Midterm 1A Soln.

6 1. $\nabla \times (\nabla \phi)$ $\qquad \varepsilon_{ijk} \frac{\partial}{\partial x_j}\left(\frac{\partial \phi}{\partial x_k}\right)$ 1st order (one free index)

6 2. Laminar vs. Turbulent – Reynolds No.
inviscid vs. viscid – Reynolds No.
compressible vs. incomp. – Mach No.
others.

6 3. $h \to$ position change along a line parallel with body force; grad. is zero in direction $\perp$ to body force

6 4. $\nabla^2 \psi = 0$ if irrotational

5. $\psi = 4x^2 - 4y^2 + C$; $\psi = 2$ at $(0,0)$ so $C = 2$

10 a. $u = \frac{\partial \psi}{\partial y} = -8y$; $v = -\frac{\partial \psi}{\partial x} = -8x$

8 b. $\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 8 - 8 = 0$ so irrotational

6. $q = 10(x^2 - y^2)i - (20xy)j$

8 a. $\frac{\partial u_i}{\partial x_i} = 0$ if incomp.: $\frac{\partial u}{\partial x} = 20x$ $\quad \frac{\partial v}{\partial y} = -20x$ yes incompressible

10 b. check if irrotational: $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -20y + 20y$ yes.

so $-\frac{dP}{dx} = \rho \frac{Du_x}{Dt} = \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y}$ (no body force, no visc. force).

$$\frac{dP}{dx} = -\rho(10(x^2 - y^2))(20x) - \rho(-20xy)(-20y)$$

2 7. a. yes; rotation causes asymmetry in the flow & pressure between top & bottom

2 b. yes: rotation modelled with vortex that needs circulation

2 c. as rotation increases stag. pts. move up/down along surface

10 8. a. $\psi = \psi_{vor} + \psi_{source} = +\mu_v \ln r + \mu_s \theta + C$; $\mu_v = \Gamma/2\pi$ $\quad \mu_s = Q/2\pi$ (superposition)

need to know or set ψ at a particular $(r, \theta)_1$ then insert into eqn. $\psi = -\mu_v \ln r_1 + \mu_s \theta_1 + C$, solve for C

8 b.  ψ & ϕ are $\perp$ flow spirals outward

8 c. $v_\theta = -\frac{\partial \psi}{\partial r} = \frac{\mu_v}{r}$ $\quad v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\mu_s}{r}$ $\quad q = \left(\left(\frac{\mu_v}{r}\right)^2 + \left(\frac{\mu_s}{r}\right)^2\right)^{1/2}$

where $\mu_v = \Gamma/2\pi$ $\quad \Gamma = \int_0^{2\pi} v_\theta r \, d\theta = 2\pi \omega R^2$; $\mu_s = Q/2\pi$

14 d. $P = P_A + \frac{1}{2}\rho q_A^2 + \gamma h_A - \frac{1}{2}\rho q^2 - \gamma h$; $\quad q_A^2 = \left(\frac{\mu_v}{R_A}\right)^2 + \left(\frac{\mu_s}{R_A}\right)^2$; $q^2 = \left(\frac{\mu_v}{r}\right)^2_{r=1.5m} + \left(\frac{\mu_s}{r}\right)^2_{r=1.5}$

(neglect t, r terms) $\int \frac{dP}{\rho} \to \frac{\Delta P}{\rho}$ $\qquad h = 1.0$

6 9. $\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0 = \frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial u_i}{\partial x_i}\right) + u_i \frac{\partial \rho}{\partial x_i} = \frac{D\rho}{Dt} + \rho \frac{\partial u_i}{\partial x_i}$