1. Consider a harmonic function $f(x) = e^{ikx}$ where k is the wavenumber (or frequency) and can take on any of the following

$$k = \frac{2\pi}{L}n, \qquad n = 0, 1, 2, ..., N/2. \tag{1}$$

Find the modified wavenumber for f'_j using forward differencing.

1 Significance of the real and imaginary parts of modified wavenumbers

Consider the wave equation,

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0; \quad c > 0 \tag{2}$$

Here u is the dependent variable and t and x are independent variables. If we seek periodic solutions, with the initial condition $u(x, 0) = e^{ikx}$, then solution to this equation is of the form $u(t, x) = e^{ikx} f(t)$, where $f(t)$ is just function of t. Substituting this solution into the PDE, we get,

2 Solution of First Order Ordinary Differential Equations

1. Derive a finite difference equation for the differential equation given below using a forward differencing approach.

$$\frac{dy}{dy} = -0.5y; \quad y(0) = 1. \tag{3}$$

2. Derive a finite difference equation for the differential equation given below using a backward differencing approach.

$$\frac{dy}{dy} = -0.5y; \qquad y(0) = 1. \tag{4}$$