ME 4/560 Eqn. Sheet

Continuity: $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0$; Mat'l Der. $\frac{D}{Dt} = \frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x_j}$

Vorticity: $\underset{\sim}{\zeta} = \nabla \times \underset{\sim}{q} = \varepsilon_{ijk} \frac{\partial u_k}{\partial x_j}$

2D Cartesian $\zeta_z = \left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \right)$

2D Cylindrical $\zeta_z = \frac{1}{r}\left[\frac{\partial (r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta} \right]$

Circulation: $\Gamma = \int_A \underset{\sim}{\zeta} \cdot \hat{n}\, dA = \oint \underset{\sim}{q} \cdot dl$

Stream function (2D): Cartesian: $u = \frac{\partial \psi}{\partial y}$ $v = -\frac{\partial \psi}{\partial x}$

cylindrical: $u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$ $v_\theta = -\frac{\partial \psi}{\partial r}$

Velocity Potential (2D): cartesian: $u = -\frac{\partial \phi}{\partial x}$ $v = -\frac{\partial \phi}{\partial y}$

cylindrical: $u_r = -\frac{\partial \phi}{\partial r}$ $v_\theta = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}$

Bernoulli Eqn. $\int_l \frac{\partial \underset{\sim}{q}}{\partial t} \cdot dl + \Delta_l \frac{q^2}{2} + \Delta_l gh + \int_l \frac{dP}{\rho} - \pi = f(t)$

where $\nabla \pi = \underset{\sim}{q} \times \underset{\sim}{\zeta}$

Basic Flows:

uniform $\psi = Uy = Ur\sin\theta$ $\phi = -Ux$

source/sink $\psi = \pm \mu_s \theta$ $\phi = \mp \mu_s \ln r$

$\mu_s = Q'/2\pi$ $Q' =$ flow rate per length

vortex $\psi = -\mu_v \ln r$ $\phi = \mu_v \theta$ $\mu_v = \Gamma/2\pi$

Doublet $\psi = \mu_d \frac{\sin\theta}{r}$ $\phi = \mu_d \frac{\cos\theta}{r}$

Flow over cylinder : uniform + doublet : $\mu_d = Ua^2$

$a =$ cylinder radius

Lift per length $F' = L' = -\rho U \Gamma$

Euler's Eqn: $\rho \frac{Du_i}{Dt} = -\frac{\partial P}{\partial x_i} - \rho g \frac{dh}{dx_i}$

Irrotational Flow: $\nabla^2 \psi = 0$ $\nabla^2 \phi = 0$

Tensor operators: $\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

$$\varepsilon_{ijk} = \begin{cases} 1 & \text{for } (1,2,3)(2,3,1)(3,1,2) \\ -1 & \text{for } (1,3,2)(3,2,1)(2,1,3) \\ 0 & \text{if } i=j,\ i=k \text{ or } j=k \end{cases}$$