

# An Optimal Multi-User, Multi-Service Algorithm for Dynamic Spectrum Management in DSL

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**Abstract**—Crosstalk between neighbouring lines is the major limiting factor in the performance of modern DSL systems. Dynamic Spectrum Management (DSM) techniques have been proposed to mitigate the effects of crosstalk. Many of these techniques address the problem from a multi-user perspective, considering crosstalk effects during bit-loading. Previous work has considered the case where each DSL line carries one constant bit-error-rate (BER) service. However, to date the multi-user, multi-QoS scenario has not been considered. This work considers the case where each line may carry more than one service with different BERs. Reducing the BER requirements for a given service reduces the power requirements and thus reduces crosstalk, improving the performance of all users in the binder group. The optimal approach for the two service case is derived and simulation results are presented.

**Index Terms**—xDSL, DSM, dual-QoS.

## I. INTRODUCTION

Crosstalk is a major limiting factor in xDSL systems. In fact, crosstalk noise is the dominant noise source in DSL. Recently, new techniques which fall under the category of Dynamic Spectrum Management (DSM) have been proposed which seek to mitigate the effects of crosstalk. Although it is possible to almost eliminate crosstalk [1], it is often not practical due to local loop unbundling, as co-ordination is required between co-located modems. Where crosstalk cancellation is not possible, other techniques have focused on new bit-loading techniques which are multiuser aware [2] [3] [4] [5].

Previous work has considered the multiuser bit-loading problem where each line carries one constant BER service [2] [3] [4] [5]. In a typical DSL system this is set at a value of approximately  $1 \times 10^{-7}$ . However, further performance gains are possible if the DSL lines adjust the BER requirements depending on the service being carried. For example, if a DSL line carries a single data service at a BER requirement of  $1 \times 10^{-7}$  and a voice service with a BER requirement of  $1 \times 10^{-3}$ , then less power will be required to carry the voice service, thus reducing the output power on that line and subsequently reducing crosstalk into neighbouring lines.

Some work has looked at dual-QoS (i.e. two BERs) loading for the traditional, single user scenario [6]. This paper investigates the dual-QoS bit loading problem from a multi-user perspective.

## II. SYSTEM MODEL

The system model adopted in this paper is an xDSL system based on DMT modulation, with  $K$  users and  $N$  tones per user. For simplicity, it is assumed that NEXT is eliminated by frequency division duplexing (FDD) and transmission is considered in the downstream direction only. Crosstalk noise is purely from FEXT coupling and occurs on a tone by tone basis. Background noise is included according to noise model A in [7].

The Signal-Noise Ratio for user  $k$  on tone  $n$  is computed as follows:

$$SNR_k^n = \frac{p_k^n g_{kk}^n}{\sum_{j \neq k} p_j^n g_{kj}^n + \sigma_k^n} \quad (1)$$

Where  $p_k^n$  is the transmit power spectral density (PSD) of user  $k$  on tone  $n$ . The term  $g_{kk}^n$  represents the direct channel gain of the DSL line for user  $k$  on tone  $n$ . The term  $g_{kj}^n$  represents the crosstalk gain (FEXT in this case) of user  $j$  into user  $k$  on tone  $n$ . The crosstalk gains are calculated according to standard models [8].  $\sigma_k^n$  is the received background noise power for user  $k$  on tone  $n$ . It includes thermal noise plus background noise from other systems (e.g. ISDN, HDSL). It is assumed that each modem has a maximum bit-loading of  $b_{max} = 15$ . It is further assumed that modems can only support integer bit-loading, which is the case in current implementations. The achievable bit-loading on tone  $n$  for user  $k$  is given by

$$b_k^n = \log_2 \left( 1 + \frac{SNR_k^n}{\Gamma} \right) \quad (2)$$

where  $\Gamma$  is the ‘SNR-gap’ which is a function of the line code and target BER [9].

## III. THE SPECTRUM BALANCING PROBLEM

The spectrum balancing problem is expressed in a number of different ways, depending on rate and power constraints. They are further categorised into Rate Adaptive (RA), Margin Adaptive (MA) and Fixed Margin (FM) [9]. Rate adaptive algorithms attempt to maximise users bit rates under power budget constraints. Margin Adaptive algorithms maximise the user performance margin based upon a target bit rate. Fixed Margin algorithms minimise the power required to transmit a target bit rate at a fixed performance margin. Margin Adaptive algorithms are not so interesting as they always use the

entire power budget to maximise the performance margin of a particular line. This is wasteful of power and is very harmful to the performance of other lines in the binder.

In the single user case, the rate adaptive problem is stated as follows:

$$\begin{aligned} \max R_k &= \sum_{n=1}^N b_k^n \\ \text{s.t. } \sum_{n=1}^N p_k^n &\leq P_{budget} \end{aligned} \quad (3)$$

Similarly, the fixed margin problems is stated as:

$$\begin{aligned} \min \sum_{n=1}^N p_k^n \\ \text{s.t. } \sum_{n=1}^N b_k^n &\geq B_{target} \end{aligned} \quad (4)$$

In the context of multiuser algorithms, the problem is normally stated in a slightly different way and there are a variety of possibilities depending on requirements. For example:

$$\begin{aligned} \max R_1 \text{ s.t. } R_k &\geq R_k^{target} \\ \text{s.t. } \sum_{n=1}^N p_k^n &\leq P_{budget} \end{aligned} \quad (5)$$

Equation (5) states that the rate of line 1 is to be maximised, subject to all other lines meeting or exceeding their rate targets and that all lines are within their respective power budgets.

#### A. Multi-User Multi-Service Spectrum Balancing

Consider the situation where a DSL user requires a constant bit-rate channel to carry a service that can tolerate a reasonably high BER (e.g. voice). The user also requires a data service with a more stringent BER constraint which does not require a particular data rate. If this problem is considered from a multiuser perspective it can be written as:

$$\begin{aligned} \max R_k^{s0} \text{ s.t. } R_k^{s1} &\geq R_{k_{target}}^{s1} \\ \text{s.t. } \sum_{n=1}^N p_k^n &\leq P_{budget} \end{aligned} \quad (6)$$

Equation (6) states that the rates of service 0 on all lines is to be maximised (i.e. the sum rate), subject to all lines meeting the data rate requirement for service 1 and with all lines within their power budgets. In this case, service 0 is the low BER service (e.g. data) and service 1 is the higher BER service (e.g. voice).

#### B. Related Work

Other DSM techniques have been proposed which address the single service problem, with varying levels of co-ordination required between modems. Iterative Waterfilling (IW) [10] is a DSM technique which is autonomous, whereby each modem calculates its own power spectral densities. In IW, each modem executes a discrete waterfilling algorithm in succession until equilibrium is reached.

Optimal Spectrum Balancing (OSB) [2] is a centralised DSM algorithm, where the power spectral densities are calculated centrally by a Spectrum Management Centre (SMC). OSB provides better performance than IW at the cost of increased co-ordination and a higher computational complexity. The inner loop of OSB requires an exhaustive search over all bit-loading combinations on each tone, resulting in a running time that is exponential in the number of users  $K$ . Iterative Spectrum Balancing (ISB) [5] is a sub-optimal approach based on OSB, which reduces the computational complexity of the inner loop to  $O(K^2)$  at the cost of the optimality of the solution. The exhaustive search of the inner loop is replaced by an iterative search on each individual line which converges to at least a local optimum solution, and often to the global optimum for a tone.

The SCALE [3] algorithm takes a different approach, attempting to maximise the per tone lagrangian through an iterative convex relaxation procedure. SCALE has been shown to achieve close to optimal performance in networks with a moderate number of DSL lines and has a complexity similar to ISB [11].

All of the algorithms mentioned above apply to the single service scenario. Algorithms have been developed for the single user case that address the multi-service problem [6] [12]. However, they have not been extended to address the multi-user problem. The algorithm presented here incorporates techniques from OSB to calculate the optimal spectrum for 2-user 2-service case.

#### IV. OPTIMAL MULTI-USER MULTI-SERVICE SPECTRUM BALANCING

The Optimal Spectrum Balancing algorithm [2] utilises a dual decomposition method which transforms a power constrained optimisation into an unconstrained optimisation of a Lagrangian. This results in a complexity which is linear in the number of tones  $N$ , instead of an exhaustive search across all tones, which is exponential in  $N$ . Proper choice of the Lagrangian multipliers and weights enforces the power and rate constraints respectively. These co-efficients are found by bisection or gradient search methods. OSB is provably optimal and it is expected that the same proofs are applicable to the 2-service problem. Simulation results indicate that this assumption is reasonable.

To extend this algorithm to the multi-service case a new Lagrangian is formulated. It is assumed that service 0 is a low BER data service, with no fixed rate requirement that will be maximised. Service 1 is a higher BER service which requires a specified rate. For the example case of a 2-service

scenario of the optimisation problem shown in equation (6), the Lagrangian on tone  $N$  is as follows:

$$L_n = \sum_{k=1}^K w_{ks1} b_{ks1}^n + \sum_{k=1}^K b_{ks0}^n - \sum_{k=1}^K \lambda_k p_k^n(b_0^n, \dots, b_k^n) \quad (7)$$

In the single service case, the lagrangian function is non-convex and an exhaustive search over all bit-loading combinations on each tone is required. In the dual service case, it was found that the lagrangian function (7) was also generally non-convex and that an exhaustive search over all bit-loading and service combinations is necessary. This means that the complexity is exponential in the number of tones  $N$  and the number of services. As the computational complexity is very high, it is not tractable for more than 3-4 lines with more than one service. Although complex, the algorithm produces the optimal solution for the two service case and thus is useful for assessing the performance of lower complexity algorithms. An example algorithm is shown in algorithm 1, for the 2-user 2-service case.

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**Algorithm 1** Dual QoS Optimal Spectrum Balancing

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repeat
  for  $n = 1 \dots N$  do
    for all  $b_0, b_1, s_0, s_1$  do
       $b_n^0, b_n^1 = \max L_n(b_n^0, b_n^1, s_0, s_1, w_0, w_1, \lambda_1, \lambda_2)$ 
       $p_n^0 = p_n^0(b_n^0, b_n^1, s_0, s_1)$ 
       $p_n^1 = p_n^1(b_n^0, b_n^1, s_0, s_1)$ 
    end for
  end for
  for all  $k$  do
    if  $\sum_N p_n^k > P_{budget}$  then
       $\lambda_k^{max} = \lambda_k$ 
    else
       $\lambda_k^{min} = \lambda_k$ 
    end if
  end for
  for all  $k$  do
    if  $R_k^{s1} > R_{k_{target}}^{s1}$  then
       $w_k^{max} = w_k$ 
    else
       $w_k^{min} = w_k$ 
    end if
  end for
until  $|R_k^{s1} - R_{k_{target}}^{s1}| < e$ 

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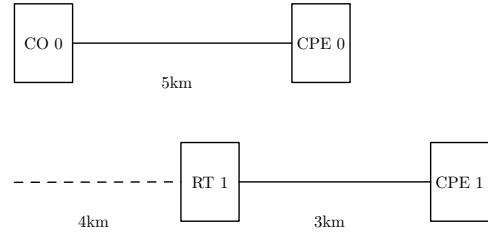


Fig. 1. DSL network configuration

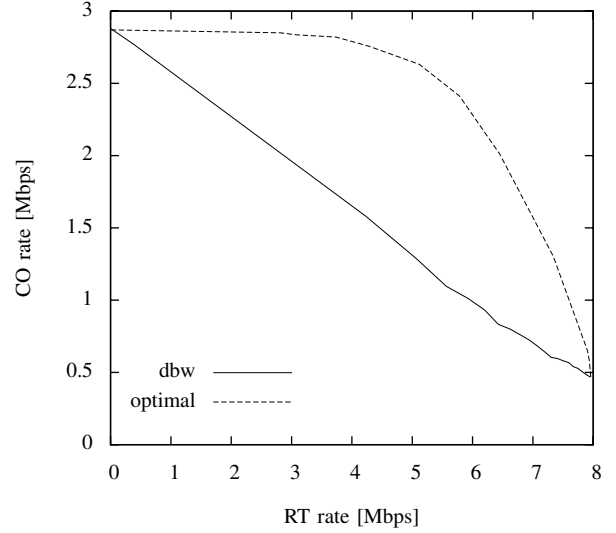


Fig. 2. Rate Region of Low BER service with High BER service set at 400kbps on each line

## V. SIMULATION RESULTS

In this section the new Dual QoS algorithm is tested by simulation. The network configuration is as shown in figure 1. The node named CO represents a DSL line termination in the central office. The node named RT represents a DSL line deployed from a remote terminal. This is a highly unbalanced crosstalk situation which is known to exhibit poor performance with static spectrum management techniques [13]. All line diameters are assumed to be 0.5mm (24-AWG). The coding gain is set at 3dB and noise margin at 6dB. A power budget of 100mW is assumed on each line.

Each line in the network is configured to load a high BER service ( $p_e = 1 \times 10^{-3}, \Gamma = 6.02dB$ ) with 100 bits per frame (approx. 400kbps), and maximise the data rate of the low BER service ( $p_e = 1 \times 10^{-7}, \Gamma = 9.95dB$ ). The most fully developed dual service algorithm in current literature is the Disjoint Bandwidth (DBW) algorithm presented in [6]. This exhibits very close to optimal performance for the single user case when the number of tones  $N$  is large. The DBW algorithm is used as a baseline for comparison to the new algorithm described here. This DBW algorithm is executed iteratively on each line until an equilibrium point is reached.

The resulting rate region for this scenario is shown in figure 2. It is obvious that the size of the rate region is greatly increased with the optimal method. Table I shows the achievable downstream rates when a rate target of 5Mbps is set for the low BER service on the RT line. The optimal spectrum allocation allows a rate of 2.7Mbps to be achieved on the CO line compared to a rate of 1.3Mbps with the iterative DBW loading method, an increase of over 100%.

The bit allocations and power spectral densities for each case are shown in figures 3, 4, 5 and 6. It is noted that for DBW loading, each line has a relatively flat PSD. This is characteristic of a waterfilling algorithm. It is however, suboptimal, as the RT line is crosstalking heavily into the CO line at low frequencies rather than utilising its own higher frequencies, which are usable due to the relatively short length of the RT line.

In the optimal loading case, the RT line's PSD is split into two bands. As the CO line is 5km long, the channel attenuation is very high at the top end of the ADSL band. At frequencies below channel 100, the RT line yields to the CO line, i.e. it's PSD is vastly reduced on these frequencies to reduce crosstalk into the CO line. Thus, the CO line is allowed better use of its strongest, or most 'useful' tones. The RT line makes up for losing bits on its low frequencies by utilising a much higher PSD in the tones above channel 100, where the heavy crosstalk generated into the CO line is not detrimental to its performance, as the CO line is inactive at these frequencies.

The resulting PSD graphs are very similar to the single service case for the same network scenario. In the single service case, the RT PSD is also split into two bands where the RT line yields to the CO line in a similar fashion. In the dual service case, it is noted that service 1 (High BER) is carried entirely within the low frequency band of the RT line. Intuitively, this is expected as service 1 requires a lower PSD and hence produces less crosstalk than service 0 otherwise would had it been carried in the low frequency tones.

Future work will examine the possibility of new loading algorithms which account for the 'usefulness' of a particular tone in an attempt to approximate the results of the optimal solution without performing an exhaustive searches on each tone.

## VI. CONCLUSION

In this paper, an algorithm has been presented to calculate the optimal spectrum allocation for a DSL network with more than one service class. Simulation results show that the algorithm outperforms the single user equivalent algorithm. The algorithm is computationally complex and is thus not practical to implement. However, it is useful to use as a performance indicator for less complex, sub-optimal algorithms.

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TABLE I  
ACHIEVABLE RATES OF DOWNSTREAM ADSL WITH DUAL-QoS SERVICE

	CO Line		RT Line	
	Low BER	High BER	Low BER	High BER
DBW	1.3 Mbps	400 kbps	5 Mbps	400 kbps
Optimal	2.7 Mbps	400 kbps	5 Mbps	400 kbps

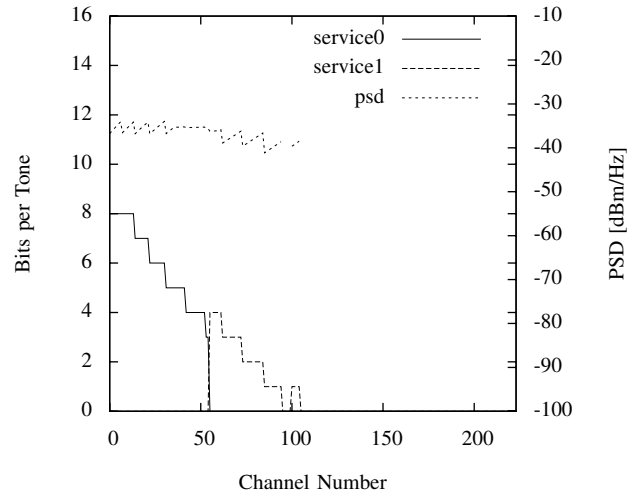


Fig. 3. CO Line - Bit allocation and PSD with DBW loading

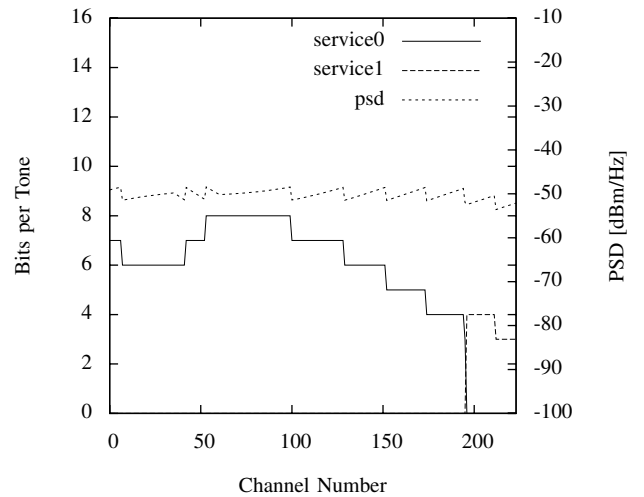


Fig. 4. RT Line - Bit allocation and PSD with DBW loading

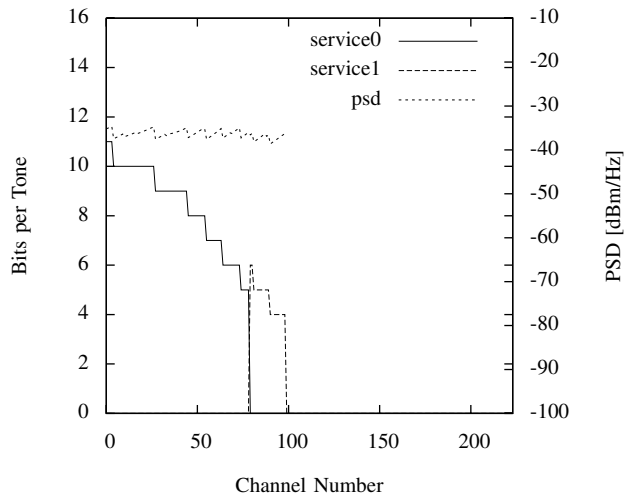


Fig. 5. CO Line - Bit allocation and PSD with optimal loading

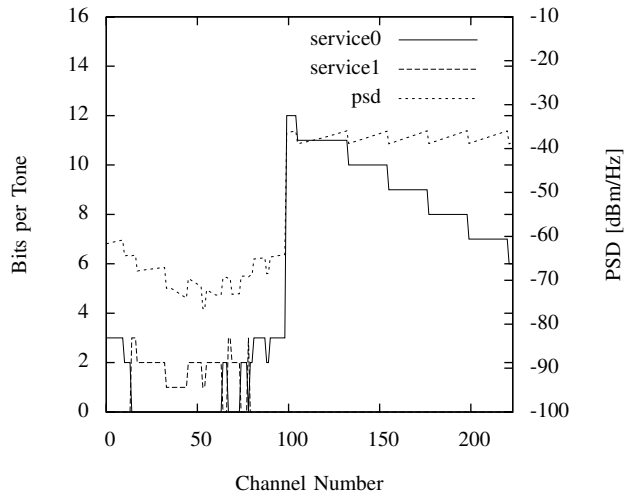


Fig. 6. RT Line - Bit allocation and PSD with optimal loading

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