Chapter IV

§17 Formation Rules

Definition 1. A *term* is:

1. 0 is a term
2. A variable is a term

3–5. If s and t are terms, then the following are terms:
(s)+(t), (s)·(t), and (s)'

6 The only terms are those defined by these rules

Definition 2. Given terms s and t, a *formula* is:

1. (s)=(t)

2–5. If A and B are formulas, then the following are formulas:
(A)⊃(B), (A)&(B), (A)∨(B), and ¬(A)

6–9. For term x and formula A, the following are formulas:
∀x(A) and ∃x(A)

10 The only formulas are those defined by these rules

Definition 3. Some helpful definitions are as follows:

- $\supset, \&, \vee, \neg, \forall x, \exists x, =, +, \cdot, '$ are all *operators*
- The *scope* of an operator is the one or two formulas associated with it
- The following symbols are all *logical operators*
- *Propositional connectives* are the symbols $\supset, \&, \vee, \neg$
- *quantifiers* are $\forall x, \exists x$, with $\forall x$ as the *universal quantifier* and $\exists x$ as the *existential quantifier*

Lemma 4. Uniqueness of Operator Scope in an Expression
For a given term or formula (with defined operator scopes), there exists a proper pairing of parentheses such that the scope satisfies the following two rules:

(a) **One-Expression Operators**: The scope of the operator is within paired parenthesis, and (depending on the operator) the operator is either immediately to the right of the right parenthesis or to the left of the left parenthesis.

(b) **Two-Expression Operators**: The scope of the operator is immediately within two pairs of parentheses such that the right parenthesis of the left expression and the left parenthesis of the right expression border the operator.

§18 Free and Bound Variables

Definition 4. An occurence of a variable x in a formula A is *bound* if x is in a quantifier or scope of a quantifier ∃x or ∀x. Otherwise, the occurence is *free*.

If x occurs in A as a free variable, we say x *is a free variable of* A or A *contains* x *as a free variable.*

Definition 5. The *substitution* of a term t for a variable x in a formula or term A means that t will relpace every free occurence of x in A.