Numerical Solution of Ordinary Differential Equations
(MTH 452/552)
Homework due Friday, January 18, 2017

Problem 1. (50 points) Consider the initial value problem (IVP)

$$u''(t) + t^2 u'(t) + (u(t))^2 = t^6 + 3t^4 + 6t, \quad u(0) = u'(0) = 0.$$

a) Check that $u(t) = t^3$ solves this IVP.

b) Convert the differential equation to a first order system.

c) Modify the posted code for the Euler method so that it solves the above IVP for the interval $0 \le t \le 2$, including the graphs of the computed and exact solutions. Submit your source code and the output for NSTEP $= 100$.

d) Compare the maximum errors for NSTEP $= 50, 100, 200$. What can you say about the apparent order of convergence?

e) Change the initial conditions to $u(0) = 1$, $u'(0) = 2$. use your code for the Euler method to find a numerical approximation for $u(2)$ with error at most 0.001. Estimate the error without using an exact solution. Explain why you think that your answer has the required accuracy.