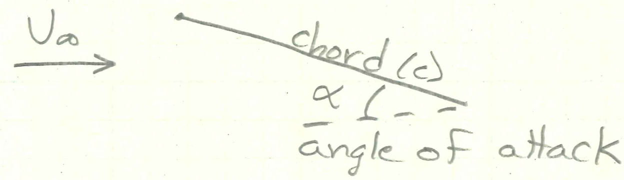
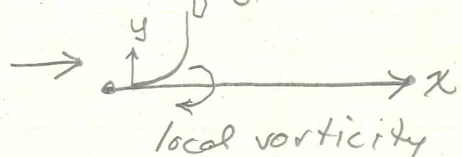
Flat plate airfoils



- viscous flow causes a thin boundary layer which generates vorticity: $\zeta \cong -\frac{\partial u}{\partial y}\Big|_{\text{near surface}}$



define γ as local measure of circulation: $\Gamma = \int_0^c \gamma(x)\,dx$

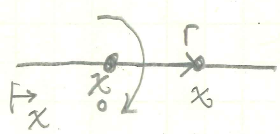
This "vortex" can not cause flow into the airfoil, so any flow down by the vortex, dv_v, must be balanced by the flow caused by the uniform approach velocity:

$$dv_v = U_\infty \sin\alpha$$

By integrating along the entire chord length:

$$\int_0^c dv_v = \int_0^c \frac{d\Gamma}{2\pi r} = \int_0^c \frac{\gamma(x)\,dx}{2\pi r}$$

where r is the distance along x from the location of the "vortex".



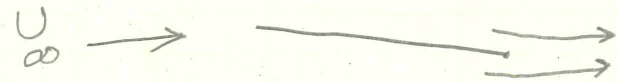
So the above integral must be evaluated for a line of vortices along x; that is x_0 varies along x.

So we have the following:

$$U_\infty \sin\alpha = \int_0^c \frac{\gamma(x)\,dx}{2\pi(x_0 - x)}$$

To solve this we need to know a boundary condition: in comes the "<u>Kutta Condition</u>":

Flow leaves the trailing edge smoothly:



There is no tendency for flow to curl up, or down, around the trailing edge.

For this to happen the pressure difference across the flow at this point $= 0$. So we can say that there is <u>no</u> tendency for the flow to "curl" or "rotate" so $\underline{\underline{\gamma(c) = 0}}$

This becomes the boundary condition.

So it turns out <u>if</u> $\gamma(x) = 2U_\infty \sin\alpha \left(\frac{c}{x} - 1\right)^{1/2}$

then the above integral equation (top of the page) is satisfied. This tells us how the local circulation must be distributed (or could be distributed).

Now we use the relationship: $L = \rho U_\infty \Gamma s$

(L = lift and s is the span)

Taking $\Gamma = \int_0^c \gamma(x)\,dx$ and using our expression for $\gamma(x)$ found above we get:

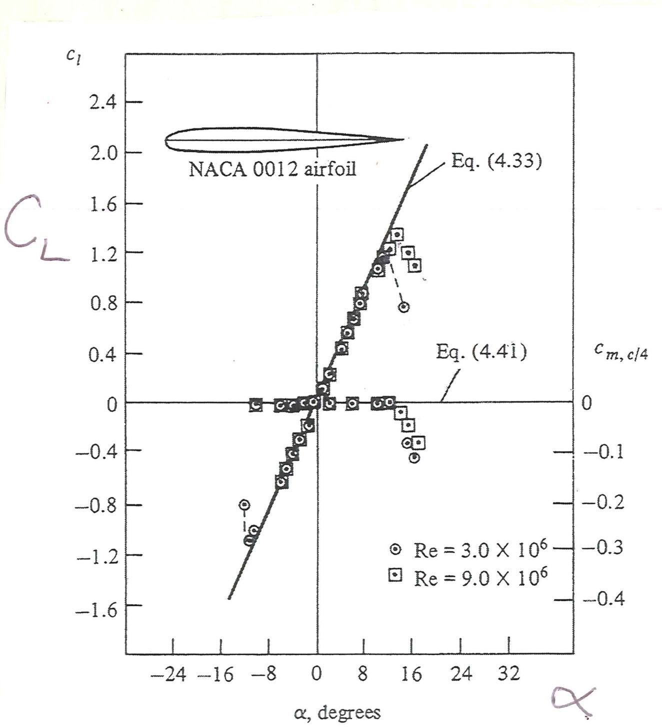
$$L = \rho U_\infty b \int_0^c \gamma(x)\,dx$$

$$C_L = \frac{L}{\frac{1}{2}U_\infty^2 cb} = \frac{\int_0^c \gamma(x)\,dx}{\frac{1}{2}U_\infty c}$$

Do the integration and get:

$$\boxed{C_L = 2\pi \sin\alpha}$$

This is for a "flat" airfoil at angle of attack α that satisfies the Kutta Condition.



The solid line passing through the data (4.33) is $C_L = 2\pi \sin\alpha$

Note C_m is the moment coef. about the quarter cord.

For finite span wing:

$$C_L = \frac{2\pi \sin\alpha}{1 + \frac{2c}{b}}$$

with "camber" "h" $\quad C_L = \frac{2\pi \sin(\alpha + \beta)}{1 + 2c/b} \quad \beta = \tan^{-1}\left(\frac{2h}{c}\right)$