**ME560 Assignment 3**

Group 8

Andrew Alferman

Nathan Schorn

1. Plot the surface profile η vs distance *x*. Describe the shape relative to a typical airfoil you might find.



The above figure was created using the attached Python code and shows the shape of the surface profile, stretched in the *y* direction. The shape is thin and has a long chord length, but is otherwise similar to the top of an airfoil shape, though it is not mirrored across the positive *x* axis.

2. Select flat panels, using at least 10 along the length of the surface. Draw a sketch showing your panel distribution. Create a table listing your (*x, z*) coordinates at the beginning and end of each panel as well as the locations for vortex placement and collocation points along the surface.



|  |  |  |
| --- | --- | --- |
|  | Vertex Locations | |
|  | x | z |
| 1 | 0.0000 | 0.0000 |
| 2 | 0.0500 | 0.0146 |
| 3 | 0.1000 | 0.0205 |
| 4 | 0.1500 | 0.0206 |
| 5 | 0.2000 | 0.0173 |
| 6 | 0.2500 | 0.0125 |
| 7 | 0.3000 | 0.0077 |
| 8 | 0.3500 | 0.0038 |
| 9 | 0.4000 | 0.0013 |
| 10 | 0.4500 | 0.0002 |
| 11 | 0.5000 | 0.0000 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Panel | Collocation Points | | Vortex Points | |
|  | x | z | x | z |
| 1 | 0.0375 | 0.0109 | 0.0125 | 0.0036 |
| 2 | 0.0875 | 0.0190 | 0.0625 | 0.0161 |
| 3 | 0.1375 | 0.0206 | 0.1125 | 0.0205 |
| 4 | 0.1875 | 0.0181 | 0.1625 | 0.0198 |
| 5 | 0.2375 | 0.0137 | 0.2125 | 0.0161 |
| 6 | 0.2875 | 0.0089 | 0.2625 | 0.0113 |
| 7 | 0.3375 | 0.0048 | 0.3125 | 0.0067 |
| 8 | 0.3875 | 0.0019 | 0.3625 | 0.0032 |
| 9 | 0.4375 | 0.0005 | 0.4125 | 0.0010 |
| 10 | 0.4875 | 0.0000 | 0.4625 | 0.0001 |

The points were selected using the attached Python code by evenly spacing 11 points from x = 0 to x = 0.5, inclusive of 0 and 0.5, to create 10 panels. The z value was found using the given formula. The vortex locations were placed at L = ¼ for each panel, and the collocation points were placed at L = ¾ for each panel.

3. Evaluate the matrix aij (as defined in class which is the matrix for the condition of vortex circulation values = 1.0) and put the results in the form of a table for each angle of attack.

The aij matrix was evaluated in the attached Python code by first finding the u and w values of flow velocity at each point. The formula were derived in class and can be found in the online handout (Additional Notes on the Panel Method, available on Canvas) as follows:



These formula were evaluated at each collocation point using the distance between each collocation point and each vortex point. Gamma was equal to one in each instance for the purpose of creating the aij matrix. Next, the u and w values were placed in a vector q, and the dot product of that vector and a vector n denoting the direction normal to each panel was obtained. Because this process assumes that the circulation at each panel is equal to one, the uniform flow velocity U was not taken into account, and thus the angle of attack was not considered. Therefore, there is only a single aij matrix for any angle of attack. The 10 x 10 matrix, with i values increasing horizontally and j values increasing downwards is as follows:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| aij: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | -6.11 | 6.22 | 2.08 | 1.24 | 0.88 | 0.68 | 0.55 | 0.47 | 0.40 | 0.36 |
| 2 | -2.07 | -6.32 | 6.34 | 2.11 | 1.26 | 0.90 | 0.70 | 0.57 | 0.48 | 0.42 |
| 3 | -1.25 | -2.11 | -6.37 | 6.36 | 2.11 | 1.27 | 0.90 | 0.70 | 0.58 | 0.49 |
| 4 | -0.90 | -1.27 | -2.12 | -6.35 | 6.34 | 2.11 | 1.27 | 0.91 | 0.71 | 0.58 |
| 5 | -0.70 | -0.91 | -1.27 | -2.11 | -6.34 | 6.34 | 2.11 | 1.27 | 0.91 | 0.71 |
| 6 | -0.57 | -0.71 | -0.91 | -1.27 | -2.11 | -6.34 | 6.34 | 2.12 | 1.27 | 0.91 |
| 7 | -0.49 | -0.58 | -0.71 | -0.91 | -1.27 | -2.11 | -6.35 | 6.35 | 2.12 | 1.27 |
| 8 | -0.42 | -0.49 | -0.58 | -0.70 | -0.91 | -1.27 | -2.12 | -6.36 | 6.36 | 2.12 |
| 9 | -0.37 | -0.42 | -0.49 | -0.58 | -0.70 | -0.91 | -1.27 | -2.12 | -6.36 | 6.37 |
| 10 | -0.34 | -0.37 | -0.42 | -0.49 | -0.58 | -0.71 | -0.91 | -1.27 | -2.12 | -6.37 |

Note: Indices of i and j are included in the first row and column of the above table, respectively.

4. Obtain the distribution of Γ, that is determine Γ for each panel, and plot this versus x for each α. Show all work.

The value of Γ for each panel was found using the attached Python code by first obtaining the dot product of x and z components of U for each angle of attack with the x and z components of a unit vector normal to the surface of each panel (named ni in the attached code):

for i in range(len(ni)):

umatrix.append(-1 \* np.dot([u \* np.cos(np.radians(aoa)),

u \* np.sin(np.radians(aoa))],

[np.cos(ni[i]), np.sin(ni[i])]))

Next, the dot product between the inverse of the aij matrix and the previously obtained vector to obtain a vector of Γ values for each panel:

aiji = np.linalg.inv(aij)

gammas = np.dot(aiji, umatrix)

return gammas

The value of Γ on each panel can be found in the figure and table below for each angle of attack α. The plot shows that the circulation decreases across the length of the hydrofoil after the first few panels.



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| AoA | Panel 1 | Panel 2 | Panel 3 | Panel 4 | Panel 5 | Panel 6 | Panel 7 | Panel 8 | Panel 9 | Panel 10 |
| 0 | -0.0197 | 0.0145 | 0.0187 | 0.0159 | 0.0107 | 0.0053 | 0.0009 | -0.0019 | -0.0028 | -0.0021 |
| 2 | -0.0001 | 0.0237 | 0.0252 | 0.0209 | 0.0147 | 0.0087 | 0.0037 | 0.0003 | -0.0012 | -0.0011 |
| 5 | 0.0292 | 0.0373 | 0.0347 | 0.0283 | 0.0208 | 0.0136 | 0.0077 | 0.0035 | 0.0012 | 0.0005 |

5. Find the local pressure difference across the airfoil, Δpk, for each element, k, based on the local lift force, Lk, and plot this versus *x* for each α in terms of the local pressure coefficient, ΔCp. Show all work.

The pressure difference across each panel Δpk was found by dividing the local lift of each panel per 1m of span length by the width of each panel. The local lift force was found using the formula Lk’ = ρU∞Γk where Lk’ is the lift per 1m span length for each panel, U∞ is the uniform flow velocity, ρ is the density of the fluid, and Γk is the circulation for each panel which was obtained in step 4. The local pressure coefficient was then found using the formula:

Where P0 is the pressure far away from the hydrofoil and P is the pressure at the point to be evaluated, thus P – P0 is equal to Δpk. Combining this formula with Lk’ = ρU∞Γk and Δpk = Lk’/wk where wk is the width of each panel, we get:

This cp value was calculated at each panel:

for k in range(len(gammas)):

cp.append(2 \* gammas[k] / (u \* plm[k]))

Which produces the following figure:



6. Determine the total lift force on the surface for each α. Compare this to the expected lift coefficient for a two dimensional flat plate at the same angle of attack.

The total lift coefficients were computed in the attached Python code by adding together the circulation from each panel, then multiplying by the uniform flow velocity U, and the density ρ (1000 kg/m3) using the equation L’ = ρUΓ as follows:

for j in range(len(gammas)):

gammat += gammas[j]

tliftm.append(gammat \* u \* 1000)

The angle of attack was taken into account in each case when the Γ value of each plate was computed, as discussed in step 4 above. Using this formula, the attached Python code produced the following output:

For 1m Span Length:

Lift with AoA = 0.0: 39.64

Flat Plate Lift with AoA = 0.0: 0.00

Lift with AoA = 2.0: 94.60

Flat Plate Lift with AoA = 2.0: 54.82

Lift with AoA = 5.0: 176.80

Flat Plate Lift with AoA = 5.0: 136.90

The output demonstrates that the hydrofoil produces approximately 40 N of lift force per 1m span length greater than a two dimensional flat plate for the same angle of attack, thus the hydrofoil shape does have some effect on lift.

7. Now to get a feel for what the influence of viscous forces may be compare these results to flow over a flat plate of length *c*, for zero angle of attack. Look up the solution for the viscous force on the surface (assume boundary layer flow along the length c) and compare this in magnitude to the lift force at zero angle of attack. Indicate the conditions and equations that you use.

This problem was clarified as follows:

You are to compare the lift force at zero angle of attack of the surface described by “eta(x)” to the viscous force predicted for flow over a flat plate which is also at zero angle of attack. Discuss the relative magnitudes.

The viscous force discussed is the drag force. The drag force can be found using an equation on page 221 of Nuun:

Where Df is the drag force, b is the plate width (span length), L is the plate length (chord length), and Cf is as follows:

Where the Reynolds number ReL is defined as Re = U∞L/υ, with υ equal to the kinematic viscosity, equal to 1.003 x 10-6 m2/sec for water at 20 degrees C. Combining these two formulas and substituting in values, we find that the drag force is 0.47 N per 1m of span length. This is compared to a lift force of approximately 40 N per 1m of span length for the hydrofoil evaluated in this assignment. The hydrofoil, being a relatively flat, very thin shape, will have a drag force roughly similar to the drag force of the flat plate. The lift will be roughly 2 orders of magnitude greater than the drag force in this case, which is an excellent lift/drag ratio.

Below is the code used to obtain the solutions to all of the above problems:

#!/usr/bin/env python3

# -\*- coding: utf-8 -\*-

"""

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"""

import numpy as np

import matplotlib.pyplot as plt

# Global variables

c = 0.5

meshpoints = 10

aoam = [0.0, 2.0, 5.0]

u = 1.0

p\_inf = 101000

method = 1

# Create a function that uses the given equation to make vertexes for the

# panels. The function also finds the vortex points, colocation points, and

# angle outward (normal) from the panel.

def createaij(meshpoints, c):

# Setting up lists for X coordinate (xc), eta coordinate (etac), vortex

# coordinates (vcx/vcz), colocation point coordinates (clcx/clcz),

# and angle normal to panel (ni)

xc, etac, vcx, vcz, clcx, clcz, ni, plm = [], [], [], [], [], [], [], []

# Initialize a few variables to create the points.

x = 0

dx = c / meshpoints

c\_i = 1 / c

# Create all of the vertex points.

for i in range(meshpoints + 1):

xc.append(x)

etac.append(0.2 \* x \* c\_i \* (1 - x\*c\_i)\*\*3)

x += dx

# Create the vortex points, colocation points, normal angles, and lengths

for i in range(1, len(xc)):

vcx.append(xc[i] - 0.75\*(xc[i]-xc[i-1]))

vcz.append(etac[i] - 0.75\*(etac[i]-etac[i-1]))

clcx.append(xc[i] - 0.25\*(xc[i]-xc[i-1]))

clcz.append(etac[i] - 0.25\*(etac[i]-etac[i-1]))

ni.append(np.pi/2 + np.arctan((etac[i]-etac[i-1])/(xc[i]-xc[i-1])))

plm.append(np.sqrt((xc[i]-xc[i-1])\*\*2 + (etac[i]-etac[i-1])\*\*2))

# Create the aij matrix

aij = []

for i in range(len(vcx)):

aijr = []

for j in range(len(clcx)):

lr = (clcx[i] - vcx[j])\*\*2 + (clcz[i] - vcz[j])\*\*2

u = (1 / (2 \* np.pi)) \* (clcz[i] - vcz[j]) / lr

w = (-1 / (2 \* np.pi)) \* (clcx[i] - vcx[j]) / lr

n = [np.cos(ni[i]), np.sin(ni[i])]

q = [u, w]

aijr.append(np.dot(q, n))

aij.append(aijr)

aij = np.array(aij)

return xc, etac, vcx, vcz, clcx, clcz, ni, aij, plm

# Create a function that finds the circulation across all the panels

def findgammas(ni, aij, aoa, u):

umatrix = []

for i in range(len(ni)):

umatrix.append(-1 \* np.dot([u \* np.cos(np.radians(aoa)),

u \* np.sin(np.radians(aoa))],

[np.cos(ni[i]), np.sin(ni[i])]))

aiji = np.linalg.inv(aij)

gammas = np.dot(aiji, umatrix)

return gammas

def flatplate(meshpoints, c):

# Setting up lists for X coordinate (xc), eta coordinate (etac), vortex

# coordinates (vcx/vcz), colocation point coordinates (clcx/clcz),

# and angle normal to panel (ni)

xc, etac, vcx, vcz, clcx, clcz, ni, plm = [], [], [], [], [], [], [], []

# Initialize a few variables to create the points.

x = 0

dx = c / meshpoints

# Create all of the vertex points.

for i in range(meshpoints + 1):

xc.append(x)

etac.append(0)

x += dx

# Create the vortex points, colocation points, normal angles, and lengths

for i in range(1, len(xc)):

vcx.append(xc[i] - 0.75\*(xc[i]-xc[i-1]))

vcz.append(0)

clcx.append(xc[i] - 0.25\*(xc[i]-xc[i-1]))

clcz.append(0)

ni.append(np.pi/2)

plm.append(0.05)

# Create the aij matrix

aij = []

for i in range(len(vcx)):

aijr = []

for j in range(len(clcx)):

lr = (clcx[i] - vcx[j])\*\*2

u = 0

w = (-1 / (2 \* np.pi)) \* (clcx[i] - vcx[j]) / lr

n = [0, 1]

q = [u, w]

aijr.append(np.dot(q, n))

aij.append(aijr)

aij = np.array(aij)

return xc, etac, vcx, vcz, clcx, clcz, ni, aij, plm

xc, etac, vcx, vcz, clcx, clcz, ni, aij, plm = createaij(meshpoints, c)

fxc, fetac, fvcx, fvcz, fclcx, fclcz, fni, faij, fplm =\

flatplate(meshpoints, c)

tliftm, gammam, cpm = [], [], []

ftliftm, fgammam, fcpm = [], [], []

for i in aoam:

cp = []

fcp = []

gammas = findgammas(ni, aij, i, u)

gammam.append(np.ndarray.tolist(gammas))

gammat = 0

fgammas = findgammas(fni, faij, i, u)

fgammam.append(np.ndarray.tolist(fgammas))

fgammat = 0

for j in range(len(gammas)):

gammat += gammas[j]

fgammat += fgammas[j]

tliftm.append(gammat \* u \* 1000)

ftliftm.append(fgammat \* u \* 1000)

for k in range(len(gammas)):

cp.append(2 \* gammas[k] / (u \* plm[k]))

fcp.append(2 \* fgammas[k] / (u \* fplm[k]))

cpm.append(cp)

fcpm.append(fcp)

plt.figure(1)

plt.title('Circulation per Panel')

plt.plot(vcx, gammas, label='AoA = {}'.format(i))

plt.legend(bbox\_to\_anchor=(1, 1), loc=2)

plt.figure(2)

plt.title('Local Pressure Coefficient per Panel')

plt.plot(clcx, cp, label='AoA = {}'.format(i))

plt.legend(bbox\_to\_anchor=(1, 1), loc=2)

gammam = np.array(gammam)

fgammam = np.array(fgammam)

plt.figure(3)

plt.scatter(xc, etac, color='blue')

plt.xlabel('Length X (meters)', fontsize=12)

plt.ylabel('Width Z (meters)', fontsize=12)

plt.title('Panel Vertex Locations (Stretched in Y Direction)', fontsize=14)

plt.xlim(0, 0.5)

plt.figure(4)

plt.plot(xc, etac, color='blue')

plt.xlabel('Length X (meters)', fontsize=12)

plt.ylabel('Width Z (meters)', fontsize=12)

plt.title('Hydrofoil Shape (Stretched in Y Direction)', fontsize=14)

plt.xlim(0, 0.5)

for i in range(1, 5):

plt.figure(i)

plt.grid(b=True, which='both')

plt.show()

print('For 1m Span Length:')

for i in range(len(aoam)):

print('Lift with AoA = {}: {:.2f}'.format(aoam[i], tliftm[i]))

print('Flat Plate Lift with AoA = {}: {:.2f}'.format(aoam[i], ftliftm[i]))

# The code below just outputs the location of points in an easy to read format

vertexes, clc, vc = [], [], []

for i in range(len(clcx)):

clc.append([clcx[i], clcz[i]])

vc.append([vcx[i], vcz[i]])

for i in range(len(xc)):

vertexes.append([xc[i], etac[i]])

vertexes = np.array(vertexes)

clc = np.array(clc)

vc = np.array(vc)