12.2

Find velocity components at "A"

– Use Blasius Soln: $\eta = y\left(\frac{U_\infty}{\nu x}\right)^{1/2}$

$$v = -\frac{1}{2}(Re_x)^{1/2} U_\infty (f - \eta f')$$

pt. A: $y = .01'$ $\quad u = U_\infty f'$

$x = 1'$

$$\eta = (.01)\left(\frac{32}{1.57\times10^{-4}\times1'}\right)^{1/2} = 4.515 \quad \left(\text{near edge of B.L.}\right.$$

from Fig 12.7 : $\frac{v}{U_\infty}(Re)^{1/2} \simeq .78$

so $\underline{v = 5.5\times10^{-2}\ ft/s}$

also $\frac{u}{U_\infty} \sim .98$ or $\underline{u = 31.36\ ft/s}$

12.4

Find angle of inclination of the boundary layer at the end of the plate

$\underline{\delta = 5x\, Re_x^{-1/2}}$ so $\frac{d\delta}{dx} = \frac{5}{2}\left(\frac{\nu}{U_\infty}\right)^{1/2} x^{-1/2}$

at $x = 1'$: $\frac{d\delta}{dx} = 5.5\times10^{-3} = \tan\theta$

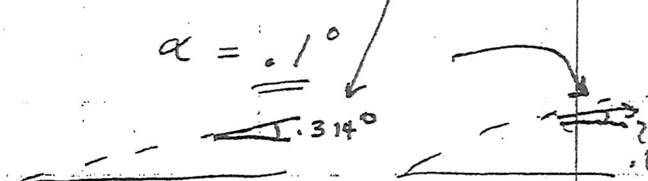
$\theta \simeq \underline{\underline{.314^\circ}}$

also: $\frac{v}{U_\infty}(Re)^{1/2} \simeq .8$ so $v = 5.7\times10^{-2}\ ft/s$

streamline makes an angle of:

$$\tan\alpha = \frac{v}{u_\infty} = \frac{5.7\times10^{-2}}{32}$$

$\alpha = \underline{\underline{.1^\circ}}$



12.7

$$\frac{u}{U_\infty} = a + b\eta - \eta^2 \qquad \eta = y/\delta$$

find a & b : use B.C. ① $\frac{u}{U_\infty} = 0$ at $\eta = 0$

② $\frac{u}{U_\infty} = 1$ at $\eta = 1$

① $0 = a + b(0) - (0)^2$

or $\boxed{a = 0}$

② $1 = 0 + b(1) - (1)^2$

or $\boxed{b = 2}$

$$\boxed{\frac{u}{U_\infty} = 2\eta - \eta^2}$$

end

↓

extra $\left.\frac{\partial u}{\partial y}\right|_{\eta=0} = \frac{U_\infty}{\delta}\left.\frac{\partial(u/U_\infty)}{\partial \eta}\right|_{\eta=0} = 2\frac{U_\infty}{\delta}$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{\eta=0} = \frac{2\mu}{\delta} U_\infty$$

12.8 Given $\frac{u}{U_\infty} = a + b\eta - \eta^2$ $\eta = \frac{y}{\delta}$

$\boxed{\text{show } \frac{\delta_2}{\delta} = \frac{2}{15}}$ and $\boxed{\text{find } \delta(x)}$

- need to find a & b:

use B.C. $u(0) = 0$ so $a = 0$

$u(\delta) = 1$ so $1 = b - 1$; $b = 2$

$$\frac{u}{U_\infty} = 2\eta - \eta^2$$

$$\frac{\delta_2}{\delta} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\eta$$

$$= \int_0^1 (2\eta - \eta^2)(1 - 2\eta + \eta^2)\, d\eta$$

$$= \int_0^1 (2\eta - \eta^2 - 4\eta^2 + 2\eta^3 + 2\eta^3 - \eta^4)\, d\eta$$

$$= \int_0^1 (2\eta - 5\eta^2 + 4\eta^3 - \eta^4) dy = \left(\eta^2 - \frac{5\eta^3}{3} + \eta^4 - \frac{\eta^5}{5}\right)\Big|_0^1$$

$$= 1 - \frac{5}{3} + 1 - \frac{1}{5}$$

$$\frac{\delta_2}{\delta} = 2 - \frac{28}{15} = \frac{2}{15}$$

Find $\delta(x)$:

$$\tau_w = \rho U^2 \frac{d\delta_2}{dx} = \rho U^2 \frac{2}{15}\frac{d\delta}{dx} = \mu \frac{du}{dy}\Big|_{y=0}$$

from vel. profile: $\frac{\partial u}{\partial y}\Big|_{y=0} = U_\infty\left(\frac{2}{\delta} - \frac{2y}{\delta^2}\right)\Big|_{y=0} = \frac{2U_\infty}{\delta}$

$$\rho U^2 \frac{2}{15}\frac{d\delta}{dx} = 2\mu U/\delta \rightarrow \delta\, d\delta = \frac{2\nu(15)}{U\,2} = \frac{15\nu}{U}dx$$

$$\frac{\delta^2}{2} = \frac{15\nu x}{U} \rightarrow \boxed{\frac{\delta}{x} = 5.48/Re_x^{1/2}}$$

12.9 find C_f : use a 2nd order vel. profile.

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu \frac{2U_\infty}{\delta} = \frac{2\mu U}{5.48\sqrt{\frac{\nu x}{U}}}$$

$$C_f = \frac{2\mu U}{5.48\sqrt{\frac{\nu x}{U}}} \frac{2}{\rho U^2} = \underline{.73 / Re_x^{1/2}}$$

$$\text{and } \cancel{C_{fD}} = \cancel{\frac{1.46}{Re_{xL}^{1/2}}}$$

Define $C_D = \frac{1}{\frac{1}{2}\rho U^2 \cdot L \cdot \text{width}} \int \tau_w \, dx \cdot \text{width}$

$$= \frac{2}{\rho U^2 \ell \cdot \cancel{w}} \int \frac{2\mu U}{5.48\sqrt{\frac{\nu x}{U}}} \, dx \cdot \cancel{\text{width}}$$

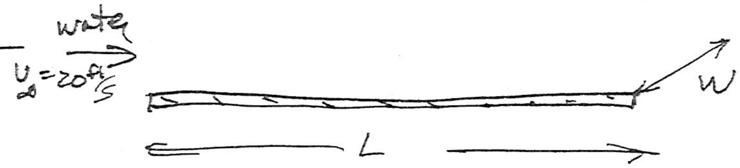
$$= \frac{.730\,\nu}{\rho \ell U^{1/2} \nu^{1/2}} \int_0^L \frac{dx}{x}$$

$$\int_0^L x^{-1/2} dx = 2x^{1/2}\Big|_0^L$$

$$= .730 \frac{\nu^{1/2} 2\ell^{1/2}}{U^{1/2} \ell}$$

$$C_D \doteq 1.46 \left(\frac{1}{Re_L}\right)^{1/2} = 2C_f(x=L)$$

12.13



$$\boxed{\begin{array}{l} \text{Find Drag.} \\ L = 2',\ 4' \\ W = 4',\ 2' \\ \quad (a)\quad (b) \end{array}}$$

We will use the Blasius soln.

$$C_f = \frac{1.328}{Re_L^{1/2}} \quad \text{(needs to be laminar)} \quad Re_L = \frac{20\,(2 \text{ or } 4)}{\nu_w = 1.2\times10^{-5}\ \frac{ft^2}{s}}$$

$\therefore$ not laminar? $= 3.3\times10^{+6} \rightarrow 6.6\times10^{+6}$

(a) $$Re_L = \frac{(20)(2)}{\cancel{1.12\times10^{-6}}\ 1.21\times10^{-5}} = \cancel{3.5,714,286}\ 3.3\times10^6 = \cancel{3.57\times10^7}\ !$$ not laminar but assume so

so $$C_f = \cancel{2.22}\ 7.30\times10^{-4} = \frac{F_d}{\frac{1}{2}\rho V^2 (L\times W)} \qquad \rho_w = 1.94\ \frac{slugs}{ft^3}$$

$$F_d = \cancel{355}\ \underline{2.27\ lb}$$

(b) $$Re_L = \frac{20(4)}{1.21\times10^{-5}} = 6.6\times10^6$$

$$C_f = 5.17\times10^{-4}; \quad F_d = \underline{\underline{1.6\ lb}}$$

$\boxed{\text{why is there a diff.??}}$ % reduction: $\sim 30\%$!!

when positioned with L > W the end of the plate has a lower shear stress so the total drag is reduced.

13.5 repeat 12.13 for turbulent boundary layer

(a) $Re_L = 3.3 \times 10^6$; $L = 2$

$$C_f = \frac{.072}{Re_L^{1/5}} = .0036$$

$$F_D = \frac{1}{2}\rho V^2 (LW) C_f = \frac{1}{2}(1.94)(20)^2(8)(.0036)$$
$$= 11.1 \text{ lb.}$$

(b) $Re = 6.6 \times 10^6$; $L = 4$

$$C_f = \frac{.072}{Re_L^{1/5}} = .0031$$

$$F_D = \frac{1}{2}\rho V^2 (LW) C_f = 9.67 \text{ lb.}$$

% reduction using $L=4$: 13%

Compare with laminar results 12.13 (2.27 & 1.61 lb).

(why does turb. case increase drag?)

13.6 Again back to 12.13 but account for laminar region at beginning of the plate (most realistic!).

$$C_f = C_{f_{tt}} - \frac{x_{crit}}{L}\left(C_{f_{tx}} - C_{f_{lx}}\right)$$

or $$C_f = C_{f_{tt}} - \frac{A}{Re_L}$$

$$A = Re_{crit}\left(C_{f_{tx}} - C_{f_{lx}}\right)$$

or $$A = 1.328\, Re_{crit}^{1/2}\left[0.056\, Re_{crit}^{.3} - 1\right]$$

where $Re_{crit} = 5\times10^5$ so $A \approx 1750$

$$\Rightarrow \quad C_f = C_{f_{tt}} - \frac{1750}{Re_L}$$

& $C_{f_{tt}}$ from 13.5

(a) $$C_f = .0036 - \frac{1750}{3.3\times10^6} = .0031 \quad (\sim 15\% \text{ correction}).$$

$$F_D = \tfrac{1}{2}\rho V^2 (LW)\, C_f = 9.56 \text{ lb.}$$

(b) $$C_f = .0031 - \frac{1750}{6.6\times10^6} = .0028 \quad (8.6\% \text{ correction})$$

$$F_D = 8.73 \text{ lb.}$$