Assign #1 Soln. Guide – F '16

1. N-S eqn. tensor notation: (incompressible, μ = const)

(+5) $$\rho\left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}\right) = +\rho g_i - \frac{\partial P}{\partial x_i} + \mu\left(\frac{\partial^2 u_i}{\partial x_j \partial x_j}\right)$$

- scales: vel. = U

press. = $\frac{\mu U}{L}$

L/U = time, t

replace $u_i = U u_i^*$

$P = \frac{\mu U}{L} P^*$

$t = \frac{L}{U} t^*$

$x_i = L x_i^*$

insert into N-S eqn. and rearrange:

(+10) result: $$\frac{\partial u_i^*}{\partial t^*} + u_j^* \frac{\partial u_i^*}{\partial x_j^*} = \frac{1}{F_r} + \frac{1}{Re}\left(-\frac{\partial P^*}{\partial x_i^*} + \frac{\partial^2 u_i^*}{\partial x_j^* \partial x_j^*}\right)$$

There are two parameters: $Re = \rho \frac{UL}{\mu} = \frac{\text{inertia}}{\text{visc. force}}$

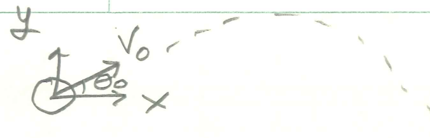
(+5) $F_r = \frac{U^2}{gL} = \frac{\text{inertia}}{\text{body force}}$

2. If you mult. by Re then let $Re \to 0$

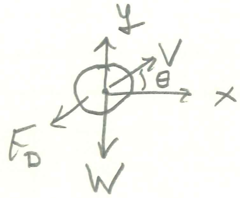
(+5) $$0 = \left(-\frac{\partial P^*}{\partial x_i^*} + \frac{\partial^2 u_i^*}{\partial x_j^* \partial x_j^*}\right)$$

(+5) eliminates inertia terms (no accel.) or body force terms; Left with balance of Press. & visc. forces.

3. Trajectory Problem



FBD



F_D opposite to V

W always in neg. y

(x. is horizontal, y vertical)

$D = .1\,m = L$

$W = 1\,kg\,(9.8\,m/s^2) = 9.8\,N$

$\rho_{fluid} = \rho = 10\,kg/m^3$

V_0 = initial vel. at 45^0

$F_D = C_D\,\rho \frac{V^2}{2} A_f \qquad A_f = \frac{\pi D^2}{4} \qquad C_D$ = const. in 3. c.

a. $\sum F = m \frac{d\bar{V}}{dt} = \bar{F}_D + \bar{W}$ (sign of F_D is shown pos. here but direction is imposed when writing component eqns.)

(+8) nondimensionalize : $m \frac{V_0^2}{D} \frac{dV^*}{dt^*} = -\left(\frac{C_D \rho A_f}{2} V_0^2\right) V^{*2} + \frac{\bar{W}}{W}$

in nondim. form : $m = W/g$

(+8) $\frac{dV^*}{dt^*} = -\left(\rho \frac{C_D A_f g D V_0^2}{2 W V_0^2}\right) V^{*2} + \frac{\bar{W}}{W}$

$$\frac{dV^*}{dt^*} = \left(\frac{1}{F_r} \cdot F_D^*\right) V^{*2} + \frac{\bar{W} D g}{W V_0^2}$$

(+4) where: $F_r = \frac{V_0^2}{gD}$ (ratio inertia/body force) $\qquad F_D^* = C_D \frac{\rho V_0^2 A_f}{2W}$ (ratio of drag coef. at initial time to weight)

b. (physical meaning) (+8) $\boxed{P_1 = \frac{F_D^*}{F_r}} = \frac{\text{initial drag coef}}{\text{body force}} \cdot \frac{\text{body force}}{\text{inertia}} \equiv \frac{\text{initial drag}}{\text{inertia}}$

$= \boxed{\frac{C_D \rho A_f g D}{2W}}$

Eqn. becomes: $\frac{dV^*}{dt} = -P_1 V^{*2} + \frac{1}{F_r}\frac{\bar{W}}{W}$

note if $P_1 \to 0$ we have no drag force

(+5) let $\boxed{P_2 = \frac{1}{F_r}} = \frac{\text{body force}}{\text{inertia}} = \frac{gD}{V_0^2}$

C.

Component Eqns.

X: (+5) $\dfrac{dV_x^*}{dt^*} = -P_1 \, V^{*2} \cos\theta = \dfrac{d^2x^*}{dt^{*2}}$ $\quad x^* = x/D$

note: P_1 is initial drag force/inertia; here we use x comp.

$(P_1 V^{*2})_x = P_1 V^{*2} \cos\theta$; use V^* not V_x^* to find F_D, then take x component

also $W_x = 0$

• Integrate (numerically) from $t=0$: $V=V_0$ $\theta_0 = 45°$ $x=0$

(+5) (note $V_x^*(t=0) = V_0 \cos 45°$) $P_1 = .001$ & $.01$

y: (+5) $\dfrac{dV_y^*}{dt} = -P_1 \, V^{*2} \sin\theta - P_2 = \dfrac{d^2y^*}{dt^{*2}}$ $\quad y^* = y/D$

where P_1 is same as in x eqn. (scalar)

$P_2 = gD/V_0^2$ (scalar)

$V_y^* = V^* \sin\theta$

$W_y = W$

at $t=0$ $V_y^* = 1 \sin 45°$, $y=0$

(+5) Integrate for 2 values of P_1 & 2 values of P_2

interpret

- as $P_1 \uparrow$ (increased drag) expect trajectory to be less (x_{end} decreases & y_{max} decreases)

(+5)

- as $P_2 \uparrow$ (less inertia) expect trajectory to be less.

Results show greater sensitivity to P_2

d. To convert to actual variables:

$x = x^* D$

$y = y^* D$

$t = t^* \frac{D}{V_0}$

if D is same for 2 cases then trajectories look the same and all values of x^* & y^* are multiplied by D=1

for $P_1 = .01 = \frac{C_D \rho A_f g D}{2W}$

+3 $C_D = P_1 2W / \rho \frac{\pi D^2}{4} g D$ $= (.01)2(9.8)/10 \; \frac{\pi (.1)^2}{4} 9.8 (.1)$

$= 2.54$ (very high)

for $P_1 = .001$ $C_D = 0.254$. $C_D \sim P_1$, all else given as constant

+3 (This is very close to the drag coef. of a sphere at high Re).

for $V_0 : 5 \rightarrow 25$ then $P_2 = \frac{gD}{V_0^2}$ ranges: $.0392 \rightarrow .0016$.

+5 So let $P_1 = .01$ (or .001) & solve for range of P_2 to find X^*_{max} & Y^*_{max} then $X_{max} = X^*_{max}(.1)$; $Y_{max} = Y^*_{max}(.1)$

Example values

$P_1 = .01$:

- $P_2 = .04$ $X^*_{max} = 21.25$ $Y^*_{max} = 5.81$
- $P_2 = .001$ " $= 61.$ " $= 20.01$
- $P_2 = .0016$ $X^*_{max} = 136$ $Y^*_{max} = 58.0$

intepret: decreasing P_2 means greater initial inertia so

+4 expect x to increase & y to increase.

Typical results: Using nondimensional variables of x* and y* for the four set of P1 and P2 values.