Assign #2 Soln. Guide 2016 ✓

+5 1. $\psi = Ur\sin\theta\left[1-\frac{a^2}{r^2}\right] \pm \frac{\Gamma}{2\pi}\ln r$

a = cylinder radius ($=.1$m)
Γ = circulation $= 2\pi a\, v_\theta (r=a)$
or $\Gamma = 2\pi a^2 \omega$
U = freestream
$\Gamma > 0 \quad v_\theta > 0$

+5 $\phi = -Ur\cos\theta\left[1+\frac{a^2}{r^2}\right] \pm \frac{\Gamma\theta}{2\pi}$

in eqn. (+) rotation is clockwise
(−) rotation is counter-clockwise

ψ - +5 2. plot ψ & ϕ : conditions $\underline{U = 10\ m/s}$
ϕ - +5
$\omega \rightarrow$ +5

$\omega \rightarrow$ select to be $.6\,\omega_{max}$ where ω_{max} is when stag. pts. at $\theta = \frac{\pi}{2}$:

or $\sin\theta = 1 = \frac{\Gamma}{4\pi U a}$

$$\omega_{max} = 2U/a = \frac{2(10)}{.1} = 200\ \frac{rad}{s}$$

so $\omega = .6(200) = 120\ \frac{rad}{s}$ and $\Gamma = 2\pi(.1)^2(120) = \underline{7.54}$

• select rot. direction (or sign of vortex term) & plot.
Stag. pts. should be at:

$$\sin\theta = \frac{7.54}{4\pi(10)(.1)} = .6 \qquad \underline{\theta = 36.9^\circ \ \&\ 143.1^\circ}$$

3. $U = 4 \rightarrow 15\ m/s$: use $\sin 90^\circ = \Gamma/2\pi U a = 1$; $\Gamma = 2\pi a^2 \omega_{max}$

+8 → • plot $\omega_{max} = \frac{2U}{a} = 20(U)$ → straight, linear

+8 → • derivation: stag. pt. at $\pi/2$: $v_\theta = 0$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -\left[U\sin\theta + U\sin\theta\,\frac{a^2}{r^2} + \frac{\Gamma}{2\pi}\frac{1}{r}\right]$$

set $r=a$: $v_\theta = -2U\sin\theta + \Gamma/2\pi a$

for stag. pt. set $v_\theta = 0$ so $\sin\theta_s = \Gamma/4\pi U a$

4. Start with general Bern. eqn. & find P_s:

$$\underset{①}{\int \frac{\partial q}{\partial t} \cdot dl} + \underset{②}{\Delta \frac{q^2}{2}} + \underset{③}{\Delta g h} + \underset{④}{\int \frac{dP}{\rho}} = \underset{⑤}{\Delta \Pi} + \underset{⑥}{f(t)}$$

steady, incomp., irrot. flows results in terms ① & ⑤ & ⑥ = 0

$$\& \int \frac{dP}{\rho} = \Delta P/\rho$$

apply eqn. between far upstream at $h=0$ (x axis through the center of the cylinder) to any pt. on surface "s."

derivation +8

$$\left(\frac{q^2}{2} + gh + \frac{P}{\rho}\right)_s = \left(\frac{q^2}{2} + gh + \frac{P}{\rho}\right)_\infty = \frac{U^2}{2} + 0 + \frac{P_\infty}{\rho}$$

$$P_s = P_\infty + \frac{U^2}{2} - \left(\frac{q^2}{2}\right)_s - g h_s$$

+5 where $q_s = v_\theta = -2U \sin\theta + \frac{\Gamma}{2\pi a}$

$$h_s = a \sin\theta$$

5. Plot P_s for U (4 → 15) this should be a number, say 4-6, values of U for P_s vs. θ (position around cylinder) on a given plot illustrating effect of U on P_s

+10 (plots) All shapes are the same, as U↑ P_s should go up

+10 (discussion) (or down) relatively

→ 2 plots for the range of U values stag. pt. at 45° → $\omega = \frac{2U}{a} \sin 45° = \frac{1.414 U}{.1}$

so $\Gamma = 2\pi a^2 \omega = .89\,U$

at 90° → $\omega = \frac{2}{a} U$; $\Gamma = 1.26\,U$

Increase rotation or Γ will increase the relative Press. (more neg. values) creating greater lift.

6. Numerically integrate P_s vs. θ to obtain $\int_0^{2\pi} P_s \sin\theta \, a \, d\theta$

+13 (integration) results in $L = \int_0^{2\pi} P_s \sin\theta \, a \, d\theta$ for each U and for 2 values of Γ

as U increases L increases; as Γ increases L increases.

discussion +5

Analytic soln. $\quad L = -\rho U \Gamma$

Compare Num. integration results with analytic

7. Use plot of C_L given: C_L vs. $a\omega/U$

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 (2a)} = \frac{\rho U \Gamma}{\frac{1}{2}\rho U^2 (2a)} = \frac{\rho U 2\pi a^2 \omega}{\frac{1}{2}\rho U^2 (2a)} = \frac{2\pi a \omega}{U}$$

(fig. eqn. for C_L is off by "2" although numbers are correct !!).

– use 2a in denominator since it is based on the frontal area per length.

+5 • Plot or make table of $L_{calc.}$ vs. L_{real} for range of U 4→15

(includes discussion)

$L_{calc.} = \int_0^{2\pi} P_s \sin\theta \, a \, d\theta$ from #6

L_{real} = get data from plot for each $a\omega/U$

Compare $\frac{L_{calc.}}{L_{real}}$ for range of U & 2 values of Γ

- since $L_{calc.} \approx \rho U \Gamma$ then this $C_{L,calc.}$ should be close to Theory in the plot. (e.g. at $\frac{a\omega}{U} = 2$ $\quad C_{L_{calc.}} \approx 12.5$)

- For fixed ω: as $U \uparrow$ $\frac{a\omega}{U} \downarrow$ and $C_{L_{calc}} \downarrow$ and $C_{L_{real}} \downarrow$ and difference $\downarrow$.

- For fixed U: as $\omega \uparrow$ $\frac{a\omega}{U} \uparrow$ and $C_{L_{calc.}} \uparrow$ and $C_{L_{real}} \uparrow$ and difference $\uparrow$.

+3 → Plot $(L/D)_{real}$ from fig. for range of U for 2 ω's.