DATA 621 - HW3

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Homework 3 - Logistic Regression

Overview:

In this homework assignment, you will explore, analyze and model a data set containing information on crime for various neighborhoods of a major city. Each record has a response variable indicating whether or not the crime rate is above the median crime rate (1) or not (0).

Your objective is to build a binary logistic regression model on the training data set to predict whether the neighborhood will be at risk for high crime levels. You will provide classifications and probabilities for the evaluation data set using your binary logistic regression model. You can only use the variables given to you (or, variables that you derive from the variables provided).

Below is a short description of the variables of interest in the data set:

Column	Description
`zn`	proportion of residential land zoned for large lots (over 25000 square feet) (*predictor variable*)
`indus`	proportion of non-retail business acres per suburb (*predictor variable*)
`chas`	a dummy var. for whether the suburb borders the Charles River (1) or not (0) (*predictor variable*)
`nox`	nitrogen oxides concentration (parts per 10 million) (*predictor variable*)
`rm`	average number of rooms per dwelling (*predictor variable*)
`age`	proportion of owner-occupied units built prior to 1940 (*predictor variable*)
`dis`	weighted mean of distances to five Boston employment centers (*predictor variable*)
`rad`	index of accessibility to radial highways (*predictor variable*)
`tax`	full-value property-tax rate per $10,000 (*predictor variable*)
`ptratio`	pupil-teacher ratio by town (*predictor variable*)
`lstat`	lower status of the population (percent) (*predictor variable*)
`medv`	median value of owner-occupied homes in $1000s (*predictor variable*)
`target`	**whether the crime rate is above the median crime rate (1) or not (0) (*response variable*)**

Data Exploration:

```
## [1] 466  13
```

The dataset consists of 466 observations of 13 variables. There are 12 predictor variables and one response variable (`target`).

```
## Rows: 466
## Columns: 13
## $ zn      <dbl> 0, 0, 0, 30, 0, 0, 0, 0, 0, 80, 22, 0, 0, 22, 0, 0, 100, 20, 0~
## $ indus   <dbl> 19.58, 19.58, 18.10, 4.93, 2.46, 8.56, 18.10, 18.10, 5.19, 3.6~
## $ chas    <int> 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,~
## $ nox     <dbl> 0.605, 0.871, 0.740, 0.428, 0.488, 0.520, 0.693, 0.693, 0.515,~
## $ rm      <dbl> 7.929, 5.403, 6.485, 6.393, 7.155, 6.781, 5.453, 4.519, 6.316,~
## $ age     <dbl> 96.2, 100.0, 100.0, 7.8, 92.2, 71.3, 100.0, 100.0, 38.1, 19.1,~
## $ dis     <dbl> 2.0459, 1.3216, 1.9784, 7.0355, 2.7006, 2.8561, 1.4896, 1.6582~
## $ rad     <int> 5, 5, 24, 6, 3, 5, 24, 24, 5, 1, 7, 5, 24, 7, 3, 3, 5, 5, 24, ~
## $ tax     <int> 403, 403, 666, 300, 193, 384, 666, 666, 224, 315, 330, 398, 66~
## $ ptratio <dbl> 14.7, 14.7, 20.2, 16.6, 17.8, 20.9, 20.2, 20.2, 20.2, 16.4, 19~
## $ lstat   <dbl> 3.70, 26.82, 18.85, 5.19, 4.82, 7.67, 30.59, 36.98, 5.68, 9.25~
## $ medv    <dbl> 50.0, 13.4, 15.4, 23.7, 37.9, 26.5, 5.0, 7.0, 22.2, 20.9, 24.8~
## $ target  <int> 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 1, 0,~
```

All of the columns in the dataset are numeric, but the predictor variable `chas` is a dummy variable, as is the response variable `target`. We re-code them as factors.

Let's take a look at the summary statistics for the variables in the dataset.

```
##        zn             indus        chas         nox               rm        
##  Min.   :  0.00   Min.   : 0.460   0:433   Min.   :0.3890   Min.   :3.863  
##  1st Qu.:  0.00   1st Qu.: 5.145   1: 33   1st Qu.:0.4480   1st Qu.:5.887  
##  Median :  0.00   Median : 9.690           Median :0.5380   Median :6.210  
##  Mean   : 11.58   Mean   :11.105           Mean   :0.5543   Mean   :6.291  
##  3rd Qu.: 16.25   3rd Qu.:18.100           3rd Qu.:0.6240   3rd Qu.:6.630  
##  Max.   :100.00   Max.   :27.740           Max.   :0.8710   Max.   :8.780  
##       age              dis              rad              tax        
##  Min.   :  2.90   Min.   : 1.130   Min.   : 1.00   Min.   :187.0  
##  1st Qu.: 43.88   1st Qu.: 2.101   1st Qu.: 4.00   1st Qu.:281.0  
##  Median : 77.15   Median : 3.191   Median : 5.00   Median :334.5  
##  Mean   : 68.37   Mean   : 3.796   Mean   : 9.53   Mean   :409.5  
##  3rd Qu.: 94.10   3rd Qu.: 5.215   3rd Qu.:24.00   3rd Qu.:666.0  
##  Max.   :100.00   Max.   :12.127   Max.   :24.00   Max.   :711.0  
##     ptratio          lstat            medv       target 
##  Min.   :12.6   Min.   : 1.730   Min.   : 5.00   0:237  
##  1st Qu.:16.9   1st Qu.: 7.043   1st Qu.:17.02   1:229  
##  Median :18.9   Median :11.350   Median :21.20          
##  Mean   :18.4   Mean   :12.631   Mean   :22.59          
##  3rd Qu.:20.2   3rd Qu.:16.930   3rd Qu.:25.00          
##  Max.   :22.0   Max.   :37.970   Max.   :50.00
```

We can see the mean, median, standard deviations, ranges, etc. for each of the variables in the dataset.

There are 229 instances where crime level is above the median level and 237 instances where crime is not above the median level.

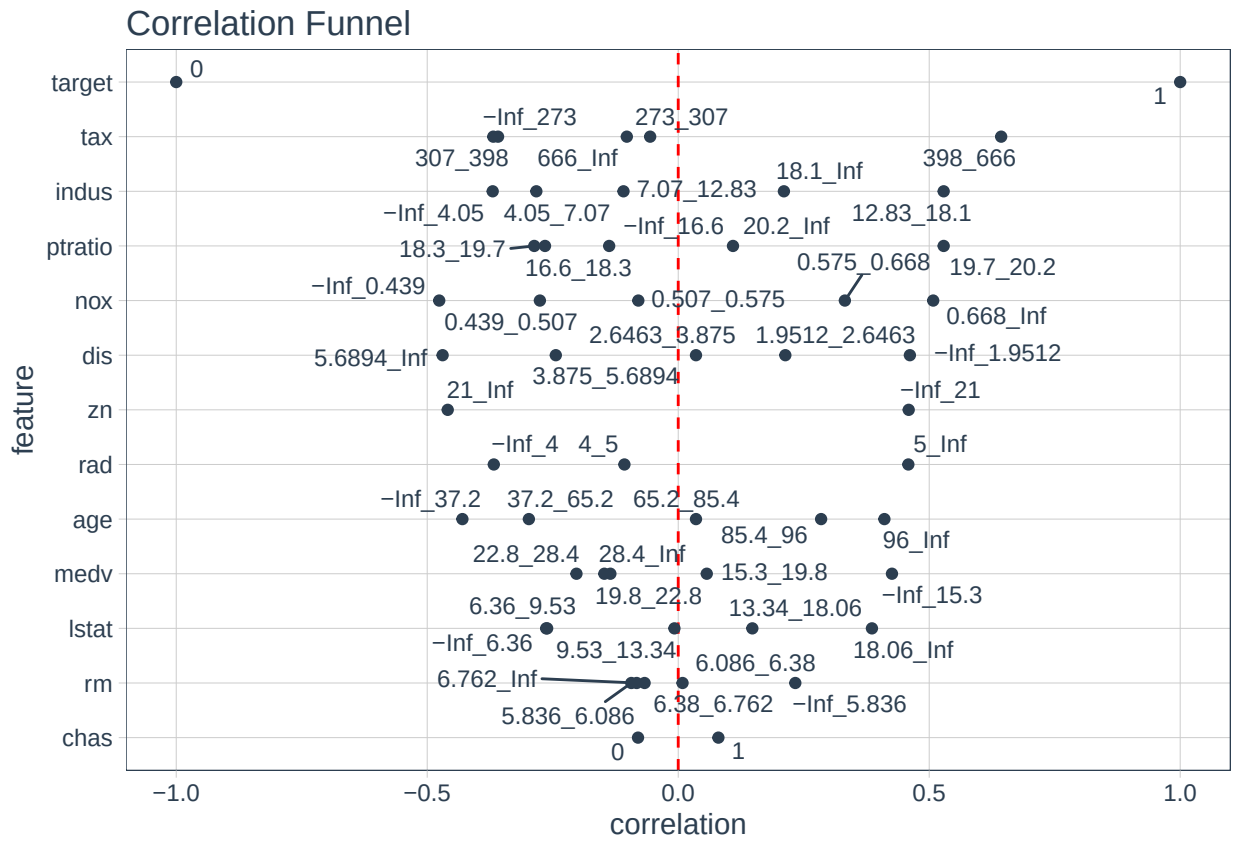
More discussion of interesting summary stats within the dataset TK

Each predictor has 466 values, which matches the number of observations in our dataset, so there do not appear to be any missing values to address. Let's validate this.

```
## [1] 0
```

There are in fact no missing values in the dataset.

To check whether the predictor variables are correlated to the target variable, we produce a correlation funnel that visualizes the strength of the relationships between our predictors and our response.



The correlation funnel plots the most important features towards the top. In our dataset, the four most important features correlated with the response variable are `tax`, `indus`, `ptratio`, and `nox`.

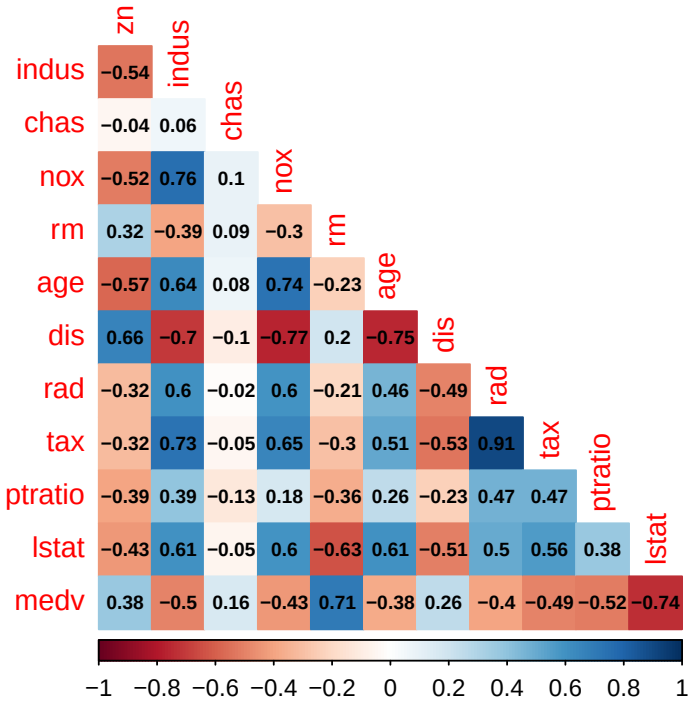
Looking at the features towards the bottom, the four least important features correlated with the response variable are `medv`, `lstat`, `rm`, and `chas`, with `chas` being the least correlated to `target`. These correlations are measured by the Pearson Correlation coefficient by default.

Since both `chas` and `target` are binary categorical variables, the correct coefficient to use to understand the strength of their relationship is actually the ϕ coefficient. If either of these categorical variables had more than two categories, we would need to calculate ϕ using the formula for Cramer's V (also called Cramer's ϕ) coefficient. However, in the special case that both categorical variables are binary, the value of Cramer's V coefficient will actually be equal to the value of the Pearson Correlation coefficient. So either formula actually results in the same value for ϕ. We prove this below.

```
## [1] TRUE
```

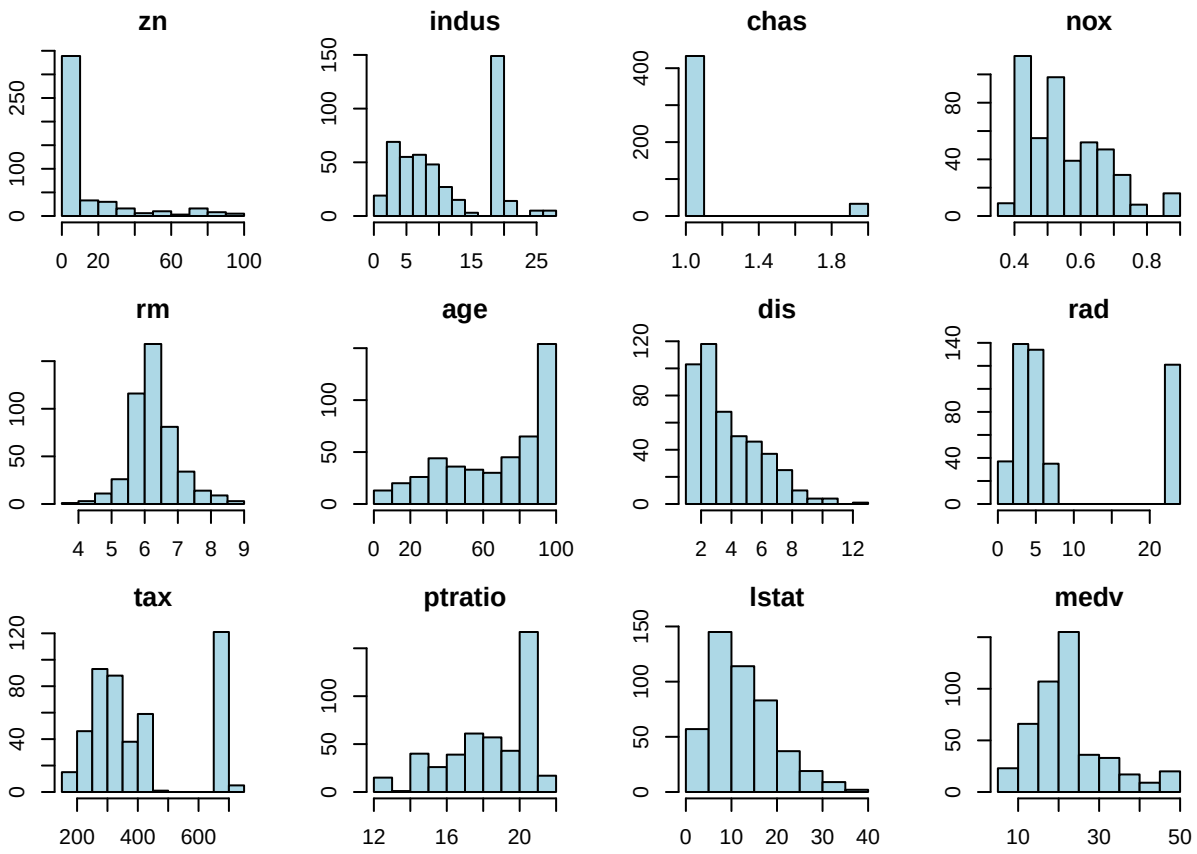
The value for ϕ is 0.08004 regardless of the formula used to calculate it, and this very low value proves the very small amount of correlation between `chas` and `target` estimated by the correlation funnel is accurate.

Now we check for multicollinearity between predictor variables.



Predictor variables: `indus` is highly correlated (more than 0.7) with `nox`, `dis`, and `tax`. `nox` is also highly correlated with `age` and `dis`. `rm` is highly correlated with `medv`. `rad` is very highly correlated with `tax`. `lstat` is highly correlted with `medv`.

Let's take a look at the distributions for the predictor variables.



The distribution for `rm` appears to be normal, and the distribution for `medv` is nearly normal. The distributions for `zn`, `dis`, `lstat`, and `nox` are right-skewed. The distributions for `age` and `ptratio` are left-skewed.

The distributions for the remaining variables are multimodal, including the distribution for `chas`, which appears degenerate at first glance. It looks like a near-zero variance predictor, which we can confirm using the `nearZeroVar` function from the `caret` package.

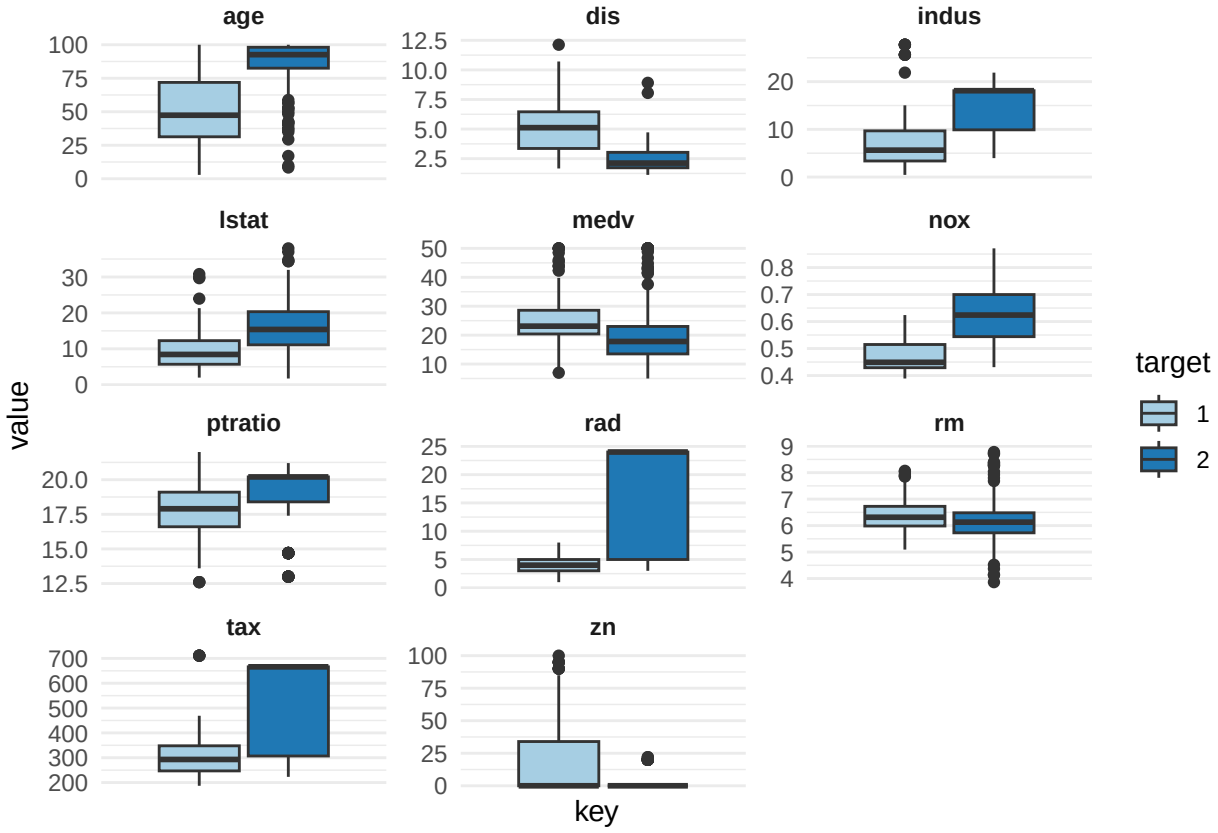
	freqRatio	percentUnique	zeroVar	nzv
zn	16.142857	5.5793991	FALSE	FALSE
indus	4.321429	15.6652361	FALSE	FALSE
chas	13.121212	0.4291845	FALSE	FALSE
nox	1.176471	16.9527897	FALSE	FALSE
rm	1.000000	89.9141631	FALSE	FALSE
age	10.500000	71.4592275	FALSE	FALSE
dis	1.000000	81.5450644	FALSE	FALSE
rad	1.110092	1.9313305	FALSE	FALSE
tax	3.457143	13.5193133	FALSE	FALSE
ptratio	4.000000	9.8712446	FALSE	FALSE
lstat	1.000000	90.9871245	FALSE	FALSE
medv	2.142857	46.7811159	FALSE	FALSE

The percentage of unique values, `percentUnique`, in the sample for this predictor is less than the typical threshold of 10 percent, but there is a second criterion to consider: the `freqRatio`. This measures the frequency of the most common value (0 in this case) to the frequency of the second most common value (1 in this case). The `freqRatio` value for this predictor is less than the typical threshold of 19 (i.e. 95 occurrences

of the most frequent value for every 5 occurrences of the second most frequent value). So it is not considered a near-zero variance predictor. Neither are any of the other predictors.

Next we analyze boxplots to determine the spread of the numeric predictor variables. This will also reveal any outliers.

Fix legend labels



For certain predictors, the variance between the two categories of the response variable differs largely: `age`, `dis`, `nox`, `rad`, and `tax`.

Data Preparation:

We check whether our predictor variables with skewed distributions would benefit from transformations.

Skewed Variable	Ideal Lambda Proposed by Box-Cox	Reasonable Alternative Transformation
zn	-0.3	log
dis	-0.15	log
lstat	0.25	log
nox	-0.95	inverse
age	1.3	no transformation
ptratio	2	square

Some of the skewed variables might benefit from common transformations, so when we train our models, we will train some without any transformations, and some with the five transformations listed above.

Modeling

Let's start with a full model and then reduce using the `stepAIC()` function from the `MASS` package.

```
##
## Call:
## glm(formula = target ~ zn + nox + age + dis + rad + tax + ptratio +
##     medv, family = "binomial", data = train_df)
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -37.415922   6.035013  -6.200 5.65e-10 ***
## zn           -0.068648   0.032019  -2.144  0.03203 *
## nox          42.807768   6.678692   6.410 1.46e-10 ***
## age           0.032950   0.010951   3.009  0.00262 **
## dis           0.654896   0.214050   3.060  0.00222 **
## rad           0.725109   0.149788   4.841 1.29e-06 ***
## tax          -0.007756   0.002653  -2.924  0.00346 **
## ptratio       0.323628   0.111390   2.905  0.00367 **
## medv          0.110472   0.035445   3.117  0.00183 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 645.88  on 465  degrees of freedom
## Residual deviance: 197.32  on 457  degrees of freedom
## AIC: 215.32
##
## Number of Fisher Scoring iterations: 9
```

The reduced model consists of 8 predictor variables and has an AIC of 215.32.

To interpret the model coefficients, we first need to exponentiate them.

Feature	Coefficient
(Intercept)	0.000000e+00
zn	9.336551e-01
nox	3.901012e+18
age	1.033499e+00
dis	1.924943e+00
rad	2.064956e+00
tax	9.922739e-01
ptratio	1.382133e+00
medv	1.116805e+00

The coefficients are now easier to interpret. Features with coefficients less than 1 indicate the odds of the crime rate being above the median crime rate decrease as that feature increases, while coefficients greater than 1 indicate the odds of the crime rate being above the median crime rate increases as that feature increases. How much the odds increase or decrease per 1 unit increase in the feature is the difference between that feature's coefficient and 1, multiplied by 100 so we can understand it as a percentage increase or decrease.

The coefficient for `nox` is extremely large. If we look back at the boxplots, we can see that the spreads for each category of the `target` value have no overlap in their interquartile ranges for this predictor. Almost all

measures greater than 0.5 are associated with above median crime rates, and since this variable is measured on a scale less than 1, an increase of 1 in its value makes any measurement a large enough value to associate it with above median crime rates.

Now that we understand that, let's exclude `nox` from the features so that we can look at the rest of their coefficients more closely. The `(Intercept)` coefficient is practically zero, so we will exclude it as well.

```
remove <- c("nox", "(Intercept)")
beta_exp <- beta_exp |>
    filter(!Feature %in% remove) |>
    mutate(diff = round(Coefficient - 1, 3) * 100) |>
    arrange(desc(diff))
cols <- c("Feature", "Coefficient", "Percentage Change in Odds Crime Rate Above Median")
colnames(beta_exp) <- cols
knitr::kable(beta_exp, format = "simple")
```

Feature	Coefficient	Percentage Change in Odds Crime Rate Above Median
rad	2.0649560	106.5
dis	1.9249425	92.5
ptratio	1.3821333	38.2
medv	1.1168050	11.7
age	1.0334994	3.3
tax	0.9922739	-0.8
zn	0.9336551	-6.6

Here, we see that increasing `rad` by 1 unit increases the odds of being above the median crime rate by 106.5%, increasing `zn` by 1 unit decreases the odds of being above the median crime rate by 6.6%, and so forth.

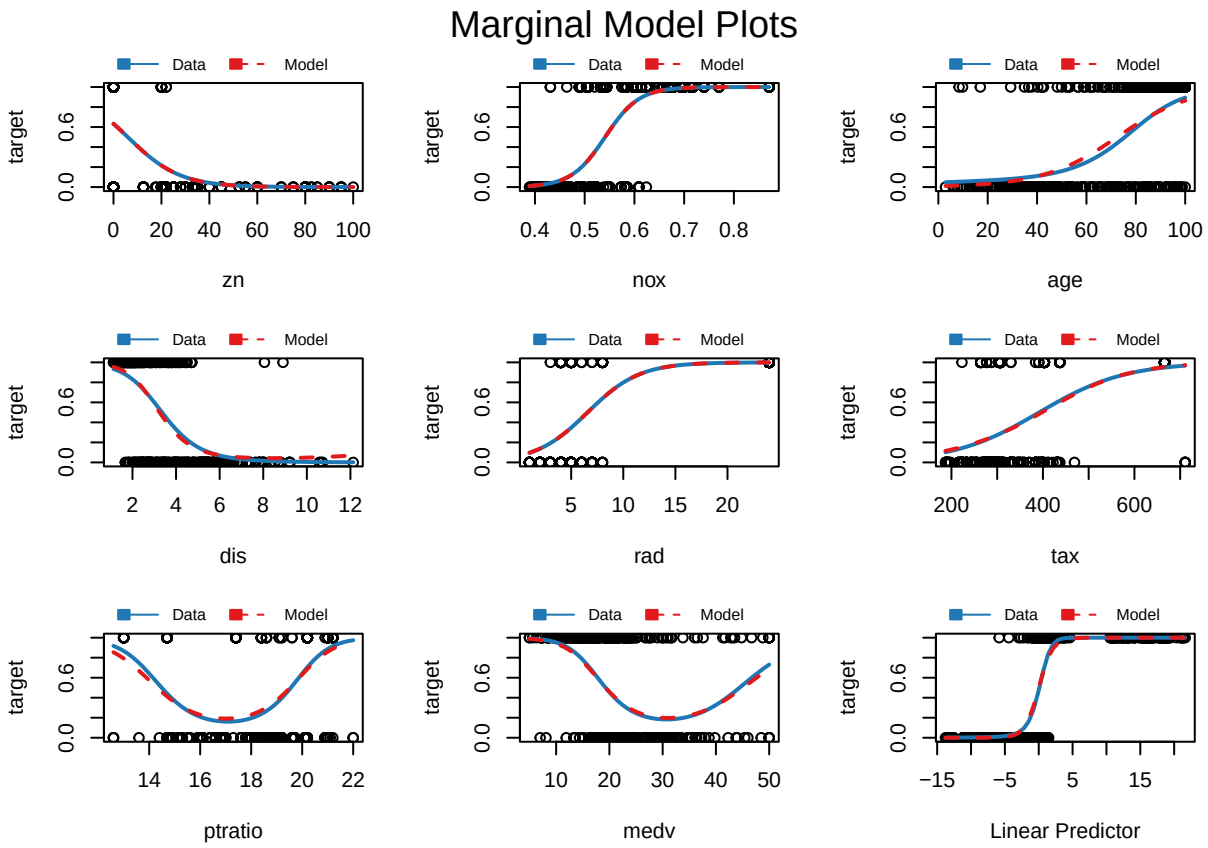
More Discussion of These Coefficients/Inference TK

Let's check for possible multicollinearity within this model.

```
##       zn      nox      age      dis      rad      tax  ptratio     medv
## 1.789037 3.172660 1.701974 3.595939 1.697110 1.754274 1.865085 2.193689
```

All of the variance inflation factors are less than 5 so there are no issues of multicollinearity within this model.

To check for goodness of fit, we create marginal model plots for the response and each predictor in this model.



There is very good agreement between the two fits in each of the marginal model plots.

We calculate the Hosmer-Lemeshow statistic to further check for lack of fit.

```
hlstat <- hltest(model_1)
```

```
##
##    The Hosmer-Lemeshow goodness-of-fit test
##
##  Group Size Observed     Expected
##      1   47        0  0.005688599
##      2   47        1  0.105069358
##      3   47        2  0.810426844
##      4   47        4  6.174181007
##      5   47       12 15.325035907
##      6   47       32 28.339890838
##      7   47       41 41.240137655
##      8   47       47 46.999577116
##      9   47       47 46.999993110
##     10   43       43 42.999999565
##
##          Statistic =  12.57643
## degrees of freedom =  8
##            p-value =  0.12728
```

The moderate p-value here suggests no lack of fit. However, this test provides no insight about whether there is overfitting in the model.

We create a different model using transformed variables. TK

Appendix

```
knitr::opts_chunk$set(echo = TRUE, warning=FALSE)

library(tidyverse)
library(modelr)
library(DataExplorer)
library(correlationfunnel)
library(caret)
library(knitr)
library(confintr)
library(psych)
library(car)
library(corrplot)
library(RColorBrewer)
library(MASS)
select <- dplyr::select
library(glmtoolbox)

train_df <- read.csv('https://raw.githubusercontent.com/ShanaFarber/businessAnalyticsDataMiningDATA621/m

dim(train_df)

train_df |>
    glimpse()

train_df <- train_df |>
    mutate(chas = as.factor(chas), target = as.factor(target))

summary(train_df)

sum(is.na(train_df))

train_df_binarized <- train_df |>
    binarize(n_bins = 5, thresh_infreq = 0.01, name_infreq = "OTHER",
           one_hot = TRUE)
train_df_corr <- train_df_binarized |>
    correlate(target__1)
train_df_corr |>
    plot_correlation_funnel()
rm(train_df_binarized, train_df_corr)

factors <- c("chas", "target")
cramersv <- round(cramersv(train_df |> select(all_of(factors))), 5)
pearson <- round(cor(as.numeric(train_df$chas), as.numeric(train_df$target), method = "pearson"), 5)
(cramersv == pearson)

train_df$chas <- as.numeric(train_df$chas)
train_df$target <- as.numeric(train_df$target)
```

```r
corrplot(cor(train_df |> select(-target)), method="color",
         diag=FALSE,
         type="lower",
         addCoef.col = "black",
         number.cex=0.70)

train_df$chas <- as.factor(train_df$chas)
train_df$target <- as.factor(train_df$target)
par(mfrow=c(3,4))
par(mai=c(.3,.3,.3,.3))

variables <- names(train_df)
for (i in 1:(length(variables)-1)) {
    if (variables[i] %in% factors){
        hist(as.numeric(train_df[[variables[i]]]), main = variables[i],
             col = "lightblue")
    }else{
        hist(train_df[[variables[i]]], main = variables[i], col = "lightblue")
    }
}

nzv <- nearZeroVar(train_df |> select(-target), saveMetrics = TRUE)
knitr::kable(nzv)

train_df |>
    dplyr::select(-chas) |>
    gather(key, value, -target) |>
    mutate(key = factor(key),
           target = factor(target)) |>
    ggplot(aes(x = key, y = value)) +
    geom_boxplot(aes(fill = target)) +
    scale_x_discrete(labels = NULL, breaks = NULL) +
    facet_wrap(~ key, scales = 'free', ncol = 3) +
    scale_fill_brewer(palette = "Paired") +
    theme_minimal() +
    theme(strip.text = element_text(face = "bold"))

skewed <- c("zn", "dis", "lstat", "nox", "age", "ptratio")
train_df_trans <- train_df
for (i in 1:(length(skewed))){
    #Add a small constant to columns with any 0 values
    if (sum(train_df_trans[[skewed[i]]] == 0) > 0){
        train_df_trans[[skewed[i]]] <-
            train_df_trans[[skewed[i]]] + 0.001
    }
}
for (i in 1:(length(skewed))){
    if (i == 1){
        lambdas <- c()
    }
    bc <- boxcox(lm(train_df_trans[[skewed[i]]] ~ 1),
                 lambda = seq(-2, 2, length.out = 81),
                 plotit = FALSE)
```

```
    lambda <- bc$x[which.max(bc$y)]
    lambdas <- append(lambdas, lambda)
}
lambdas <- as.data.frame(cbind(skewed, lambdas))
adj <- c("log", "log", "log", "inverse", "no transformation", "square")
lambdas <- cbind(lambdas, adj)
cols <- c("Skewed Variable", "Ideal Lambda Proposed by Box-Cox", "Reasonable Alternative Transformation
colnames(lambdas) <- cols
kable(lambdas, format = "simple")
rm(train_df_trans)

glm_full <- glm(target~., family='binomial', data=train_df)
model_1 <- stepAIC(glm_full, trace=0)
summary(model_1)

beta <- coef(model_1)
beta_exp <- as.data.frame(exp(beta)) |>
    rownames_to_column()
cols <- c("Feature", "Coefficient")
colnames(beta_exp) <- cols
knitr::kable(beta_exp, format = "simple")

remove <- c("nox", "(Intercept)")
beta_exp <- beta_exp |>
    filter(!Feature %in% remove) |>
    mutate(diff = round(Coefficient - 1, 3) * 100) |>
    arrange(desc(diff))
cols <- c("Feature", "Coefficient", "Percentage Change in Odds Crime Rate Above Median")
colnames(beta_exp) <- cols
knitr::kable(beta_exp, format = "simple")

vif(model_1)

palette <- brewer.pal(n = 12, name = "Paired")
mmps(model_1, layout = c(3, 3), grid = FALSE, col.line = palette[c(2,6)])

hlstat <- hltest(model_1)
```