Midterm 1B Soln.

6 1. $\nabla \cdot (\nabla \times \bar{v}) = \frac{\partial}{\partial x_i}\left(\varepsilon_{ijk} \frac{\partial u_k}{\partial x_j}\right)$ 0 order tensor (scalar)

6 2. Laminar vs. Turbulent – Re #
Inviscid vs. viscid – Re #
Compressible vs. incompressible – Mach #

6 3. $\nabla^2 \phi = 0$ irrotational flow, incompressible, inviscid

6 4. $\frac{\partial h}{\partial x_i}$ – h is position change along a line parallel with the body force; grad is zero & in direction $\perp$ to body force

5. $\psi = -2x^2 + y^2 + C$; $\psi = 4$ at $(0,0)$ so $C = 4$

10 a. $u = \frac{\partial \psi}{\partial y} = 2y$ $v = -\frac{\partial \psi}{\partial x} = 4x$

8 b. $\nabla^2 \psi = 0$? $= -4 + 2 \neq 0$ so rotational

10 6. a. $\psi = \psi_{vor} + \psi_{sink} = -\mu_v \ln r - \mu_s \theta + C$ (superposition)
C is determined knowing or settin ψ at some (r, θ)

8 b.  spiral inward ϕ lines $\perp$ to ψ lines

8 c. $q = (v_r^2 + v_\theta^2)^{1/2}$ $v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{-\mu_s}{r}$ $v_\theta = -\frac{\partial \psi}{\partial r} = -\mu_v / r$
$\mu_v = \Gamma / 2\pi$ $\mu_s = Q' / 2\pi$

14 d. $P = P_A + \frac{1}{2}\rho q_A^2 + \gamma h_A - \frac{1}{2}\rho q^2 - \gamma h$ $q_A^2 = \left(\frac{\mu_v}{R_A}\right)^2 + \left(\frac{\mu_s}{R_A}\right)^2$
$(h_A = 1)$ $(h = .5)$ $q^2 = \left(\frac{\mu_v}{r_{=1}}\right)^2 + \left(\frac{\mu_s}{r_{=1}}\right)^2$
(neglect t, K terms) $\left(\int \frac{dP}{\rho} \to \Delta P / \rho\right)$

7. $q = -20xy\, i + 10(y^2 - x^2)\, j$

8 a. $\frac{\partial u_i}{\partial x_i} \overset{?}{=} 0 = -20y + 20y = 0$ so is incompressible

10 b. check is irrotational: $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -20x - (-20x) = 0$ yes
so can use Euler's eqn. (inviscid).
$$-\frac{\partial P}{\partial x} = \rho \frac{Du_x}{Dt} = \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y}$$ (no body or visc. force)
$$= -\rho(-20xy)(-20y) - \rho(10(y^2 - x^2))(-20x)$$

2 8. a. yes rotation causes asymmetry in the flow & pressure between the top & bottom

2 b. yes rotation is modelled with a vortex that has circulation

2 c. if rotation increases stag. pts. move up/down along surface

6 9. $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 = \frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial u_i}{\partial x_i}\right) + u_i \frac{\partial \rho}{\partial x_i} = \frac{D\rho}{Dt} + \rho \frac{\partial u_i}{\partial x_i}$