Delay and Discontinuity

Nonlinear and Complex Systems Group Research Programme

Balancing a long stick is easier than balancing a short stick. This well-known fact is due to the human reaction time, which introduces the effect of a time delay into this control problem. This reaction time is about 100 ms for eye-hand coordination, which is not negligible for a short stick (about 30 cm) since the inherent time scale for the stick motion is of the same order. Delay-induced instabilities have been studied extensively in other systems with delay, for example, in coupled neurons, lasers subject to optical feedback due to external reflections, and in cutting processes.

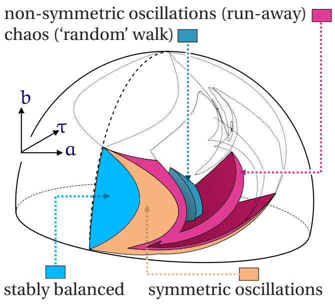

Figure 1: Bifurcation diagram of inverted pendulum with delayed control, charting practically relevant long-time behaviour. Equation of motion: $\ddot{x} - \sin x = -ax(t-\tau) - b\dot{x}(t-\tau)$.

Mathematically, dynamical systems with delay are often modelled using delay-differential equations (DDEs), which have an infinite-dimensional phase space. In applications one is typically interested in bifurcation diagrams, charting the long-time behaviour of the system depending on system parameters. We recently proved (based on a construction by R. Szalai) that it is possible to construct a low-dimensional characteristic matrix $\Delta(1/\lambda)$ for periodic liner delay-differential equations: $\Delta(1/\lambda)$ is singular if and only if λ is a Floquet multiplier of the periodic problem. The technique is potentially applicable also to nonlinear problems with state-dependent delays where questions about smooth dependence of periodic orbits on system parameters are still open.

A related area of research is the interaction between delay and discontinuities of the right-hand-side as occurs, for example, in system with delayed switching. Paradoxically, the study of dynamics near periodic orbits becomes simpler as long as all switching events satisfy certain transversality conditions. When these conditions are violated (at so-called *discontinuity-induced bifurcations*) new phenomena may occur, for example, instant transition from stable periodic orbits to chaos, period-adding cascades, or emergence of invariant polygons.

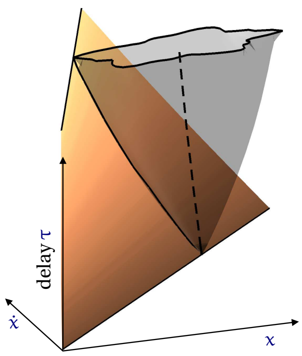

Figure 2: Family of grazing invariant tori in an unstable oscillator with delayed switching. Equation of motion: $\ddot{x} - \gamma\dot{x} + x = \mathrm{sign}[\cos\alpha x(t-\tau) + \sin\alpha\dot{x}(t-\tau)]$.

The Nonlinear and Complex Systems Group welcomes enquiries regarding job vacancies, Ph.D. and Postdoctoral study, and academic and industrial collaboration on its research programmes.

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