# Describing Motion: Kinematics in One Dimension

## Displacement

"the change in x," or "change in position"
$\Delta x = x_2 - x_1$
The change
in any quantity means the final value of that quantity, minus the initial value.

## Average Speed

is defined as the total distance
travelled along its path divided by the time it takes to travel this distance.

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{time elapsed}}$$

## Average Velocity

is defined in terms of displacement, rather than total distance travelled.

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{time elapsed}}$$

is defined as the displacement divided by the elapsed time

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

## Instantaneous Velocity

Instantaneous velocity at any moment
is defined as the average velocity over an infinitesimally short time interval.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

## Average Acceleration

is defined
as the change in velocity divided by the time taken to make this change.

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}$$

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

**Instantaneous Acceleration**

Instantaneous acceleration,
$a$, can be defined in analogy to instantaneous velocity as the average acceleration over an infinitesimally short time interval at a given instant.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

**Deceleration**

is whenever the magnitude of the velocity is decreasing.
is when velocity and acceleration point in opposite directions.

**Motion at Constant Acceleration**

We now examine motion in a straight line when the magnitude of the acceleration is constant. In this case, the instantaneous and average accelerations are equal.
We use the definitions of average velocity and acceleration to derive a set of valuable equations that relate $x$, $v$, $a$, and $t$ when $a$ is constant, allowing us to determine any one of these variables if we know the others. We can then solve many interesting Problems.
First we choose the initial time in any discussion to be zero, and we call it $t_0$. That is, $t_1 = t_0 = 0$. (This is effectively starting a stopwatch at $t_0$ .) We can then let $t_2 = t$ be the elapsed time.
The initial position $x_1$ and the initial velocity $v_1$ of an object will now be represented by $x_0$ and $v_0$ , since they represent $x$ and $v$ at $t = 0$. At time $t$ the position and velocity will be called $x$ and $v$ (rather than $x_2$ and $v_2$). The average velocity during the time interval $t - t_0$ will be

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$$

since we chose $t_0 = 0$

$$a = \frac{v - v_0}{t}$$

The velocity of an object after any elapsed time t

$v = v_0 + at$

Calculating the Position x of an object after a time t

$\bar{v} = \frac{x - x_0}{t}$

becomes

$x = x_0 + \bar{v}t$

Because the velocity increases at a uniform rate $\bar{v}$ will be midway between the initial and final velocities

$\bar{v} = \frac{v_0 + v}{2}$

Combining the last three equations these become

$x = x_0 + \bar{v}t$

$x = x_0 + (\frac{v_0 + v}{2})t$

$x = x_0 + (\frac{v_0 + (v_0 + at)}{2})t$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

Three of the four most useful equations for motion at constant acceleration

The velocity of an object after any elapsed time t

$v = v_0 + at$

Average velocity

$\bar{v} = \frac{v_0 + v}{2}$

Position x of an object after a time t

$x = x_0 + v_0 t + \frac{1}{2}at^2$

Situations where time $t$ is not known

$x = x_0 + (\frac{v_0 + v}{2})t$

solve for $t$ $v = v_0 + at$

$t = \frac{v - v_0}{a}$

Substituting this into the previous equation we get

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)\left(\frac{v - v_0}{a}\right) = x_0 + \frac{v^2 - v_0^2}{2a}$$

solve for $v^2$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Kinematic equations for constant acceleration

$[a = \text{constant}]$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\bar{v} = \frac{v + v_0}{2}$$

**Solving Problems**

1. Read and reread the whole problem carefully before trying to solve it.
2. Decide what object (or objects) you are going to study, and for what time interval. You can often choose the initial time to be t = 0.
3. Draw a diagram or picture of the situation, with coordinate axes wherever applicable. [You can place the origin of coordinates and the axes wherever you like to make your calculations easier. You also choose which direction is positive and which is negative. Usually we choose the x axis to the right as positive.]
4. Write down what quantities are "known" or "given," and then what you want to know. Consider quantities both at the beginning and at the end of the chosen time interval. You may need to "translate" language into physical terms, such as "starts from rest" means v0 = 0.
5. Think about which principles of physics apply in this problem. Use common sense and your own experiences. Then plan an approach.
6. Consider which equations (and/or definitions) relate the quantities involved. Before using them, be sure their range of validity includes your problem (for example, Eqs. 2–11 are valid only when the acceleration is constant). If you find an applicable equation that involves only known quantities and one desired unknown, solve the equation algebraically for the unknown. Sometimes several

sequential calculations, or a combination of equations, may be needed. It is often preferable to solve algebraically for the desired unknown before putting in numerical values.

7. Carry out the calculation if it is a numerical problem. Keep one or two extra digits during the calculations, but round off the final answer(s) to the correct number of significant figures (Section 1–4).

8. Think carefully about the result you obtain: Is it reasonable? Does it make sense according to your own intuition and experience? A good check is to do a rough estimate using only powers of 10, as discussed in Section 1–7. Often it is preferable to do a rough estimate at the start of a numerical problem because it can help you focus your attention on finding a path toward a solution.

9. A very important aspect of doing problems is keeping track of units. An equals sign implies the units on each side must be the same, just as the numbers must. If the units do not balance, a mistake has been made. This can serve as a check on your solution (but it only tells you if you're wrong, not if you're right). Always use a consistent set of units.

**Freely Falling Objects**

with upwards as positive
solving for t

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

We rewrite our y equation just above in standard form,

$$at^2 + bt + c = 0$$

$$0 = (y_0 - y) + v_0 t + \frac{1}{2} a t^2$$

$$\frac{1}{2} a t^2 + v_0 t + (y_0 - y) = 0$$

$$(\frac{1}{2} a) t^2 + (v_0) t + (y_0 - y) = 0$$

Using the quadratic formula, we find as solutions

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$g = 9.80 m/s^2$

**Stuff about graphing**

**Right angled triangles**

$\sin\theta = Opp/Hyp$
$\cos\theta = Adj/Hyp$
$\tan\theta = Opp/Adj$
$\csc\theta = Hyp/Opp = 1/\sin\theta$
$\sec\theta = Hyp/Adj = 1/\cos\theta$
$\cot\theta = Adj/Opp = 1/\tan\theta$

**Resolving vectors**

$\mathbf{v}_x = r\cos\theta$
$\mathbf{v}_y = r\sin\theta$
$r = |\mathbf{v}| = \sqrt{x^2 + y^2}$ (Magnitude)
$r = |\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$ (Magnitude for 3-space)
$\theta = \tan^{-1}(\frac{\mathbf{v}_y}{\mathbf{v}_x})$

$\theta = \begin{array}{|c|c|} \hline 180 - \theta & \theta \\ \hline 180 + \theta & 360 - \theta \\ \hline \end{array}$ ???

**Kinematics in Two Dimensions; Vectors**

**Solving Projectile Motion Problems**

General Kinematic Equations for Constant Acceleration in Two Dimensions

| x component (horizontal) | y component (vertical) |
|---|---|
| $v_x = v_{x0} + a_x t$ | $v_y = v_{y0} + a_y t$ |
| $x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$ | $y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$ |
| $v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$ | $v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$ |

We can simplify Eqs to use for projectile motion because we can set
$a_x = 0$, $a_y = -g$

Kinematic Equations for Projectile Motion
(y positive upward; $a_x = 0, a_y = -g = 9.80 m/s^2$)

| Horizontal Motion ($a_x = 0, v_x =$ constant) | Vertical Motion ($a_y = -g =$ constant) |
|---|---|
| $v_x = v_{x0}$ | $v_y = v_{y0} - gt$ |
| $x = x_0 + v_{x0}t$ | $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ |
| | $v_y^2 = v_{y0}^2 - 2g(y - y_0)$ |

**Projectile Motion Is Parabolic**

We now show that the path followed by any projectile
is a parabola, if we can ignore air resistance and can assume that g is constant.
for simplicity we set $x_0 = y_0 = 0$

$$x = v_{x0}t$$

$$y = v_{y0}t - \frac{1}{2}gt^2$$

From the first
equation, we have $t = \frac{x}{v_{x0}}$ , and we substitute this into the second one to obtain

$$y = v_{y0}\left(\frac{x}{v_{x0}}\right) - \frac{1}{2}g\left(\frac{x}{v_{x0}}\right)^2$$

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \frac{1}{2}g\left(\frac{x^2}{v_{x0}^2}\right)$$

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \left(\frac{g}{2v_{x0}^2}\right)x^2$$

We see that
$y$ as a function of $x$ has the form $y = Ax - Bx^2$, where $A$ and $B$ are constants for any specific projectile motion. This is the standard equation for a parabola.

Relative Velocity

We now consider how observations made in different frames of reference are related to each other.

For example, consider two trains approaching one another, each with a speed of 80 kmh with respect to the Earth. Observers on the Earth beside the train

tracks will measure 80 kmh for the speed of each of the trains. Observers on either one of the trains (a different frame of reference) will measure a speed of 160 kmh for the train approaching them.
Similarly, when one car traveling 90 kmh passes a second car traveling in the same direction at 75 kmh, the first car has a speed relative to the second car of 90 kmh - 75 kmh = 15 kmh.
Use a diagram and a careful labeling process. Each velocity is labeled by two subscripts: the first refers to the object, the second to the reference frame in which it has this velocity.
$\vec{v}_{OR}$
Example, suppose a boat heads directly across a river
let
$\vec{v}_{BW}$ be the velocity of the Boat with respect to the Water.
$\vec{v}_{BS}$ be the velocity of the Boat with respect to the Shore,
$\vec{v}_{WS}$ be the velocity of the Water with respect to the Shore
$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}$
By writing the subscripts using this convention, we see that the inner subscripts (the two W's) on the right-hand side of Eq. 3-7 are the same; also, the outer subscripts on the right of Eq. 3-7 (the B and the S) are the same as the two subscripts for the sum vector on the left, vBS . By following this convention (first subscript for the object, second for the reference frame), you can write down the correct equation relating velocities in different reference frames.
Equation 3-7 is valid in general and can be extended to three or more velocities.
example,
$\vec{v}_{FB}$ is the velocity of the fisherman relative to the boat
his velocity relative to the shore is $\vec{v}_{FS} = \vec{v}_{FB} + \vec{v}_{BW} + \vec{v}_{WS}$
The equations involving relative velocity will be correct when
there is no vector subtraction
adjacent inner subscripts are identical
and when the outermost ones correspond exactly to the two on the velocity on the left of the equation.
It is often useful to remember that for any two objects or reference frames,

A and B, the velocity of A relative to B has the same magnitude, but opposite direction, as the velocity of B relative to A:
$\vec{v}_{AB} = -\vec{v}_{BA}$

## Work and Energy

### Kinetic Energy, and the Work-Energy Principle

To obtain a quantitative definition for kinetic energy, let us consider a simple rigid object of mass $m$ (treated as a particle) that is moving in a straight line with an initial speed $v_1$. To accelerate it uniformly to a speed $v_2$ a constant net force $F_{net}$ is exerted on it parallel to its motion over a displacement $d$, Fig. 6–7. Then the net work done on the object is $W_{net} = F_{net} d$. We apply Newton's second law, $F_{net} = ma$ and use Eq. 2–11c $(v_2^2 = v_1^2 + 2ad)$ which we rewrite as $a = \frac{v_2^2 - v_1^2}{2d}$
where $v_1$ is the initial speed and $v_2$ is
the final speed. Substituting this into $F_{net} = ma$, we determine the work done:
$$W_{net} = F_{net} d = mad = m\left(\frac{v_2^2 - v_1^2}{2d}\right)d = m\left(\frac{v_2^2 - v_1^2}{2}\right)$$
or
$$W_{net} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$
We define the quantity to be the translational kinetic energy (KE) of the object:
$$KE = \frac{1}{2}mv^2$$
so
$$W_{net} = KE_2 - KE_1$$
or
work-energy principle
$$W_{net} = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$
It can be stated in words:
The
net work done on an object is equal to the change in the object's kinetic energy.
Thus, the work-energy principle is valid only if W is the net work
done on the object—that is, the work done by all forces acting on the object.

Potential Energy Defined in General
In general, the change in potential energy associated with a particular force is equal to the negative of the work done by that force when the object is moved from one point to a second point (as in Eq. 6–7b for gravity). Alternatively, we can define the change in potential energy as the work required of an external force to move the object without acceleration between the two points
FS =–kx.
where k is a constant, called the spring stiffness constant (or simply spring constant), and is a measure of the stiffness of the particular spring.
spring either
stretched or compressed an amount x from its natural (unstretched) length
spring equation and
also as Hooke's law, and is accurate for springs as long as x is not too great.
elastic potential energy

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