10.7

expand: $\frac{\partial}{\partial x_j}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)$

$$= \frac{\partial u_j}{\partial x_j \partial x_i} + \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}\right)$$

reverse order of derivatives in 1st term:

2 $$= \underbrace{\frac{\partial}{\partial x_i}\left(\frac{\partial u_j}{\partial x_j}\right)}_{= 0 \text{ (incompressible)}} + \underbrace{\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}\right)}_{= \frac{\partial^2 u_i}{\partial x_j^2} = \nabla^2 u_i}$$

for $\mu = \text{const.}$

1 $$\rho \frac{Du_i}{Dt} = -\gamma \frac{\partial h}{\partial x_i} - \frac{\partial P}{\partial x_i} + \mu \frac{\partial}{\partial x_i}\left(\frac{\partial u_j}{\partial x_j}\right) + \mu \nabla^2 u_i - \frac{2}{3}\frac{\partial}{\partial x_i}\left(\frac{\partial u_m}{\partial x_m}\right)$$

combine 3rd term with 5th term on right to get $\frac{1}{3}\mu \frac{\partial}{\partial x_i}\left(\frac{\partial u_m}{\partial x_m}\right)$ (recall that repeated index is summation and j can be replaced with m)

Now we have Eqn. (10.18)

10.9

(a) $u = cx \quad v = cy \quad w = -2cz$

$\nabla \cdot \underline{q} = c + c - 2c = 0$ (incomp.)

$\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = \frac{1}{2}(0)$ no strain rate by shear $(i \neq j)$

$\frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right) = \frac{1}{2}(0)$ irrotational

for $i=j$ (linear deformation) $\frac{\partial u_1}{\partial x_1} = c; \frac{\partial u_2}{\partial x_2} = c; \frac{\partial u_3}{\partial x_3} = -2c$

so $\frac{\partial u_i}{\partial x_i} = 0$ (incompress.)

(b) $u = c \quad v = w = 0$

all $= 0$

(c) $u = 2cy \quad v = w = 0$

$\nabla \cdot \underline{q} = 0$

rotation $= c$

strain rate $= c$

(d) 2-D Flow

$\nabla \cdot \underline{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

$$\left.\begin{aligned} \text{rotation} &= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\ \text{strain rate} &= \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \end{aligned}\right\} \text{one term}$$

$\hookrightarrow$ could include $\left.\begin{matrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \end{matrix}\right\}$ linear strain rates.

10.11 $\vec{q} = 3x^2\hat{i} - 6xy\hat{j} + 16xy^2\hat{k}$

– satisfy conservation of mass?

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \stackrel{?}{=} 0$$

$$6x - 6x + 0 = 0 \quad \underline{\text{yes!}}$$

– Find normal & shear stresses at (20,10,5)

$$\mu = 4.5\times10^{-5}\ \frac{lb-s}{ft^2}$$

given $\tau_{zz} = 0.100\ lb/ft^2$

Note: text includes Press. in defn. of τ

so τ includes <u>all</u> surface stresses.

$$\tau_{ij} = 2\mu e_{ij} - p\delta_{ij} - \frac{2}{3}\mu\underbrace{(\text{div}\cdot\vec{q})}_{=0 \text{ from above.}}\delta_{ij}$$

$$\tau_{zz} = 0.1\frac{lb}{ft^2} = -P \quad \text{since } e_{zz} = \frac{1}{2}\left(\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z}\right) = 0$$

so:

$$\tau_{xx} = 2\mu\frac{\partial u}{\partial x} - P = 2\mu 6x - P = .1108\ lb/ft^2$$

$$\tau_{yy} = 2\mu\frac{\partial v}{\partial y} - P = 2\mu(-6x) - P = .0892\ lb/ft^2$$

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu(0 - 6y) = .0027\ lb/ft^2$$

$$\tau_{xz} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = \mu(16y^2 + 0) = .072\ lb/ft^2$$

$$\tau_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \mu(0 + 32xy) = .288\ lb/ft^2$$