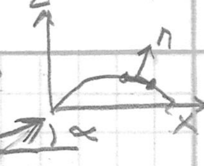
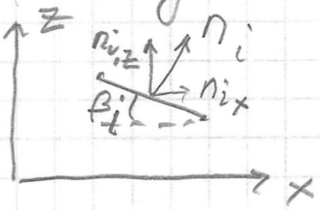


A slightly more general form: cambered plate

(X is along chord line)



- if n_i is on a tilted panel relative to Z:
- β is angle from chord line.

So here the outward normal, n_i, is a vector with x comp. $= \sin\beta_i$

z comp $= \cos\beta_i$

if NO camber $\beta = 0$ & n_i is in Z direction

Our general B.C. eqn. at the collocation pt.

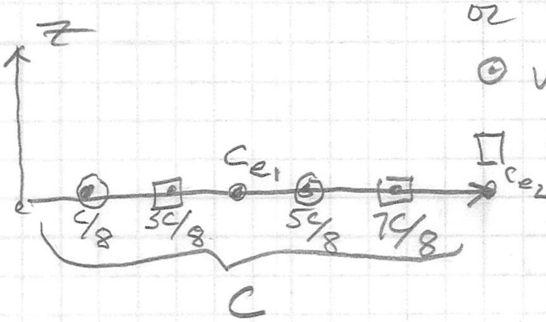
$$\boxed{\underbrace{q \cdot n}_{\text{induced flow by all vortices.}} = \underbrace{-U \cdot n}_{\text{Flow from free stream}}}$$

(here q is induced flow by vortices)

Two panels — no camber; each panel length $= \frac{c}{2}$ or $c_e = c/2$

⊙ vortex centers

□ collocation pts



Write eqn. for q in general:

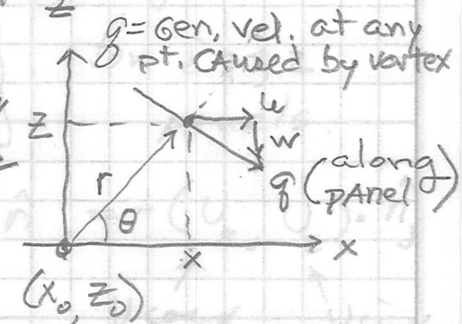
Note here: $\eta(x)$ = surface location $= 0$ ($Z=0$)

In general the surface may have some x_s, z_s curve: $\eta(x) = f(x)$ for curve:

$\boxed{\text{Gen. eqn. For } (u, w)}$

$$u = \frac{\Gamma}{2\pi}\,\frac{(z-z_0)}{(x-x_0)^2+(z-z_0)^2} \qquad \frac{r\sin\theta}{r^2} = \frac{\text{"}z\text{"}}{r^2}$$

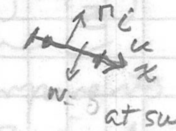
$$w = \frac{-\Gamma}{2\pi}\,\frac{(x-x_0)}{(x-x_0)^2+(z-z_0)^2} \qquad \frac{r\cos\theta}{r^2} = \frac{\text{"}x\text{"}}{r^2}$$



at vortex center x_0 & z_0 on surface $z_0 = 0$.

We are interested in pts. on surface where $z = z_0$ or

$$\begin{pmatrix} u \\ w \end{pmatrix} = \frac{\Gamma}{2\pi r^2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x-x_0 \\ z-z_0 \end{pmatrix} \Big\} \text{ on surface:}$$



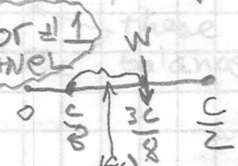
We write $\begin{pmatrix} u \\ w \end{pmatrix}$ for "unit gamma" $\Gamma = 1$ at $z = z_0$ (on surface).

with vortex located at $c/4$ of each element

<u>Note</u>: Vortex generates no x comp. of vel. on surface !!

Define u_{ij} & w_{ij} $\left(\begin{matrix}\text{for}\\ \Gamma=1\end{matrix}\right)$ i = vel. at collocation i; j = vel. from vortex at j

$$u_{11},\ w_{11} = \left(0,\ -\frac{1}{2\pi \frac{c}{4}}\right) = \left(0,\ -\frac{2}{\pi c}\right) \quad \text{(see below)}$$



$u_{11} = 0$ since $z = z_0$ (see above eqn.)

$$\longrightarrow w_{11} = \frac{-1\,(x-x_0)}{2\pi(x-x_0)^2} = \frac{-1}{2\pi(x-x_0)} = \frac{-1}{2\pi\left(\frac{3c}{8}-\frac{c}{8}\right)} = \frac{-1}{2\pi\frac{c}{4}} = \frac{-2}{\pi c}$$

$$w_{21} = \frac{-2}{3\pi c}$$

$$w_{12} = \frac{2}{\pi c} \quad \leftarrow \text{positive because } (x-x_0) \text{ is neg. along } x \text{ axis.}$$

$$w_{22} = \frac{-2}{\pi c}$$

define: $\boxed{a_{ij} = q_{ij}(\Gamma_j = 1)\cdot \hat{n}_i}$

We use this in our B.C. at the surface:

define: $\boxed{\bar{q}\cdot\hat{n} = \sum_{j=1}^{2} a_{i_j}\,\Gamma_j}$ for $i=1,2$ panels.

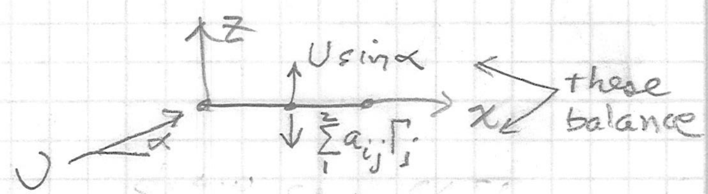
This is set equal to $-U\cdot\hat{n} = -(\underbrace{U_x}_{U\cos\alpha},\ \underbrace{U_z}_{U\sin\alpha})\cdot\hat{n}_j$
to satisfy B.C. of zero flow across the boundary

So we end up with:

$$\sum_{j=1}^{2} a_{i_j}\,\Gamma_j = -(U_x, U_z)\cdot(\underbrace{n_{i_x}, n_{i_z}}_{\text{components of } n_i \text{ in } x,z \text{ directions}})$$

For our 2 panels with $\eta = 0$

$$\begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix}\begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix} = \underbrace{U\sin\alpha\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{\text{for the case of outward normal in Z direction.}}$$



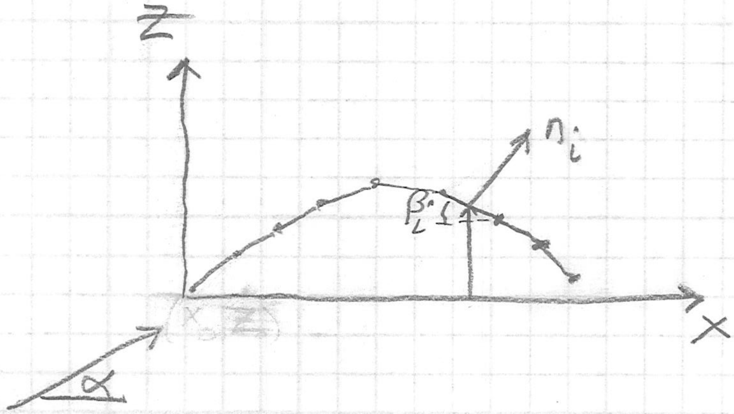
Solve for $\Gamma_1 \ldots \Gamma_N$

Lift from each panel: $\Delta L_i = \rho U_\infty \Gamma_i$
here Γ (+) so no neg. sign.

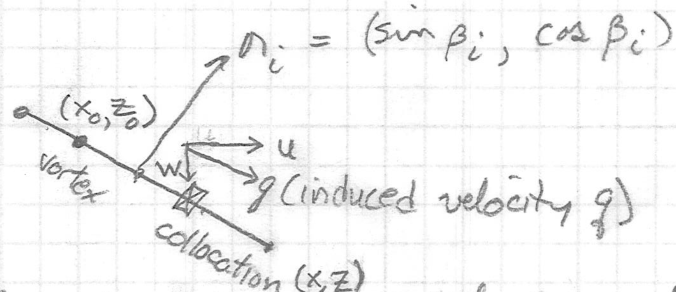
Total Lift. $L = \sum_{i=1}^{N} \Delta L_i$

and $C_L = \dfrac{L}{\frac{1}{2}\rho U^2 c}$ also $M_0 = -\sum_{i=1}^{N} \Delta L\, x_{0_i}$

General Diagram for B.C.



- define panel end pts in terms of (x, z)
- locate vortex & collocation pts for each panel



at panel 1 = i

$$a_{1j} = (u, w)_{1j} \cdot n_1$$ → vel. normal to surface by induced vortex flow at collocation 1 by vortex element j.

(x_o, z_o) is at vortex location which is ¼ length of panel from start of panel.

(x, z) is at collocation pt. (where velocity is evaluated)

So, must determine $(u, w)_{ij}$ for $\Gamma = 1$, then take dot product with n_i to find a_{ij}