air_1=100[kPa]                "Initial Pressure of Air"
K=4.5[kN/m]                     "Spring Constant"
l_he_1=0.30[m]                  "Length that helium occupies in the cylinder at initial state"
L=1.2[m]                        "Total length of cylinder"
th=0.005[m]                     "Thickness of piston"
d=0.2[m]                        "Diameter of the piston"


"Calculation"
l_air_1=(L-th)-l_he_1           "Length that air occupies in the cylinder at initial state"
A=(pi/4)*d^2                    "Area of the piston"
V_He_1=A*l_he_1                 "Volume that helium occupies in the cylinder at initial state"
V_Air_1=A*l_air_1               "Volume that air occupies in the cylinder at initial state"
P_He_1*V_He_1=P_He_2*V_He_2     "Formula Isothermal process of helium relating pressure and volume"
P_Air_1*V_Air_1=P_Air_2*V_Air_2 "Formula Isothermal process of air relating pressure and volume"
P_He_2*A=P_Air_2*A+K*l_he_2     "Force balance at final state"
L=l_he_2+l_air_2+th             "Total length of cylinder"
V_He_2=A*l_he_2                 "Volume that helium occupies in the cylinder at final state"
V_Air_2=A*l_air_2               "Volume that air occupies in the cylinder at final state"
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Q3.C-1

part c

knowns

$P_{He,1} = 400$ [kPa] *Initial Pressure of Helium*

$P_{Air,1} = 100$ [kPa] *Initial Pressure of Air*

$K = 4.5$ [kN/m] *Spring Constant*

$l_{he,1} = 0.3$ [m] *Length that helium occupies in the cylinder at initial state*

$L = 1.2$ [m] *Total length of cylinder*

$th = 0.005$ [m] *Thickness of piston*

$d = 0.2$ [m] *Diameter of the piston*

Calculation

$l_{air,1} = L - th - l_{he,1}$ *Length that air occupies in the cylinder at initial state*

$A = \frac{\pi}{4} \cdot d^2$ *Area of the piston*

$V_{He,1} = A \cdot l_{he,1}$ *Volume that helium occupies in the cylinder at initial state*

$V_{Air,1} = A \cdot l_{air,1}$ *Volume that air occupies in the cylinder at initial state*

$P_{He,1} \cdot V_{He,1} = P_{He,2} \cdot V_{He,2}$ *Formula Isothermal process of helium relating pressure and volume*

$P_{Air,1} \cdot V_{Air,1} = P_{Air,2} \cdot V_{Air,2}$ *Formula Isothermal process of air relating pressure and volume*

$P_{He,2} \cdot A = P_{Air,2} \cdot A + K \cdot l_{he,2}$ *Force balance at final state*

$L = l_{he,2} + l_{air,2} + th$ *Total length of cylinder*

$V_{He,2} = A \cdot l_{he,2}$ *Volume that helium occupies in the cylinder at final state*

$V_{Air,2} = A \cdot l_{air,2}$ *Volume that air occupies in the cylinder at final state*

SOLUTION

Unit Settings: SI C kPa kJ mass deg

$A = 0.03142$ [m^2]	$d = 0.2$ [m]	$K = 4.5$ [kN/m]
$L = 1.2$ [m]	$l_{air,1} = 0.895$ [m]	$l_{air,2} = 0.6442$ [m]
$l_{he,1} = 0.3$ [m]	$l_{he,2} = 0.5508$ [m]	$P_{Air,1} = 100$ [kPa]
$P_{Air,2} = 138.9$ [kPa]	$P_{He,1} = 400$ [kPa]	$P_{He,2} = 217.8$ [kPa]
$th = 0.005$ [m]	$V_{Air,1} = 0.02812$ [m^3]	$V_{Air,2} = 0.02024$ [m^3]
$V_{He,1} = 0.009425$ [m^3]	$V_{He,2} = 0.01731$ [m^3]	

No unit problems were detected.

KEY VARIABLES

$P_{He,2} = 217.8$ [kPa]	*part c*
$l_{he,2} = 0.5508$ [m]	*part c*
$V_{Air,2} = 0.02024$ [m^3]	*part c*
$V_{He,2} = 0.01731$ [m^3]	*part c*
$P_{Air,2} = 138.9$ [kPa]	*part c*