

# Control-based Continuation in Experiments

## Nonlinear and Complex Systems Group Research Programme

The aim of this research is to extend continuation methods, which are a well-known and successful tool for numerical bifurcation analysis, to feedback-controllable real-life experiments. This will enable experimenters to observe phenomena that remain hidden in conventional experiments due to their dynamical instability, or their sensitivity to disturbances. Since control-based continuation does not require the ability to initialise the system's state 'at will', or to do numerical computations in real-time, it is in principle suitable for complex experiments such as inclined cables, fast rotating machinery, or dynamically clamped neurons (these are planned experiments).

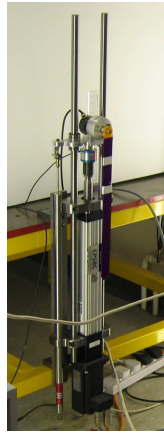
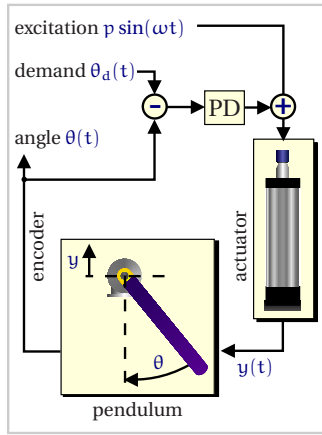


Figure 1: Set-up of mechanical pendulum experiment: schematic diagram of real-time feedback control loop (left), and photo (right). Figure 2 shows the results of pseudo-arclength continuation for the 'equation'  $\theta_d - \theta = 0$ .

The environment of a controlled experiment features much larger disturbances than the well-researched numerical roundoff error (2 significant digits at best, instead of 16). This requires a redesign of the current numerical methods for solving boundary-value problems. The algorithms will be developed and

tested using computer simulations, tunable electrical circuits and table-top sized mechanical experiments. Figure 1 shows the first prototype experiment, a vertically excited pendulum. When the pivot of the pendulum is excited ( $p > p_{\text{critical}}$  in Figure 1) the pendulum can rotate periodically:  $\theta(t) = \pm\omega t + \phi(t)$  where  $\phi$  has period  $2\pi/\omega$ . In a conventional experiment one could observe only the upper, stable (green), part of the branch of rotations shown in Figure 2. Using a combination of feedback control and Newton iterations we have tracked the rotations through a region that is too sensitive to detect in a conventional experiment (yellow) into the dynamically unstable part of the branch (red).

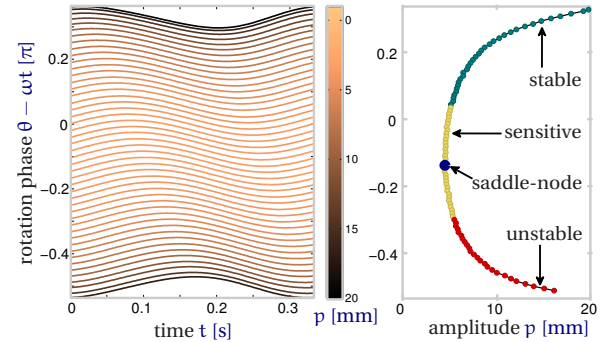


Figure 2: Experimental measurements: time profiles of stable and unstable rotating periodic orbits (left), and bifurcation diagram near the saddle-node bifurcation (right).

The Nonlinear and Complex Systems Group welcomes enquiries regarding job vacancies, Ph.D. and Postdoctoral study, and academic and industrial collaboration on its research programmes.

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