

## Assignment 4

**Due: Saturday 12/03 (4:00 pm on Canvas)**

The determination of drag forces on relatively flat surfaces is ubiquitous, and part of many overall engineering evaluations of forces on objects moving in fluids or moving fluids over objects. The transition to turbulence greatly affects the magnitude of the drag force – this transition may be useful in delaying separation if the surface is slightly rounded and thereby influences total drag and lift force components. Also the effect of roughness on overall drag forces is very important in many applications and considerable effort is spent reducing roughness.

So here we are to evaluate the drag force on a flat surface that forms the top of a vehicle that moves horizontally in a body of water. We want to know the power needed by the friction drag force on the surface. The surface is 5m in the flow direction and 8 m wide. We can use fresh water properties for this problem.

Roughness effects are to be evaluated. To do this see the attached figure from White that shows  $C_D$  versus  $Re_L$  for a range of roughness values  $L/\epsilon$  (the ratio of plate length to average roughness). Select two velocities before and two velocities after transition to the Fully Rough regimes, all for turbulent flow, for roughness values of  $L/\epsilon = 2000, 5000, 10^4$  and  $5 \times 10^4$ . That is, we want to compare smooth versus rough conditions, but they will be at different velocities.

- (i) First, assuming a smooth flat plate select two velocities that result in laminar flow and two that result in turbulent flow. For the turbulent conditions be sure to account for the laminar starting edge. Indicate specifically all equations used.
- (ii) For the smooth plate of (i) plot the required power to overcome drag versus the four velocities selected. Discuss results.
- (iii) For the effect of roughness using the four roughness values above determine and plot the effect of roughness on turbulent flow on the power required to overcome drag versus vehicle speed for the four velocities selected for the four roughness values. Plot these results and discuss effect of roughness on power.
- (iv) Discuss your results of (iii) and make recommendations as to the need to smooth the surface. To estimate total cost assume that the vehicle is used for 8 hours a day for a year, operates at an efficiency of 25% using gasoline (lower heating value of 43 MJ/kg and the specific gravity of the gas is 0.85) and the fuel costs \$3.00/gal. Assume a 10 year life of operation. Also assume a cost of \$500/m<sup>2</sup> to smooth the surface from  $L/\epsilon = 5 \times 10^4$  to totally smooth conditions; \$1000/m<sup>2</sup> to go from  $L/\epsilon = 10^4$  to smooth; \$1500/m<sup>2</sup> to go from  $L/\epsilon = 5000$  to smooth; and \$2000/m<sup>2</sup> to go from  $L/\epsilon = 2000$  to smooth.
- (v) To obtain further details of the turbulent part of the flow for smooth conditions (neglect roughness here) look at the four velocities you selected in the turbulent regime above and assume a velocity profile that has a 1/8 power law and that the flow goes turbulent right at the beginning of the surface (no laminar starting region). Show all work and discuss results.
  - a. Determine the ratio of the momentum thickness to boundary layer thickness.
  - b. Find and plot the boundary layer thickness and momentum thickness versus distance along the surface.
  - c. Find and plot the local skin friction coefficient. Then find the overall drag coefficient and plot the total power required for the four velocities.
- (vi) Now considered the surface to be at a slightly tilted angle,  $\alpha$ , from the horizontal, that is the leading edge is higher than the trailing edge. Using the relationship:  $\frac{c_f}{2} = \frac{d\delta_2}{dx} + (2 + \frac{\delta_1}{\delta_2}) \frac{\delta_2}{U} \frac{dU}{dx}$  and estimate the U variation with x to be:

$U=U_0(1-x\sin\alpha)$ , where  $U_0$  is the velocity at the leading edge. Find the force and power required to move the plate for the four turbulent velocities selected. Assume that the  $\delta_2$  and  $\delta_1$  dependences on  $x$  do not change from a flat plate with  $\alpha$  equal to zero – comment on this.

- (vii) ME 560 only: Starting from the integral momentum equation:  $\frac{d(U^2\delta_2)}{dx} + \delta_1 U \frac{dU}{dx} = \frac{\tau_w}{\rho}$  show how to obtain the following equation for the local kin friction coefficient:

$$\frac{c_f}{2} = \frac{d\delta_2}{dx} + \left(2 + \frac{\delta_1}{\delta_2}\right) \frac{\delta_2}{U} \frac{dU}{dx}$$

**Fig. 7.6** Drag coefficient of laminar and turbulent boundary layers on smooth and rough flat plates. This chart is the flat-plate analog of the Moody diagram of Fig. 6.13.

