1. Compact Differencing and Pade' approximations Consider a continuous real function $f(x)$, discretized on a uniform mesh of points $x_j = jh$, where $j = 0, 1, 2, ...$ Find an expression for the first dervative f'_j that uses the functional values f_j, f_{j+1}, f_{j-1} and the derivatives f'_{j-1}, f'_{j+1} and gives the best possible accuracy.

2. Consider a harmonic function $f(x) = e^{ikx}$ where k is the wavenumber (or frequency) and can take on any of the following

$$k = \frac{2\pi}{L} n, \qquad n = 0, 1, 2, ..., N/2. \tag{1}$$

Find the modified wavenumber for f'_j using central differencing.

3. Consider a harmonic function $f(x) = e^{ikx}$ where k is the wavenumber (or frequency) and can take on any of the following

$$k = \frac{2\pi}{L}n, \qquad n = 0, 1, 2, ..., N/2. \tag{2}$$

Find the modified wavenumber for f'_j for the Pade' scheme.

4. Consider a harmonic function $f(x) = e^{ikx}$ where k is the wavenumber (or frequency) and can take on any of the following

$$k = \frac{2\pi}{L}n, \qquad n = 0, 1, 2, ..., N/2. \tag{3}$$

Find the modified wavenumber for f_j'' using central differencing.