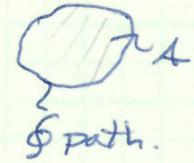
Circulatory Lift

- ## 2D approach:

- Related to momentum balance : loss of mom caused by Force

"Circulation" $\Gamma = \oint u \cdot ds = \int_A \xi_\perp dA$



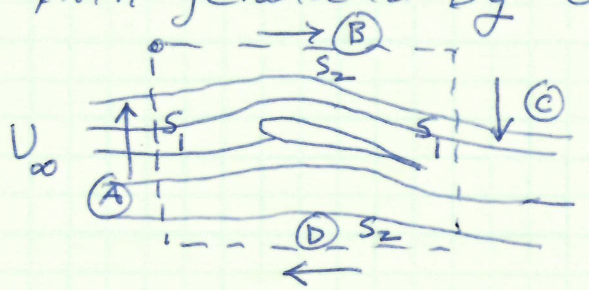
$\oint$ contains area S_A

$\xi_\perp$ is normal to A

$u \cdot ds \rightarrow$ component of velocity along path

note Γ depends on selected area.

Flow path generated by object (air foil):



- integrate around the box along $\rightarrow$.
- integrate by sections Ⓐ, Ⓑ, Ⓒ, Ⓓ

Ⓐ : $= 0$

Ⓑ : $\approx U_\infty S_2$

Ⓒ : $\approx w S_1$ (w is "downwash" caused by air foil)

Ⓓ : $\approx -U_\infty S_2$ (neg. since in $-S_2$ direction. or U_∞ in opposite direction of ds. so $u \cdot ds$ is neg. #)

- Net result: $\Gamma \approx w S_1$
- We have caused an accel. downward by the wing

circulatory Lift - cont. (2)

- Lift force : on ~~wing~~ foil (or opposite on fluid) is $\perp$ to flow (U_∞) - or in downwash direction

- Added momentum rate of fluid going down is $= (\dot{m} W)$

$$\dot{m} = (\rho A_x V) W \rightarrow \rho (sh) U_\infty W \quad \begin{cases} s = \text{span} \\ sh = A_x \end{cases}$$

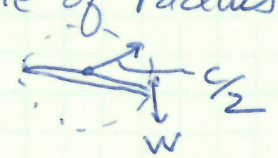
so $L = \rho s h U_\infty W$

$L = \rho U_\infty s \Gamma$ or $\boxed{L' = \rho U_\infty \Gamma}$ Lift per span.

So here Γ represents the effect of any mechanism that sets up a downwash of the flow - we can show that this is caused by vorticity along the surface!!

In terms of $C_L = \dfrac{L}{\frac{1}{2}\rho U_\infty^2 s c}$

Select area to be a circle of radius $c/2$ around the ~~a~~ foil with a "net vel." $= W$



$$\Gamma = \oint u \cdot ds = W \oint ds = W \, 2\pi \frac{c}{2} = \pi c W$$

and $L = \rho U_\infty s (\pi c W)$

plug this L into C_L expression:

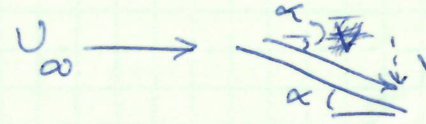
$$C_L = \frac{2\pi W}{U_\infty}$$

Alternatively we can write: $C_L = \dfrac{\rho U_\infty s \Gamma}{\frac{1}{2}\rho U_\infty^2 s c}$

$$C_L = \frac{2\Gamma}{U_\infty c}$$

Circulatory Lift - cont (3)

- Approximate foil as ~ flat plate that flow is deflected at angle α (A.A)



$\sin\alpha \cong \frac{w}{U_\infty}$

For small α: $\alpha \simeq \frac{w}{U_\infty}$

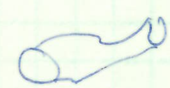
so: $C_L \cong 2\pi\alpha$ (or $2\pi \sin\alpha$).

and as before: $C_L = \frac{2\Gamma}{U_\infty c}$

: Suppose we have a finite span wing - what are end effects?

- Kelvin's theorem: "Conservation of circulation"
  → without derivation: imagine cons. of mom. (vel) per m.
  (true is no visc. forces causing vorticity to die away).

Also: some constraints on vortex tubes:
1. can't start/end in fluid.



2.

show pics. of starting airfoil & shed circulation (vorticity) that matches that around air when in steady motion (opposite rotation) (when foil stops get reverse rotation)