ME 460/560 Midterm #1A (WHITE)

Fall 2016

NAME Soln. **Are you taking this for credit (Yes/no)__________**

COURSE NUMBER__________

Answer each question with short concise statements or equations or sketches as requested. Be sure to provide explanations or discussions if requested and show your work for partial credit.

1. Which terms in the Navier-Stokes equations are linear, explain why and what they represent physically? (5)

linear → $\frac{\partial u_i}{\partial t}$; $\frac{\partial P}{\partial x_i}$; ρg_i ; $\mu \nabla^2 u$, could also include $\frac{\partial}{\partial x_i}\left(\frac{\partial u_m}{\partial x_m}\right)$ (+3)

local accel ; press force ; body force ; visc.

(+2) none have product of variable times variable or function of variable

2. If a flow is inviscid, explain if or if not the flow needs to be irrotational. (5)

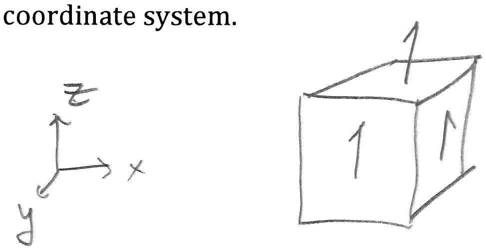
(+3) does not have to be irrotational; viscous terms come from symmetric part of $\frac{\partial u_i}{\partial x_j}$.

(+2) there is rotational part to convective accel.

3. The use of suction at a wall is often used to control the boundary layer from separating for flow over an object, For a constant suction velocity explain how the Navier Stokes equations become linear. (5)

(+2) $v\frac{\partial u}{\partial y}$ becomes $v_{suction}\frac{\partial u}{\partial y}$ & since $v_{suction}$ = constant then linear term assuming $u\frac{\partial u}{\partial x} = 0$ (+3)

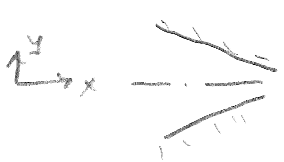
4. Draw a cubic fluid element. Also include a Cartesian coordinate system for you cube. Identify all surface forces on the cube that occur in the "z" direction based on your coordinate system. (5)



3 forces (+3) in z on x, y, z faces (+2)

5. Consider the linear deformation contributions to the strain rate tensor in a fluid flow situation such as along the centerline of a 2D (x,y) converging channel (the flow contracts in the y direction going downstream).

(8) a. Write out the components of the linear deformation of a fluid element.



$\left(\frac{\partial u}{\partial x}\right)$ & $\left(\frac{\partial v}{\partial y} = 0\right)$

(+4) (+4)

b. Sketch a square fluid element entering the channel upstream at the centerline and show
(6) how it changes going further downstream.

expands in x (+3)
contracts in y (+3)

c. For flow along the centerline is there vorticity – show mathematically and explain your
(6) answer physically.

(+3) at centerline $\frac{\partial u}{\partial y} = 0$ & $\frac{\partial v}{\partial x} = 0$ so $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$

(+3) no tendency for "rotation" of fluid element

6. There is steady, incompressible flow between two parallel flat plates, oriented horizontally. The flow is driven by pressure forces. Take the x axis along the center between the plates in the flow direction.

   a. Reduce the Navier-Stokes equations in the flow direction for these conditions, indicate
(8) why.

$0 = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$ (+5)

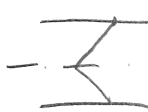
(+3)
- if fully developed no conv. accel.
- no body force
- visc. term - derivatives only $\frac{\partial^2 u}{\partial y^2}$ since fully dev. & 2D

   b. Explain in words the expected shape of the velocity profile.
(8)

parabolic (+4) since 2nd. der. = constant = $\frac{\partial P}{\partial x}$ (+4)

   c. Sketch the shear stress distribution across the flow.
(6)

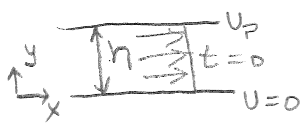
$\tau \sim \frac{\partial u}{\partial y}$ = linear (+3)



(+3) at ℄ $\tau = 0$

7. Two horizontal, wide, parallel plates are both moving at velocity U_p, the gap between the plates has thickness h and is filled with oil with viscosity ν. At time t=0 the bottom plate suddenly stops.

   a. Start from the full Navier-Stokes equation in the direction of motion of the top plate and
(8) reduce the equation by eliminating all zero terms. Be sure to state what the physical condition is for the term to be zero. Write out the final form of the equation along with the needed boundary and initial conditions.



(+3) $\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$

B.C. (+2) $y = h : u = U_p$ $\quad t = 0 \; u = U_p$
$y = 0 : u = 0$

- fully developed so no conv. accel. & only 1 visc. term.

(+3)
- no press gradient driving the flow
- horiz. no body force

b. Assume that the similarity solution for very short times for the flow next to the bottom plate is given below where t=0 is when the bottom plate stops: ,

$$f''+2\eta f'=0 \qquad \text{where} \qquad \eta=\frac{y}{2\sqrt{\nu t}} \quad f=u/U_w$$

u is the oil velocity and y is measured from the bottom plate into the oil, (prime is the η derivative) and y is measured from the bottom plate into the oil.

(10) i. Find the mathematical solution for f in terms of any needed integration constants.

(+2) let $f' = g$ $\qquad \frac{dg}{d\eta} + 2\eta g = 0$

$$\frac{dg}{g} = -2\eta\, d\eta$$

(+3) $\ln g = -\eta^2 + C_1$

(+2) $g = \frac{df}{d\eta} = C_1 e^{-\eta^2}$

$$\int df = \int C_1 e^{-\eta^2} d\eta$$

(+3) $f = C_2 + \int C_1 e^{-\eta^2} d\eta$

C_1 & C_2 are integration constants

(8) ii. Determine the boundary conditions in terms of f and η – show your work. <u>Also</u>, show how to obtain the final solution for f.

(+2) $y=0$: $\eta=0$ $\quad$ so $\quad f=0$

(+2) $t=0$: $\eta\to\infty$ $\quad$ so $\quad f=1$ (can also say $\frac{\partial u}{\partial y}=0$)

evaluate C_1 & C_2

(+2) 1. $\eta=0$ $\quad f=0 \Rightarrow \underline{\underline{C_2=0}}$

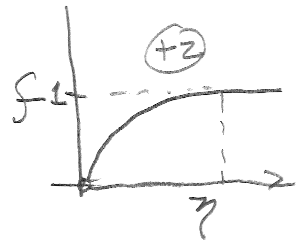
(+2) 2. $\eta\to\infty$ $\quad f=1=\int C_1 e^{-\infty^2} d\eta$ $\quad$ or $\quad C_1 = \frac{1}{\int_0^{\infty} e^{-\infty} d\eta} = \frac{1}{\overbrace{[erf(\infty)]}^{\text{not needed}}\frac{\sqrt{\pi}}{2}}$

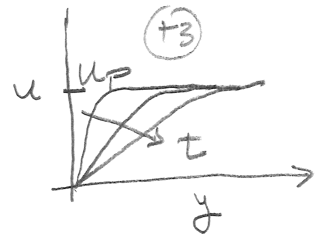
<u>note</u>: $erf(\eta) = \frac{2}{\sqrt{\pi}}\int_0^{\eta} e^{-\eta^2} d\eta$

(don't need to include this however for full pts.)

$$C_1 = \frac{2}{\sqrt{\pi}\underbrace{\int e^{-\infty} d\eta}_{=\frac{\sqrt{\pi}}{2}}}$$

(8) iii. Sketch the expected graphical form of f vs η in the region near the bottom plate for short times after the bottom plate stops, also on a separate plot show the expected velocity profile, $u(y)$, for three different times, t. Based on this show how to determine the time, T_t, it takes for the top moving plate to start to feel a frictional force from the bottom plate stopping.





+3 T_t time for viscous effects to reach $y = h$

- find $\eta = "\infty"$ which is when $f \cong 1$
- using this $\eta_\infty = \dfrac{h}{2\sqrt{\nu T_t}}$ solve for T_t for value of η_∞.

(6) iv. Show how to determine the circulation per length of plate, Γ/l, associated with the flow affected by the stopping of the bottom plate. Explain if this is a function of time or not.

+1 $\Gamma = \oint u\,ds$

$= U_p\, l$

+3 $\dfrac{\Gamma}{l} = U_p$

+2 • not a function of time, need to take integration region up to U_p.