1. What is a Differential Equation? How is its order defined?

2. How do we classify a Differential Equation?

3. Consider motion of a fluid flow through a small gap of height H between two parallel plates. The bottom plate is fixed and the top plate is moving with a constant uniform velocity U_0. If the flow is assumed steady, laminar and fully developed, the governing equation for fluid velocity is given as

$$\frac{d^2u}{dy^2} = \frac{(P_2 - P_1)}{\mu L} \tag{1}$$

where P_1, P_2, μ and L are constants. Assume that plates have unit width.

Classify the differential equation (type, linear/non-linear, homogeneous/inhomogeneous.).

4. Identify the order of the following differential equation and also state whether it is a linear or non-linear equation.

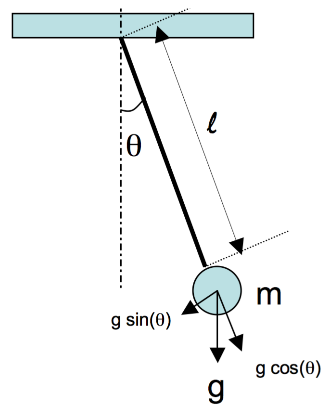
$$t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y - 5 = 0. \tag{2}$$

5. For the following differential equation:

$$f''' = ff'' \tag{3}$$

Find the order of the differential equation. Indicate if it is homogeneous or non-homogeneous equation. Also indicate if it is linear or non-linear.

6. Angular motion of a pendulum is given by the following differential equation



$$\ddot{\theta} + \omega^2 \sin(\theta) = 0, \tag{4}$$

where θ is the angle the pendulum string makes with the vertical axis, $\omega = \sqrt{g/\ell}$ where g is gravitational acceleration and ℓ is the length of the string. At $t = 0$, $\theta = \pi/6$ and $\dot{\theta} = 0$.

Convert the above differential equation into a system of first-order differential equations.

7. Finite Difference Approximation. Consider a continuous real function $f(x)$, discretized on a uniform mesh of points $x_j = jh$, where $j = 0, 1, 2, ...$ Find an approximate formula for $f'(x)$ at point x_j with best possible accuracy, using two function values at adjacent points x_j and x_{j+1}. Identify the order of the error (also termed as truncation error) and its form in the formula.

8. Finite Difference Approximation. Consider a continuous real function $f(x)$, discretized on a uniform mesh of points $x_j = jh$, where $j = 0, 1, 2, ...$ Find an approximate formula for $f'(x)$ at point x_j with best possible accuracy, using two function values at adjacent points x_j and x_{j-1}. Find the complete form of the truncation error.

9. Finite Difference Approximation. Consider a continuous real function $f(x)$, discretized on a uniform mesh of points $x_j = jh$, where $j = 0, 1, 2, ...$ Find an approximate formula for $f'(x)$ at point x_j with best possible accuracy, using three function values at adjacent points x_j, x_{j+1} and x_{j-1}. Find the complete form of the truncation error.