Worksheet 7, Oct 29, 2020

Computing Eigenvalues

1. **Power method.** Given is the following matrix A together with its eigenvalues λ_i and eigenvectors $\boldsymbol{v}_i$, i.e., $A\boldsymbol{v}_i = \lambda_i \boldsymbol{v}_i$ for $i = 1, 2, 3$.

$$A = \begin{bmatrix} -2 & 1 & 4 \\ 1 & 1 & 1 \\ 4 & 1 & -2 \end{bmatrix},$$

The eigenvalues and eigenvectors are

$$\lambda_1 = 0,\ \boldsymbol{v}_1 = \begin{bmatrix} 0.41 \\ -0.82 \\ 0.41 \end{bmatrix}, \quad \lambda_2 = -6,\ \boldsymbol{v}_2 = \begin{bmatrix} 0.71 \\ 0.0 \\ -0.71 \end{bmatrix}, \quad \lambda_3 = 3,\ \boldsymbol{v}_3 = \begin{bmatrix} -0.58 \\ -0.58 \\ -0.58 \end{bmatrix}.$$

   (a) Calculate the first iterate of the power method when $\boldsymbol{x}_0 = (0, 1, 1)^T$ (you don't have to normalize).

   (b) Which eigenvalue direction will the sequence defined in (a) converge to?

   (c) Give an initialization vector such that the power method does *not* converge to the direction of the largest (in absolute value) eigenvalue.

   (d) Write (or adapt) a simple program implementing the power method for the matrix A.

2. **Inverse power method.** Let $\theta \in \mathbb{R}$ and let $\boldsymbol{x}_0 \in \mathbb{R}^3$.

   (a) Define the *Inverse Iteration* (also called *Inverse Power Method*) to calculate eigenvectors of A near θ.

   (b) If $\theta = 2$, where will the sequence defined in (i) converge to and why?

   (c) If $\theta = -2$, where will the sequence defined in (i) converge to and why?

   (d) Write (or adapt) a simple program implementing the inverse power method.