Chapter 1

Introduction

1.1 DSL Technologies

Since the beginning of the Internet age, new applications have continually demanded more bandwidth.

Recently this trend has been driven by high-definition video streaming and file sharing applications such as bittorrent. Digital Subscriber Lines (DSL) technologies have played an important role in the delivery of higher bandwidth broadband Internet access because of the relatively low cost of deploying DSL networks over existing telephone lines originally used for dial-up access.

Although fibre optic networks are a much more attractive technology in terms of achievable data rates, DSL is still much more prevalent today than fibre due to the significantly lower capital expenditure required. It is predicted that by 2012, only 4% of access connections in Western Europe will be from FTTx, with almost 80% still provided over xDSL[4].

Since their inception, DSL technologies have matured from ADSL (8 Mbits downstream) through to ADSL2/2+ (24 Mbits downstream) and most recently to VDSL2. VDSL2 can deliver up to 100Mbps symmetrically to the customer premises over short loop lengths ($<$ 250m). As the frequency range utilised by the DSL technology is increased (up to 30MHz for VDSL2), the depth of fibre penetration into the access network must be increased to shorten the loop lengths or the maximum possible speeds will not increase. In this manner, xDSL provides an evolutionary pathway to increased data rates through the progressive deployment of fibre optic cables closer to the customer. It is for these reasons that DSL is likely to be a major provider of broadband Internet access for some time to come.

1.2 Motivation

As DSL technologies utilise the existing copper plant, which was originally designed for transmission in the 0-4kHz band used by the Plain Old Telephone Service (POTS) , they suffer from the limitations of the existing medium. In particular, at the high frequencies used by DSL (up to 30 MHz in VDSL2) there is significant attenuation of the transmitted signal and leakage of signal power into adjacent lines through electromagnetic induction. This leakage of signal power is known as crosstalk.

Crosstalk is a major limiting factor in the achievable data rates of DSL deployments and there is thus far no standard method to mitigate its effects. The modulation scheme used in the most prominent xDSL technologies is Discrete Multi-Tone (DMT). The frequency spectrum is divided up into multiple independent sub-channels which can be individually modulated with a number of bits per symbol based on the Signal-Noise Ratio (SNR). Crosstalk reduces the SNR on the DMT sub-channels in the victim DSL line, reducing the amount of bits per symbol that can be transmitted on that sub-channel at the same error rate.

Over recent years, much research has been undertaken into techniques to reduce the data rate degradations caused by crosstalk. These techniques are known as Dynamic Spectrum Management (DSM). DSM techniques improve the performance in DSL systems by more intelligently allocating signal power on the DMT sub-channels. The purpose of DSM algorithms is to increase the possible data rates in a bundle of DSL lines through mitigation of the effects of crosstalk. DSM techniques may also allow a DSL network to support the same data rates to each user at a lower total power. This is known as “Green DSL” and is currently an active topic of research in DSM applications [5] [6] [7]. Although it has been demonstrated that DSM techniques can produce large performance gains, many of the currently known algorithms suffer from either intractable solution times or convergence problems.

This thesis concentrates on the design and performance of level 2 DSM algorithms because it is expected that if they can be designed to operate in a tractable time frame, they provide certain unique advantages over level 1 and level 3 techniques. In this case tractable is defined as a time that makes it practically applicable to a real DSL system, on the order of hours or less.

A brief definition of a level 1 DSM algorithm is one which is distributed, that is, DMT sub-channel bit and power assignment is primarily handled by an algorithm within the CP modem with some guidance from a Spectrum Management Centre (SMC). Popular examples of these types of algorithms are Iterative Water-Filling (IWF) [8], Autonomous Spectrum Balancing [9] and Band Preference algorithms [10]. In all of these algorithms, it is required that the modems execute their bit-loading algorithms individually in an iterative fashion. At each iteration, the modem sees the new background noise power generated by the crosstalk from all other modems in the previous iteration. Each modem should approximately converge to the desired performance margin γ_m. This iterative behaviour is illustrated in Figure 1.1.

This distributed, iterative behaviour means that the speed of the DSL network initialisation as a whole is limited by algorithm execution speed on each modem multiplied by the number of iterations required.

In level 3 DSM, crosstalk pre-compensation enables partial or complete cancellation of crosstalk [11]. While this is the ideal case for mitigating crosstalk, it requires extensive deployment of new hardware with significantly increased complexity. It also requires that the signals are co-generated, i.e. on the same hardware. This restricts the market for level 3 DSM solutions to scenarios where it is possible to co-locate modems at the customer premises, for example, at newly constructed shared accomodation. This requirement also makes level 3 DSM difficult and costly to retrofit into existing DSL networks.

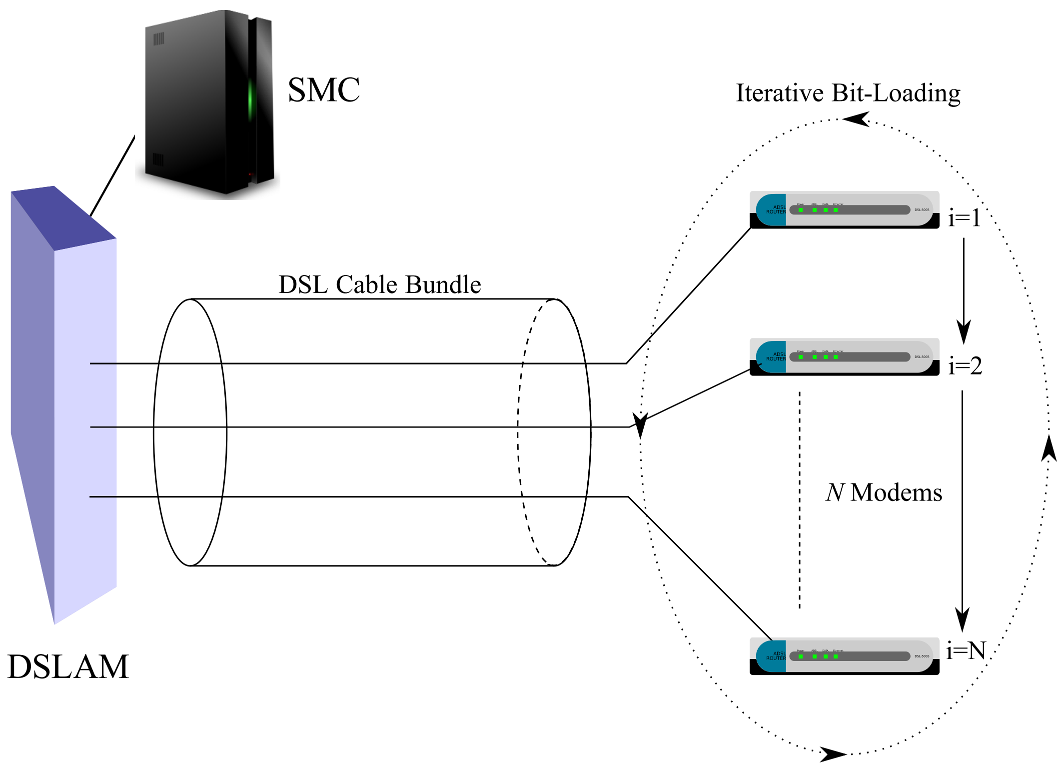
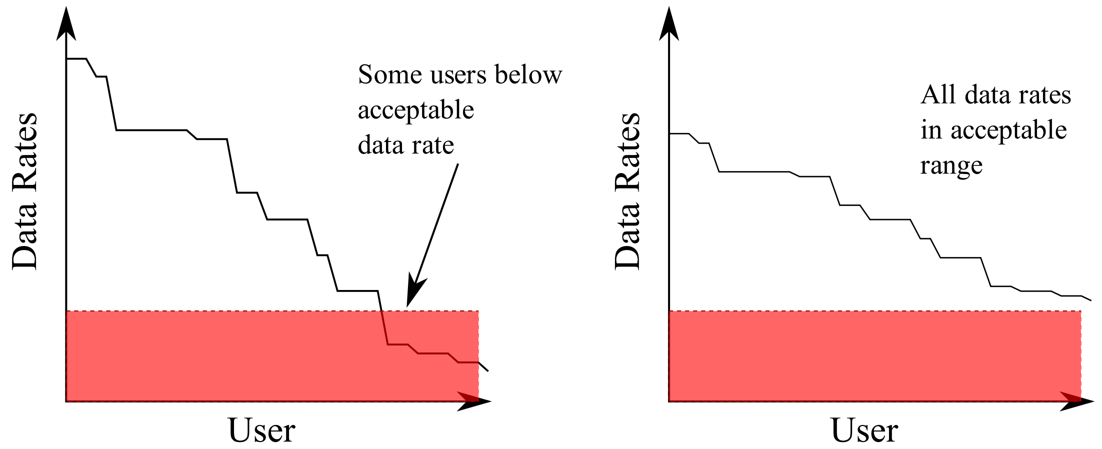


Figure 1.1: Illustration of iterative behaviour in Level 1 DSM operation

In level 2 DSM, all the bit and power allocations are centrally calculated and then distributed to the modems. This approach has several advantages over level 1 and level 3 DSM. Firstly, the performance margins γ_m will be tight to the desired figures when the network is initialised, which is not the case for level 1 DSM. Secondly, there is no settling time for network initialisation as in level 1 DSM, as soon as the bit and power allocations are calculated, the network can begin operation. The limiting factor is now the speed of the level 2 DSM algorithm. Level 2 DSM does not require the addition of new costly and computationally expensive DSP functionality which is required for level 3 DSM.

Figure 1.2 shows an example of the capabilities of a DSM level 2 algorithm in practise. In Figure 1.2(a), the data rates in the bundle are shown in descending order. It may be decided by the network operator that some of these lines are below a minimum level of service, illustrated by the red box. By setting appropriate data rate targets on the lowest data rate lines, a DSM level 2 algorithm can bring these lines up to an acceptable level of service, whilst slightly reducing



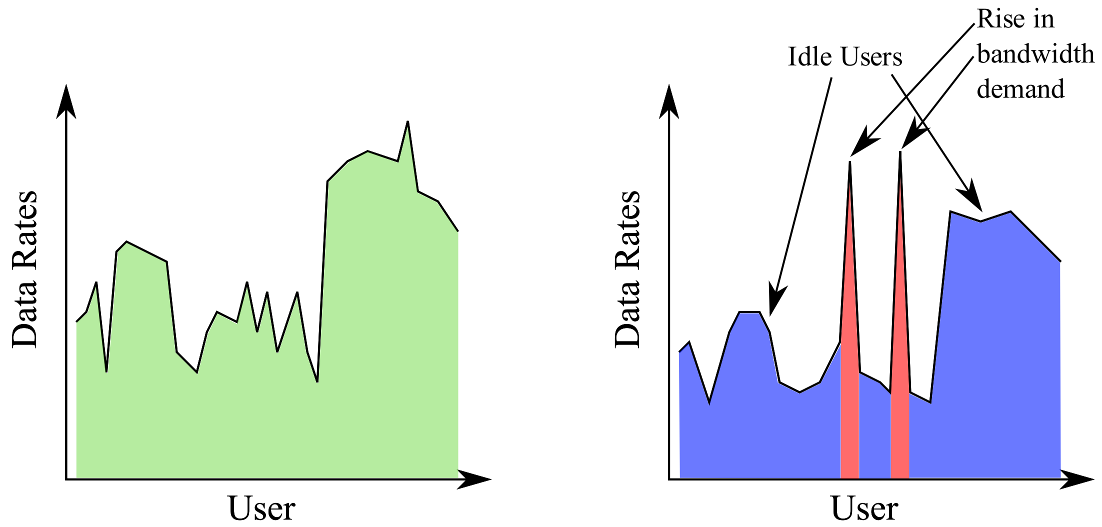
(a) Before DSM

(b) After DSM

Figure 1.2: Data rates ordered from highest to lowest in a DSL bundle before and after DSM is utilised to raise the lowest data rates in the bundle

the data rates of the other lines in the bundle. This is illustrated in Figure 1.2(b). This operation could be completed seamlessly without any service interruption as in level 2 DSM the required spectra are calculated off line in the SMC.

Another possible application of level 2 DSM rate adaption is shown in Figure 1.3. Figure 1.3(a) shows the data rates in the DSL bundle during normal operation. At some point two users in the network are utilising a service (e.g. a large FTP transaction) which could utilise higher bandwidth if it was available. In response to this, the SMC allocates new spectra to all the bundle users which deliver much higher data rates to those users at the expense of the other users' data rates in the bundle. This is shown in Figure 1.3(b). If the other users in the bundle are either idle or have a low bandwidth requirement at that time, there will be no drop in performance perceived by these users. Providing the DSM level 2 algorithm can be executed in a time frame on the order of the typical service time then this rate adaption could be used to further increase the efficiency of the available bandwidth in the entire DSL bundle by constantly adjusting the data



(a) Steady state rates in a DSL network (b) Rate changes after "Real-Time" DSM adaption

Figure 1.3: Data rates before and after a "Real-Time" DSM rate adjustment

rates in the bundle to better match the demand of each user.

The major disadvantage of level 2 DSM is that the channel and crosstalk coupling gains must be known for the algorithm to function. Fortunately, there has been much recent research into crosstalk and channel estimation and it is now possible with good accuracy using slightly modified hardware [12] [13] [14].

At present, known level 2 DSM algorithms are too computationally expensive to be used on realistic DSL bundle sizes (50+ cables) and are merely of theoretical interest. The major theme of this thesis is therefore to find level 2 DSM algorithms which are fast, tractable for large bundle sizes and still provide good data rate performance.

1.3 Thesis Overview and Contributions

Chapter 2 introduces the fundamentals of DMT technology as well as the channel and crosstalk models required for the DSL system model employed in this thesis. The concept of bit-loading in DMT is introduced and the spectrum allocation problem defined. A variety of existing DSM algorithms are presented and their relative merits discussed.

In Chapter 3, a new DSM level 2 algorithm known as Multi-User Incremental Power Balancing (MIPB) is derived. This is shown through simulation results to achieve performance very close to that of the optimal algorithm when used with "equal line weightings" in a variety of different DSL network configurations. Equal line weightings refers to the values of w_n in the OSB algorithm. When all the values of w_n are set to unity, the algorithm produces a relatively fair allocation of data rates betweens the DSL users. The complexity of the algorithm is investigated and it is found to have a vastly reduced complexity compared to other well known DSM level 2 algorithms. Later in this chapter, the parallelisation of DSM level 2 algorithms including MIPB is investigated which exploit modern multicore hardware to further reduce the execution times of MIPB.

In Chapter 4 of this thesis, it is shown that a greedy algorithm combined with a new rate search algorithm can produce nearly optimal spectrum allocations for a variety of DSL networks. In this case, an optimal solution is defined as a point on the rate region boundary (see Section 2.9.1). A number of different rate search algorithms are derived and simulations are used to examine their complexity. A Power Spectral Density (PSD) vector caching algorithm is proposed and the potential performance improvements are demonstrated through simulation. For a 7 User ADSL2+ network, the best of the new algorithms reaches a solution 3850 times faster on commodity x86 hardware than ISB.

The work presented in chapters 3 and 4 is based on previous work published on greedy algorithms in [15] and [16]. A patent application is currently being pursued for the work in Chapter 4 and will be submitted for publication in the near future.

Finally, in Chapter 5 conclusions are drawn and a section for possible future work is included.

Chapter 2

Background

2.1 Digital Subscriber Lines

Digital Subscriber Lines (DSLs) have become a very common means of providing high speed Internet connectivity to both residential and business customers. In Western Europe there were an estimated 85 million residential broadband connections by the end of 2007 [4]. Approximately 80% of these were provided by DSL.

As DSL is delivered over existing telephone lines, it has been instrumental in enabling the "broadband revolution" as it provides a technology for dramatically increasing bit rates compared to dial-up access, without the need for laying new cables. The original ADSL standard enabled downstream bit-rates of up to 8 Mbits to be obtained on short loops (approx. 1km), which previously only supported 56kbits over a V.92 voice band connection.

As the demand for high bandwidth applications has increased, new DSL technologies have emerged to fulfil the requirements. Upgrading to higher bit-rates is often simply a matter of upgrading the DSL equipment in the Central Office (CO) and Customer Premises (CP), although sometimes loop lengths need to be shortened to achieve significant improvement.

2.2 Transmission Impairments in xDSL

The original telephone loop plant was intended for transmission of voice signals in the 0-4kHz band. DSL systems utilise the spectrum from 25kHz up to 30MHz, depending on the particular flavour of DSL in use. As a result of using the copper medium well outside its original design limits, data transmission in DSL can be problematic.

Signals travelling over the copper telephone lines are generally subject to high levels of attenuation, which increase with increasing frequency. The unshielded copper pairs are also susceptible to noise from various sources including crosstalk from neighbouring copper pairs, radio frequency ingress and impulsive noise. The dominant noise source in DSL is crosstalk from neighbouring lines and is typically at least 10-20dB larger than the background noise [17]. The management of crosstalk in DSL is the main subject of this thesis.

In general, the effects of impulsive noise and RF ingress are not affected by the bit-loading or spectrum balancing techniques investigated in this thesis. Recently, some investigation has been carried out into noise-margin optimisation which may help to mitigate the effects of impulse noise when its characteristics vary with freqeuncy [18]. However, in this thesis, impulse noise and RF ingress are considered to be part of the background noise and only the nature and effects of crosstalk noise are considered.

2.2.1 Crosstalk

Crosstalk noise is caused by leakage of signal power from one DSL line into another. The coupling between neighbouring DSL lines is caused by electromagnetic induction and occurs where DSL cables overlap within a bundle of DSL cables. The signals in a particular line create an electromagnetic field which in turn induces a current in neighbouring lines.

There are two major types of crosstalk known as Near-End Crosstalk (NEXT) and Far-End Crosstalk (FEXT), which are illustrated in Figure 2.2.1. NEXT is caused by leakage of a user's signal into a receiver at the same end as the

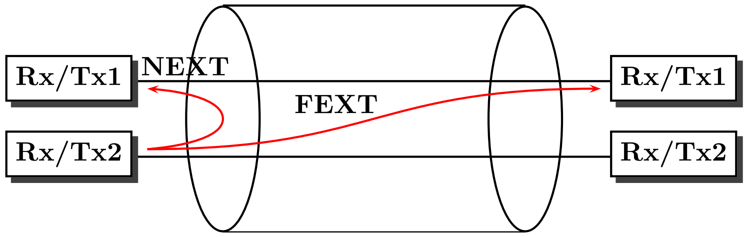


Figure 2.1: NEXT and FEXT in DSL

transmitter. Conversely, FEXT is caused by leakage of of signal power from a transmitter at one end of a DSL into a receiver on the other side.

In the majority of DSL systems, NEXT can be eliminated by employing Frequency-Division-Duplexing (FDD). In FDD, the frequency bands used for downstream and upstream transmission are separated.

The magnitude of crosstalk coupling varies greatly with frequency but is typically stationary over time. It is possible that the coupling functions may change in areas with large changes in humidity or temperature but these effects are usually negligible.

2.3 DSL Channel Models

Throughout this thesis, models for both the direct channel and crosstalk gains will be utilised in order to simulate a variety of DSL network configurations. Models are utilised rather than real data because measurement of these values is at present a time consuming process, although there is a large effort currently being undertaken to standardise the measurement process for the purposes of DSM.

The direct gain channel models used in this thesis are calculated by the long line approximation for the channel shown in [19]. The approximation is that for a long DSL line the input line impedance V_1 is equal to the characteristic impedance V_0. Given this assumption, for a line of length d with characteristic impedance Z_0, load impedance Z_l, source impedance Z_s and propagation constant γ, the voltage transfer function is given by

$$H = \frac{Z_0 \cdot sech(\gamma d)}{Z_s \cdot [\frac{Z_0}{Z_l} + tanh(\gamma d)] + Z_0 \cdot [1 + \frac{Z_0}{Z_l} \cdot tanh(\gamma d)]} \tag{2.1}$$

The characteristic impedance Z_0 of the DSL line is given by the relationship

$$Z_0 = \sqrt{\frac{Z}{Y}} \tag{2.2}$$

Where Z and Y are the impedance and admittance per unit length respectively.

The value of the propagation constant γ is given by

$$\gamma = \sqrt{Z \cdot Y} \tag{2.3}$$

2.3.1 RLCG Parameter Curve Fitting

In order to calculate the frequency dependent values of R,C,L and G for a particular length of DSL line, parameterised models are used to ensure the models follow smooth curves with frequency [19]. These models are particularly useful as practical measurements suffer from relatively large margins of error.

The parameterised model for $R(f)$ is expressed as

$$R(f) = \frac{1}{\frac{1}{\sqrt[4]{r_{0c}^4 + a_c \cdot f^2}} + \frac{1}{\sqrt[4]{r_{0s}^4 + a_s \cdot f^2}}} \tag{2.4}$$

where r_{0c} is the copper DC resistance and r_{0s} is the steel resistance. The values a_c and a_s are constants which characterise the increase in resistance with frequency due to the skin effect.

The smoothed model for $L(f)$ is as follows

$$L(f) = \frac{l_0 + l_\infty (\frac{f}{f_m})^b}{1 + (\frac{f}{f_m})^b} \tag{2.5}$$

where l_0 and l_∞ are the low and high frequency inductance respectively and the values b and f_m are chosen to characterise the transition from low to high frequencies.

For the smoothed capacitance $C(f)$, the model is

$$C(f) = c_\infty + c_0 \cdot f^{-c_e} \tag{2.6}$$

Finally, the smoothed conductance $G(f)$ is given by

$$G(f) = g_0 \cdot f^{g_e} \tag{2.7}$$

Throughout this thesis, the majority of results will be calculated using models for AWG 24 wires. Table 2.1 shows the parameters used to model the RCLG parameters for AWG 24. Parameter values for various cable types are available in [20]. For the purposes of analsying DSM algorithms, the specific cable model used is not particularly important, as long as the same model is used for all comparisons.

Figure 2.2 shows the gain over the ADSL frequency range for different lengths of AWG 24 wire given by the smoothed RCLG models, when these values are used to calculate the transfer functions given by equation 2.1.

2.4 Crosstalk Models

The basis for the crosstalk models used in this thesis are the 1% worst case crosstalk models from [19]. The 1% worst case models produce crosstalk coupling functions that follow smooth curves with frequency. Although this is not generally

r_{0c}	r_{0s}	a_c	a_s
174.55888 ohms/km	∞ ohms/km	0.0530734	0.0
l_0	l_∞	b	f_m
617.295 μH/km	478.97 μH/km	1.1529	553.76 kHz
c_∞	c_0	c_e	
50 nF/km	0.0 nF/km	0.0	
g_0	g_e		
234.874 fS/km	1.38		

Table 2.1: Characteristics of AWG 24 UTP [1]

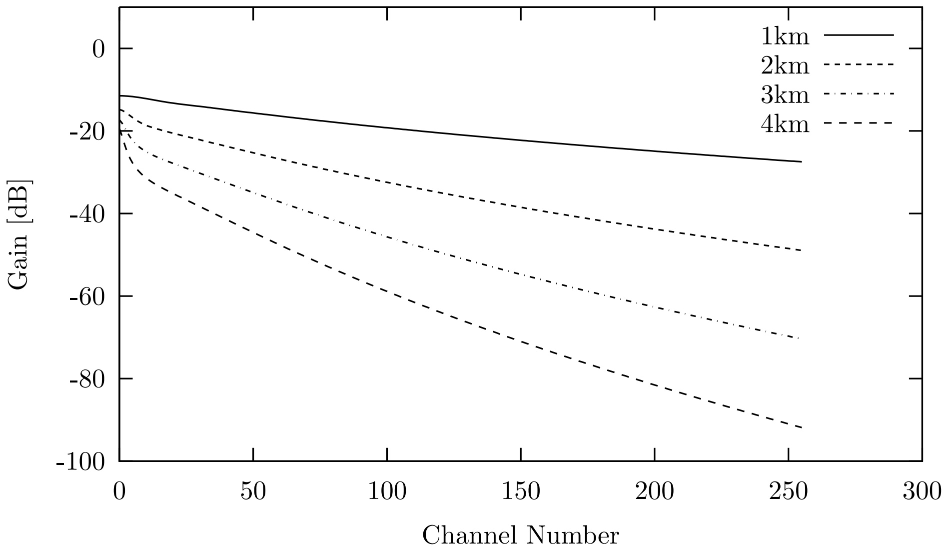


Figure 2.2: Gain vs DSL channel number for varying lengths of AWG 24 UTP

the case in practise, these models ensure that the calculated crosstalk coupling will be worse than in 99% of real life twisted pair scenarios.

2.4.1 FEXT Modelling

The simplified 1% worst case ETSI FEXT model [21] is given by:

$$|H_{FEXT}(f)|^2 = N^{0.6} K_{FEXT} f^2 |H_{channel}(f, L)|^2 \tag{2.8}$$

Where f is the frequency and L is the length of overlapping cable in the bundle. In a practical DSL system, the number of cables that overlap and their overlap distance vary along the length of the cable between the Central Office (CO) end and the Customer Premises (CP). To model this, equation 2.8 is used in conjunction with the rules in [22] to calculate the crosstalk coupling functions in a more complex scenario such as a mixed CO and Remote Terminal (RT) deployment.

2.4.2 Statistical Crosstalk Model

The 1% worst case model tends to over-estimate the crosstalk coupling functions between DSL lines [18]. Additionally it assumes that the crosstalk coupling functions are independent of the relative positions of the cables in the bundle.

In order the improve the realism of the 1% worst case model, it was recently extended with a statistical approach [23]. This model was developed by the DSL industry through the analysis of raw FEXT coupling data measured in a 100x100 DSL binder [24] consisting of four sub-binders. From the model deduced from the coupling data, a adjustment is applied to the 1% worst case model to produce more realistic FEXT coupling values. The modified crosstalk coupling function between any two lines j and k is now given by:

$$|H_{FEXT}^{j,k}(f)|^2 = |H_{FEXT}^{j,k}(f)|^2 \times 10^{\frac{X_{dB}}{10}} \tag{2.9}$$

Where the value X_{dB} is a value picked from a Beta distribution. A random draw from this distribution for a 100 pair DSL bundle is made available in [25].

2.5 Discrete Multi-Tone Modulation

The modulation scheme that has been standardised for DSL is Discrete Multi-Tone (DMT). It was first described in [26] and [27]. DMT is designed to be able to operate over channels with significantly different channel and noise characteristics over the transmission bandwidth, such as those encountered in DSL systems.

In DMT, the channel bandwidth is divided up into a set of independent sub-channels also known as tones. The sub-channel bandwidth is chosen to be sufficiently narrow so that the transfer function is approximately flat across its bandwidth. This means that each sub-channel is free from ISI and thus complex time domain equalisation is not required. Figure 2.3 illustrates the division of channels in a FDD DMT system.

Each DMT sub-channel can be viewed as an independently M-QAM modu-

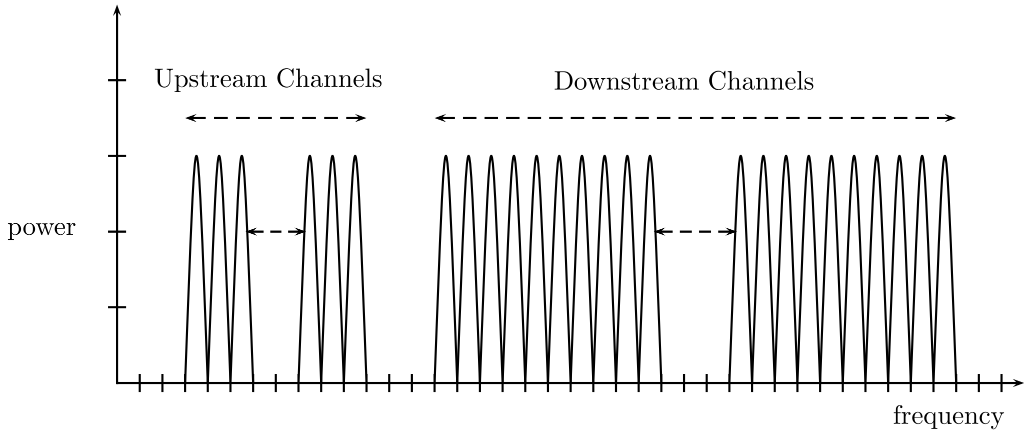


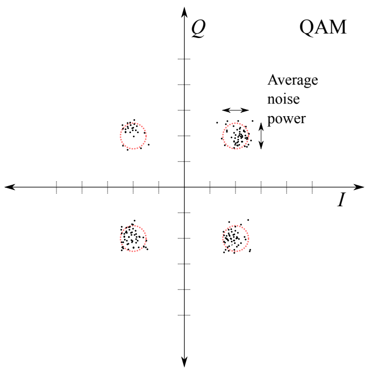
Figure 2.3: Illustration of a Frequency Division Duplex DMT system

lated channel for data transmission. The number of bits loaded on each DMT sub-channel is determined by the constellation size. The symbol error probability P_e is dependent on the constellation size and the power allocated to that sub channel. This is roughly illustrated in Figure 2.4. Figure 2.4(a) shows a square QAM constellation, which encodes 2 bits per symbol. The red circles approximately represent the average noise power and the dots represent detection of received symbols which all fall within the correct detection windows, resulting in error free transmission. In Figure 2.4(b), the result of increasing the constellation size to 16-QAM is illustrated. The transmit signal power and noise power remain unchanged but because of the smaller detection windows, some of these symbols may be decoded incorrectly, resulting in symbol errors. This is remedied in Figure 2.4(c) which illustrates the result of increasing the transmit signal power on this tone. The signal-noise ratio is increased and all of the decoded symbols fall within the correct detection windows resulting in error free transmission.

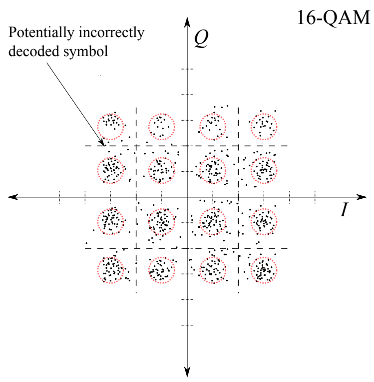
The process of assigning the number of bits per symbol (constellation size) and power to each DMT sub-channel is known as "bit-loading".

2.5.1 Bit-Loading

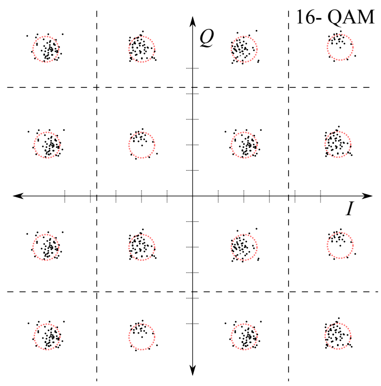
In DMT systems, the process of assigning bits to individual sub-channels is known as "bit-loading". When considering a single DMT user, the maximum capacity



(a) 4-QAM



(b) 16-QAM with low SNR



(c) 16-QAM with high SNR

Figure 2.4: Illustration of the relationship between constellation size, noise power and symbol error probability for M-QAM

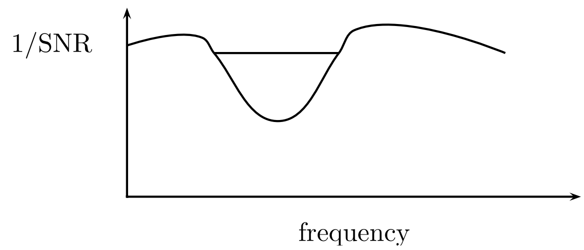


Figure 2.5: Illustration of the Water-Filling concept. The depth of the water level indicates the amount of energy allocated at that frequency

of a system with a fixed power budget is well known. It is given by the so called "Water-Filling" solution first introduced by Claude Shannon in 1948 [28]. The water filling solution is so named because the mathematical solution resembles the concept of "pouring" energy in the channels with the best Signal-Noise-Ratio (SNR). This is illustrated in Figure 2.5 which depicts the inverse signal to noise ratio graphed against frequency.

In an ideal system, the waterfilling solution gives the maximum capacity of the system. A practical communications system must implement a data rate lower than capacity to be reliable. The so called "SNR-gap" or "Shannon-gap" is a measure of the gap from capacity for a communications system. The derivation for a DSL system based on M-QAM is shown in the following Section [18].

2.5.2 The SNR-Gap Approximation

The Shannon capacity of an Additive White Gaussian [1] Noise (AWGN) channel is given by the following formula:

$$C(k) = \log_2(1 + SNR(k)) \tag{2.10}$$

This is the maximum rate at which error free transmission is possible, given unlimited coding complexity and decoding delay. Assuming that a DSL system utilises an uncoded square M-QAM constellation, the probability of a symbol

[1] Crosstalk approaches gaussian behaviour as the number of crosstalkers increases, the central limit theorem loosely applies [29]

error on a particular sub-channel is given by

$$P_e \approx N_e Q\left(\sqrt{\frac{3}{M-1}SNR}\right) \tag{2.11}$$

Where the $Q(x)$ function gives the probability that a Gaussian variable with a unit mean and variance exceeds the value x and N_e is the number of nearest neighbours in the M-QAM constellation.

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du \tag{2.12}$$

The $Q(x)$ function can also be written in terms of the the standard error function $erf(x)$.

$$Q(x) = \frac{1}{2}\left(1 - \text{erf}\left(\frac{x}{\sqrt{2}}\right)\right) \tag{2.13}$$

$$Q^{-1}(x) = \sqrt{2}\,\text{erf}^{-1}(1-2x) \tag{2.14}$$

The number of bits that can encoded by an M-QAM constellation is given by

$$b = \log_2(M) \tag{2.15}$$

By rearranging equation 2.11 for M, we obtain the following expression for the number of bits encoded on a sub-channel

$$b = \log_2\left(1 + \frac{SNR}{\frac{1}{3}\left(Q^{-1}(\frac{P_e}{N_e})\right)^2}\right) \tag{2.16}$$

By comparing this expression with equation 2.10, we can define the uncoded channel gap $\Gamma_{uncoded}$ as

$$\Gamma_{uncoded} = \frac{1}{3}\left(Q^{-1}(\frac{P_e}{N_e})\right)^2 \tag{2.17}$$

Thus, the equation for the number of bits that can be loaded on a sub-channel is given by

$$b = \log_2\left(1 + \frac{SNR}{\Gamma_{uncoded}}\right) \tag{2.18}$$

The SNR-gap Γ represents the distance from the theoretical maximum capacity at a particular symbol error probability. In a practical system, a performance margin is employed to ensure the system always performs at least as well as its designed symbol error probability under unforeseen noise conditions.

The SNR-gap can also be reduced by employing forward error correction. The reduction in gap is called the coding gain γ_{cg}. The resulting complete formula for Γ is shown below.

$$\Gamma = \Gamma_{uncoded} + \gamma_m - \gamma_{cg} \tag{2.19}$$

In a typical DSL system used for data transmission, the target symbol error probability is set at $1e^{-7}$ which corresponds to an uncoded gap of $9.8dB$ as calculated by equation 2.17. Typical values of γ_{cg} and γ_m are $3dB$ and $6dB$ respectively which results in a value of $\Gamma = 12.8dB$ at $P_e = 1e^{-7}$.

2.6 DSL System Model

In this section, the system of equations which describe a multi-user DSL system will be derived. In essence, this system generates the vector of Power Spectral Densities (PSDs) on each line which are required to support a particular vector of bits on that tone, with a given performance margin.

It is assumed in this thesis that crosstalk coupling happens on a per tone basis, which is true when DMT blocks are received synchronously or when "Zipper" [30] DMT is used. Zipper DMT adds a cyclic suffix and pulse shaping to the transmitted DMT frames along with windowing of the received signal to minimise the sidelobes of the received signal, reducing out of band leakage to a negligible level.

In a multi-user DSL system with N users and K tones with FEXT coupling, the achievable bit-loading on line n tone k is as follows [31]:

$$b_n(k) = \log_2\left(1 + \frac{p_n(k)|h_{nn}(k)|^2}{\Gamma\left(\sigma_n^2(k) + \sum_{j\neq n}^{N} p_j(k)|h_{jn}(k)|^2\right)}\right) \tag{2.20}$$

Re-arranging the formula (2.20) letting $f(b_n(k)) = \Gamma(2^{b_n(k)} - 1)$ we obtain:

$$p_n(k) - f(b_n(k)) \sum_{j \neq n}^{N} p_j(k) \frac{|h_{jn}(k)|^2}{|h_{nn}(k)|^2} = f(b_n(k)) \frac{\sigma_n^2(k)}{|h_{nn}(k)|^2} \tag{2.21}$$

For a particular tone k, equation (2.21) is an N-dimensional linear system of equations, where N is the number of lines. This can be written in matrix form:

$$A(k)P(k) = B(k) \tag{2.22}$$

where

$$A(k)_{ij} = \begin{cases} 1, & \text{for } i = j \\ \frac{-f(b_i(k))|h_{ji}|^2}{|h_{ii}|^2}, & \text{for } i \neq j \end{cases} \tag{2.23}$$

$$P(k) = [p_1(k) \ldots p_i(k) \ldots p_N(k)]^T \tag{2.24}$$

$$B(k) = \left[\frac{f(b_1(k))\sigma_1^2}{|h_{11}|^2} \ldots \frac{f(b_i(k))\sigma_i^2}{|h_{ii}|^2} \ldots \frac{f(b_N(k))\sigma_N^2}{|h_{NN}|^2} \right]^T \tag{2.25}$$

This can be solved for $P(k)$ via direct inversion or by LU/QR decomposition. $P(k)$ is a vector which contains the power value required on each line to support a vector of bits $b(k)$ on tone k. (Note $B(k)$ is a function of $b(k)$).

2.7 Discrete Waterfilling

The waterfilling algorithm provides the optimal solution to maximise the capacity of the channel for a single user with a continuously variable bit distribution. In a real communications system, each channel must transmit an integer number of bits. When the waterfilling analogy is adapted for a real DMT system, it is known as discrete waterfilling.

There are two variations of the discrete waterfilling problem, rate-adaptive (RA) and fixed-margin (FM). In RA mode, the objective is to maximise the bit-rate for a given power budget. Formally, the single-user RA problem is stated as follows

$$max \ \ R = \sum_{k=1}^{K} b(k) \tag{2.26}$$

$$s.t \sum_{k=1}^{K} p(k) \quad \leq P_{budget} \tag{2.27}$$

Where k is the tone index and K is the number of tones.

In FM mode, the objective is to minimise the energy required to transmit at a particular bit rate. The single-user FM problem is stated as follows

$$min \quad \sum_{k=1}^{N} p(k) \tag{2.28}$$

$$s.t \quad \sum_{k=1}^{N} b(k) \quad \geq B_{target} \tag{2.29}$$

The most widely used discrete waterfilling algorithm is known as the Levin-Campello (LC) Algorithm [32].

The solution to the single-user RA and FM modes is surprisingly simple. In RA mode, the LC algorithm adds a bit to that tone which requires the least energy to do so until the output is equal to the power budget. In FM mode, the algorithm adds bits the lowest energy tone until the target data rate is met.

The RA LC algorithm is shown is Algorithm 1. $|H(k)|$ is the gain of channel k and $\sigma(k)$ is the background noise power on channel k. The FM version of the LC algorithm is shown in Algorithm 2.

Algorithm 1 Levin-Campello Algorithm in Rate Adaptive Mode

repeat

  $\text{argmin}_k \frac{\Gamma(2^{b(k)}-1)\sigma(k)}{|H(k)|} - \frac{\Gamma(2^{b(k)+1}-1)\sigma(k)}{|H(k)|}$

  Increment $b(k)$

until $\sum p(k) == P_{budget}$

Algorithm 2 Levin-Campello Algorithm in Fixed Margin Mode

repeat

  $\text{argmin}_k \frac{\Gamma(2^{b(k)}-1)\sigma(k)}{|H(k)|} - \frac{\Gamma(2^{b(k)+1}-1)\sigma(k)}{|H(k)|}$

  Increment $b(k)$

until $\sum b(k) == B_{target}$

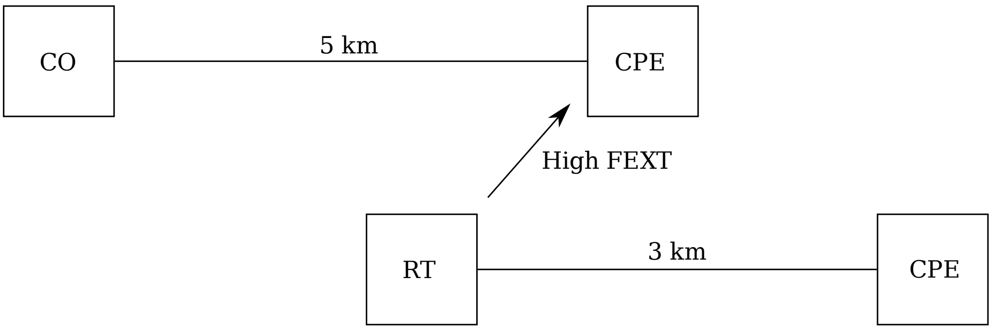


Figure 2.6: A two user DSL network which exhibits the near-far effect in the downstream direction

2.8 Spectrum Management

Although the discrete waterfilling algorithm provides the optimal solution for a single user DSL system, it performs very sub-optimally in a multi-user scenario. In a real DSL system a number of users (typically 50-100 in a bundle) will generate crosstalk into one another. If each modem executes a LC algorithm independently, the overall performance of the whole bundle will be sub-optimal. The level of sub-optimality depends on the network configuration. A particularly bad case is shown in Figure 2.6.

In the downstream direction, the modem in the RT causes a large amount of crosstalk into the CO line. As the CO line is long, it already has a limited achievable data rate due to attenuation. The large amount of crosstalk from the CO line causes a significant reduction in its performance.

To improve this situation, techniques known as Spectrum Management can be employed. The early forms of Spectrum Management are now known as Static Spectrum Management (SSM) methods. They are quite simplistic and involve setting limits on the transmit PSD in certain bands. SSM techniques help improve performance somewhat but are now considered obsolete in favour of more complex methods known as Dynamic Spectrum Management (DSM)[33] which is now a very active research topic [8] [17] [34] [9] [10] [35] [2].

2.9 Dynamic Spectrum Management

A DSL system employing Dynamic Spectrum Management (DSM) will utilise a Spectrum Management Centre (SMC) which monitors line conditions. The level of monitoring required depends on the type of DSM in use. The SMC will attempt to optimise the performance of the DSL system based on the conditions in the bundle. It may adjust parameters such as margin, power budget, power-spectral-density (PSD) masks and target data rates, depending on the DSM level employed by the SMC.

In essence, a DSM technique will attempt to solve the Spectrum Balancing problem and achieve a result as close to the optimum as possible. The Spectrum Balancing problem for a 2-user situation can be stated as shown in equation 2.30.

$$\max R_2 \quad \text{s.t.} \quad R_1 \geq R_1^{target} \tag{2.30}$$

More generally, the spectrum management problem is a maximisation of a weighted sum of the data rates in a bundle, where the weights are chosen to select the data rates of each user. The full expression for the weighted sum is shown in equation 2.31.

$$\max \sum_{n=1}^{N} w_n \sum_{k=1}^{K} \log_2 \left(1 + \frac{p_n(k)|h_{nn}(k)|^2}{\Gamma \left(\sigma_n^2(k) + \sum_{m \neq n}^{N} p_m(k)|h_{jm}(k)|^2 \right)} \right) \tag{2.31}$$

$$\text{s.t.} \sum_{k=1}^{K} p_n(k) \leq P_n \forall n \tag{2.32}$$

DSM techniques are divided into four categories, which are illustrated in Table 2.2. Each new level increases the achievable performance gains over the previous level, at the cost of increased co-ordination between DSL lines.

Level 0 DSM means that no DSM is employed and each modem will operate completely autonomously, which is the way traditional DSL systems operate. In level 1 DSM, an SMC will adjust DSL lines parameters individually, in order to improve performance and reduce crosstalk. This usually means adjusting the parameters of the discrete water-filling Algorithm [32] such as power budget. Iterative waterfilling [8] and Autonomous Spectrum Balancing (ASB) [9] are examples of level 1 DSM techniques.

In level 2 DSM, the spectra of managed DSL lines are jointly optimised and allocated by the SMC. This requires knowledge of the crosstalk coupling gains between the individual lines in the bundle. There are many examples of level 2 DSM algorithms developed in literature. The optimal performance is given by Optimal Spectrum Balancing (OSB) [17].

Iterative Spectrum Balancing [34] is based on the same framework as OSB with modifications to significantly reduce it's complexity. SCALE [35] is a spectrum balancing algorithm based on a successive convex relaxation of the original spectrum balancing problem. It achieves very close to the optimal result with a significantly lower complexity than OSB. It is not entirely clear at this stage precisely how it compares to OSB and ISB, as their implementation and complexity analysis are non-trivial and to date there has been no detailed analysis of it's complexity published.

Level 3 DSM requires signal level co-ordination of DSL modems. This is known as "vectored" [11] transmission. With vectored transmission, crosstalk can be, in principle, effectively cancelled. This results in the highest possible performance of the DSL bundle. Level 3 DSM is less flexible than level 1 and level 2 DSM as it requires significantly modified hardware with increased cost, due to computationally complex nature of crosstalk pre-compensation. It also necessitates physical co-location of transmitting modems which is particularly problematic in the upstream direction.

Table 2.2: Table of Dynamic Spectrum Management Levels

DSM Level	Description
0	No DSM
1	Some control of individual lines bit loading parameters
2	Joint spectral optimisation of multiple lines
3	Signal level co-ordination between multiple lines

2.9.1 Rate Regions

A rate region illustrates the region of possible rate vectors for a particular DSL network and spectrum balancing algorithm. In general, the larger the rate region that can be traced by a spectrum balancing algorithm, the better its performance.

Figure 2.7 shows a rate region for a two user DSL network with two potential rate vectors marked. The solid and dashed lines represent two different spectrum balancing algorithms. From this it can be deduced that the algorithm which traces the solid line performs better in terms of possible data rates than the algorithm which traces the dashed line.

Rate regions like those shown in Figure 2.7 can only be visualised in two dimensions. With more than two lines, it becomes very difficult or perhaps intractable to trace the entire rate region. When there are more than two lines, the usual method of analysis is to fix the rates of all other lines in the bundle and draw the rate region for the two remaining lines.

2.9.2 Iterative Water-Filling

IWF was one of the early developments in DSM and showed that large performance gains were possible over traditional SSM methods. IWF has many advantageous features as a DSM algorithm such as low-complexity, de-centralised control and relative ease of implementation. IWF utilises a traditional discrete water-filling algorithm on each modem such as the Levin-Campello Algorithm [32] with some parameters of the algorithm controlled by the SMC.

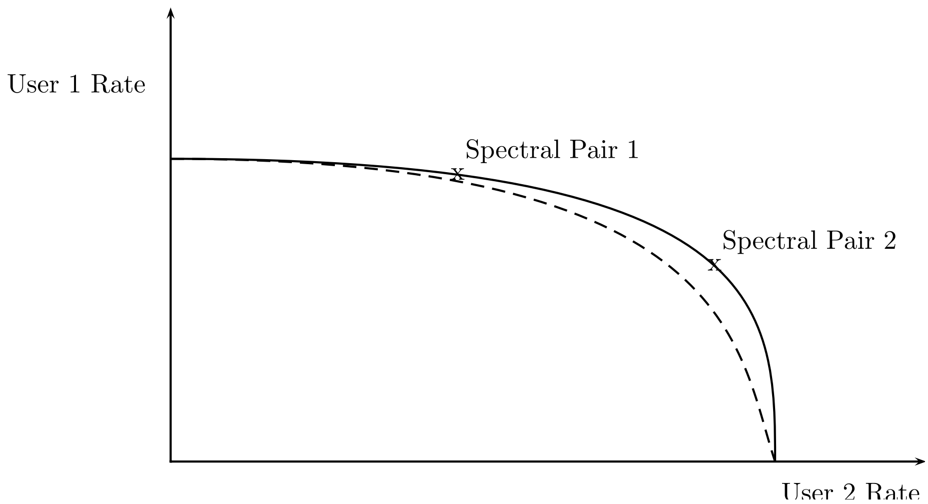


Figure 2.7: Two User Rate Region with two different spectral pairs shown

The basic idea of the algorithm is to iteratively adjust the power constraints of each modem so that they use just enough power to obtain their target data rate. To a certain extent, this helps to protect modems from the so called ”near-far” effect which is illustrated in Figure 2.6.

In the downstream direction, the modem in the RT causes a large amount of crosstalk into the CO line. As the CO line is long, it already has a limited achievable data rate due to attenuation. The large amount of crosstalk from the CO line causes a significant reduction in its performance. This is a classic scenario where any form of DSM will produce a large increase in performance in comparison to SSM. Even though IWF provides a performance increase here, it is actually highly suboptimal in this scenario as other DSM techniques can outperform IWF greatly. The algorithm for IWF is shown in Algorithm 3.

Another problem with IWF is that it has been shown that the performance margins γ_m on each tone will not necessarily converge. In Section 3.4.3 two scenarios are shown in which margin convergence does not occur.

2.9.3 Optimal Spectrum Balancing

The optimal solution to the spectrum balancing problem was found by Cendrillon et al. in 2004 [17] and is known as the Optimal Spectrum Balancing (OSB)

Algorithm 3 Iterative Water-Filling

repeat
  for $n = 1 \dots N$ **do**
    Execute LC algorithm with power budget P_n on line n
    if $R_n > R_n^{target}$ **then**
      $P_n = P_n - \delta$
    else
      $P_n = P_n + \delta$
    end if
  end for
until convergence

algorithm. Their solution is based on a Lagrangian dual decomposition of the spectrum balancing problem given in equation 2.31.

The innovation is to form the Lagrangian dual problem of the optimisation in equation 2.31.

$$L = \sum_{n=1}^{N} w_n \sum_{k=1}^{K} \log_2 \left(1 + \frac{p_n(k)|h_{nn}(k)|^2}{\Gamma \left(\sigma_n^2(k) + \sum_{m \neq n}^{N} p_m(k)|h_{jm}(k)|^2 \right)} \right) - \lambda_n \left(\sum_{k=1}^{K} p_n(k) \right) \tag{2.33}$$

The Lagrangian multipliers are chosen so that the power constraints on each line are tight. It was shown by Cendrillon et al. in [17] that the unconstrained maximisation of the Lagrangian is approximately equivalent to the constrained maximisation of the original problem. This was generalised by Yu and Lui in [36] who also showed that this approximation is exact for multicarrier systems with a large number of channels.

By examining equation 2.33 it can be seen that the Lagrangian can be decoupled on each tone to form the per-tone Lagrangian shown in equation 2.34. Thus by optimising the per tone Lagrangian and searching the λ space the original spectrum balancing problem can be solved. The lambda values can be found by bisection or more efficiently via sub-gradient or ellipsoid methods.

Users	Execution Time
2	95.14 Seconds
3	27.93 Minutes
4	7.97 Hours
5	5.95 Days
6	110 Days
7	4.85 Years (Projected)
8	82.6 Years (Projected)

Table 2.3: OSB execution times for Downstream ADSL as listed in [2]

$$L(k) = \sum_{n=1}^{N} w_n b_n(k) - \sum_{n=1}^{N} \lambda_n p_n(k) \tag{2.34}$$

Unfortunately, the per-tone Lagrangian is not convex, so in order to find the optimum solution, an exhaustive search over all possible bit combinations is required. The number of possible bit combinations on each tone is given by $(b_{max} + 1)^N$. This means that the per-tone optimisation and hence the OSB algorithm itself is exponential in the number of users N.

This is the most serious disadvantage of OSB, as it is intractable for more than 4-5 users as illustrated in Table 2.3. The OSB algorithm utilising a sub-gradient λ update method is shown in Algorithm 4.

Algorithm 4 Optimal Spectrum Balancing

```
repeat
    repeat
        for k = 1...K do
            argmax_{b(k)} L(k) = Σ_{n=1}^{N} w_n b_n(k) − Σ_{n=1}^{N} λ_n p_n(k)
            (Solve by N-d exhaustive search)
        end for
        λ_n = λ_n + ε(Σ_{k=1}^{K} p_n(k) − P_n)
    until convergence
    w_n = w_n + ε(Σ_{k=1}^{K} b_n(k) − R_n^{target})
until convergence
```

2.9.4 Iterative Spectrum Balancing

As the complexity of OSB makes the algorithm intractable for even a moderate number of users, Iterative Spectrum Balancing (ISB) was developed by Cendrillon and Moonen [34] in an attempt to address this problem. It was later more thoroughly investigated by Yu and Lui in [36]. ISB is based around the same Lagrangian decomposition as OSB, the difference being the method of maximising the per tone Lagrangian. Instead of an exhaustive search over all bit-loading combinations, the Lagrangian is optimised on each line in turn. In OSB, the optimisation is a "grid search" whereas in ISB the optimisation is an iterative "line search". This iterative line search often leads to the global optimum, but it not guaranteed to do so [34]. In practise the results are similar to OSB, with a slight performance reduction. ISB achieves a significant complexity reduction compared to OSB. In terms of the number of users N, OSB is $O(2^N)$ where ISB is $O(N^2)$. This means that ISB can optimise spectra for entire bundles of 50-100 lines. The ISB algorithm is shown in Algorithm 5.

Algorithm 5 Iterative Spectrum Balancing

---

repeat
    for $k = 1 \dots K$ **do**
        repeat
            for $n = 1 \dots N$ **do**
                Fix $b_m(k) \forall m \neq n$
                $\text{argmax}_{b(k)} L(k) = \sum_{n=1}^{N} w_n b_n(k) - \sum_{n=1}^{N} \lambda_n p_n(k)$
                (Solve by 1-d exhaustive search)
            end for
            $\lambda_n = \lambda_n + \epsilon(\sum_{k=1}^{K} p_n(k) - P_n)$
        until convergence
    end for
    $w_n = w_n + \epsilon(\sum_{k=1}^{K} b_n(k) - R_n^{target})$
until convergence

2.9.5 Optimal Spectrum Balancing With Branch and Bound

OSB with Branch and Bound [2] (BBOSB) is another spectrum balancing algorithm based on the Lagrangian dual decomposition. The key difference between OSB and BBOSB is the method of solving the per tone Lagrangian in equation

2.34, which is solved more efficiently with a branch and bound procedure rather than an exhaustive search.

Branch and bound is a generic technique for solving optimisation problems which consists of a branch operation and a bounding operation. The branching operation splits the original solution set into smaller sub regions. The bounding operation then finds the upper and lower bound of each sub regions. Any region which has an upper bound lower than the current maximum lower bound can be safely discarded from the search. This can have a significant effect on the complexity of the algorithm. In [2] the authors note that the BBOSB algorithm calculates the optimum spectra for an eight line scenario in one four thousandth of the execution time of OSB.

Although BBOSB is much more efficient than OSB, it is still exponential in the number of users N. However, where OSB is only tractable for 4-5 lines, BBOSB is tractable up to approximately 10 lines. This fact makes the BBOSB algorithm very useful for testing the performance of other spectrum balancing algorithms for moderate size scenarios. The branch and bound procedure is shown in Algorithm 6.

Algorithm 6 Branch and Bound Procedure for OSB

```
repeat
    repeat
        for k = 1 ... K do
            Start with initial region which contains all solutions
            repeat
                Split each region in half in each dimension into 2^N sub regions
                Calculate Upper and Lower Bound of each sub region
                Discard regions with an upper bound lower than max_low
            until Region converges to one single point
        end for
        λ_n = λ_n + ε(Σ_{k=1}^{K} p_n(k) − P_n)
    until convergence
    w_n = w_n + ε(Σ_{k=1}^{K} b_n(k) − R_n^{target})
until convergence
```

2.9.5.1 Efficient Lambda Update Methods

One of the problems with Lagrangian dual based spectrum balancing methods is to select the appropriate update method for λ. The most reliable method is bisection of each individual λ_n in turn, but this is unacceptably slow to converge.

The sub-gradient method is the most straightforward multi-user λ_n update method but includes the problem of selecting an appropriate step size. Choosing a step size that is too low will result in a very slow convergence time. Choosing a step size that is too large may result in oscillation of the solution about the power constraints resulting in a failure to converge.

The authors in [37] presented a λ_n update method utilises a gradient search with a dynamic step size. Briefly stated, the step size is increased as long as the Euclidean distance of the current solution from the system power constraints decreases. When it is discovered that the Euclidean distance is increasing, a new search is initiated started from the closest point that has currently been reached. This algorithm is illustrated in Algorithm 7. This algorithm is utilised in many of the results obtained in this thesis, however it is noted that there are often problems with this approach resulting from a tendency to get stuck in local minima in the power plane.

Algorithm 7 Efficient Lagrangian Update Method

```
while |P(n) − P_budget| < p_tolerance do                                  ▷ ∀n
    Calculate_Loading(λ)          ▷ e.g.Exhaustive/Iterative/Branch and Bound
    distance = |P_budget − P(n)|                            ▷ Euclidean distance
    if distance < min_distance then
        λ_best = λ
        λ_n = λ_n + μ(P(n) − P_budget)
        μ = μ ∗ 2
    else
        λ = λ_best
        μ = 1
    end if
end while
```

2.9.6 Multi-User Greedy Spectrum Balancing

In [38], [3] and [39], Lee, Sonalkar and Cioffi presented a spectrum balancing algorithm known as the Multi-User Greedy Algorithm. It is based on a heuristic extension of the single user Levin-Campello Algorithm.

In the Levin-Campello Algorithm, bits are added to the channel which requires the least energy to do so, until the power budget of the line is reached. It is a conceptually simple algorithm, which produces the optimal discrete solution for the single user case. However, as noted in Section 2.9.2, when it is applied to the multi-user case in IWF, it produces sub-optimal results.

The Multi-User Greedy Algorithm attempts to extend the Levin-Campello algorithm to the multi-user case. The entire binder group is considered as a whole and bits are added incrementally to the channel and line with the lowest energy cost. The cost metric is defined as the power required to add one bit to a particular line/tone and the sum of the power increases required on all other lines on that tone in order to bring those lines back within the performance margin given the increased crosstalk. The objective of the multi-user greedy algorithm is to minimise the total power required to transmit a total rate-sum.

The cost function is calculated as shown in equation 2.35. $C(m, k)$ is the cost to add a bit to user m on tone k. $p(k)$ is a vector of the power spectral density on each line on tone k, for a given bit-vector $b(k)$. In equation 2.35 the vector e_m is a vector of zeroes with element m set to 1. This represents adding a bit to line m. Thus, the total power increase is calculated by the sum of the differences in the vector $p(k)$ before and after adding a bit to line m.

$$C(m,k) = \sum_{n=1}^{N} \left(\underset{b(k)+e_m}{p_n(k)} - \underset{b(k)}{p_n(k)} \right) \tag{2.35}$$

A simplified version of the Multi-User Greedy Algorithm is illustrated in Algorithm 8.

In [3] the authors proposed an algorithm to find arbitrary data rate points with the addition of a per line weight w_n and an algorithm to adjust the values of w_n on each iteration of the algorithm. Unfortunately, this algorithm is potentially

Algorithm 8 Multi-user Greedy Algorithm

repeat

$$\operatorname{argmin}_{m,k} C(m,k) = \sum_{n=1}^{N} \left(\underset{b(k)+e_m}{p_n(k)} - \underset{b(k)}{p_n(k)} \right)$$

Increment $b_m(k)$

until All tones full

very slow or unstable as it relies on a arbitrarily chosen factor.

2.10 Conclusions

In this chapter, some popular DSM algorithms that have been developed in recent years were reviewed. It was shown that none of the current known algorithms are a perfect solution for implementing DSM. In particular, the level 2 DSM algorithms [17] and [34] discussed in this chapter are far too slow or intractable to be of practical use in realistic DSL deployments.

Level 2 DSM has two inherent advantages over level 1 techniques such as IWF. Firstly, as long as the algorithm is fast enough (i.e. less than the typical service time), the behaviour of users in the DSL network can be changed instantaneously, for example in response to changing data rate requirements or the addition of new users to the network. Secondly the performance margins can be set with exact precision in order to guarantee a consistent probability of error for each user in the network.

The remainder of this thesis investigates the development of new level 2 DSM algorithms with good data rate performance and lower complexity and execution times than currently known methods. In particular, the performance of the Multi-User Greedy Algorithm is investigated further in chapters 3 and 4 to form new algorithms which can produce solutions in a tractable time with good data rate performance.

In Chapter 3, a new level 2 DSM algorithm known as MIPB is developed and is shown to produce results comparable to the optimal solution given by OSB for a range of DSL scenarios, with a low enough complexity to be utilised on large bundle sizes (50-100 DSL lines). However, it currently has a limitation whereby only one data rate point can be obtained, which is similar to that given by OSB with equal line weights.

In Chapter 4 another algorithm is developed which overcomes the limitation of MIPB and is able to calculate arbitrary points on the rate region. It is shown through simulation of various DSL network configurations that this algorithm produces rate regions that are practically as large as the optimal rate region given by OSB. Several rate search techniques for this algorithm are derived and it is shown that it can deliver a massive speed increase compared to ISB, up to 3850 times faster for a 7-User ADSL2+ downstream scenario.