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An optimal spectrum-balancing algorithm for digital subscriber lines based on particle swarm optimization

Meiqin Tang, Chengnian Long and Xinping Guan*,†

Center for Networking Control and Bioinformatics, Department of Electrical Engineering, Yanshan University, Qinhuangdao 066004, People's Republic of China

SUMMARY

This paper presents a new algorithm for optimal spectrum balancing in modern digital subscriber line (DSL) systems using particle swarm optimization (PSO). In DSL, crosstalk is one of the major performance bottlenecks, therefore various dynamic spectrum management algorithms have been proposed to reduce excess crosstalks among users by dynamically optimizing transmission power spectra. In fact, the objective function in the spectrum optimization problem is always nonconcave. PSO is a new evolution algorithm based on the movement and intelligence of swarms looking for the most fertile feeding location, which can solve discontinuous, nonconvex and nonlinear problems efficiently. The proposed algorithm optimizes the weighted rate sum. These weights allow the system operator to place differing qualities of service or importance levels on each user, which makes it possible for the system to avoid the selfish-optimum. We can show that the proposed algorithm converges to the global optimal solutions. Simulation results demonstrate that our algorithm can guarantee fast convergence within a few iterations and solve the nonconvex optimization problems efficiently. Copyright © 2008 John Wiley & Sons, Ltd.

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KEY WORDS: digital subscriber line (DSL); dynamic spectrum management (DSM); PSO; nonconvex optimization

1. INTRODUCTION

In recent years, digital subscriber line (DSL) technology has received increasing popularity as it can build high-speed data connections to homes via ordinary telephone twisted pairs. Worldwide DSL equipment market is forecast to grow at a compound annual growth rate (CAGR) of 12%

*Correspondence to: Xinping Guan, Center for Networking Control and Bioinformatics, Department of Electrical Engineering, Yanshan University, Qinhuangdao 066004, People's Republic of China.
†E-mail: xpguan@ysu.edu.cn

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from 2000 to 2007. The dominant source of performance degradation in DSL systems is the *crosstalk*, which is the interference generated among different lines in the same cable binder. Near-end crosstalk is caused by transmitters interfering with receivers on the same side of the bundle and is often avoided by using nonoverlapping transmit and receive spectra or disjoint time intervals. Far-end crosstalk is caused by transmitters on opposite sides of the bundle. The crosstalk is typically 10–20 dB larger than the background noise. It could be canceled when the receivers and transmitters are coordinated. On the other hand, they could be reduced by dynamic spectrum management (DSM) when no coordination is allowed. Without crosstalk cancelation or proper spectrum management, DSL systems suffer from a near–far problem: when the user is much closer to the central office than the others, the interference coming from the closer users overwhelms the signals from the farther users [1]. If all users employ equal power, the crosstalk from a short loop can severely degrade the performance of a long loop.

Several crosstalk cancelers [2–5] have been proposed to solve this problem. Unfortunately, in many scenarios it is not feasible because of error propagation, high complexity and long latency. In this case, the effects of crosstalk must be mitigated through spectrum management, where the transmit spectra of all modems are limited in some way to minimize crosstalk. Static spectrum management (SSM) is the traditional approach and employs identical spectral masks for all modems. The performance projection under SSM is based on the levels of worst-case crosstalk interference. As a result, they can be overly restrictive and lead to poor performance.

DSM, a new paradigm, overcomes this problem by designing the spectra of each modem to match the specific topology of the network. These spectra are adjusted based on the direct and crosstalk channels seen by the different modems. They are customized to match each modem in each particular situation. Recently, various DSM algorithms have been proposed to address this frequency-selective interference problem. They can be applied to dynamically optimize transmission power spectra of different modems in DSL networks. These methods can be classified into two categories, i.e. distributed [6–8] methods and centralized [9–13] (semi-coordinated and fully coordinated) methods. Iterative waterfilling (IW) [6] is the first low-complexity multiuser spectrum optimization technique that takes advantage of the ability of DSL modems to perform spectral shaping. In this algorithm, each user iteratively maximizes its own achievable rate by performing a single-user waterfilling with the crosstalk interference from all other users treated as noise. Although IW can achieve suboptimal performance in weak interference channels, it is highly suboptimal in the widely encountered near–far scenarios, such as mixed central office and remote terminal deployments of ADSL and upstream VDSL. Recently, Yu and Lui [7] apply the dual theory to the nonconvex spectrum optimization problem. The proposed algorithm for DSLs can be interpreted as dual algorithm. Their paper makes progress toward solving optimization problems of this type by showing that under a certain condition called the time-sharing condition, the duality gap of the optimization problem is always zero, in spite of the convexity of the objective function. In fact, in some practical situations, the time-sharing condition cannot be guaranteed. As the duality gap is not equal to zero, the primal and the dual optimization problems cannot yield the same solution. An autonomous algorithm with linearity complexity called autonomous spectrum management (ASB) is proposed in [8]. The authors use the concept of reference line to mimic a 'typical' victim line in the current binder, which sets the power spectrum level to protect the reference line. However, the algorithm can only achieve close optimal performance. When a spectrum management center (SMC) is responsible for setting the spectra of the modems within the network, the centralized algorithm is suitable. A related work [11] is proposed to formulate a centralized solution to the optimal-balancing problem based on convex optimization techniques. Unfortunately,

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this is possible only when the crosstalk channels are sufficiently weak. In this case, the objective function is convex. More recently, two other centralized optimal DSM algorithms are proposed, i.e. the optimal spectrum-balancing (OSB) algorithm [9] and the iterative spectrum-balancing (ISB) algorithm [10]. OSB is proved to be optimal and can be used to achieve the best possible balance between the rates of the different modems in the network, allowing operation at any point on the rate region boundary. It is based on a weighted rate sum, which forces each modem to account for the damage done to other modems in the network when deciding its own transmit spectra. OSB has an exponential complexity in terms of the number of users, which makes it intractable for DSL network with more than five lines. As an improvement over the OSB algorithm, ISB is proposed to implement the weighted rate sum optimization in an iterative fashion over the users. This leads to a quadratic complexity in terms of the number of users, which makes the ISB feasible for networks with a relatively large number of users. In [12, 13] two semi-centralized DSM algorithms are proposed. SCALE [12] algorithm achieves better performance than IW with comparable complexity. However, the algorithm is not autonomous as explicit message passing among users is required. The algorithm proposed in [13] is based on multiuser hybrid time divisioning (HTD) scheme for power allocation in DSL systems. The proposed HTD scheme considers the presence of a centralized scheduler for calculating a threshold γ_t^*, which is difficult to obtain in theory.

This paper considers the spectrum management problem in the multiuser DSL system. In this case, the objective function is always nonconcave, and the optimization problem becomes computationally difficult to solve. As we know, it is difficult to solve the nonconvexity problem by using the classical nonlinear programming theory. Even if it can be used, enormous computational efforts and time consumption are usually needed. With the development of artificial intelligence, some alternative methods for the nonlinear, nonconvex and discontinuous optimization problems are revealed. In this paper, we apply one of the evolutionary computation techniques, i.e. the particle swarm optimization (PSO) algorithm [14] to solve the nonconvexity issues. PSO is a newly proposed population based stochastic optimization algorithm inspired by the social behaviors of animals, and the distinct advantages of it are as follows: it has a simple concept and an easy implementation; it does not need to calculate the gradients of the objective function, even the form of the objective function; it has comparable or even superior search performance for some hard optimization problems with faster convergence rates. And it has been used to deal with very complicated optimization problems [15–17] in recent years.

In this paper, we apply the penalty function to the spectrum-balancing problem, which transforms the constrained problem into an unconstrained one. The simulation study shows that our approach is efficient in finding the optimal power. The remainder of this paper is organized as follows. In the following section, we give the system model and the basic principle of PSO. A new spectrum-balancing management algorithm is then proposed based on PSO. Moreover, the convergence of the algorithm is also proved. Examples are provided to show the efficiency of the proposed algorithm in Section 3 and concluding remarks are given in Section 4.

2. FORMULATION OF SPECTRUM-BALANCING OPTIMIZATION

2.1. System description

We consider the centralized spectrum management framework where a SMC is responsible for setting the spectra of the modems within the network. Each modem has knowledge of the noise

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and crosstalk channels of all modems in the binder, which increases the overhead required for communication with the SMC. However, as the twisted-pair channel is slowly time varying, the additional overhead is trivial. In this paper, we will present a centralized algorithm for OSB in the DSL interference channel. When the DMT technique is used with synchronized receivers, a DSL channel can be modeled as N-independent frequency nonselective subchannels, each of which is an interference channel of M users. Without crosstalk cancelation, the signal-to-interference-plus-noise ratio (SINR) of user i in subchannel n is expressed as

$$S_i(n) = \frac{H_{i,i}^2(n)P_i(n)}{N_i(n) + \sum_{j=1, j\neq i}^{M} H_{i,j}^2(n)P_j(n)} \tag{1}$$

where $P_i(n)$ and $N_i(n)$ are the signal power and the background noise power of user i in the subchannel n, respectively. $H_{i,i}(n)$ represents the direct channel gain of user i in the subchannel n, whereas $H_{i,j}(n)$ represents the crosstalk channel gain from user j to user i. It is assumed that each modem treats interference from other modems as noise. When the number of interfering modems is large, the interference is well approximated by a Gaussian distribution. Under this assumption, the number of bits transmittable with quadrature amplitude modulation is approximated by

$$b_i(n) = \log_2\left(1 + \frac{S_i(n)}{\Gamma}\right) \tag{2}$$

where Γ is the signal-to-noise ratio (SNR) gap that depends on the probability of symbol error, the noise margin, and the coding gain.

The data rate of user i is then given by

$$R_i = \sum_{n=1}^{N} b_i(n) \tag{3}$$

in bits per symbol. A rate region is defined as the union of all the rate sets $(R_1, \ldots, R_N)$ that can be achieved from the following power constraints:

$$P_i \leqslant P_{\max} \quad \text{for } i = 1, \ldots, M \tag{4}$$

where

$$P_i = \sum_{n=1}^{N} P_i(n) \tag{5}$$

and $P_{\max}$ is the maximum power for the user i.

The spectrum optimization problem in a multiuser DSL system is formulated as the maximization of a weighted sum rate of all participating users subjecting to power constraints. It then makes it possible to avoid the selfish-optimum. Thus, we have

$$\begin{aligned} \max \quad & \sum_{i=1}^{M} \varpi_i R_i \\ \text{s.t.} \quad & P_i \leqslant P_{\max} \end{aligned} \tag{6}$$

The weights $\varpi_i \geqslant 0, i = 1, 2, \ldots, M$, are chosen so that $\sum_{i=1}^{M} \varpi_i = 1$, which can be interpreted as the priority given to user i in the optimization. If the data rate of user i is below its target, ϖ_i is increased to allocate more priority to user i.

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As the objective function is not concave in P_i, numerical optimization is difficult. This is only possible when the crosstalk channels are sufficiently weak such that the objective function is concave. This approach is not valid in the general case. In the past studies [7–10], the authors consider the nonconvex optimization problem under some conditions, and then use the dual-based sub-gradient algorithm or general dual decomposition approaches to solve this problem. It may lead to suboptimal or high computational complexity. In this paper, we propose a PSO algorithm solving this nonconvex optimization problem directly.

2.2. *The PSO method*

The main difficulty in the optimal design of multiuser DSL spectra is the nonconvexity and the computational complexity associated with the optimization problem. In this paper, we apply the PSO method to the nonconvex problems, so that the complexity will be reduced at the same time. PSO, an evolutionary computation technique, was first proposed by Kennedy and Eberhart [14]. This method is derived based on the social-psychological theory and has been found to be robust for solving problems featuring nonlinearity and nondifferentiability multiple optima, as well as high dimensionality through adaptation. The PSO method is studied extensively in swarms' activities, such as fish schooling and bird flocking. Instead of using evolutionary operators to manipulate the individual, as in other evolutionary computational algorithms, each individual in PSO flies in the search space with a velocity that is dynamically adjusted according to its own flying experience and its companions' flying experience. The individual is treated as a volume-less particle in a d-dimensional search space. Each particle keeps track of its coordinates in the space of interest, which are associated with the best solution (fitness) it has achieved so far. This value is called *Pbest*. Another best value that is tracked by the global version of the particle swarm optimizer is the overall best value, and its location, obtained so far by any particle in the population. This location is called *Gbest*.

In a physical d-dimensional space, the position and the velocity of the individual a are represented as $X_a = (x_{a1}, \ldots, x_{ad})$ and $V_a = (v_{a1}, \ldots, v_{ad})$, respectively, in the PSO algorithm. Let $Pbest_a = (x_{a1}^{Pbest}, \ldots, x_{ad}^{Pbest})$ and $Gbest = (x_1^{Gbest}, \ldots, x_d^{Gbest})$, respectively, be the best position of individual a and its neighbors' best position so far. Using the information, the velocity of individual a is updated by the following law:

$$V_a^{k+1} = \omega V_a^k + c_1 * \mathscr{R}_1 * (Pbest_a^k - X_a^k) + c_2 * \mathscr{R}_2 * (Gbest^k - X_a^k) \tag{7}$$

where V_a^k is the velocity of individual a at the iteration step k, ω is the inertia weight factor, c_1 and c_2 are the acceleration constants, $\mathscr{R}_1$ and $\mathscr{R}_2$ are the uniform random numbers between 0 and 1, X_a^k is the current position of individual a at the iteration step k, $Pbest_a^k$ and $Gbest^k$ are the best position of the individual a and the group until iteration k, respectively.

Each individual moves from the current position to the next position according to the updated velocity in (7) by the following law:

$$X_a^{k+1} = X_a^k + V_a^{k+1} \tag{8}$$

The search mechanism of PSO using the modified velocity and position of individual a is based on (7) and (8). In the aforementioned procedures, the particle velocity is limited by the maximum value $V^{\max}$. The parameter $V^{\max}$ is determined by the fitness with which regions are to be searched between the present position and the target position. In practice, $V^{\max}$ is normally set to 10–20%

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of the dynamic range of the variable on each dimension. The constants c_1 and c_2 represent the weights of the stochastic acceleration terms that pull each particle toward *Pbest* and *Gbest*.

As the spectrum-balancing problem is subjected to the inequality constraint in the algorithm, we use PSO with the penalty functions in our algorithm. In the penalty method, the fitness function is defined as follows:

$$F_f = \begin{cases} f(X) & \text{the solution is feasible} \\ f(X)+h(k)H(x) & \text{otherwise} \end{cases} \tag{9}$$

where $f(x)$ is the original objective function to be optimized, $h(k)$ is a penalty value and $H(x)$ is a penalty factor.

2.3. *The nonconvex maximization algorithm using PSO*

In this paper, we remove the strict assumptions (such as continuity and differentiability) from the objective function and consider the nonconvexity directly using the PSO method. The important notations of the proposed PSO algorithm are defined as follows:

Particle: Each particle in PSO represents a group of solutions of the DSM problems; for each particle, X_a, x_{ab} $(b=1,2,\ldots,d)$ represents one user's power in DSL systems, so the bth dimension of the particle corresponds to the ith user in DSL, x_b $(b=1,2,\ldots,d)$ for the particle a in PSO corresponds to $P_i(n)$ $(i=1,2,\ldots,M)$.

Velocity: V_a, the velocity in PSO, is the auxiliary variable for the algorithm to find the optimal value.

Fitness function: We know that the objective is to maximize aggregate source rates subjecting to power constraints in the DSL framework; therefore, the fitness function here is given according to (9) as

$$F_f = \begin{cases} \sum_{i=1}^{M} \varpi_i R_i & \text{if } P_i \leqslant P_{\max} \\ \sum_{i=1}^{M} \varpi_i R_i + \mu_i (P_{\max} - P_i) & \text{otherwise} \end{cases} \tag{10}$$

where the penalty value $\mu_i\,(\mu_i > 0, \forall i)$ can be interpreted as the prices for power. If allocated power for user i is below its total power budget, the price for power is decreased. Thus, the power constraint is violated, and we pay an extra charge proportional to the amount of violation with price μ_i.

In all, the algorithm is to find the optimal value (the optimal power for the user) according to the fitness function (the objective function) through the velocity (the auxiliary variable).

The PSO Algorithm can be described in the following steps:

Step 1: Initialize the variables and the parameters. Let 'k' be zero and set the maximum number of iterations as K. Initialize positions of particles X_a, as mentioned before, in the DSL system. The particle X_a corresponds to a group of users' powers, which must be one of the feasible candidate solutions satisfying the practical power constraints. Initialize velocity V_a, which is an auxiliary variable in our algorithm. Initialize the PSO parameters (ω, c_1, c_2).

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Step 2: Calculate the fitness. The current searching points are evaluated by using the objective functions of the target problem, i.e. F_f in Equation (10). F_f is, in fact, the fitness function in our algorithm, which can be obtained through the message $C_i(n)$ passed to user i from SMC. $C_i(n)$ is defined as follows:

$$C_i(n) = \sum_{j=1, j\neq i}^{M} H_{i,j}^2(n) P_j(n) \tag{11}$$

$Pbest_a^k$ is then set to be the initial searching point. The initial best-evaluated value among $Pbest_a^k$ is $Gbest^k$.

Step 3: Update the searching points. Compare the fitness value of $Pbest_a^k$ with the fitness value F_f. If the fitness value of $Pbest_a^k$ is better than F_f, then set $Pbest_a^k$ to the current position X_a. Choose the best value of $Pbest_a^k$ of the whole swarm as the value of $Gbest^k$. All the $Gbest^k$ are candidates for the final control strategy. Update velocities and positions according to Equations (7) and (8).

Step 4: Set the stop criterion. As mentioned before, when the power constraint is violated, we pay an extra charge proportional to the amount of violation with the penalty value. When the number of iterations reaches the maximum, the searching procedure can be stopped. The last $Gbest^k$ can be drawn as the solution, as $Gbest = (x_1^{Gbest}, \ldots, x_d^{Gbest})$, and the corresponding $(x_b^{Gbest})^k$ is the optimal power $P_i^*(n)$ in the DSL systems.

In the procedure of the algorithm, steps 2 and 3 describe the power control using message passing. Each user updates its power according to the PSO principle while taking the messages C_i. When the allocated power of the current time slot is below its maximum power limit, it will update its power until the algorithm gets the optimum. The complete PSO spectrum-balancing algorithm is listed as Algorithm 1. It is obvious that the computational complexity of the proposed algorithm is very low, which is $O(MN)$.

Algorithm 1. PSO Spectrum Balancing

```
repeat
  for each user i = 1, ..., M
    repeat
      for each tone n = 1, ..., N
       (the PSO Algorithm)
            find P_i^*(n)
    until convergence
  end
until convergence
```

2.4. Convergence analysis

The convergence property of the proposed algorithm is investigated in this subsection. In the PSO algorithm, the base parameters are ω, c_1 and c_2. Appropriate choices of these parameters can ensure the convergence of the algorithm. Before the discussion goes further, a definition of convergence should be presented here for clarification.

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Definition 1
A sequence $\{P_{ai}^k\}_{k=1}^{\infty}$ is convergent if

$$\lim_{k\to\infty} P_{ai}^k = P_i^*(n)$$

for $i = 1, \ldots, M$, where $P_i^*(n)$ is referred to as the optimum.

Theorem 1
The proposed PSO algorithm is convergent if and only if $\max(\|\alpha\|, \|\beta\|) \leqslant 1$.

Proof
See Appendix A.

From this theorem, we can obtain the exact range of parameters of the algorithm. In terms of (A7) mentioned in Appendix A, substituting the expectation ϕ $(\frac{1}{2}c_k)$, we have

$$\omega \geqslant \tfrac{1}{2}(c_1 + c_2) - 1$$

If we set $\omega = 1$, $c_1 = 2$, $c_2 = 2$, the equation holds.

3. NUMERICAL EXAMPLE

In this section, we aim to show the effectiveness of the proposed power spectrum algorithm. In the simulation, all DSL lines are 26-AWG twisted pairs with a background noise level of -140 dBm/Hz. The target symbol error probability is 10^{-7} or less. The specified coding gain, noise margin and the probability of error target lead to a transmission gap of 12 dB. Users are assumed to be symbol synchronized so that the sidelobe interference does not take effect. Also, no spectral masks are enforced.

Example 1 (*Two-user ADSL*)
In the first scenario, the system is set in a two-user ADSL downstream distributive environment with both users having a loop length of 12 k feet and with a crosstalk distance of 3 k feet. The loop topology is shown in Figure 1. The power constraint for each user is set to 20.4 dBm as defined in [18]. The parameters of PSO are given in Table I.

Figures 2 and 3 show the convergence of the algorithm and the power spectral densities (PSDs) of the two users generated by the proposed algorithm, respectively. In Figure 3, user 1 corresponds

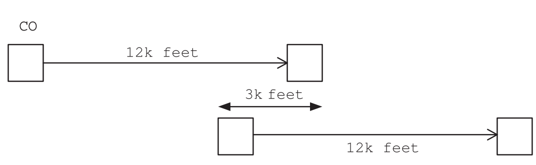

Figure 1. Loop topology for two users.

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Table I. The parameters of PSO.

Swarm size	20
c_1	2
c_2	2
ω	1
$V_{\max}$	4
Maximum iterations	50

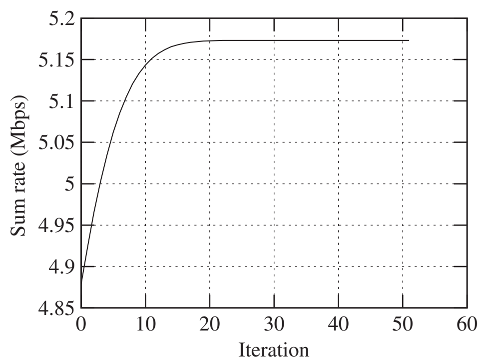

Figure 2. The convergence behavior of the proposed algorithm for two users.

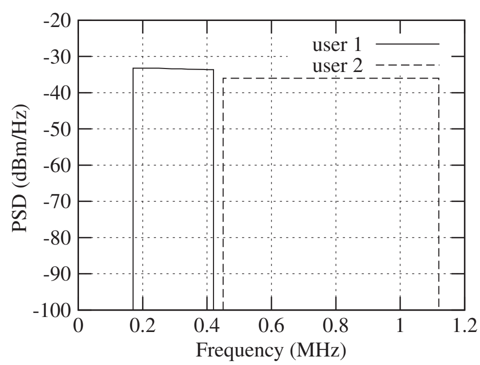

Figure 3. The optimal PSDs on the CO line (user 1) and on the RT line (user 2).

to the CO line, whereas user 2 is the RT line. It can be seen from the figures that the objective function converges to the optimum very quickly and the algorithm is very efficient in the mixed CO/RT scenario.

Example 2 (*Five-user VDSL*)
When the number of users is larger than two, the proposed algorithm can also obtain good performance. A five-user scenario with the same loop length (3 k feet long) in the same binder has been simulated to evaluate the convergence of the proposed algorithm and performance in

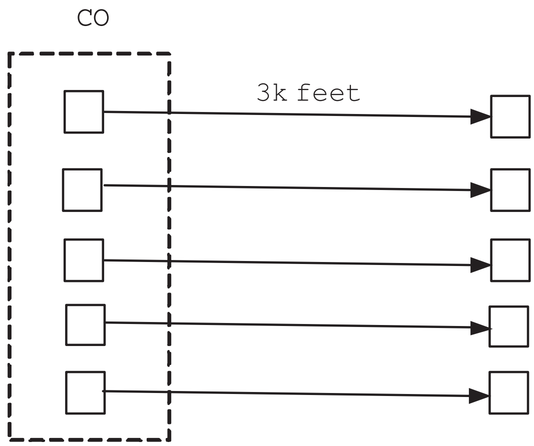

Figure 4. Loop topology for five users.

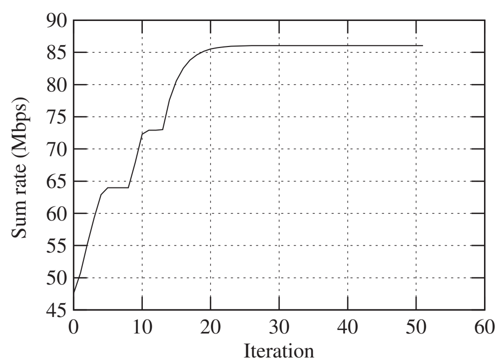

Figure 5. The convergence behavior of the proposed algorithm for five users.

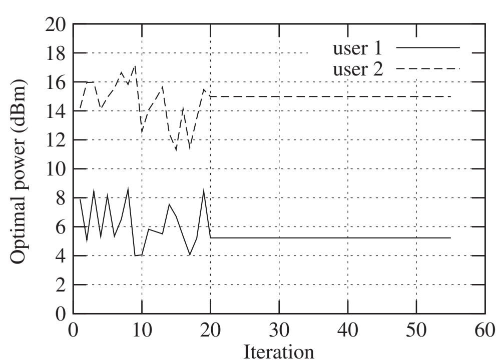

Figure 6. The optimal powers of two users for $\varpi_1 = 0.9$, $\varpi_2 = 0.1$.

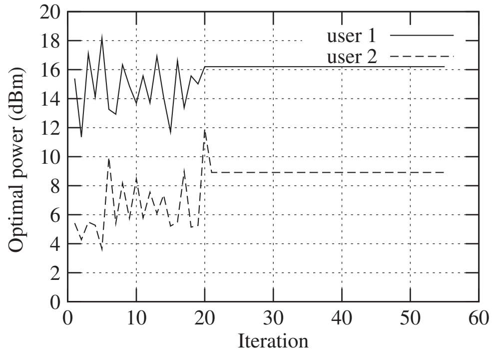

Figure 7. The optimal powers of two users for $\varpi_1 = 0.1$, $\varpi_2 = 0.9$.

Table II. Comparison of the performance of the algorithms.

Algorithm	Complexity	Performance
IW	$O(KN)$	Suboptimal
OSB	$O(Ke^N)$	Optimal
ISB	$O(KN^2)$	Near optimal
ASB	$O(KN)$	Near optimal
Ours	$O(KN)$	Optimal

large networks with the topology as shown in Figure 4. Perfect echo cancelation is assumed. The downstream transmission has a power constraint of 11.5 dBm, and the upstream transmission has a power constraint of 14.5 dBm, in accordance with [19]. The data-rate convergence of this scenario has been shown in Figure 5. As can be seen, the sum rate of the algorithm converges within 20 iteration steps.

Example 3 (*The users' optimal powers with different weights*)
The optimal transmit powers with different weights on users of two-user scenario as in Example 1 are shown in Figures 6 and 7. When ϖ_1 (the weight on user 1) is set to 0.9 and ϖ_2 (the weight on user 2) is 0.1, the optimal powers we can get are 5.23 and 14.9 dBm for users 1 and 2, respectively; when ϖ_1 is equal to 0.1 and ϖ_2 is 0.9, the optimal powers we can get are 16.2 and 8.9 dBm for users 1 and 2, respectively. It has been well demonstrated that the weights $\varpi_i \geqslant 0$ are the priorities given to user i in the optimization. If the allocated power to the user needs to be increased, we should reduce ϖ_i; otherwise, ϖ_i should be raised. This allows the selfish-optimum to be avoided and leads to significantly improved performance.

Providing a low complexity, convergent and optimal solution is crucial for the utilization of the DSL problem. This paper presents a new iterative algorithm for a weighted rate sum spectrum management such as OSB and ISB. However, the proposed algorithm based on PSO has a lower complexity. Table II compares the performance of different DSM algorithms with ours. From the table we can find that the proposed algorithm using PSO can provide an efficient solution to the nonconvex optimization problem.

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4. CONCLUSION

In this paper, we studied the nonconvex optimization of spectrum-balancing control in multiuser DSL systems and proposed a new algorithm based on PSO. Our algorithm optimizes the objective function including the joint rates of all users; therefore, we avoid the so-called selfish-optimum. By the simulation results of different cases, it is shown that the sum rate can converge to the optimal point very quickly by the proposed algorithm.

APPENDIX A: PROOF OF THEOREM 1

As mentioned in Section 2.2, the basic equation of PSO can be rewritten as

$$v_{ai}^{k+1} = \omega v_{ai}^{k} + \phi_1((P_{ai}^{Pbest})^k - P_{ai}^k) + \phi_2((P_i^{Gbest})^k - P_{ai}^k), \quad i = 1, \ldots, M \tag{A1}$$

$$P_{ai}^{k+1} = P_{ai}^k + v_{ai}^{k+1}, \quad i = 1, \ldots, M \tag{A2}$$

where $\phi_1 = c_1\mathscr{R}_1$, $\phi_2 = c_2\mathscr{R}_2$.

Substituting (A1) into (A2) yields

$$P_{ai}^{k+1} = P_{ai}^k + \omega v_{ai}^k + \phi_1((P_{ai}^{Pbest})^k - P_{ai}^k) + \phi_2((P_i^{Gbest})^k - P_{ai}^k) \tag{A3}$$

Equation (A3) can be rewritten as

$$P_{ai}^{k+1} = (1-\phi_1-\phi_2)P_{ai}^k + \phi_1(P_{ai}^{Pbest})^k + \phi_2(P_i^{Gbest})^k + \omega v_{ai}^k \tag{A4}$$

In terms of (A2), we have $v_{ai}^k = P_{ai}^k - P_{ai}^{k-1}$. It then follows from (A4) that

$$P_{ai}^{k+1} = (1+\omega-\phi_1-\phi_2)P_{ai}^k - \omega P_{ai}^{k-1} + \varphi \tag{A5}$$

where $\varphi = \phi_1(P_{ai}^{Pbest})^k + \phi_2(P_i^{Gbest})^k$.

Moreover, Equation (A5) can be rewritten as

$$\begin{bmatrix} P_{ai}^{k+1} \\ P_{ai}^k \\ 1 \end{bmatrix} = \begin{bmatrix} 1+\omega-\phi_1-\phi_2 & -\omega & \varphi \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_{ai}^k \\ P_{ai}^{k-1} \\ 1 \end{bmatrix} \tag{A6}$$

The eigenpolynomial of the eigenmatrix in (A6) is

$$(1-\lambda)(\omega-\lambda(1+\omega-\phi_1-\phi_2)+\lambda^2)$$

Then we can obtain the latent roots as follows:

$$\begin{aligned} \lambda &= 1 \\ \alpha &= \frac{1+\omega-\phi_1-\phi_2+\gamma}{2} \\ \beta &= \frac{1+\omega-\phi_1-\phi_2-\gamma}{2} \end{aligned} \tag{A7}$$

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where

$$\gamma = \sqrt{(1+\omega-\phi_1-\phi_2)^2-4\omega}$$

Therefore, we have the following solution:

$$P_{ai}^k = m_1 + m_2\bar{\alpha} + m_3\bar{\beta} \tag{A8}$$

where m_1, m_2 and m_3 are maintained to be constant in the process of iteration.

If $(1+\omega-\phi_1-\phi_2)^2 \geqslant 4\omega$, γ, α and β are all real numbers; else if $(1+\omega-\phi_1-\phi_2)^2 < 4\omega$, γ, α and β are all complex numbers, then

$$\gamma = \mathrm{i}\sqrt{4\omega-(1+\omega-\phi_1-\phi_2)^2}$$

The polar coordinates of α and β are as follows:

$$\begin{aligned} \alpha_{ai} &= \cos(\theta_{ai}) + \mathrm{i}\sin(\theta_{ai}), \quad i=1,\ldots,M \\ \beta_{ai} &= \cos(\theta_{ai}) - \mathrm{i}\sin(\theta_{ai}), \quad i=1,\ldots,M \end{aligned} \tag{A9}$$

where $\theta_{ai} = \arctan(\|\gamma\|(1+\omega-\phi_1-\phi_2))$.

If γ, α and β are all real numbers, $\|\alpha\|$ and $\|\beta\|$ denote the absolute value of α and β, respectively. If γ, α and β are all complex numbers, $\|\alpha\|$ and $\|\beta\|$ denote the modular values of α and β, respectively.

It follows from (A8) that

$$\begin{aligned} \lim_{k\to\infty} P_{ai}^k &= \lim_{k\to\infty}(m_1+m_2\bar{\alpha}+m_3\bar{\beta}) \\ &= m_1 + m_2\lim_{k\to\infty}\bar{\alpha} + m_3\lim_{k\to\infty}\bar{\beta} \end{aligned}$$

If $\|\alpha\|>1$ or $\|\beta\|>1$, $\lim_{k\to\infty} P_{ai}^k$ does not exist. Then we can conclude that the algorithm does not converge. If $\|\alpha\|<1$ and $\|\beta\|<1$, we have $\lim_{k\to\infty} P_{ai}^k = m_1$. Therefore, the algorithm converges. Given $\max(\|\alpha\|, \|\beta\|)=1$, if the value of $\lim_{k\to\infty} P_{ai}^k$ is $m_1+m_2+m_3$, m_1+m_2 or m_1+m_3, the algorithm also converges. Therefore, the proof is completed.

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AUTHORS' BIOGRAPHIES

Meiqin Tang received the BS degree in computer science and technology, and MS degree in operational research and control theory from Yantai Normal University, China, in 2002 and 2005, respectively. She is currently pursing the PhD degree in control theory and engineering at Yanshan University, China. Her research interests include optimization and wireless communication theory.

Chengnian Long received the BS, MS, and PhD degrees from Yanshan University, China, in 1999, 2001, and 2004, respectively, all in control theory and engineering. He is an associate professor at Yanshan University. He visited Department of Computer Science and Engineering, Hongkong University of Science and Technology in 2006. Now he is a Killam postdoctoral fellow at the Department of Electrical and Computer Engineering, University of Alberta. His current research interests are in the area of noncooperative behavior and incentive mechanism design in wireless multi-hop networks, energy-efficiency protocol design in wireless sensor networking, and pricing mechanism in the Internet. Dr Long is a member of the IEEE.

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Xinping Guan (M'02–SM'04) received the BS degree in mathematics from Harbin Normal University, Harbin, China, and the MS degree in applied mathematics and the PhD degree in electrical engineering, both from Harbin Institute of Technology, in 1986, 1991, and 1999, respectively. He is currently a Professor and Dean of the Institute of Electrical Engineering, Yanshan University, Qinhuangdao, China. He is the (co)author of more than 100 papers in mathematical, technical journals, and conferences. As (a)an (co)-investigator, he has finished more than 19 projects supported by the National Natural Science Foundation of China (NSFC), the National Education Committee Foundation of China, and other important foundations. His current research interests include wireless sensor networks, congestion control of networks, robust control and intelligent control for nonlinear systems, chaos control, and synchronization.

Dr Guan is serving as an Associate Editor of *IEEE Transaction on Systems, Man and Cybernetics* (Part C), a Reviewer of *Mathematic Review of America*, a Member of the Council of Chinese Artificial Intelligence Committee, and Vice-Chairman of Automation Society of Hebei Province, China. Dr Guan is 'Cheung Kong Scholars Programme' Special appointment professor, Ministry of Education, 2005, and the National Science Fund for Distinguished Young Scholars of China, 2005.

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