Practice Prob. #4 Soln (Chapt. 4 rand)

4.5) Flow: Source at (1, 0), Source at (−1, 0) } same strength.

at $x = 0$ find eqn. for velocity. —

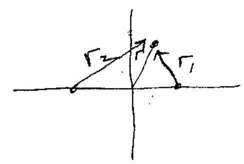
$\psi_{source} = \mu_s \theta$ if displaced +1 : $\psi_1 = \mu_s \tan^{-1}\frac{y}{x-1}$

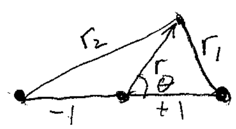
" " −1 : $\psi_2 = \mu_s \tan^{-1}\frac{y}{x+1}$

$$\psi = \psi_1 + \psi_2 = \mu_s\left[\tan^{-1}\left(\frac{y}{x-1}\right) + \tan^{-1}\left(\frac{y}{x+1}\right)\right]$$

or $$\phi = \phi_1 + \phi_2 = -\mu_s\left[\ln\left[\left((x-1)^2+y^2\right)^{1/2}\right] + \ln\left[\left((x+1)^2+y^2\right)^{1/2}\right]\right]$$

or : $\phi = -\mu_s \ln(r_1 r_2)$





also $r = \left(\frac{r_1^2}{2} + \frac{r_2^2}{2} - 1\right)^{1/2}$ (see law of cosines)

at $x=0$ $r_1 = r_2$ so $r = (r_1^2 - 1)^{1/2}$ or $r_1^2 = r^2 + 1$

so : $\boxed{\phi = -\mu_s \ln(r^2+1)}$

and $\boxed{v_r = -\frac{\partial\phi}{\partial r} = \mu_s\left(\frac{1}{r^2+1}\right)(2r) = \mu_s \frac{2r}{r^2+1}}$

along x axis this is also $\boxed{v_y = \mu_s \frac{2y}{y^2+1}}$

also $\boxed{v_\theta = 0}$

Apply Bern. Eqn (irrot. flow, steady)

at $r \to \infty$ $v \to 0$, $P = P_\infty$ so $P_{(x=0)} = P_\infty + \frac{\rho v_r^2}{2} + \rho g r$

$$P = P_\infty + \frac{\rho}{2}\left(\mu_s^2 \frac{4r^2}{(r^2+1)^2}\right) + \rho g r \quad \text{where } r = y \text{ (vertical)}$$

Force/b (neglect P_∞ & $\rho g r$) $= \int_{-\infty}^{+\infty} P\,dy = 2\int_0^\infty \frac{\rho}{2}\left(\mu_s^2 \frac{4r^2}{(r^2+1)^2}\right) dr = 4\rho\mu_s^2\left(\frac{\pi}{4}\right)$

$$\boxed{F/b = \pi\rho\mu_s^2}$$

4.6) where is max. vel. on surface of 4.5;

$$\frac{\partial v_r}{\partial r} = 0 = \frac{\partial}{\partial r}\Big(\underbrace{\mu_s \frac{2r}{r^2+1}}_{=v_r \text{ from } 4.5}\Big) = \mu_s\left[\big((r^2+1)2 - 2r(2r)\big)\big/(r^2+1)^2\right] = 0$$

or $\dfrac{2(r^2+1) - 4r^2}{(r^2+1)^2} = 0$; $(r^2+1) - 2r^2 = 0$

$$-r^2+1 = 0$$

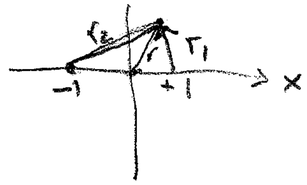
$$\underline{r = \pm 1}$$

max. vel. is at $\underline{(0,1) \ \& \ (0,-1)}$

Since the vel. is a max. then pres. should be a minimum.

Note: to get v_r in 4.5

find $\phi = \phi_1 + \phi_2 = -\mu_s\left[\ln((x-1)^2+y^2)^{1/2} + \ln((x+1)^2+y^2)^{1/2}\right]$



or $\phi = -\mu_s \ln(r_1 \cdot r_2)$

to find r_1 & r_2 in terms of r : $\left\{\begin{array}{l}\text{at } x=0 \ \ r_1 = r_2 \\ \text{so } r = (r_1^2-1)^{1/2}\end{array}\right.$

we can write $\phi = -\mu_s \ln(r^2+1)$

$$v_r = -\frac{\partial \phi}{\partial r} = \mu_s\left(\frac{2r}{r^2+1}\right)$$

4.12) — Hurricane ≡ sink + source.

a)

$\psi = \psi_1 + \psi_2$

$\psi_1 = -\mu_s \theta \rightarrow$ sink at origin

$\psi_2 = -\mu_v \ln(r) \rightarrow$ counterclockwise vortex at origin

$\boxed{\psi = -\mu_s \theta - \mu_v \ln(r)}$

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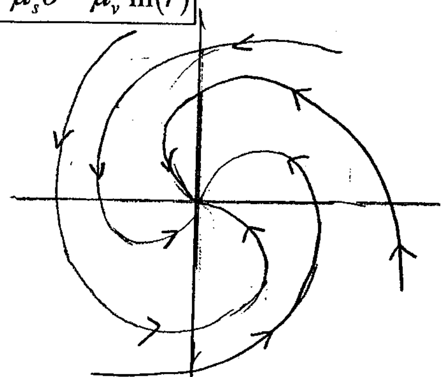
b)



c)

$r = 20m \qquad v_\theta = 45 \, m/s \qquad v_r = -15 \, m/s$

$$v_\theta = -\frac{\partial \psi}{\partial r} = -\frac{\partial}{\partial r}\left(-\mu_s \theta - \mu_v \ln(r)\right) = \frac{\mu_v}{r}$$

$$\mu_v = r v_\theta = (20m)(45 \, m/s)$$

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{1}{r}\frac{\partial}{\partial \theta}\left(-\mu_s \theta - \mu_v \ln(r)\right) = -\frac{\mu_s}{r}$$

$$\mu_s = -r v_r = -(20m)(-15 \, m/s)$$

$\boxed{\mu_v = 900 \, m^2/s}$ $\qquad$ $\boxed{\mu_s = 300 \, m^2/s}$

d) (ΔP between $r = 20m$ & $r = 40m$):

from Bernoulli's (irrot., steady, incompressible).

$$P_r = P_\infty - \rho g h - \frac{1}{2}\rho q^2 \quad (\text{at } \infty: q = 0 \quad h = 0)$$

assume $h = 0$

$$q^2 = v_r^2 + v_\theta^2 = \left(\frac{\mu_v}{r}\right)^2 + \left(\frac{\mu_s}{r}\right)^2 = \frac{\mu_v^2 + \mu_s^2}{r^2}$$

$$P_{20} = P_\infty - \frac{1}{2}\rho \frac{(900 \, m^2/s)^2 + (300 \, m^2/s)^2}{(20m)^2} = P_\infty - (1125 \, m^2/s^2)\rho$$

$$P_{40} = P_\infty - \frac{1}{2}\rho \frac{(900 \, m^2/s)^2 + (300 \, m^2/s)^2}{(40m)^2} = P_\infty - (281.25 \, m^2/s^2)\rho$$

$$\Delta P = P_{40} - P_{30}$$
$$= P_\infty - (281.25 \, m^2/s^2)\cdot\rho - P_\infty + (1125 \, m^2/s^2)\cdot\rho$$
$$= 844 \, (m^2/s^2)\cdot\rho$$

Assume: $\rho_{air} = 1.2 \, kg/m^3$

$\Delta P = 844 \, m^2/s^2 \cdot 1.2 \, kg/m^3$

$= 1012.8 \, N/m^2$

$\approx 1 \, kPa$

4.14 $U = 10$ m/s
cylinder radius $= a = 105$ mm
stagnation pts. at 30° up from ℄.
find: Γ :

$$q = -2U\sin\theta + \frac{\Gamma}{2\pi a} = 0 \quad \text{at stagnation pt.}$$

$$\text{so} \quad \sin\theta = \frac{\Gamma}{4\pi U a}$$

$$\text{if } \theta = 30° \qquad \Gamma = 4\pi U a \sin 30°$$
$$= \underline{6.597 \ m^2/s}$$

Using air, cylinder 0.5 m long $F_L = ?$

$$F/\text{length} = \rho U \Gamma + \underbrace{W_f/\text{length}}_{\text{included buoy. force.}}$$

$$= 1.23 \tfrac{kg}{m^3} \times 10 \times 6.597 + W_f/\text{length}$$

$\ell = .5$ so:
$$F = \underline{\underline{-40.57 \ N}} + (W_f/\ell)$$

W_f/ℓ = weight of fluid displaced.
$$= \rho(\pi a^2 \ell) g$$
$= .209$ N (small part compared to circulation).