Equation Sheet

1. Taylor series expansion of function $f(x)$ around x_a:

$$f(x) = f(x_a) + (x - x_a)f'(x_a) + \frac{(x - x_a)^2}{2!}f''(x_a) + \frac{(x - x_a)^3}{3!}f'''(x_a) + ... \tag{5}$$

2. For $i = \sqrt{-1}$,

$$\begin{aligned} e^{i\theta} &= \cos(\theta) + i\sin(\theta) && (6)\\ e^{-i\theta} &= \cos(\theta) - i\sin(\theta) && (7)\\ \cos(\theta) &= \frac{e^{i\theta} + e^{-i\theta}}{2} && (8)\\ \sin(\theta) &= \frac{e^{i\theta} - e^{-i\theta}}{2i} && (9)\\ \sin^2(\theta) + \cos^2(\theta) &= 1 && (10)\\ & && (11) \end{aligned}$$

3. Power series,

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + ... \tag{12}$$

4. Runge Kutta 2 (RK2) for $y' = f(y, t)$ is given as

$$\begin{aligned} y^*_{n+1/2} &= y_n + \frac{\Delta t}{2}f(y_n, t_n) && (13)\\ y_{n+1} &= y_n + \Delta t f(y^*_{n+1/2}, t_{n+1/2}) && (14)\\ & && (15) \end{aligned}$$

5. Forward Euler Scheme for $y' = f(y, t)$ is given as

$$y_{n+1} = y_n + \Delta t f(y_n, t_n) \tag{16}$$

6. Leap Frog Scheme for $y' = f(y, t)$ is given as

$$y_{n+1} = y_{n-1} + 2\Delta t f(y_n, t_n) \tag{17}$$

7. Centered differencing for $y' = f(y, t)$ is given as

$$y_{n+1} = y_n + \frac{\Delta t}{2}[f(y_n, t_n) + f(y_{n+1}, t_{n+1})] \tag{18}$$

8. Backward differencing for $y' = f(y, t)$ is given as

$$y_{n+1} = y_n + \Delta t f(y_{n+1}, t_{n+1}) \tag{19}$$

9. Centered differencing for $y''(x_j)$ is given as

$$y''_j = \frac{y_{j+1} - 2y_j + y_{j-1}}{\Delta x^2} \tag{20}$$