ME560 Assignment 2

Group 8

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1.

$$\psi = Ur\sin\theta - \frac{M_d \sin\theta}{r} - M_v \ln r + C$$

$$\underset{\text{uniform flow}}{\downarrow} \qquad \underset{\text{doublet}}{\downarrow} \qquad \underset{\text{vortex}}{\downarrow}$$

$$\psi = U\sin\theta\left(r - \frac{a^2}{r}\right) - M_v \ln r + C$$

$\psi = 0$ at $r = a \longrightarrow C = M_v \ln a$

$$\boxed{\psi = U\sin\theta\,(r - a^2/r) - M_v \ln(r/a)}$$

where U is stream velocity

Θ is angle at which it rotates

r is radius to stream function

a is radius of cylinder

M_v is vortex strength $= \frac{\Gamma}{2\pi}$ Γ is circulation

$$\phi = -Ur\cos\Theta - M_d\frac{\cos\theta}{r} - M_v\Theta + C$$

$M_d = Ua^2$

$$\phi = -U\cos\theta\left(r + \frac{a^2}{r}\right) - M_v\Theta + C$$

$\phi = 0$ at stag $a = r$

$$0 = -U\cos\theta\left(a + \frac{a^2}{a}\right) - M_v\theta + C$$

$$C = U\cos\theta\,(2a) + M_v\theta$$

$$\boxed{\phi = U\cos\Theta\left(2a - r - \frac{a^2}{r}\right)}$$

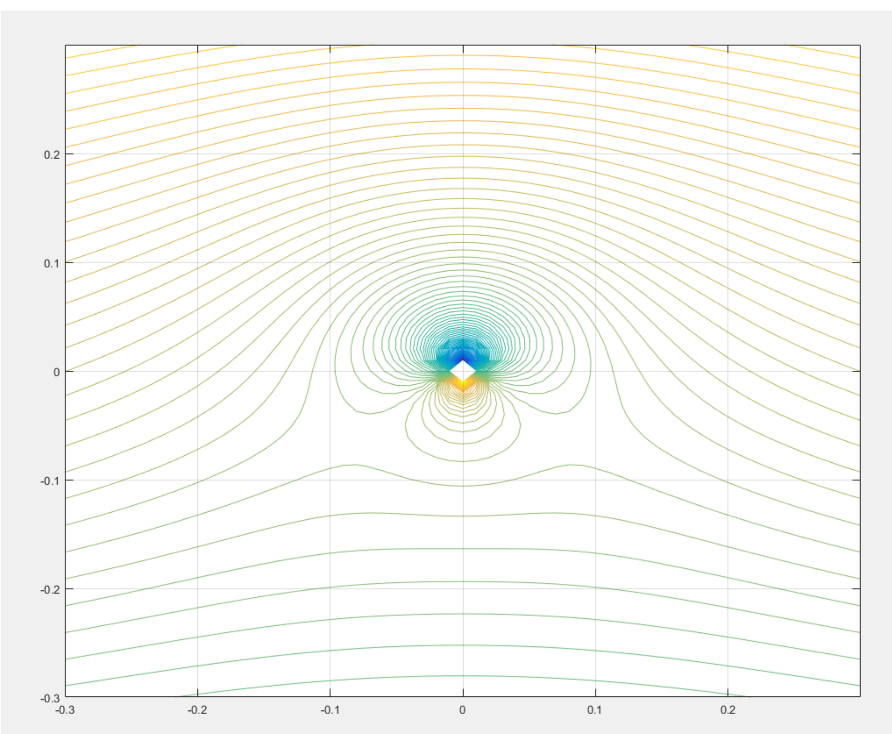
Problem 2

Code:

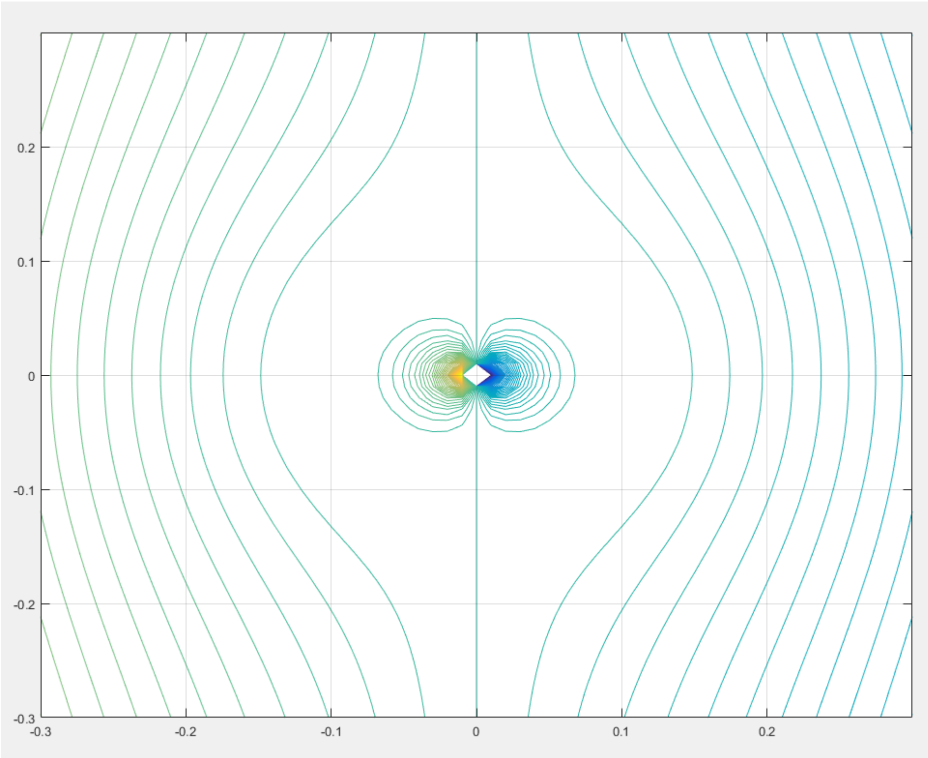
```
clear
clc
close all

U= 10;
a= 0.1;
x=[-.3:.01:.3];
[X,Y] = meshgrid(x);
theta= atan2(Y,X);
r= sqrt(X.^2+Y.^2);
w= -120;
psi= U*sin(theta).*(r-a^2./r)-a^2*w*log(r./a);

contour(X,Y,psi,101)
grid on

phi= U*cos(theta).*(2*a-r-a^2./r);

figure
contour(X,Y,phi,101)
grid on
```

Solution:

Streamfunction:



Velocity Potential:



3.

Surface: $V_r = 0$, Stag pts: $V_\theta = 0$

$$V_\theta = -\frac{\partial \psi}{\partial r} = -U \sin\theta \left(1 + a^2/r^2\right) + \frac{\mu_v}{r}$$

at $r = a$

$$0 = -U \sin\theta\,(2) + \frac{\mu_v}{a}$$

$$\sin\theta = \frac{\mu_v}{2Ua} \qquad \mu_v = \frac{\Gamma}{2\pi} = a^2\omega$$

$$\sin\theta = \frac{a^2\omega}{2U\cancel{a}}$$

$$\sin\theta = \frac{a\omega}{2U} \qquad \text{at } \theta = 90^\circ$$

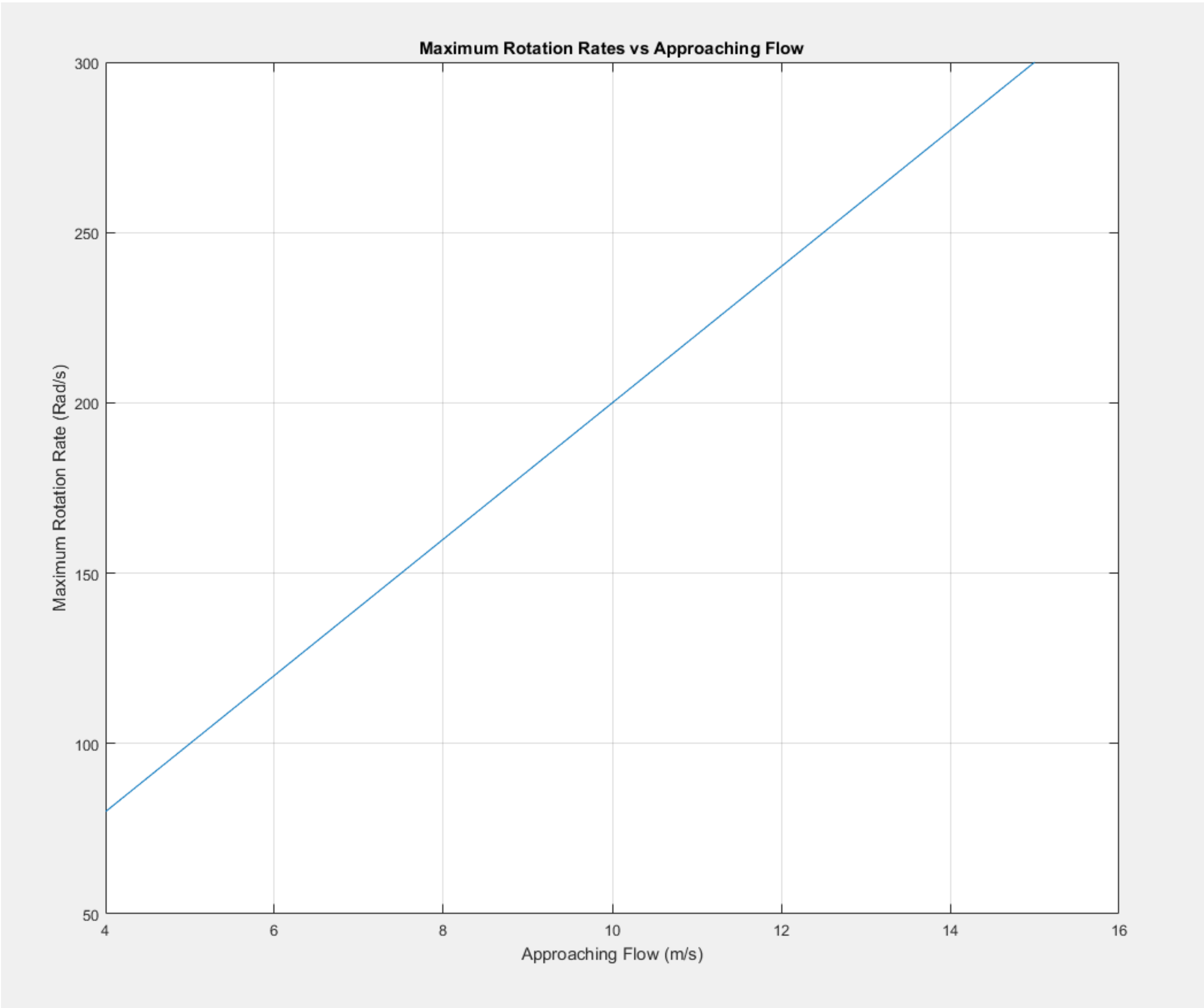
$$\boxed{\omega = \frac{2U}{a}}$$

Problem 3

Code:

```
clear
clc
close all

U= [4:.01:15];
a= 0.1;
w= 2*U/a;

plot(U,w)
```

Solution:



4. $$\int \cancel{\frac{\partial q}{\partial t}}^{0} ds + \Delta \frac{q^2}{2} + \Delta gh + \int \frac{dP}{\rho} - \cancel{\Delta \pi}^{0} = \cancel{f(t)}^{0}$$

$$\Delta \frac{q^2}{2} + \Delta \frac{P}{\rho} + \Delta gh = 0$$

$$P = P_\infty + \frac{\rho U_\infty^2}{2} - \frac{\rho q_s^2}{2} + \rho g h_s$$

$$q_s = -2U\sin\theta + \frac{\Gamma}{2\pi a} \quad , \quad U_\infty = U$$

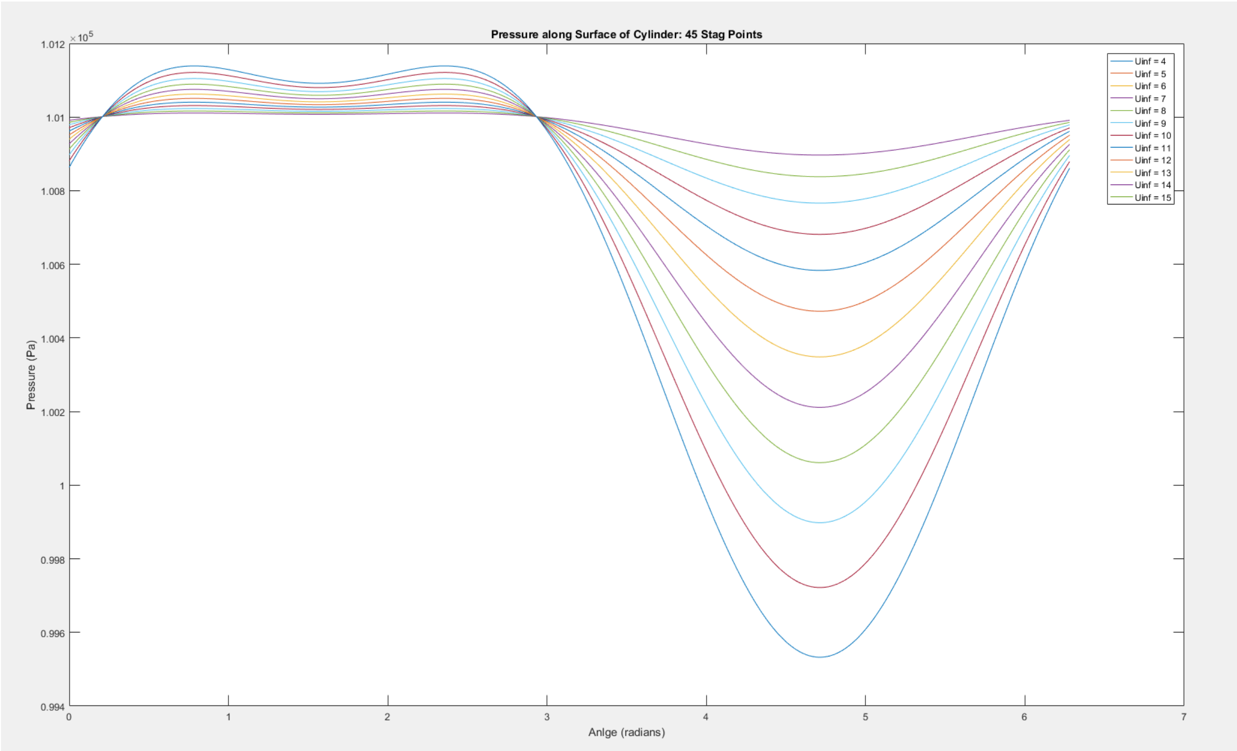
$$\boxed{P_s = P_\infty + \frac{\rho U^2}{2} - \frac{\rho(-2U\sin\theta + a\omega)^2}{2} + \rho g a}$$

Problem 5

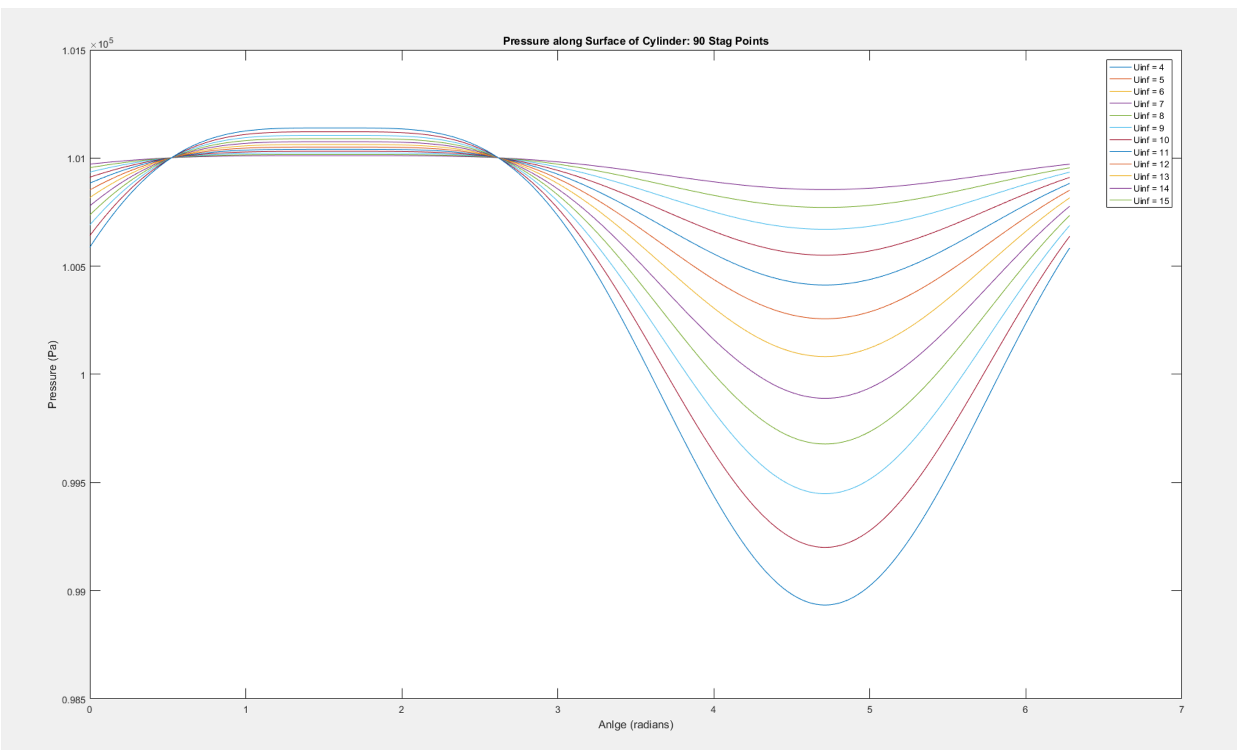
Code:

```
clear
clc
close all

a= 0.1;
U= [4:1:15];
theta= [0:.01:2*pi];
Pinf= 101000; %kPa
rho= 1.225;
g= 9.81;

for i=1:length(U)
   w45= sind(45)*2*U(i)/a;
   w90= sind(90)*2*U(i)/a;
   Ps45(i,:)= Pinf+rho*U(i)^2/2-rho*(-2*U(i)*sin(theta)+a*w45).^2/2+rho*g*a;
   Ps90(i,:)= Pinf+rho*U(i)^2/2-rho*(-2*U(i)*sin(theta)+a*w90).^2/2+rho*g*a;

   figure(1)
   plot(theta,Ps45)
   hold on
   title('Pressure along Surface of Cylinder: 45 Stag Points')
   ylabel('Pressure (kPa)')
   xlabel('Anlge (radians)')

   figure(2)
   plot(theta,Ps90)
   hold on
   title('Pressure along Surface of Cylinder: 90 Stag Points')
   ylabel('Pressure (kPa)')
   xlabel('Anlge (radians)')
end
figure(1)
legend('Uinf = 4','Uinf = 5','Uinf = 6','Uinf = 7','Uinf = 8','Uinf =
9','Uinf = 10','Uinf = 11','Uinf = 12','Uinf = 13','Uinf = 14','Uinf = 15')
figure(2)
legend('Uinf = 4','Uinf = 5','Uinf = 6','Uinf = 7','Uinf = 8','Uinf =
9','Uinf = 10','Uinf = 11','Uinf = 12','Uinf = 13','Uinf = 14','Uinf = 15')
```

Solution:



Pressure along Surface of Cylinder: 45 Stag Points
Pressure (Pa)
Anlge (radians)
×10^5
1.012
1.01
1.008
1.006
1.004
1.002
1
0.998
0.996
0.994
0
1
2
3
4
5
6
7
Uinf = 4
Uinf = 5
Uinf = 6
Uinf = 7
Uinf = 8
Uinf = 9
Uinf = 10
Uinf = 11
Uinf = 12
Uinf = 13
Uinf = 14
Uinf = 15



Pressure along Surface of Cylinder: 90 Stag Points
Pressure (Pa)
Anlge (radians)
×10^5
1.015
1.01
1.005
1
0.995
0.99
0.985
0
1
2
3
4
5
6
7
Uinf = 4
Uinf = 5
Uinf = 6
Uinf = 7
Uinf = 8
Uinf = 9
Uinf = 10
Uinf = 11
Uinf = 12
Uinf = 13
Uinf = 14
Uinf = 15

6. Numeric Solution:

$$F_y' = \int_0^{2\pi} P_s \sin\theta \, a \, d\theta$$

$$\boxed{F_y \approx P_s \sin\theta \, a \, \Delta\theta} \qquad \theta : 0 \le \theta \le 2\pi$$

$$P_s = P_\infty + \frac{\rho U^2}{2} - \rho \frac{(-2U\sin\theta + a\omega)^2}{2} + \rho g a$$

7. Based on White graph

$2 \cdot \sin(45) \xrightarrow{C_L} 2.1 \xrightarrow{C_D} 0.62$

$2 \cdot \sin(90) \xrightarrow{C_L} 3.8 \xrightarrow{C_D} 0.5$ approx.

$$\boxed{\begin{aligned} F_L'(\text{real}) &= 1/2 \rho U^2 D \cdot C_L \\ &= \rho U^2 a \cdot C_L \\ F_D'(\text{real}) &= 1/2 \rho U^2 D C_D \\ &= \rho U^2 a C_D \end{aligned}}$$

$$\boxed{\text{Ratio} = F_L/F_D}$$

Problem 6 and Problem 7

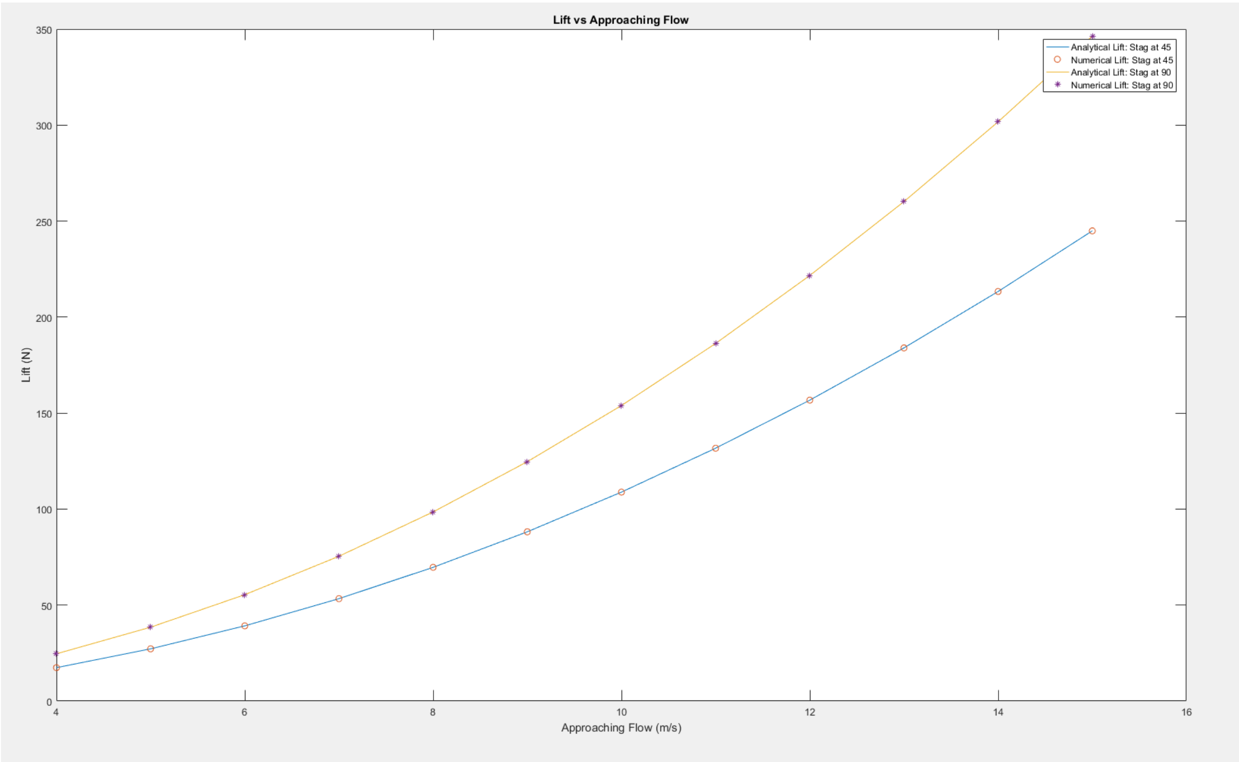
Code:

```
clear
clc
close all

a= 0.1;
U= [4:1:15];
delta= .01;
theta= [0:delta:2*pi];
Pinf= 101; %kPa
rho= 1.225;
g=9.81;

for i=1:length(U)
    w45= sind(45)*2*U(i)/a;
    w90= sind(90)*2*U(i)/a;
    Ps45= (Pinf+rho*U(i)^2/2-rho*(-
2*U(i)*sin(theta)+a*w45).^2/2+rho*g*a).*sin(theta)*delta*a;
    Ps90= (Pinf+rho*U(i)^2/2-rho*(-
2*U(i)*sin(theta)+a*w90).^2/2+rho*g*a).*sin(theta)*delta*a;
    Fy45(i)= sum(Ps45);
    Fy90(i)= sum(Ps90);
end

FortyFive= sind(45)*2*U/a;
Ninety= sind(90)*2*U/a;
LiftActual45= rho*U*2*pi*a^2.*FortyFive;
LiftActual90= rho*U*2*pi*a^2.*Ninety;

plot(U,LiftActual45)
hold on
plot(U,Fy45,'o')
hold on
plot(U,LiftActual90)
hold on
plot(U,Fy90,'*')
legend('Analytical Lift: Stag at 45','Numerical Lift: Stag at 45','Analytical
Lift: Stag at 90','Numerical Lift: Stag at 90')
title('Lift vs Approaching Flow')
ylabel('Lift (N)')
xlabel('Approaching Flow (m/s)')

% % Problem 7

Cl45= 2.1;
Cl90= 3.8;

White45= rho*U.^2*a*Cl45*2;
White90= rho*U.^2*a*Cl90*2;

figure(2)
plot(U,White45,'-o')
```

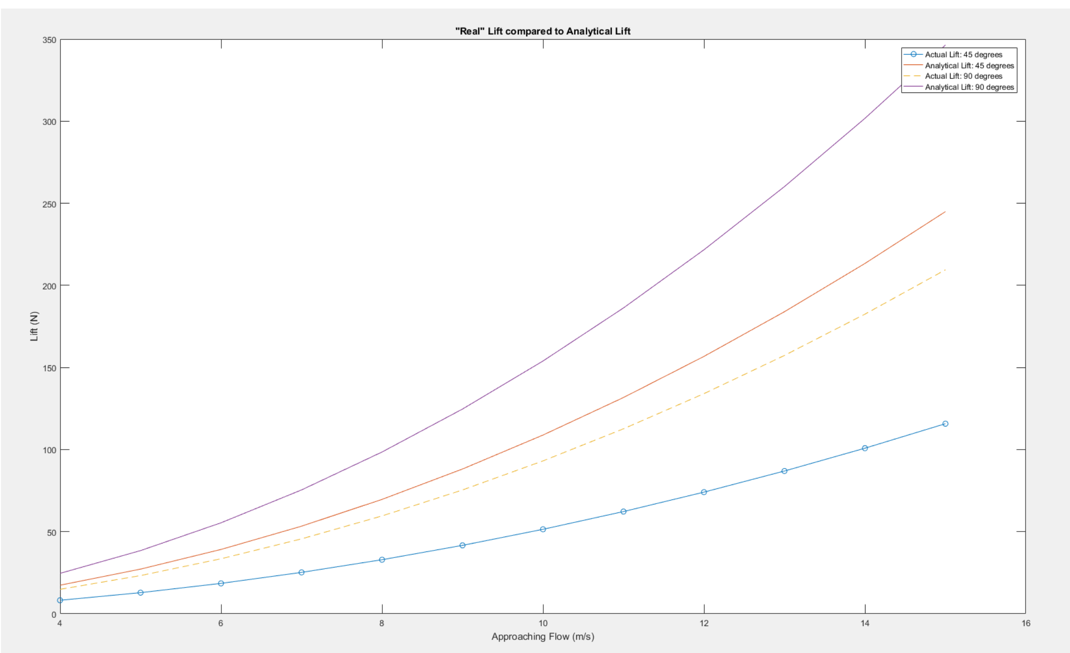
```
hold on
plot(U,LiftActual45)
hold on
plot(U,White90,'--')
hold on
plot(U,LiftActual90)
legend('Actual Lift: 45 degrees','Analytical Lift: 45 degrees','Actual Lift:
90 degrees','Analytical Lift: 90 degrees')
title('"Real" Lift compared to Analytical Lift')
ylabel('Lift (N)')
xlabel('Approaching Flow (m/s)')

Cd45= .62;
Cd90= .5;
WhiteDrag45= rho*U.^2*a*Cd45*2;
WhiteDrag90= rho*U.^2*a*Cd90*2;
ratio45= White45./WhiteDrag45;
ratio90= White90./WhiteDrag90;

figure(3)
plot(U,ratio45,'-o')
hold on
plot(U,ratio90)
legend('Ratio: 45','Ratio: 90')
title('Ratio of "Real" Lift and Drag')
ylabel('Fl/Fd')
xlabel('Approaching Flow (m/s)')
plot(U,ratio90)
```

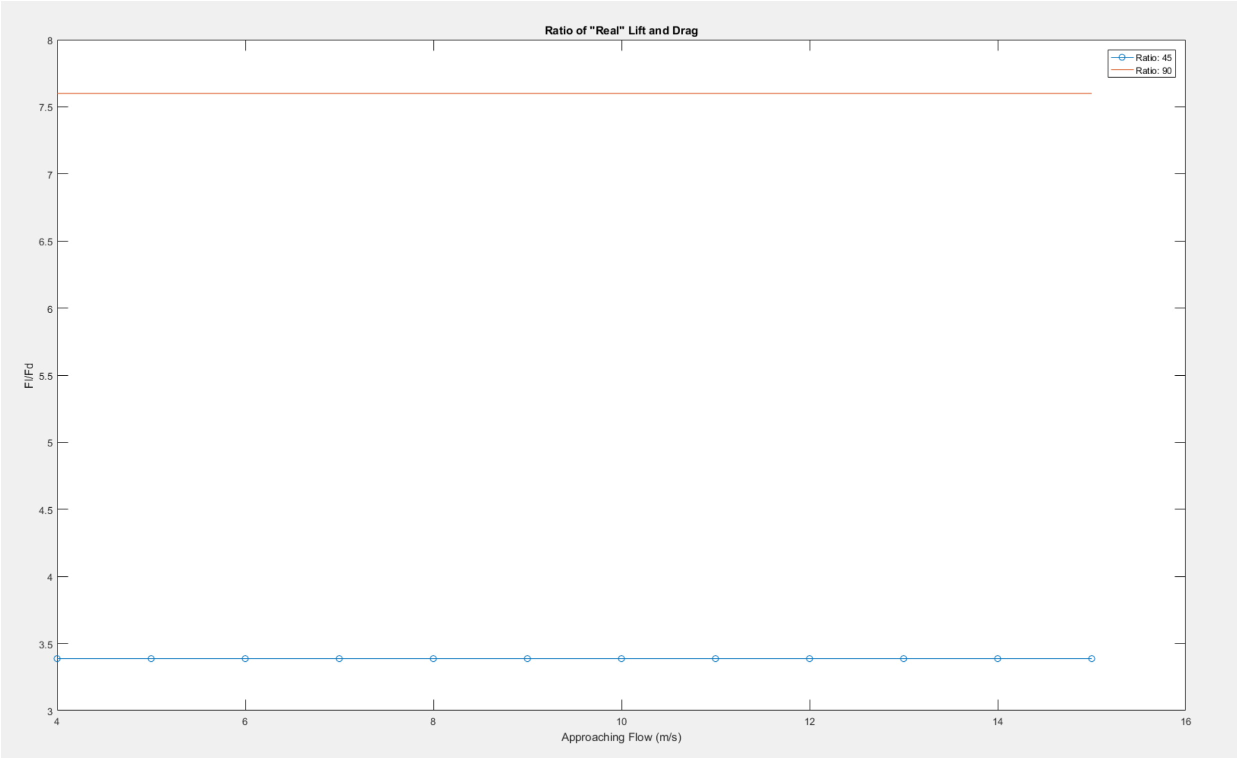
Solution:

Problem 6:



Problem 7:




Ratio of "Real" Lift and Drag
Ratio: 45
Ratio: 90
Fl/Fd
Approaching Flow (m/s)
8
7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
4
6
8
10
12
14
16