1 Taylor Series Expansion

Consider a continuous function $f(x)$. Then, the Taylor Series expansion of the function around a point x_j is:

$$f(x) = f(x_j) + (x - x_j)f'(x_j) + \frac{(x - x_j)^2}{2!}f''(x_j) + \frac{(x - x_j)^3}{3!}f'''(x_j) + \frac{(x - x_j)^4}{4!}f''''(x_j) + ... \quad (1)$$

- In the above series, notice that ALL derivatives are at the point x_j. Also, $x - x_j$ represents the step size, which can be denoted as h, for example.
- If $x > x_j$, $h = x - x_j > 0$. All terms in the expansion will then have '+' sign.
- However, if $x < x_j$, $h = x - x_j < 0$. Then, the terms with odd derivatives (first, third, fifth etc.) will have '−' sign.

2 Fourier Series

Different textbooks will use different formulae to define coefficients in the Fourier Series. You need to be aware of this and use consistent definitions of the Euler Formulae for evaluating Fourier Series coefficients.

2.1 Periodic function of period 2π

If $f(x)$ is a periodic function of period 2π; then it can be represented as

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx)\right); \tag{2}$$

where the coefficients a_0, a_n and b_n are given by the following formulae (called as Euler formulae):

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)dx; \tag{3}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\cos(nx)dx; \tag{4}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\sin(nx)dx; \tag{5}$$

2.2 Periodic function of arbitrary period T

If $f(t)$ is a periodic function of period T; then it can be represented as

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left[\frac{2\pi n}{T}t\right] + b_n \sin\left[\frac{2\pi n}{T}t\right]\right); \tag{6}$$

where the coefficients a_0, a_n and b_n are given by the following formulae (called as Euler formulae):

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} f(t)dt; \tag{7}$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t)\cos\left[\frac{2\pi n}{T}t\right] dt; \tag{8}$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t)\sin\left[\frac{2\pi n}{T}t\right] dt; \tag{9}$$