Numerical Solution of Ordinary Differential Equations
(MTH 452/552)
Homework due Wednesday, February 15, 2017

Problem 1. (40 points). Review the (updated) lecture notes from Jan. 30, 2017 on automatic stepsize control. Write a MATLAB program that implements automatic stepsize control for a general explicit Runge-Kutta method with Butcher array A,b,c, using Richardson extrapolation. You may make use of the MATLAB code provided with the lecture notes. Include a global variable that counts how often the function f is being evaluated. Apply your code to solve the IVP about the astronomical orbit that was given in problem 3 of Assignment 2. Report how many evaluations of f are needed to get a qualitatively correct plot of the orbit for a single period (time interval from $t = 0$ to $t = 17.1$). Note that accuracy is now controlled by the value of the user-supplied tolerance. Compare results for the classical Runge-Kutta method from problem 1 of Assignment 3, the RK method from problem 2 of Assignment 2, and the Euler method.

Problem 2. (20 points). Let $f(t, u) = 0$ for $(t, u) = (0, 0)$ and $f(t, u) = 2tu/(t^2 + u^2)$ otherwise. Use your program from problem 1 with the classical RK method from problem 1 of Assignment 3 to solve the IVP $u' = f(t, u)$, $u(-1) = -0.001$ on the interval $t \in [-1, 1]$. Use tolerances equal to 10^{-k} for $k = 4, 5, \ldots, 10$. Plot all the resulting curves in one frame. On which parts of the interval do you observe convergence? Explain the results.