Equations Exam #2

Note in the equations below:

a. repeated indices imply summation

b. δ_{ij} -Kronecker delta: =0 if $i \neq j$ or =1 if i=j

Material Derivative of u_i:

$$\frac{Du_i}{Dt} = \frac{\partial u}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}$$

Euler's eqn.:

$$\rho \frac{Du_i}{Dt} = -\frac{\partial P}{\partial x_j}\delta_{ij} - \rho \frac{\partial h}{\partial x_j}\delta_{ij} \qquad \text{or} \qquad \rho \frac{Du_i}{Dt} = -\frac{\partial P}{\partial x_j}\delta_{ij} - \rho g_i$$

Navier-Stokes Eqn. for constant viscosity and compressible flow:

$$\rho \frac{Du_i}{Dt} = -\frac{\partial P}{\partial x_j}\delta_{ij} - \gamma \frac{\partial h}{\partial x_j}\delta_{ij} + \mu \frac{\partial^2 u_i}{\partial x_j^2} + \frac{1}{3}\mu \frac{\partial}{\partial x_i}\left(\frac{\partial u_m}{\partial x_m}\right)$$

Or can be written as:

$$\rho \frac{Du_i}{Dt} = -\frac{\partial P}{\partial x_j}\delta_{ij} - \gamma \frac{\partial h}{\partial x_j}\delta_{ij} + 2\mu \frac{\partial e_{ij}}{\partial x_j} - \frac{2}{3}\mu \frac{\partial}{\partial x_i}\left(\frac{\partial u_m}{\partial x_m}\right)$$

where $e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ which is the strain rate tensor.

Total Stress Tensor (includes pressure):

$$\tau_{ij} = -\frac{2}{3}\mu \frac{\partial u_m}{\partial x_m}\delta_{ij} - P\delta_{ij} + 2\mu e_{ij}$$

Vorticity:

$$\varsigma_k = \left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right)$$

x component of Navier Stokes in terms of vorticity

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial (u^2 + v^2)}{\partial x} - v\zeta_z = -\frac{1}{\rho}\frac{\partial P}{\partial x} + g_x - \frac{\mu}{\rho}\frac{\partial \zeta_z}{\partial y}$$