Multi-User Discrete Bit-Loading for DMT-based DSL Systems

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***Abstract*— This paper investigates the multiuser bit and power allocation problem in discrete multi-tone digital subscriber line modems. A spectrum-management center with knowledge of direct and crosstalk-coupled channel gains allocates the bits and the available power to the subchannels for all users in a common binder. The center uses a multiuser discrete bit-loading algorithm that attempts to minimize the total transmit power given a target sum-rate. This algorithm extends the greedy algorithm for the single-user channel to the multi-user channel. Simulation results for the upstream transmission in very high-speed digital subscriber line show that the total power can be reduced using the multi-user discrete bit-loading algorithm instead of applying the single-user greedy algorithm iteratively.**

I. Introduction

Discrete multi-tone (DMT) modulation partitions a channel into N independent additive white Gaussian noise (AWGN) subchannels, each with no inter-symbol interference (ISI). DMT-based digital subscriber line (DSL) modems thus require an algorithm that allocates the bits and the power to these subchannels in order to optimize the system performance. For the single-user case, the data rate is maximized by the well-known "water-filling" of available power. However, the number of bits in each subchannel should be an integer or a multiple of some base unit in practice. Several optimal algorithms for this discrete bit-loading problem have been developed [1], [2], [3], [4] in addition to suboptimal algorithms with reduced complexity [5], [6], [7], [8]. The problem of maximizing the sum of the upstream and downstream data rates has also been considered [9].

When the level of crosstalk is high, the power allocation of a user changes the noise experienced by the other users in the same binder. In this case, the performance of DSL modems can be improved by jointly considering the bit and power allocation of all users. In [10], the problem of minimizing the total power is formulated as a nonlinear optimization problem with a simulated annealing method used to find the optimal solution numerically. The iterative water-filling algorithm in [11] can be implemented autonomously without any coordination between users, and its performance is compared with the optimal scheme in [12].

This paper considers the discrete bit-loading problem for the multi-user case with the objective of minimizing the total power necessary to achieve a target sum-rate. A spectrum-management center with knowledge of the channel gains allocates the bits and the power to the subchannels for users in a common binder This method clearly requires a certain amount of overhead to DSL systems. Specifically, each modem should estimate the channel and crosstalk transfer functions, and then transmit the channel information to the spectrum-management center. With this channel information, the center determines the bit and power allocation for every modem using the multi-user discrete bit-loading algorithm. Finally, the center transmits the allocation result back to each modem. Since the DSL channel varies slowly, the bit and power allocation does not need to be repeated often, resulting in a relatively small overhead.

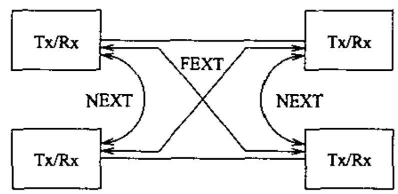

Fig. 1. An illustration of near-end crosstalk and far-end crosstalk

The remainder of this paper is organized as follows. Section II models DSL loops as interference channels and describes the multi-user discrete bit-loading problem, and Section III proposes a multi-user discrete bit-loading algorithm with discussion of its computational complexity. The simulation results for very high-speed digital subscriber line (VDSL) upstream transmission are presented in Section IV, and concluding remarks are given in Section V.

II. Problem Formulation

In DSL, a binder consists of up to 50 subscriber loops, each of which generates the electromagnetic field that causes crosstalk signals to the other loops [13]. Figure 1 shows the DSL crosstalk environment. The near-end crosstalk (NEXT) is the crosstalk generated by transmitters located at the same side of the receiver, while the far-end crosstalk (FEXT) is induced by transmitters at the opposite side of the receiver. Frequency division duplexing scheme is used to make the NEXT negligible in this paper.

Due to the FEXT, the DSL channel with M users is an interference channel with ISI. By employing the DMT technique, the overall channel can be modelled as N independent ISI-free subchannels, each of which is an interference channel of M users. Figure 2 shows the channel model of subchannel n with $H_{i,i}(n)$ representing the direct channel gain of user i

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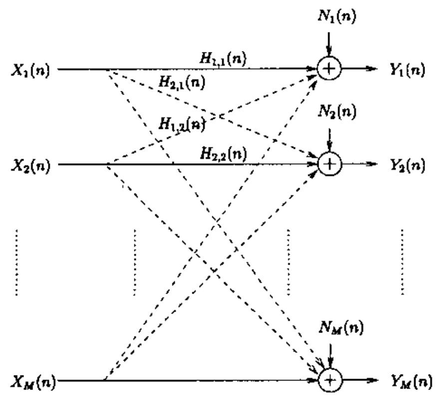

Fig. 2. Subchannel n with M transmitters and M receivers

in subchannel n and $H_{i,j}(n)$ for $i \neq j$ the crosstalk channel gain from user j to user i.

The signal-to-interference-plus-noise ratio (SINR) of user i in subchannel n is then expressed as

$$S_i(n) = \frac{H_{i,i}^2(n)P_i(n)}{N_i(n) + \sum_{j=1,j\neq i}^{M} H_{i,j}^2(n)P_j(n)}, \quad (1)$$

where $P_i(n)$ and $N_i(n)$ are the signal power and the background noise power of user i in subchannel n, respectively. The SINR necessary for user i to transmit $b_i(n)$ bits in subchannel n is expressed as

$$\gamma_i(b_i(n)) = f_i(b_i(n), \overline{P_{e,i}}), \quad (2)$$

where $\overline{P_{e,i}}$ is the average probability of symbol error of user i and the function f_i depends on the modulation and coding scheme. Different users can have different quality-of-service requirements and different coding and modulation schemes. These different requirements change the function f and the probability of symbol error P_e for each user. From (1) and (2), the power $P_i(n, \mathbf{b}_n)$ of user i in subchannel n for a bit distribution $\mathbf{b}_n = [b_1(n)\ b_2(n)\ \cdots\ b_M(n)]^T$ should satisfy

$$\frac{H_{i,i}^2(n)P_i(n,\mathbf{b}_n)}{N_i(n) + \sum_{j=1,j\neq i}^{M} H_{i,j}^2(n)P_j(n,\mathbf{b}_n)} \geq f_i(b_i(n), \overline{P_e}) \quad \text{for } i = 1, \cdots, M. \quad (3)$$

One interesting problem is minimizing the necessary power in order to achieve the target data rate, which is commonly referred to as the margin maximization [13], [8]. The multi-user analog of the single-user margin maximization is the minimization of the total power given a target sum-rate. This minimization with a finite granularity constraint on the number of bits can be mathematically formulated as follows:

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^{M}\sum_{n=1}^{N} P_i(n) \\ \text{subject to} & \sum_{i=1}^{M}\sum_{n=1}^{N} b_i(n) \geq B \\ & b_i(n) \in \mathbf{Z}_0^{\infty}, \end{array} \quad (4)$$

where B is the target number of bits and $\mathbf{Z}_0^{\infty}$ is the set of non-negative integers. The goal of this problem is to find the bit and power allocation requiring the least amount of total power to transmit a target number of bits. In practice, a bit cap is used to limit the number of bits allowed for each user in each subchannel. Similarly, a power mask constraint is often employed to prevent the power from exceeding the maximum power allowed in each subchannel. The total power minimization problem is modified to account for these additional constraints:

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^{M}\sum_{n=1}^{N} P_i(n) \\ \text{subject to} & \sum_{i=1}^{M}\sum_{n=1}^{N} b_i(n) \geq B \\ & b_i(n) \in \mathbf{Z}_0^{\bar{b}}, \\ & P_i(n) \leq \overline{P(n)} \end{array} \quad (5)$$

where $\bar{b}$ is the bit cap, $\mathbf{Z}_0^{\bar{b}}$ is $\{0, 1, \cdots, \bar{b}\}$, and $\overline{P(n)}$ is the power mask. It should be mentioned here that another interesting problem is to find the achievable data rate region given an individual power constraint for every user. In order to solve this problem, the rate and power control algorithm in [14] uses asli ght modification of the multi-user discrete bit-loading algorithm developed in this paper.

The following notation is used. Let A and B be real matrices of the same size. We write $A \geq B$ ($A > B$) if the weak (strict) inequality holds element-wise, respectively. We say that A is positive (nonnegative) if $A > 0$ ($A \geq 0$), respectively.

III. Multiuser Discrete Bit-Loading

The total power minimization problem in (5) can be reformulated as an integer linear-programming problem with the optimal solution found using the branch-and-bound method [9]. To reduce the complexity of the optimal algorithm, we propose a suboptimal multi-user discrete bit-loading algorithm that heuristically extends the optimal greedy algorithm for a single user in [1] and the algorithm for a frequency overlapped duplex operation in [9].

The single-user greedy algorithm assigns one bit in the subchannel that requires the least cost, i.e., the least amount of power increase, in order to transmit this additional bit. Similarly, the multi-user discrete bit-loading algorithm allocates one bit to the user and subchannel where adding one bit minimizes the cost. However, the cost $J(n, i)$ to allocate one more bit to user i and subchannel n is not simply the power increase of user i. The interference to the other users forces them to increase their power in order to maintain their SINRs. Thus, the cost $J(n, i)$ is chosen as the minimum total incremental power of all users. The following subsection derives the following function which represents the minimum power sum for a given bit distribution:

$$P_{\Sigma}^{*}(n, \mathbf{b}_n) = \min \sum_{i=1}^{M} P_i(n, \mathbf{b}_n). \quad (6)$$

The cost $J(n, i)$ can then be calculated using this function.

A. Minimum Power for a Given Bit Allocation

The power $P_i(n, \mathbf{b}_n)$ should satisfy (3), which can be rearranged in a matrix form:

$$(I - A_n)\mathbf{x}_n(\mathbf{b}_n) \geq \mathbf{y}_n, \tag{7}$$

where

$$\{A_n\}_{i,j} = \begin{cases} \frac{\gamma_i(b_i(n))H_{i,j}^2(n)}{H_{i,i}^2(n)} & , \text{for } i \neq j, \\ 0 & , \text{for } i = j, \end{cases} \tag{8}$$

$$\mathbf{x}_n(\mathbf{b}_n) = \begin{bmatrix} P_1(n, \mathbf{b}_n) & \cdots & P_M(n, \mathbf{b}_n) \end{bmatrix}^T, \tag{9}$$

$$\mathbf{y}_n = \begin{bmatrix} \frac{\gamma_1(b_1(n))N_1(n)}{H_{1,1}^2(n)} & \cdots & \frac{\gamma_M(b_M(n))N_M(n)}{H_{M,M}^2(n)} \end{bmatrix}^T, \tag{10}$$

and $\gamma_i(b_i(n))$ is given by (2).

When A_n is irreducible[1], the *Perron eigenvalue* of A_n, i.e., the maximum modulus eigenvalue $\lambda(A_n)$, is real and positive with a positive eigenvector by the Perron-Frobenius theorem for irreducible matrices [15]. In [16], it was shown that the following three statements are equivalent:

1) There exists a solution to (7),
2) $\lambda(A_n)$ is less than 1,
3) $(I - A_n)^{-1}$ exists and is positive.

In addition, $\mathbf{x}_n^*(\mathbf{b}_n) = (I - A_n)^{-1}\mathbf{y}_n$ is the Pareto optimal solution to (7). In other words, any $\mathbf{x}_n(\mathbf{b}_n)$ that satisfies (7) is greater than or equal to $\mathbf{x}_n^*(\mathbf{b}_n)$ element-wise.

If the target SINR $\gamma_i(n)$ is zero for some user i or equivalently some users transmit no bits in subchannel n, A_n is not irreducible. However, the results in [16] can be easily extended to the case where A_n is not necessarily irreducible but nonnegative. In Appendix, it is proven that the following lemma and proposition hold.

Lemma 1: Suppose that A_n is nonnegative. If a nonnegative vector $\mathbf{x}_n(\mathbf{b}_n)$ satisfying (7) exists, $\lambda(A_n)$ is less than 1, $(I - A_n)^{-1}$ exists, and $(I - A_n)^{-1}$ can be expressed as $\sum_{k=0}^{\infty}(A_n)^k$.

Proposition 1: Pareto Optimality
For a nonnegative matrix A_n, the vector

$$\mathbf{x}_n^*(\mathbf{b}_n) = (I - A_n)^{-1}\mathbf{y}_n \tag{11}$$

is the Pareto optimal solution to (7) when a nonnegative $\mathbf{x}_n$ satisfying (7) exists.

Proposition 1 enables us to find the minimum power sum $P_\Sigma^*(n, \mathbf{b}_n)$ necessary for the bit allocation $\mathbf{b}_n$. Using the Pareto optimal solution $\mathbf{x}_n^*(\mathbf{b}_n)$, $P_\Sigma^*(n, \mathbf{b}_n)$ can be calculated as follows:

$$\begin{aligned} P_\Sigma^*(n, \mathbf{b}_n) &= [1\ 1\ \cdots\ 1]^T \mathbf{x}_n^*(\mathbf{b}_n) & (12) \\ &= [1\ 1\ \cdots\ 1]^T (I - A_n)^{-1}\mathbf{y}_n, & (13) \end{aligned}$$

[1] A matrix A is irreducible if and only if there exists a nonnegative integer k for every (i, j) such that the (i, j) element of A^k is nonzero.

where A_n and $\mathbf{y}_n$ are determined by $\mathbf{b}_n$ using (2), (8), and (10). The cost function $J(n, i)$ is then expressed as

$$J(n, i) = P_\Sigma^*(n, \mathbf{b}_n + \mathbf{e}_i) - P_\Sigma^*(n, \mathbf{b}_n) \tag{14}$$

where $\mathbf{e}_i$ is a unit vector whose ith element is 1 and all the other elements are 0.

B. Multiuser Discrete Bit-Loading Algorithm

The multi-user discrete bit-loading algorithm for (5) can be described as follows.

Initialization:

1) Initialize $\mathbf{b}_n = [0\ \cdots\ 0]^T$ and $\mathbf{x}_n^*(\mathbf{0}) = [0\ \cdots\ 0]^T$ for $n = 1, \cdots, N$.
2) Initialize the matrix F of the flags:

$$F = O_{N,M}, \tag{15}$$

where $O_{N,M}$ is a zero matrix of size $N \times M$. $\{F\}_{n,i} = 0$ indicates that it is possible to add more bits to the user i and subchannel n, whereas $\{F\}_{n,i} = 1$ means that the subchannel n of user i is saturated.

3) Calculate the cost to transmit one bit. For $n = 1, \cdots, N$ and $i = 1 \cdots, M$,
   a) calculate $\mathbf{x}_n^*(\mathbf{e}_i)$ using (11),
   b) calculate $J(n, i)$ using (14),
   c) check the power mask constraint: if $P_i(n, \mathbf{e}_i) > \overline{P(n)}$, set $F(n, i) = 1$.
4) Find the user $K(n)$ that requires the minimum cost in each subchannel and the corresponding cost $\widetilde{J}(n)$: for $n = 1, \cdots, N$,

$$K(n) = \arg\{\min_{\{i: F(n,i)=0\}} J(n, i)\},$$

$$\widetilde{J}(n) = \min_{\{i: F(n,i)=0\}} J(n, i),$$

Bit-Loading Iterations:

5. Find the subchannel m with the minimum cost $\widetilde{J}(m)$:

$$m = \arg\{\min_{n=1,\cdots,N} \widetilde{J}(n)\}.$$

6. Add one bit to user $K(m)$ in subchannel m:

$$\mathbf{b}_m = \mathbf{b}_m + \mathbf{e}_{K(m)}.$$

7. Update the cost of adding one more bit to subchannel m. For $i = 1, \cdots, M$,
   a) calculate $\mathbf{x}_m^*(\mathbf{b}_m + \mathbf{e}_i)$ using (11),
   b) calculate $J(m, i)$ using (14),
   c) check the bit cap constraint: if $b_i(m) = \bar{b}$, set $F(m, i) = 1$,
   d) check the power mask constraint: if $\max_k\{P_k(n, \mathbf{b}_m + \mathbf{e}_i)\} > \overline{P(n)}$, set $F(m, i) = 1$.

8. Find the user $K(m)$ that requires the minimum cost in subchannel m and the corresponding cost $\widetilde{J}(m)$:

$$K(m) = \arg \{ \min_{\{i:F(m,i)=0\}} J(m,i)\},$$

$$\widetilde{J}(m) = \min_{\{i:F(m,i)=0\}} J(m,i),$$

9. Check whether the target number of bits has been reached. Go to step 5 if $\sum_{i=1}^{M} \sum_{n=1}^{N} b_i(n) < B$.

Calculating $\mathbf{x}_n^*(\mathbf{b}_n)$ requires the inversion of $I - A_n$. The complexity in the calculation of $(I - A_n(\mathbf{b}_n + \mathbf{e}_i))^{-1}$ can be reduced using $(I - A_n(\mathbf{b}_n))^{-1}$. Let $C_n = I - A_n$ for notational convenience. The construction of $C_n^{-1}(\mathbf{e}_i)$ in step 3 is straightforward:

$$\{C_n^{-1}(\mathbf{e}_i)\}_{a,b} = \begin{cases} \frac{\gamma_i(1)N_i(n)}{H_{i,i}^2(n)} & \text{, for } a = b = i, \\ 0 & \text{, otherwise .} \end{cases} \tag{16}$$

The subsequent C_n^{-1}'s can be easily updated using (7) and the *matrix inversion formula*[2],

$$C_n^{-1}(\mathbf{b}_n + \mathbf{e}_i) = C_n^{-1}(\mathbf{b}_n) - \frac{1}{1 + \mathbf{h}^T C_n^{-1}(\mathbf{b}_n)\mathbf{e}_i}(C_n^{-1}(\mathbf{b}_n)\mathbf{e}_i)(\mathbf{h}^T C_n^{-1}(\mathbf{b}_n)) \tag{17}$$

where

$$\mathbf{h}^T = (\gamma_i(b_i(n)) - \gamma_i(b_i(n)+1))[\frac{H_{i,1}^2}{H_{i,i}^2} \cdots 0 \cdots \frac{H_{i,M}^2}{H_{i,i}^2}]. \tag{18}$$

The average running time of this algorithm is proportional to $B \times (O(N) + O(M^2)))$; updating the cost and finding the minimum requires $O(N) + O(M^2)$ operations that should be repeated until the target number of bits B is reached. Here, $O(n)$ denotes the order of n. This running time is comparable to that of the Hughes-Hartogs algorithm [1], which has a quite high complexity compared to other efficient algorithms [5], [8], [3]. Developing an algorithm with less complexity may be necessary in the multi-user case as in the single-user case.

IV. Simulation Results

Weapp ly the proposed multi-user discrete bit-loading algorithm to the VDSL upstream transmission. The simulation parameters are taken from [17] and [18]. The North American frequency plan is used for the upstream and downstream frequency band assignment. The crosstalk noise model A is used to generate crosstalk from other services. The power mask is applied at frequencies below 1.1 MHz to protect other services such as asynchronous DSL (ADSL) and high-speed DSL (HDSL), and the bit cap $\bar{b}$ is chosen to be equal to 11. We use the gap approximation to calculate the minimum SINR in (2) [13]:

$$\gamma_i(b_i(n)) = \Gamma(2^{b_i(n)} - 1), \tag{19}$$

where Γ is the signal to noise ratio (SNR) gap, which depends on the symbol error probability, the noise margin, and the modulation and coding scheme. The SNR gap Γ for quadrature amplitude modulation is 12 dB with a 3.8 dB coding gain, a 6 dB noise margin, and 10^{-7} symbol error probability.

[2] $(A + bc^T)^{-1} = A^{-1} - \frac{1}{1+c^T A^{-1} b}(A^{-1}b)(c^T A^{-1})$

TABLE I

Target data rates

L (ft)	$R_{\text{sum, target}}$ (Mbps)	$R_1, \cdots, R_4$ (Mbps)	$R_5, \cdots, R_8$ (Mbps)
500	136.4	4.7	29.4
1000	118.4	4.7	24.9
1500	107.2	4.7	22.1
2000	86.0	4.7	16.8
2500	54.0	4.7	8.8

We compare the performance of the multi-user discrete bit-loading algorithm with the discrete version of the iterative water-filling [11], i.e., the iterative discrete bit-loading scheme which applies the single-user greedy algorithm iteratively until the bit and power allocation of every user converges to an equilibrium point. Consider the bit and power allocation of 8 loops. The length of the first four loops is equal to 3000 ft, while the length of the other four loops is equal to L ft, where L is varied from 500 ft to 2500 ft with a step size of 500 ft. Table I shows the target sum-rate and the resulting data rate of each user when the multi-user discrete bit-loading is used. The target sum-rate is deliberately chosen so that the data rate of 3000 ft loops becomes 4.7 Mbps. The single-user greedy algorithm is then used iteratively with the target rate of each user equal to the data rate in Table I. Figure 3 shows the total required power to transmit the data rates in Table Iwith the multi-user and iterative discrete bit-loading algorithm. The multi-user bit-loading performs better than the iterativebit-loading in all cases. The penalty for the total power decrease is that the bit and power allocation should be determined in a spectrum-management center using the complete channel information, which is not required by the iterative algorithm. Moreover, the iterative scheme has less complexity than the multi-user scheme since there exists efficient single-user bit-loading algorithms.

V. Conclusions

This paper has considered the multi-user extension of the margin maximization problem which was formulated as minimizing the total power for the given target sum-rate. Use of the multiuser bit-loading scheme can provide additional improvements in binder spectrum design over autonomous training with iterative water-filling methods. This is an intermediate step in improving performance between systems that are physically unbundled and no coordination is possible and systems that coordinate at a remote-terminal DSL-access-multiplexer (DSLAM) location and can co-generate signals (as well as spectrum). Such intermediate level of coordination may or may not be possible, depending on regulatory situations and public/private nature of the telephone network. Nonetheless, shared use of channel information can lead to better performance when possible from non-technical perspectives.

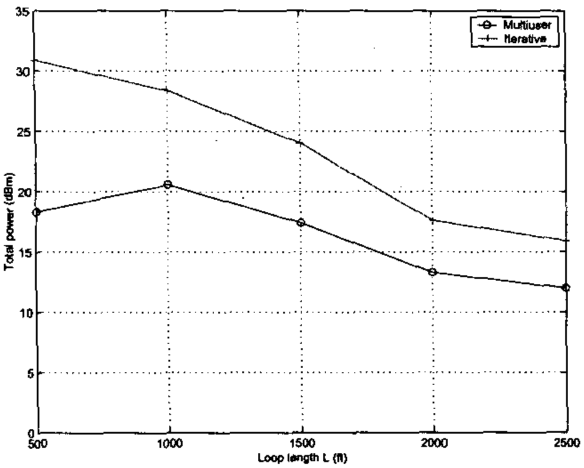

Fig. 3. Total power with iterative and multi-user discrete bit-loading

APPENDIX

The proofs of Lemma 1 and Proposition 1 are given in this appendix.

Proof of Lemma 1: Let A_n be any nonnegative matrix. By applying the same permutation on both the rows and the columns of A_n, A_n can be written in an upper block-triangular form [15]:

$$PA_nP^{-1} = C = \begin{bmatrix} C_1 & D_{1,2} & \cdots & D_{1,k} \\ O & C_2 & \cdots & D_{2,k} \\ O & O & \ddots & \vdots \\ O & O & \cdots & C_k \end{bmatrix}, \quad (20)$$

where O is a matrix whose elements are all 0, C_k are square irreducible matrices, and P is a permutation matrix. The following then holds:

$$\begin{aligned} & (I - A_n)\mathbf{x}_n \geq \mathbf{y}_n \\ \iff & P(I - A_n)P^{-1}P\mathbf{x}_n \geq P\mathbf{y}_n \\ \iff & (I - PA_nP^{-1})P\mathbf{x}_n \geq P\mathbf{y}_n \\ \iff & (I - C)\mathbf{p} \geq \mathbf{q}, \end{aligned} \quad (21)$$

where $\mathbf{p} = [\mathbf{p}_1 \cdots \mathbf{p}_k]^T = P\mathbf{x}_n$ and $\mathbf{q} = [\mathbf{q}_1 \cdots \mathbf{q}_k]^T = P\mathbf{y}_n$. Consequently, $\mathbf{x}_n \geq 0$ is equivalent to $\mathbf{p} \geq 0$, and the existence and the nonnegativity of $(I - A_n)^{-1}$ are equivalent to those of $(I - C)^{-1}$.

Suppose that a nonnegative vector $\mathbf{p}$ satisfies (21). Then $\mathbf{p}_m$ satisfies the following inequality:

$$(I - C_m)\mathbf{p}_m \geq \mathbf{q}_m \quad (22)$$

for all $m = 1, \cdots, k$. Since C_m is irreducible, $\lambda(C_m) < 1$. Thus, $\lambda(C) < 1$ by the Perron-Frobenius theorem for nonnegative matrices. In addition, $\sum_{k=0}^{\infty} C^k$ exists and is equal to $(I - C)^{-1}$. □

Proof of Proposition 1: Suppose that a nonnegative vector $\mathbf{x}_n$ satisfying (7) exists. Then $(I - A_n)^{-1} = \sum_{k=0}^{\infty}(A_n)^k$ exists by Lemma 1. Let $\mathbf{x}_n^* = (I - A_n)^{-1}\mathbf{y}_n$, and let $\mathbf{x}_n$ be any vector satisfying (7). Since $(I - A_n)^{-1} \geq 0$ and $\mathbf{y}_n \geq 0$, multiplying both sides of (7) by $(I - A_n)^{-1}$ does not change the inequality. Therefore, $\mathbf{x}_n \geq (I - A_n)^{-1}\mathbf{y}_n = \mathbf{x}_n^*$. □

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