Types of Models: WHAT's in the BOX

Conceptual.............Mathematical

Static………….Dynamic :*TIME*

Lumped.............Spatially Distributed: *SPACE*

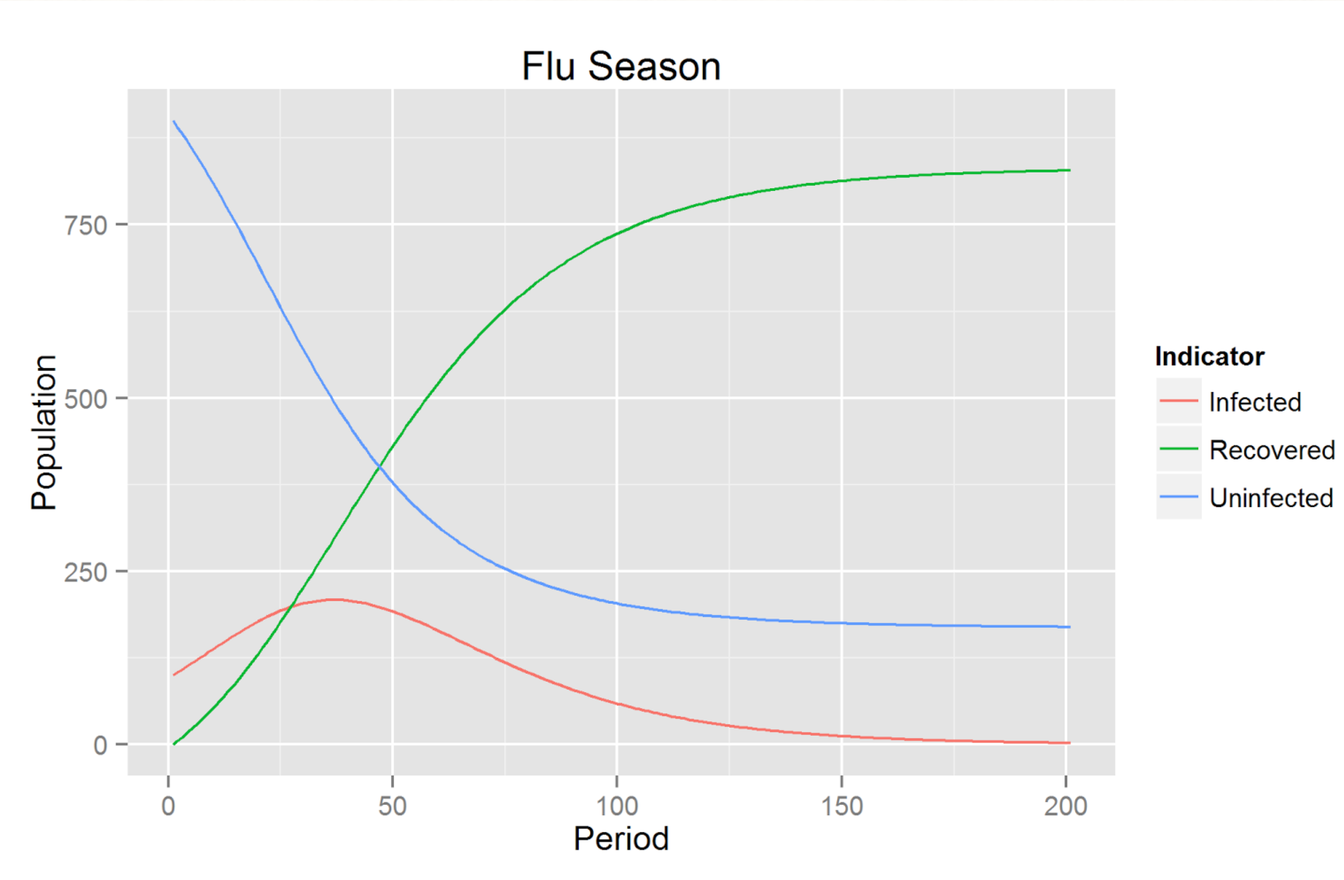
Stochastic.............Deterministic

Abstract………….Physically/Process Based

but biggest differences may often be the degree specific processes/parameters are accounted for

Static- Dynamic Time Varying

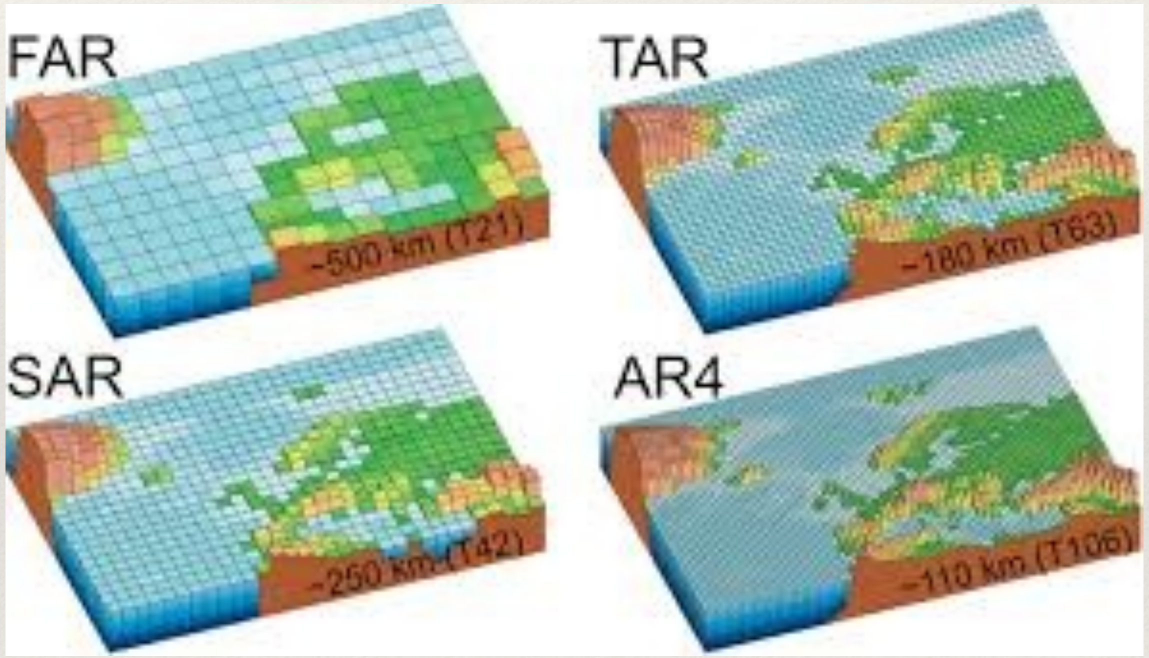
- Static - Processes or Variables modeled do not evolve with time
- Dynamic - model elements evolve through time - and variables/results at one time step typically depends on previous time step



http://www.econometricsbysimulation.com/2013/05/sir-model-flue-season-dynamic.html

Lumped …Spatially distributed

- Lumped - single point in space, or space doesn't matter
- Spatially distributed - model is applied to different "patches" in space
    - spatial units are independent
    - **spatial units interact with each other**



http://eo.ucar.edu/staff/rrussell/climate/modeling/climate_model_resolution.html

Dynamics - connection in space and time

- Dynamic modeling is common in environmental problems solving
- Similar issues: what happens at one place, depends on neighbors; what happens at one time; depends on previous time
- Space - two way; Time is usually one-way
- Dynamic system modeling - quickly becomes complex (Engineering degrees spend a lot of time on this; there are books, entire journals etc on this topic)

Dynamics models

- Many environmental problems and questions can be related to
  - Diffusion
  - Population
- Both often require dynamics models; and both often require thinking about dynamics in space and in time

Dynamic Systems

Some useful terminology

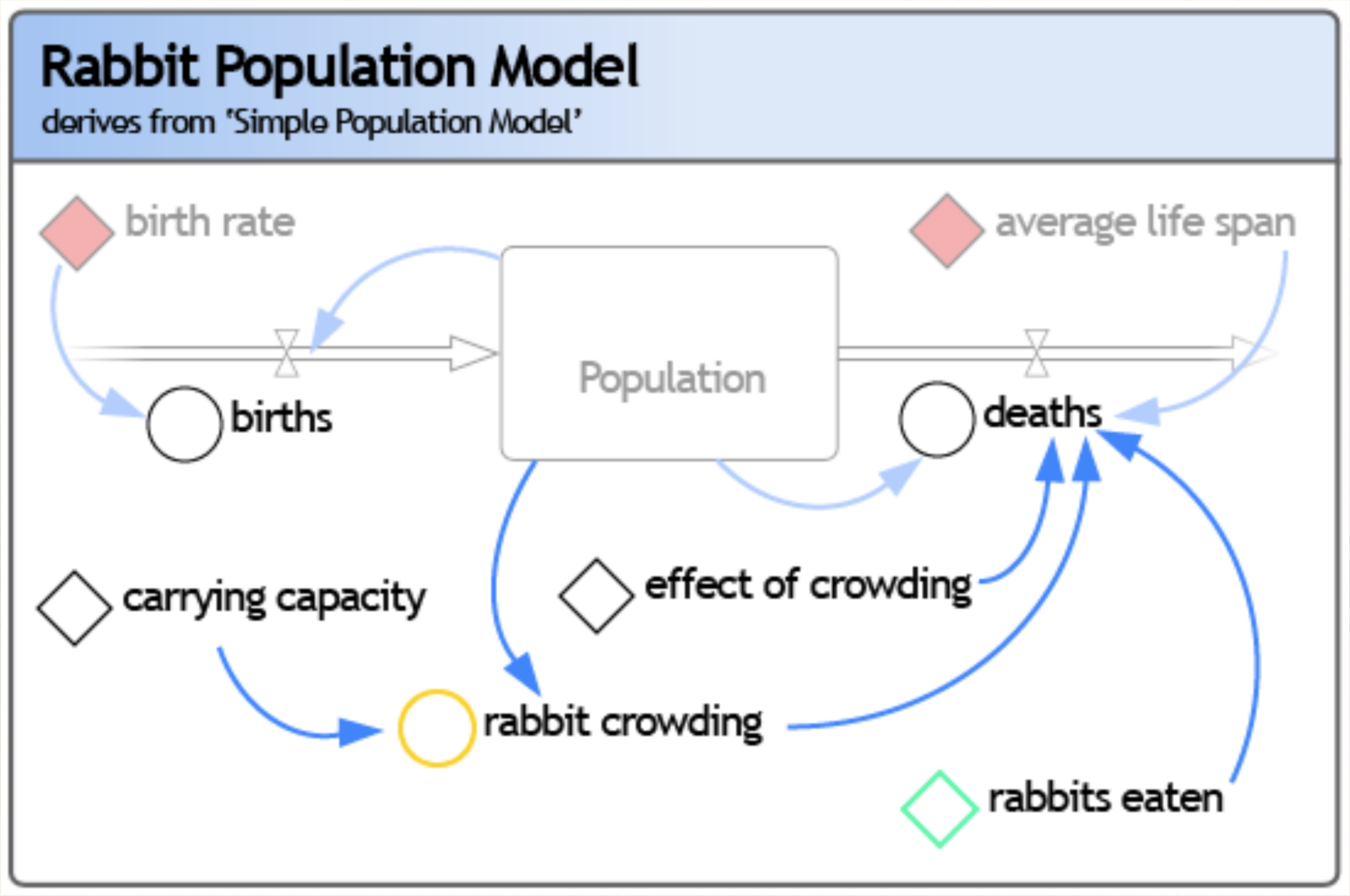
- *stocks* - variables that evolve over time
- *flows* - transfers between variables or from the system
- *parameters* - values that controls the relationship between stocks and flows
- *sink* - something that absorbs flows
- *source* - something that generates flows

Dynamic Systems

- *System state*: value of all variables need to describe the “entity that evolves through time” at a particular point in time
  - usually think of these as stores (soil moisture, bank account balance, number of individuals in a particular age class)
- *State-space*: description of the entity may require multiple variables - for a watershed this could be soil moisture, water currently in dam and water stored in trees, and for each “grid” in a watershed)
- *State-space trajectories*: how the system state evolves through time
- *Initial conditions*: values to describe the system state at the beginning

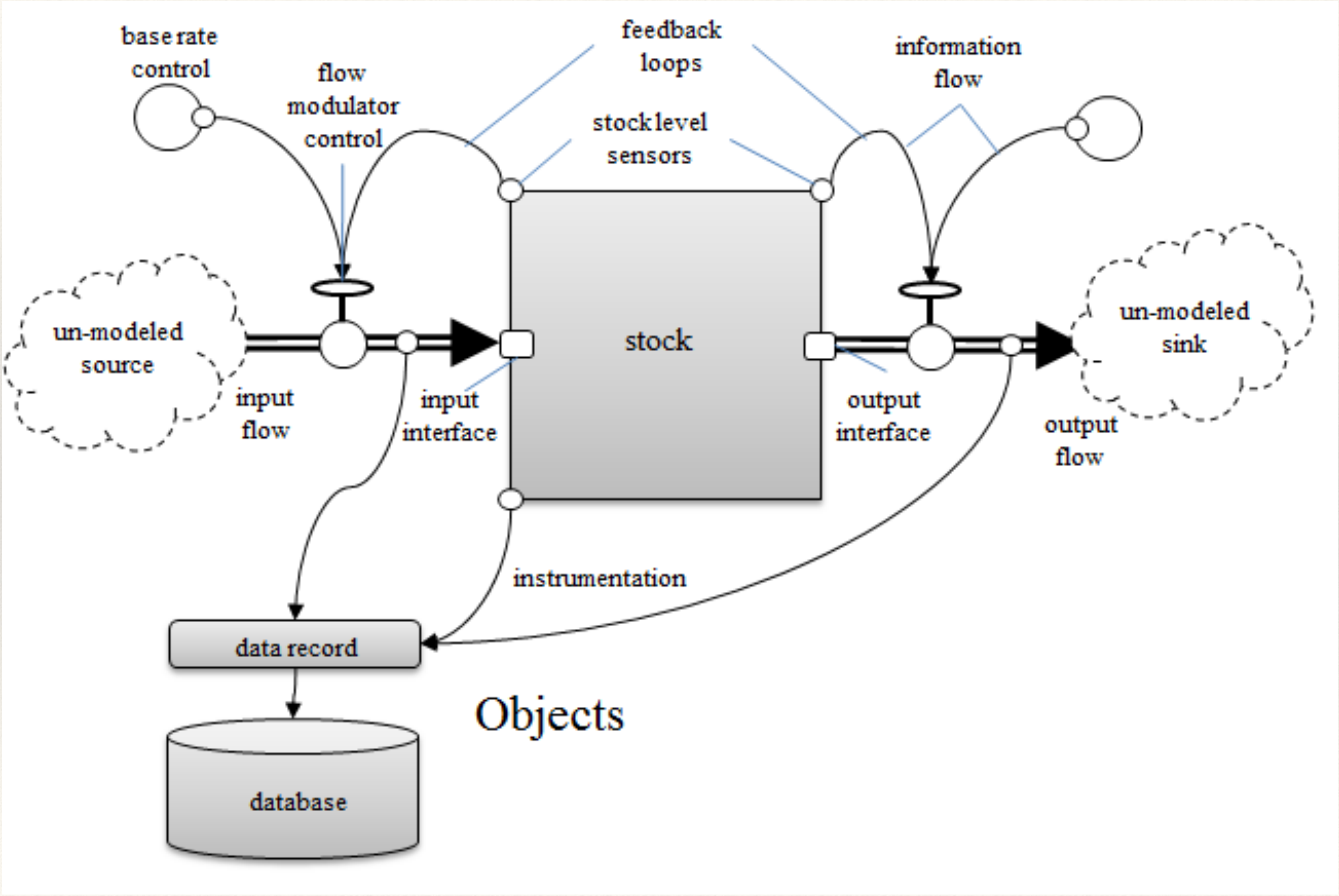
Dynamic Systems

Nature



Dynamic Systems

Human Engineered



Dynamic Systems

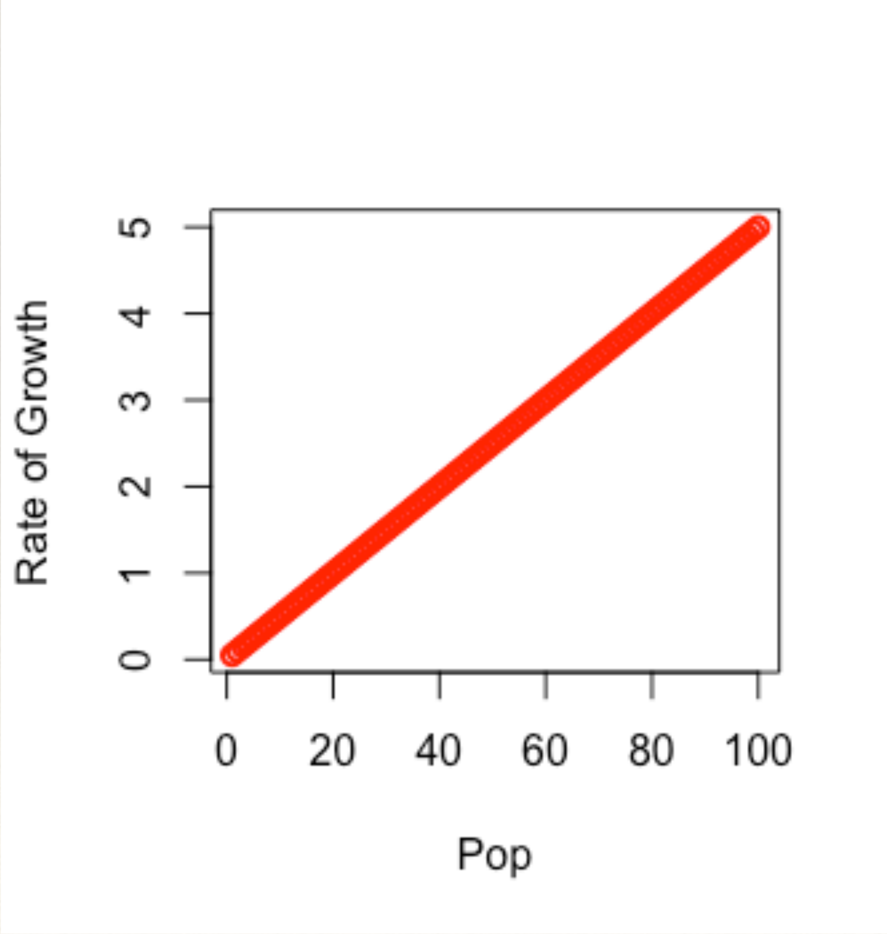
- dynamic systems often have **feedback** loops
  - positive feedback
  - negative feedback
- feedback loops often lead to highly non-linear responses
- difference and differential equations: basically describe how the state evolves through time

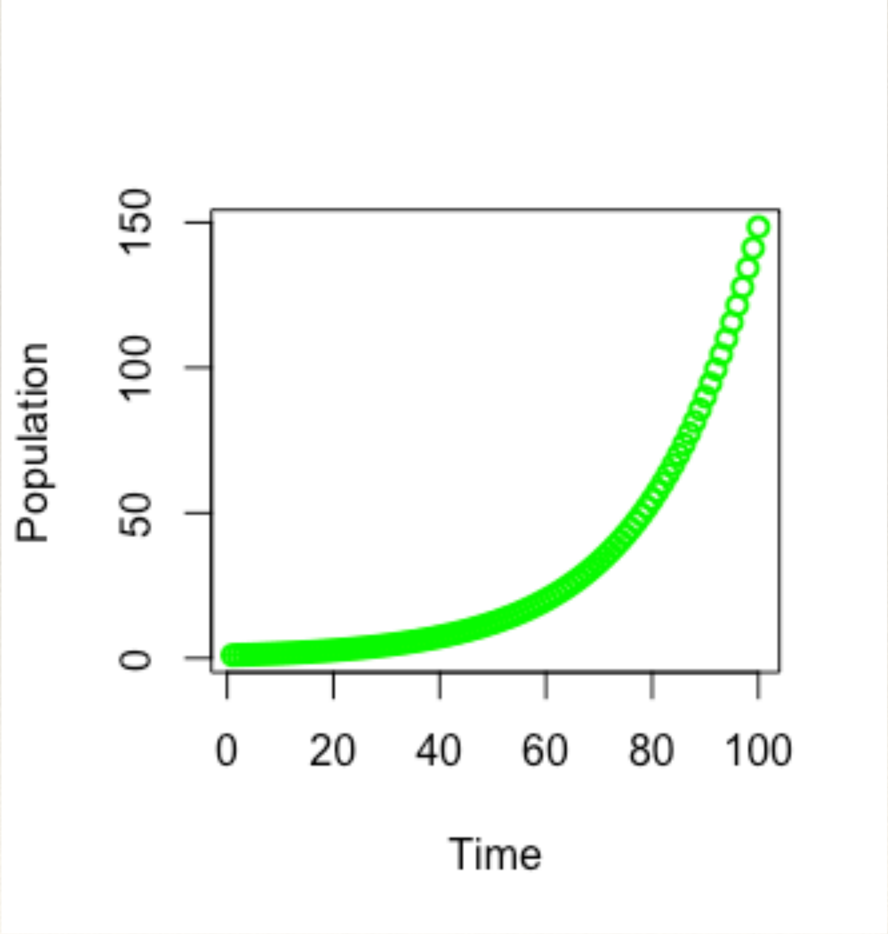
Dynamic Systems

- Dynamic system may lead to stable or unstable states over time
  - stable …converge over time to a set of values or a repeated pattern
  - unstable…grow to infinity
  - chaotic - high sensitivity to initial conditions
  - for the same dynamic system (same set of equations), whether you are stable can depend on initial conditions and parameters

Exponential Growth - Simple Dynamic System

- rate of growth(change) = r * population(density)
- differential equation
- dP / dt = rP



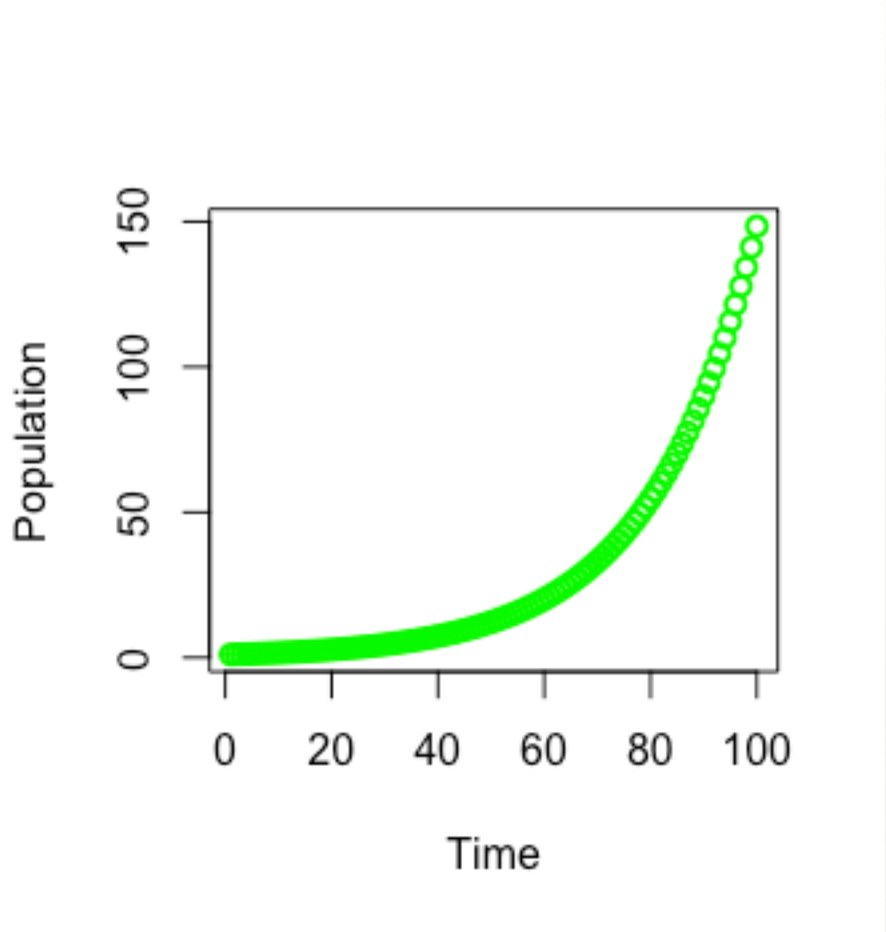


Exponential Growth - Simple Dynamic System

- differential equation
- $dP/dt = rP$
- an analytic solution exists so we can write Population as a function of time (integrating both sides)
  - $P = P0 * \exp(rt)$
  - This is a regular input-output function - that gives population after some time t

Exponential Growth - Simple Dynamic System

```
#'  Simple population growth
#' @param T  period of growth
#' @param P initial population
#' @param r intrinsic growth rate
#' @return population at time T
#'
exppop = function(T,P0,r) {
   P = P0 * exp(r*T)
   return(P)
}
```



Exponential Growth - Simple Dynamic System

But what if we couldn’t ‘solve’ it analytically ????

Integrate the differential equation step by step

Also called numerical integration!

R has tools to help you do this!

First you need to code your differential equation as a function

Integration, or Solving Differential Equations

- We want the value of the dependent variation (population) over a range of values for independent variable (time)
- We know how dependent variable is changing (that's the differential equation) dP/dt = rP
- For each P we can approximate the next P after a small time period
  - Pt+1 = P + dP/dt. Timestep
  - But as P changes dP/dt changes so we have to keep time step small (really small if possible)

Integration, or Solving Differential Equations