ME 460/560 Midterm #1B (Pink)

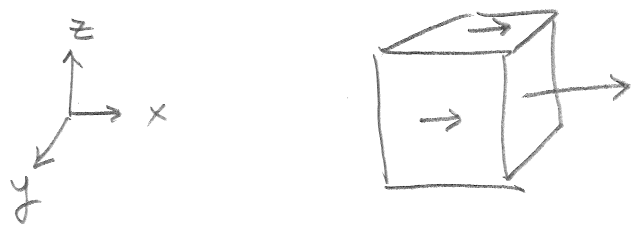
Fall 2016

NAME Soln **Are you taking this for credit (Yes/no)**__________

COURSE NUMBER__________

Answer each question with short concise statements or equations or sketches as requested. Be sure to provide explanations or discussions if requested and show your work for partial credit.

1. Draw a cubic fluid element. Also include a Cartesian coordinate system for you cube. (5) Identify all surface forces on the cube that occur in the "x" direction based on your coordinate system.



3 forces in x (+3)

on x, y, z faces (+3)

2. (5) Which terms in the Navier-Stokes equations are nonlinear, explain why and what they represent physically?

(+2) $u_j \frac{\partial u_i}{\partial x_j}$ or in x $\left\{ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right.$

(+1) • convective accel terms

(+2) • these have product of vel with function of vel

3. (5) If a flow is irrotational which terms in the Navier-Stokes equations can be modified, could this flow be classified in some other manner?

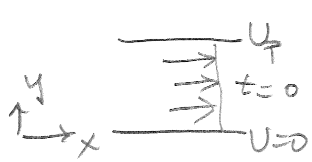
(+2) • convective accel. term

(+2) • viscous term

(+1) • irrotational obviously + inviscid.

4. Two horizontal, wide, parallel plates are both moving at velocity U_p, the gap between the plates has thickness h and is filled with oil with viscosity v. At time t=0 the bottom plate suddenly stops.

(8) a. Start from the full Navier-Stokes equation in the direction of motion of the top plate and reduce the equation by eliminating all zero terms. Be sure to state what the physical condition is for the term to be zero. Write out the final form of the equation along with the needed boundary and initial conditions.



(+3) $\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$ B.C. (+2) $y=h: u=U_p$ $t=0$ $u=U_p$

$y=0: u=0$

• fully dev. so no conv. accel & only 1 visc. term

(+3) • no press. gradient driving the flow

• horizontal so no body force

b. Assume that the similarity solution for very short times for the flow next to the bottom plate is given below where t=0 is when the bottom plate stops: ,

$$f''+2\eta f'=0 \qquad \text{where} \qquad \eta = \frac{y}{2\sqrt{\nu t}} \qquad f = u/U_w$$

u is the oil velocity and y is measured from the bottom plate into the oil,
(prime is the η derivative) and y is measured from the bottom plate into the oil.

(10) i. Find the mathematical solution for f in terms of any needed integration constants.

(+2) let $f' = g$ $\qquad \frac{dg}{d\eta} + 2\eta g = 0$

$$\frac{dg}{g} = -2\eta\, d\eta$$

(+3) $\ln g = -\eta^2 + C_1$

(+2) $g = f' = C_1 e^{-\eta^2}$

$$\int df = \int C_1 e^{-\eta^2} d\eta$$

(+3) $f = C_2 + \int C_1 e^{-\eta^2} d\eta$

C_1 & C_2 are integration constants

(8) ii. Determine the boundary conditions in terms of f and η – show your work. Also, show how to obtain the final solution for f.

(+2) $y=0$: $\eta = 0$ so $f=0$

(+2) $t=0$. $\eta \to \infty$ so $f=1$ (also $\frac{du}{dy} = 0$)

evaluate C_1 & C_2

(+2) $\eta = 0$ $\quad f=0 \Rightarrow C_2 = 0$

(+2) $\eta = \infty$ $\quad f = 1 = \int C_1 e^{-\infty^2} d\eta$ or $C_1 = \frac{1}{\int_0^\infty e^{-\infty} d\eta} = \overbrace{\frac{1}{\left[\mathrm{erf}(\infty)\frac{\sqrt{\pi}}{2}\right]}}^{\text{not needed}}$

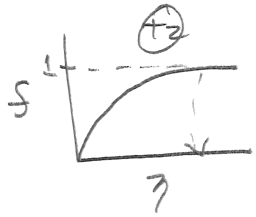
note $\mathrm{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\eta^2} d\eta$

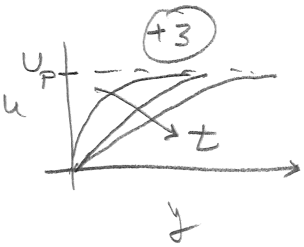
(not needed for full points).

or

$$C_1 = \frac{2}{\sqrt{\pi}\underbrace{\int e^{-\infty} d\eta}_{=\frac{\sqrt{\pi}}{2}}}$$

(8) iii. Sketch the expected graphical form of f vs η in the region near the bottom plate for short times after the bottom plate stops, also on a separate plot show the expected velocity profile, $u(y)$, for three different times, t. Based on this show how to determine the time, T_t, it takes for the top moving plate to start to feel a frictional force from the bottom plate stopping.





(+3) T_t time for viscous effects to reach $y = h$

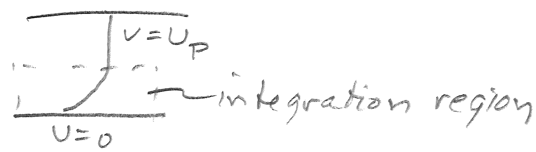
- find $\eta = $ "∞" which is when $f \cong 1$
- using this $\eta_\infty = \dfrac{h}{2\sqrt{\nu T_t}}$ solve for T_t for given value of η_∞.

(6) iv. Show how to determine the circulation per length of plate, Γ/l, associated with the flow affected by the stopping of the bottom plate. Explain if this is a function of time or not.

(+?) $\Gamma = \oint u\,ds$

$= U_p l$

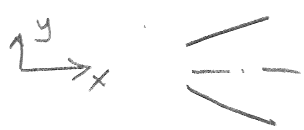
(+3) $\dfrac{\Gamma}{l} = U_p$



(+2) • not a function of time, need to take integration up to U_p

5. Consider the strain rate tensor in a fluid flow situation such as along the centerline of a diverging 2D (x,y) diffuser (where the flow is expanding in the y direction).

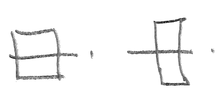
(8) a. Write out the components of the linear deformation of a fluid element.



$\dfrac{\partial u}{\partial x}$ & $\dfrac{\partial v}{\partial y}$

(+4) (+4)

(6) b. Sketch a square fluid element entering the diffuser upstream at the centerline and show how it changes going further downstream.



contracts in x (+3)

expands in y (+3)

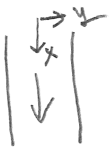
(6) c. For this same flow along the centerline is there vorticity – show mathematically and explain physically your answer.

(+3) at centerline $\dfrac{\partial u}{\partial y} = 0$ & $\dfrac{\partial v}{\partial x} = 0$ so $\zeta = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0$

(+3) no tendency for "rotation" of fluid element

6. There is steady, incompressible flow between two parallel flat plates, oriented vertically. The flow is driven by gravity only. Take the x axis as along the flow direction.

(8) a. Reduce the Navier-Stokes equations in the flow direction for these conditions, indicate why.



+5 $0 = \rho g + \mu \frac{\partial^2 u}{\partial y^2}$

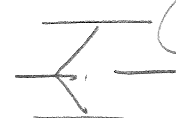
- if fully developed no conv. accel
+3 - no press. gradient driving flow
- viscous term left with $\frac{\partial^2 u}{\partial y^2}$ since fully developed & 2D

(8) b. Explain in words the expected shape of the velocity profile.

parabolic since 2nd derivative = constant
+4 +4

(6) c. Sketch the shear stress distribution across the flow.

$\tau \sim \frac{\partial u}{\partial y}$ = linear +3



+3

7. The use of suction at a wall is often used to control the boundary layer from separating (5) for flow over an object, For a constant suction velocity explain how the Navier Stokes equations become linear.

$- v \frac{\partial u}{\partial y}$ becomes $v_{suction} \frac{\partial u}{\partial y}$ & since $v_{suction}$ = constant (+2)
then this is linear
+3 assuming $u \frac{\partial u}{\partial x} = 0$