Additional Practice Problem Chapt 11

Consider a large flat plate with uniform flow approaching it in a direction normal to the plate (stagnation type flow). To solve this problem (meaning finding the velocity components as a function of position) do the following:

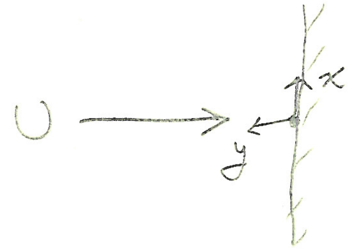
1. Assume $u=xf'(y)$ where *f* has units. As y goes to infinity (far away from the plate) the velocity behaves as (ax), where a is a constant (with units). Write the boundary conditions in terms of *f.*
2. Transform the x Navier-Stokes Eqn. using *f* but keep the pressure gradient terms as is. Note that to do this you need to find an expression for v (the y component of velocity).
3. Now assume a similarity solution of the form: $F'^2 - FF'' - F''' = 1$ Show how to obtain this equation where you can assume that the x pressure gradient behaves as $-\rho a^2 x$. To do this you need to assume a proper similarity variable η, to do this assume a length scale based on a and ν. You also need a y velocity component to scale using a and ν. Note that the u velocity component is written in terms of *f* , which we can scales with a and ν also.

Addition Prob. chapt. 11

1. $u = x f'(y) \qquad f = f(y)$ only

$u \to ax$ as $y \to \infty$

$y = 0 : \quad u = v = 0$



so $y = 0 : \quad \underline{f' = 0}$

need to find expression for v:

continuity: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$f' + \frac{\partial v}{\partial y} = 0 \quad$ or $\quad \frac{df}{dy} = -\frac{\partial v}{\partial y}$

or $\boxed{v = -f(y) + c_1}$

at $y = 0 : v = 0$ so set $\boxed{c_1 = 0}$

So at $y = 0 \quad v = 0 \quad$ or $\underline{f = 0}$

and at $y \to \infty : \quad f' = a$

integrate: $f = ay + c_2$ } actuall equivalent

$\underline{f = v = ay}$ (set $c_2 = 0$) since $\frac{df}{dy} = a$

[If you look at the inviscid flow solution for stagnation flow: (chapt. 5) p. 67 $\quad U = -2Ar\cos\theta = -2Ax$

$V = 2Ar\sin\theta = 2Ay$

(2 can be absorbed into "a")]

2. N-S (x) eqn: replace: $u = xf' \quad v = -f$

$$xf'f' + x(-f)f'' = -\frac{1}{\rho}\frac{dP}{dx} + \nu\left(\frac{\partial^2 u}{\partial x^2}^{\,0} + x f'''\right)$$

or: $x(f'f' - ff'' - \nu f''') = -\frac{1}{\rho}\frac{dP}{dx}$

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③ set $-\frac{1}{\rho}\frac{\partial P}{\partial x} = a^2 x$ eqn. becomes:

$$(f')^2 - ff'' - \nu f''' = a^2$$ (primes are derivatives relative to y).

$$f(0) = 0$$
$$f'(0) = 0$$
$$f'(\infty) = a$$

Scaling: units: a is $\frac{1}{t}$ ν is $\frac{l^2}{t}$

so length scale is $\sqrt{\nu/a}$ or $\eta = y/\sqrt{\nu/a}$

velocity scale is $\sqrt{\nu a}$ or $F = f/\sqrt{\nu a}$

substitute into above eqn.

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \eta}\cdot\frac{\partial \eta}{\partial y} \quad \text{and} \quad \frac{\partial \eta}{\partial y} = \frac{1}{\sqrt{\nu/a}}$$
$$= \frac{\partial}{\partial \eta}\left(\frac{1}{\sqrt{\nu/a}}\right)$$

then $f = F\sqrt{\nu a}$

$$f' = \frac{\partial}{\partial y}\left(F\sqrt{\nu a}\right) = \sqrt{\nu a}\,\frac{\partial F}{\partial y} = \sqrt{\nu a}\,\frac{1}{\sqrt{\nu/a}}F' = aF'$$

$$f'' = \frac{\partial}{\partial y}(f') = \frac{a}{\sqrt{\nu/a}}F''$$

$$f''' = \frac{\partial}{\partial y}(f'') = \frac{1}{\sqrt{\nu/a}}\,\frac{a}{\sqrt{\nu/a}}F'''$$

so we get:

$$a^2(F')^2 - F\sqrt{\nu a}\,\frac{a}{\sqrt{\nu/a}}F'' - \nu\left(\frac{a^2}{\nu}\right)F''' = a^2$$

or $$(F')^2 - FF'' - F''' = 1$$ (we know have eliminated ν & a)

$$F(0) = 0$$
$$F'(0) = 0$$
$$F'(\infty) = 1$$

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