

## **Describing Motion: Kinematics in One Dimension**

### **Displacement**

“the change in  $x$ ,” or “change in position”

$$\Delta x = x_2 - x_1$$

The change

in any quantity means the final value of that quantity, minus the initial value.

### **Average Speed**

is defined as the total distance travelled along its path divided by the time it takes to travel this distance.

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{time elapsed}}$$

### **Average Velocity**

is defined in terms of displacement, rather than total distance travelled.

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{time elapsed}}$$

is defined as the displacement divided by the elapsed time

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

### **Instantaneous Velocity**

Instantaneous velocity at any moment

is defined as the average velocity over an infinitesimally short time interval.

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

### **Average Acceleration**

is defined

as the change in velocity divided by the time taken to make this change.

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}$$

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

## Instantaneous Acceleration

Instantaneous acceleration,

$a$ , can be defined in analogy to instantaneous velocity as the average acceleration over an infinitesimally short time interval at a given instant.

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

## Deceleration

is whenever the magnitude of the velocity is decreasing.

is when velocity and acceleration point in opposite directions.

## Motion at Constant Acceleration

We now examine motion in a straight line when the magnitude of the acceleration is constant. In this case, the instantaneous and average accelerations are equal.

We use the definitions of average velocity and acceleration to derive a set of valuable equations that relate  $x$ ,  $v$ ,  $a$ , and  $t$  when  $a$  is constant, allowing us to determine any one of these variables if we know the others. We can then solve many interesting Problems.

First we choose the initial time in any discussion to be zero, and we call it  $t_0$ . That is,  $t_1 = t_0 = 0$ . (This is effectively starting a stopwatch at  $t_0$ .) We can then let  $t_2 = t$  be the elapsed time.

The initial position  $x_1$  and the initial velocity  $v_1$  of an object will now be represented by  $x_0$  and  $v_0$ , since they represent  $x$  and  $v$  at  $t = 0$ . At time  $t$  the position and velocity will be called  $x$  and  $v$  (rather than  $x_2$  and  $v_2$ ). The average velocity during the time interval  $t - t_0$  will be

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$$

since we chose  $t_0 = 0$

$$a = \frac{v - v_0}{t}$$

The velocity of an object after any elapsed time  $t$

$$v = v_0 + at$$

Calculating the Position  $x$  of an object after a time  $t$

$$\bar{v} = \frac{x - x_0}{t}$$

becomes

$$x = x_0 + \bar{v}t$$

Because the velocity increases at a uniform rate  $\bar{v}$  will be midway between the initial and final velocities

$$\bar{v} = \frac{v_0 + v}{2}$$

Combining the last three equations these become

$$x = x_0 + \bar{v}t$$

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)t$$

$$x = x_0 + \left(\frac{v_0 + (v_0 + at)}{2}\right)t$$

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

Three of the four most useful equations for motion at constant acceleration

The velocity of an object after any elapsed time  $t$

$$v = v_0 + at$$

Average velocity

$$\bar{v} = \frac{v_0 + v}{2}$$

Position  $x$  of an object after a time  $t$

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

Situations where time  $t$  is not known

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)t$$

solve for  $t$   $v = v_0 + at$

$$t = \frac{v - v_0}{a}$$

Substituting this into the previous equation we get

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)\left(\frac{v - v_0}{a}\right) = x_0 + \frac{v^2 - v_0^2}{2a}$$

solve for  $v^2$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Kinematic equations for constant acceleration

[ $a = \text{constant}$ ]

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\bar{v} = \frac{v + v_0}{2}$$

## Solving Problems

1. Read and reread the whole problem carefully before trying to solve it.
2. Decide what object (or objects) you are going to study, and for what time interval. You can often choose the initial time to be  $t = 0$ .
3. Draw a diagram or picture of the situation, with coordinate axes wherever applicable. [You can place the origin of coordinates and the axes wherever you like to make your calculations easier. You also choose which direction is positive and which is negative. Usually we choose the  $x$  axis to the right as positive.]
4. Write down what quantities are “known” or “given,” and then what you want to know. Consider quantities both at the beginning and at the end of the chosen time interval. You may need to “translate” language into physical terms, such as “starts from rest” means  $v_0 = 0$ .
5. Think about which principles of physics apply in this problem. Use common sense and your own experiences. Then plan an approach.
6. Consider which equations (and/or definitions) relate the quantities involved. Before using them, be sure their range of validity includes your problem (for example, Eqs. 2–11 are valid only when the acceleration is constant). If you find an applicable equation that involves only known quantities and one desired unknown, solve the equation algebraically for the unknown. Sometimes several

sequential calculations, or a combination of equations, may be needed. It is often preferable to solve algebraically for the desired unknown before putting in numerical values.

7. Carry out the calculation if it is a numerical problem. Keep one or two extra digits during the calculations, but round off the final answer(s) to the correct number of significant figures (Section 1–4).

8. Think carefully about the result you obtain: Is it reasonable? Does it make sense according to your own intuition and experience? A good check is to do a rough estimate using only powers of 10, as discussed in Section 1–7. Often it is preferable to do a rough estimate at the start of a numerical problem because it can help you focus your attention on finding a path toward a solution.

9. A very important aspect of doing problems is keeping track of units. An equals sign implies the units on each side must be the same, just as the numbers must. If the units do not balance, a mistake has been made. This can serve as a check on your solution (but it only tells you if you're wrong, not if you're right). Always use a consistent set of units.

## Freely Falling Objects

with upwards as positive  
solving for t

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

We rewrite our y equation just above in standard form,

$$a t^2 + b t + c = 0$$

$$0 = (y_0 - y) + v_0 t + \frac{1}{2} a t^2$$

$$\frac{1}{2} a t^2 + v_0 t + (y_0 - y) = 0$$

$$\left(\frac{1}{2} a\right) t^2 + (v_0) t + (y_0 - y) = 0$$

Using the quadratic formula, we find as solutions

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$g = 9.80 \text{ m/s}^2$$

## Stuff about graphing

### Right angled triangles

$$\sin \theta = \text{Opp} / \text{Hyp}$$

$$\cos \theta = \text{Adj} / \text{Hyp}$$

$$\tan \theta = \text{Opp} / \text{Adj}$$

$$\csc \theta = \text{Hyp} / \text{Opp} = 1 / \sin \theta$$

$$\sec \theta = \text{Hyp} / \text{Adj} = 1 / \cos \theta$$

$$\cot \theta = \text{Adj} / \text{Opp} = 1 / \tan \theta$$

### Resolving vectors

$$v_x = r \cos \theta$$

$$v_y = r \sin \theta$$

$$r = |\mathbf{v}| = \sqrt{x^2 + y^2} \text{ (Magnitude)}$$

$$r = |\mathbf{v}| = \sqrt{x^2 + y^2 + z^2} \text{ (Magnitude for 3-space)}$$

$$\theta = \tan^{-1} \left( \frac{v_y}{v_x} \right)$$

$\theta =$	$180 - \theta$	$\theta$	
	$180 + \theta$	$360 - \theta$	

## Kinematics in Two Dimensions; Vectors

### Solving Projectile Motion Problems

General Kinematic Equations for Constant Acceleration in Two Dimensions

x component (horizontal)

y component (vertical)

$$v_x = v_{x0} + a_x t$$

$$v_y = v_{y0} + a_y t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$$

We can simplify Eqs to use for projectile motion because we can set

$$a_x = 0, a_y = -g$$

## Kinematic Equations for Projectile Motion

(y positive upward;  $a_x = 0, a_y = -g = 9.80 \text{ m/s}^2$ )

### Horizontal Motion

( $a_x = 0, v_x = \text{constant}$ )

$$v_x = v_{x0}$$

$$x = x_0 + v_{x0}t$$

### Vertical Motion

( $a_y = -g = \text{constant}$ )

$$v_y = v_{y0} - gt$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

$$v_y^2 = v_{y0}^2 - 2g(y - y_0)$$

## Projectile Motion Is Parabolic

We now show that the path followed by any projectile

is a parabola, if we can ignore air resistance and can assume that  $g$  is constant.

for simplicity we set  $x_0 = y_0 = 0$

$$x = v_{x0}t$$

$$y = v_{y0}t - \frac{1}{2}gt^2$$

From the first

equation, we have  $t = \frac{x}{v_{x0}}$ , and we substitute this into the second one to obtain

$$y = v_{y0}\left(\frac{x}{v_{x0}}\right) - \frac{1}{2}g\left(\frac{x}{v_{x0}}\right)^2$$

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \frac{1}{2}g\left(\frac{x^2}{v_{x0}^2}\right)$$

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \left(\frac{g}{2v_{x0}^2}\right)x^2$$

We see that

$y$  as a function of  $x$  has the form  $y = Ax - Bx^2$ , where  $A$  and  $B$  are constants for any specific projectile motion. This is the standard equation for a parabola.

## Relative Velocity

We now consider how observations made in different frames of reference are related to each other.

For example, consider two trains approaching one another, each with a speed of 80 kmh with respect to the Earth. Observers on the Earth beside the train

tracks will measure 80 kmh for the speed of each of the trains. Observers on either one of the trains (a different frame of reference) will measure a speed of 160 kmh for the train approaching them.

Similarly, when one car traveling 90 kmh passes a second car traveling in the same direction at 75 kmh, the first car has a speed relative to the second car of  $90 \text{ kmh} - 75 \text{ kmh} = 15 \text{ kmh}$ .

Use a diagram and a careful labeling process. Each velocity is labeled by two subscripts: the first refers to the object, the second to the reference frame in which it has this velocity.

$\vec{v}_{OR}$

Example, suppose a boat heads directly across a river  
let

$\vec{v}_{BW}$  be the velocity of the Boat with respect to the Water.

$\vec{v}_{BS}$  be the velocity of the Boat with respect to the Shore,

$\vec{v}_{WS}$  be the velocity of the Water with respect to the Shore

$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}$

By writing the subscripts using this convention, we see that the inner subscripts (the two W's) on the right-hand side of Eq. 3-7 are the same; also, the outer subscripts on the right of Eq. 3-7 (the B and the S) are the same as the two subscripts for the sum vector on the left,  $\vec{v}_{BS}$ . By following this convention (first subscript for the object, second for the reference frame), you can write down the correct equation relating velocities in different reference frames.

Equation 3-7 is valid in general and can be extended to three or more velocities.  
example,

$\vec{v}_{FB}$  is the velocity of the fisherman relative to the boat

his velocity relative to the shore is  $\vec{v}_{FS} = \vec{v}_{FB} + \vec{v}_{BW} + \vec{v}_{WS}$

The equations involving relative velocity will be correct when  
there is no vector subtraction

adjacent inner subscripts are identical

and when the outermost ones correspond exactly to the two on the velocity on the left of the equation.

It is often useful to remember that for any two objects or reference frames,



A and B, the velocity of A relative to B has the same magnitude, but opposite direction, as the velocity of B relative to A:

$$\vec{v}_{AB} = -\vec{v}_{BA}$$

## Work and Energy

### Kinetic Energy, and the Work-Energy Principle

To obtain a quantitative definition for kinetic energy, let us consider a simple rigid object of mass  $m$  (treated as a particle) that is moving in a straight line with an initial speed  $v_1$ . To accelerate it uniformly to a speed  $v_2$  a constant net force  $F_{net}$  is exerted on it parallel to its motion over a displacement  $d$ , Fig. 6–7. Then the net work done on the object is  $W_{net} = F_{net}d$ . We apply Newton's second law,

$$F_{net} = ma \text{ and use Eq. 2-11c } (v_2^2 = v_1^2 + 2ad) \text{ which we rewrite as } a = \frac{v_2^2 - v_1^2}{2d}$$

where  $v_1$  is the initial speed and  $v_2$  is the final speed. Substituting this into  $F_{net} = ma$ , we determine the work done:

$$W_{net} = F_{net}d = mad = m\left(\frac{v_2^2 - v_1^2}{2d}\right)d = m\left(\frac{v_2^2 - v_1^2}{2}\right)$$

or

$$W_{net} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

We define the quantity to be the translational kinetic energy (KE) of the object:

$$KE = \frac{1}{2}mv^2$$

so

$$W_{net} = KE_2 - KE_1$$

or

work-energy principle

$$W_{net} = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

It can be stated in words:

The net work done on an object is equal to the change in the object's kinetic energy.

Thus, the work-energy principle is valid only if  $W$  is the net work done on the object—that is, the work done by all forces acting on the object.

## Potential Energy Defined in General

In general, the change in potential energy associated with a particular force is equal to the negative of the work done by that force when the object is moved from one point to a second point (as in Eq. 6–7b for gravity). Alternatively, we can define the change in potential energy as the work required of an external force to move the object without acceleration between the two points

$$\Delta U = -kx.$$

where  $k$  is a constant, called the spring stiffness constant (or simply spring constant), and is a measure of the stiffness of the particular spring.

spring either stretched or compressed an amount  $x$  from its natural (unstretched) length

spring equation and also as Hooke's law, and is accurate for springs as long as  $x$  is not too great.

elastic potential energy

## Phys1006 formula sheet

### DATA

Avogadro's number	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
Planck's constant	$h = 6.626 \times 10^{-34} \text{ Js}$
Stephan-Boltzmann's constant	$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Boltzmann's constant	$k_B = 1.38 \times 10^{-23} \text{ JK}^{-1}$
Gravitational constant	$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2}$
Radius of the Earth	$R_E = 6.38 \times 10^6 \text{ m}$
Mass of the Earth	$M_E = 5.98 \times 10^{24} \text{ kg}$
Gas constant	$R = 8.314 \text{ J/mol} \cdot \text{K}$
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$
Coulomb constant	$k = 9.0 \times 10^9 \text{ Nm}^2 / \text{C}^2$

acceleration due to gravity	$g = 9.80 \text{ ms}^{-2}$
index of refraction of air (STP)	$n = 1.0003$
speed of light (in vacuum)	$c = 3 \times 10^8 \text{ ms}^{-1}$
speed of sound (at $0^\circ\text{C}$ )	$v = 331 \text{ ms}^{-1}$
Pi	$\pi = 3.1416$
Volume	1 litre = $1000 \text{ cm}^3$
density of water	$\rho = 1000 \text{ kg m}^{-3}$
density of air	$= 1.29 \text{ kg m}^{-3}$
atmospheric pressure	1 atm = $1.013 \times 10^5 \text{ Pa}$
volume of air at STP	$= 22.4 \text{ L}$
coefficient of thermal conductivity for brick	$= 0.84 \text{ J m}^{-1} \text{ s}^{-1} \text{ K}^{-1}$
coefficient of thermal conductivity for glass	$= 0.84 \text{ J m}^{-1} \text{ s}^{-1} \text{ K}^{-1}$
zero Kelvin	$= -273^\circ\text{C}$
freezing point of water	$= 0^\circ\text{C} = 32^\circ\text{F}$
boiling point of water	$= 100^\circ\text{C} = 212^\circ\text{F}$
specific heat of water	$c_{\text{water}} = 4186 \text{ J kg}^{-1} \text{ C}^{\circ-1}$
specific heat of ice	$c_{\text{ice}} = 2100 \text{ J kg}^{-1} \text{ C}^{\circ-1}$
specific heat of iron	$c_{\text{iron}} = 450 \text{ J kg}^{-1} \text{ C}^{\circ-1}$
specific heat of copper	$c_{\text{cu}} = 390 \text{ J kg}^{-1} \text{ C}^{\circ-1}$
specific heat of aluminum	$c_{\text{Al}} = 900 \text{ J kg}^{-1} \text{ C}^{\circ-1}$
latent heat of vaporization of water	$= 2.26 \times 10^6 \text{ J kg}^{-1}$
latent heat of fusion of ice	$= 3.33 \times 10^5 \text{ J kg}^{-1}$
coefficient of volume expansion of petrol	$= 950 \times 10^{-6} \text{ C}^{\circ-1}$
coefficient of linear expansion of steel/iron	$= 12 \times 10^{-6} \text{ C}^{\circ-1}$
coefficient of linear expansion of brass	$= 19 \times 10^{-6} \text{ C}^{\circ-1}$
charge on an electron	$= 1.6 \times 10^{-19} \text{ C}$
1eV	$= 1.6 \times 10^{-19} \text{ J}$

## FORMULA SHEET

### MODULE 1: FUNDAMENTALS

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{Area of a circle} = \pi r^2$$

$$\text{Circumference of a circle} = 2\pi r$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Volume of a cylinder} = \pi r^2 h$$

$$\text{Density } (\rho) = \frac{\text{mass}(m)}{\text{volume}(V)}$$

$$v_{\text{average}} = \frac{v_i + v_f}{2}$$

$$(x_f - x_i) = \text{displacement/distance}$$

$$v_f = v_i + at$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

$$(x_f - x_i) = v_i t + \frac{1}{2}at^2$$

$$R = \frac{v_i^2 \sin 2\theta}{g}, t = \frac{v \sin \theta}{g}$$

$$F = ma \quad F_{\text{friction}} = \mu N \quad W = mg \quad N = mg \cos \theta$$

$$\mu_s = \tan \theta_c \quad F_x = F \cos \theta, \quad F_y = F \sin \theta, F = \sqrt{(F_x)^2 + (F_y)^2}, \tan \theta = \frac{F_y}{F_x}$$

$$\text{Work Done} = Fd \cos \theta$$

$$\text{Kinetic Energy} = \frac{1}{2}mv^2$$

$$\text{Potential Energy} = mgh$$

$$\text{Work Done} = \Delta \text{Kinetic Energy}$$

$$\text{Total Mechanical Energy} = \text{Kinetic Energy} + \text{Potential Energy}$$

$$P = \frac{\text{Work}}{\text{time}} = \frac{W}{t} = Fv$$

$$\text{Normal force in an elevator } N = mg \pm ma$$

## Propagation of Uncertainties

For Additions or Subtractions of measured values

$$C = A \pm B \quad \Delta C = \Delta A \pm \Delta B$$

For Multiplications and Divisions of measured values

$$x = \frac{k^a t^b}{m^c n^d} \quad \frac{\Delta x}{x} = a \frac{\Delta k}{k} + b \frac{\Delta t}{t} + c \frac{\Delta m}{m} + d \frac{\Delta n}{n}$$

## MODULE 2:

### Data

Charge on electron	$= 1.6 \times 10^{-19} \text{C}$
Mass of electron	$= 9.11 \times 10^{-31} \text{kg}$
Charge on proton	$= 1.6 \times 10^{-19} \text{C}$
Mass of proton	$= 1.67 \times 10^{-27} \text{kg}$
Coulomb constant	$k = 9.0 \times 10^9 \text{N.m}^2/\text{C}^2$
1eV	$= 1.6 \times 10^{-19} \text{J}$
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{N.m}^2$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{T.m/A}$
Resistivity of nichrome,	$\rho = 100 \times 10^{-8} \Omega.\text{m}$
Resistivity of copper,	$\rho = 1.68 \times 10^{-8} \Omega.\text{m}$
speed of light (in vacuum)	$c = 3 \times 10^8 \text{ms}^{-1}$

### Formula Sheet

$$F = \frac{kq_1q_2}{r^2} \quad E_{\text{isolated charge}} = \frac{F}{q} = \frac{kq_1}{r^2} \quad V = Ed \text{ (uniform E)}$$

$$V = IR$$

$$R = \frac{\rho L}{A} \quad I = Q/t$$

$V = IR$	$R = \frac{\rho L}{A}$	$I = Q/t$
$V = IR$	$W = qV$	$P = VI = I^2 R$
$C = \frac{\epsilon_0 A}{d} = \frac{k\epsilon_0 A}{d}$	$Q = CV$	$U = \frac{1}{2} CV^2$
$R_{\text{series}} = \sum_i R_i = R + R_2 + R_3$	$\frac{1}{R_{\text{parallel}}} = \sum_i \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$	
$P = VI = I^2 R = \frac{V^2}{R}$	$R_T = R_o(1 + \alpha \Delta T)$	
$P_{\text{max}} = I_o V_o$	$P_{\text{average}} = I_{\text{rms}} V_{\text{rms}} = I_o V_o / 2$	
$F = qvB \sin \theta$	$F = IlB \sin \theta$	
$F = qvB \sin \theta$	$F = IlB \sin \theta$	
$\phi_B = AB \cos \theta$	$\mathcal{E} = \frac{-N \Delta \Phi_B}{\Delta t}$	$\mathcal{E} = Blv \sin \theta$
$B = \frac{\mu_o I}{2\pi r}$	$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$	

### MODULE 3: WAVES and SOUND

Also see Fundamental Principles formulae

$F = kx$  [spring]  
 $PE = 1/2 kx^2$  [spring]  
 $KE = 1/2 mv^2$   
 Total  $E = KE + PE$   
 $T = 2\pi(m/k)^{1/2}$  [mass-spring]  
 $f = 1/T$   
 $T = 2\pi(L/g)^{1/2}$  [pendulum]  
 $\omega = 2\pi f$   
 $v = f\lambda$   
 $v = (331 + 0.6T) \text{ m/s}$   
 $x = A \cos(\omega t)$   
 $v = -A\omega \sin(\omega t)$      $a = -A\omega^2 \cos(\omega t)$

$$V_{\max} = A\omega$$

$$v = \sqrt{\frac{k}{m}(A^2 - x^2)} \quad a_{\max} = \omega^2 A$$

$$\text{Speed of wave in a string/wire, } V = \sqrt{\frac{F_T}{m/L}}$$

$$\text{Speed of wave in a solid, } v = \sqrt{\frac{E}{\rho}}$$

$$\text{Speed of wave in a liquid/gas, } v = \sqrt{\frac{B}{\rho}}$$

$$\text{Harmonics: strings/open pipes, } f_n = n\left(\frac{v}{2L}\right) \quad \text{Closed pipes, } f_n = n\left(\frac{v}{4L}\right)$$

$$\text{Law of reflection, } \theta_i = \theta_r$$

$$\text{Law of refraction, } v_1 \sin \theta_1 = v_2 \sin \theta_2$$

$$\text{Intensity level } \beta(\text{dB}) = 10 \log \left( \frac{I}{I_0} \right)$$

$$\text{where } I_0 = 1.0 \times 10^{-12} \text{ W/m}^2$$

$$\beta_2 - \beta_1 = \Delta\beta = \log \left( \frac{I_2}{I_1} \right) \quad I = \text{Power} / 4\pi r^2$$

## MODULE 4: THERMAL PHYSICS

$$T(^{\circ}\text{C}) = \frac{5}{9}[T(^{\circ}\text{F}) - 32]$$

$$\Delta L = \alpha L_o \Delta T \text{ [linear thermal expansion]}$$

$$\Delta V = \beta V_o \Delta T \text{ [volume thermal expansion]}$$

$$Q = mc\Delta T \text{ [Specific heat]}$$

$$Q = mL \text{ [Latent heat]}$$

$$\text{Rate of heat flow by conduction } \frac{\Delta Q}{\Delta t} = kA \frac{\Delta T}{L}$$

$$\text{Thermal insulation } R = \frac{L}{k}$$

$$\text{Net flow rate of heat radiation } P = \sigma A e (T_{\text{body}}^4 - T_{\text{environment}}^4)$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$PV = nRT \quad (\text{Ideal gas law})$$

$$N = nN_A \quad (N = \text{number of molecules in a gas sample})$$

$$PV = Nk_B T \quad \left( k_B = \frac{R}{N_A} \right)$$

$$\overline{KE} = \frac{3}{2} k_B T$$

(Average KE of a molecule)

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} \quad (\text{rms speed of molecule}) \quad \text{where } m \text{ is the mass of a molecule}$$

## MODULE 5: OPTICS

$$\text{Index of refraction } n = c/v \quad \text{Snell's Law, } n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\text{Mirror/Lens formula} \quad (1/f) = (1/d_o) + (1/d_i)$$

$$\text{Magnification } M = -(d_i/d_o) = (-h_i/h_o)$$

$$\text{Lens makers' formula } (1/f) = (n-1)(1/R_1 + 1/R_2)$$

$$P = \frac{1}{f(m)}$$

$$\text{Magnifying glass } M_{\text{infinity}} = N/f \quad M_N = 1 + N/f$$

$$\text{Microscope } M = M_o M_e = (N \times L) / f_o f_e$$

$$\text{Rayleigh criterion for resolution} \quad \theta_{\text{min}} = 1.22 \lambda / D$$

$$\text{Resolving power of a microscope } S = f(1.22 \lambda / D)$$

$$\text{Brewster's Law} \quad \tan \theta_p = n_2 / n_1$$

$$\text{Malus' Law: } I = I_o \cos^2 \theta$$

$$I = (1/2) I_o \cos^2 \theta$$

## MODULE 6: NUCLEAR RADIATION



charge on an electron	$= 1.6 \times 10^{-19} \text{C}$
1eV	$= 1.6 \times 10^{-19} \text{J}$
mass of neutron	$= 1.008665 \text{u}$
mass of hydrogen atom	$= 1.007825 \text{u}$
mass of $^4\text{He}$ atom	$= 4.002602 \text{u}$
atomic mass unit	$1 \text{u} = 1.66 \times 10^{-27} \text{kg}, = 931.5 \text{MeV}/c^2$
mass of electron	$= 9.11 \times 10^{-31} \text{kg}$
mass of proton	$= 1.67 \times 10^{-27} \text{kg}$

$$f\lambda = c \quad E = hf$$

$$P = \sigma A e T^4 \quad \lambda_{MAX} T = 2.89 \times 10^{-3} \text{mK}$$

$$hf = W_0 + KE_{MAX} \quad KE_{MAX} = eV_{STOPPING}$$

$$E_n = \frac{-13.6}{n^2} \text{eV} \quad E = 13.6 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \text{eV}$$

$$N = N_0 \exp(-\lambda t) \quad (\text{number of atoms}) \quad A = \lambda N \quad (\text{activity})$$

$$A = A_0 \exp(-\lambda t) \quad T_{1/2} = \frac{\ln(2)}{\lambda}$$

$$\text{Binding Energy} = (Zm_p + Nm_n - M_A) \times 931.5 \text{MeV}/u$$

$$\text{Absorbed dose: } 1 \text{Gy} = 1 \text{J/kg} \quad \text{dose} = \text{dose rate} \times \text{time}$$

$$\text{Equivalent dose} = \text{absorbed dose} \times w_R$$

$$\text{Effective dose} = \text{Equivalent dose} \times w_R$$

$$\text{kg}^{-1} \cdot \text{m}^{-1} \cdot \text{s}^{-2} \cdot \text{A}^{-1}$$

**Named units derived from SI base units**

hertz	Hz	frequency	1/s	$s^{-1}$
radian	rad	angle	m/m	1
steradian	sr	solid angle	m <sup>2</sup> /m <sup>2</sup>	1
newton	N	force, weight	kgm/s <sup>2</sup>	$kg \cdot m \cdot s^{-2}$
pascal	Pa	pressure, stress	N/m <sup>2</sup>	$kg \cdot m^{-1} \cdot s^{-2}$
joule	J	energy, work, heat	mN, CV, Ws	$kg \cdot m^2 \cdot s^{-2}$
watt	W	power, radiant flux	J/s, VA	$kg \cdot m^2 \cdot s^{-3}$
coulomb	C	electric charge or quantity of electricity	sA, FV	$s \cdot A$
volt	V	voltage, electrical potential difference, electromotive force	W/A, J/C	$kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$
farad	F	electrical capaci- tance	C/V, s/	$kg^{-1} \cdot m^{-2} \cdot s^4 \cdot A^2$
ohm		electrical resis- tance, impedance, reactance	1/S, V/A	$kg \cdot m^2 \cdot s^{-3} \cdot A^{-2}$
siemens	S	electrical conduc- tance	1/, A/V	$kg^{-1} \cdot m^{-2} \cdot s^3 \cdot A^2$
weber	Wb	magnetic flux	J/A, Tm <sup>2</sup> , Vs	$kg \cdot m^2 \cdot s^{-2} \cdot A^{-1}$
tesla	T	magnetic induction, magnetic flux density	Vs/m <sup>2</sup> , Wb/m <sup>2</sup> , N/(Am)	$kg \cdot s^{-2} \cdot A^{-1}$
henry	H	electrical inductance	Vs/A, s, Wb/A	$kg \cdot m^2 \cdot s^{-2} \cdot A^{-2}$
degree Celsius	C	temperature relative to 273.15 K	K	K
lumen	lm	luminous flux	cdsr	cd
lux	lx	illuminance	lm/m <sup>2</sup>	$cd \cdot m^{-2}$
becquerel	Bq	radioactivity (decays per unit time)	1/s	$s^{-1}$
gray	Gy	absorbed dose (of ionizing radiation)	J/kg	$m^2 \cdot s^{-2}$
sievert	Sv	equivalent dose (of ionizing radiation)	J/kg	$m^2 \cdot s^{-2}$

## Kinematic SI derived units

metre per second	m/s	speed, velocity	$\text{m} \cdot \text{s}^{-1}$
metre per second squared	m/s <sup>2</sup>	acceleration	$\text{m} \cdot \text{s}^{-2}$
metre per second cubed	m/s <sup>3</sup>	jerk, jolt	$\text{m} \cdot \text{s}^{-3}$
metre per second to the fourth	m/s <sup>4</sup>	snap, jounce	$\text{m} \cdot \text{s}^{-4}$
radian per second	rad/s	angular velocity	$\text{s}^{-1}$
radian per second squared	rad/s <sup>2</sup>	angular acceleration	$\text{s}^{-2}$
hertz per second	Hz/s	frequency drift	$\text{s}^{-2}$
cubic metre per second	m <sup>3</sup> /s	volumetric flow	$\text{m}^3 \cdot \text{s}^{-1}$

## Mechanical SI derived units

square metre	m <sup>2</sup>	area	m <sup>2</sup>
cubic metre	m <sup>3</sup>	volume	m <sup>3</sup>
newton second	Ns	momentum, impulse	mkgs1
newton metre second	Nms	angular momentum	m <sup>2</sup> kgs1
newton metre	Nm = J/rad	torque, moment of force	m <sup>2</sup> kgs <sup>2</sup>
newton per second	N/s	yank	mkgs <sup>3</sup>
reciprocal metre	m <sup>-1</sup>	wavenumber, optical power curvature, spatial frequency	m <sup>-1</sup>
kilogram per square metre	kg/m <sup>2</sup>	area density	m <sup>2</sup> kg
kilogram per cubic metre	kg/m <sup>3</sup>	density, mass density	m <sup>3</sup> kg
cubic metre per kilogram	m <sup>3</sup> /kg	specific volume	m <sup>3</sup> kg <sup>-1</sup>
joule second	Js	action	m <sup>2</sup> kgs1
joule per kilogram	J/kg	specific energy	m <sup>2</sup> s <sup>-2</sup>
joule per cubic metre	J/m <sup>3</sup>	energy density	m1kgs <sup>2</sup>
newton per metre	N/m = J/m <sup>2</sup>	surface tension, stiffness	kgs <sup>2</sup>
watt per square metre	W/m <sup>2</sup>	heat flux density, irradiance	kgs <sup>3</sup>
square metre per second	m <sup>2</sup> /s	kinematic viscosity, thermal diffusivity, diffusion coefficient	m <sup>2</sup> s <sup>-1</sup>
pascal second	Pas = Ns/m <sup>2</sup>	dynamic viscosity	m1kgs1
kilogram per metre	kg/m	linear mass density	m1kg
kilogram per second	kg/s	mass flow rate	kgs1
watt per steradian square metre	W/(sr m <sup>2</sup> )	radiance	kgs <sup>3</sup>
watt per steradian cubic metre	W/(sr m <sup>3</sup> )	spectral radiance	m1kgs <sup>3</sup>
watt per metre	W/m	spectral power	mkgs <sup>3</sup>
gray per second	Gy/s	absorbed dose rate	m <sup>2</sup> s <sup>-3</sup>
metre per cubic metre	m/m <sup>3</sup>	fuel efficiency	m <sup>-2</sup>
watt per cubic metre	W/m <sup>3</sup>	spectral irradiance, power density	m1kgs <sup>3</sup>
joule per square metre second	J/(m <sup>2</sup> s)	energy flux density	kgs <sup>3</sup>
reciprocal pascal	Pa <sup>-1</sup>	compressibility	mkg1s <sup>2</sup>
Joule per square metre	J/m <sup>2</sup>	radiant exposure	kgs <sup>2</sup>
kilogram square metre	kg m <sup>2</sup>	moment of inertia	m <sup>2</sup> kg

## Molar SI derived units

mole per cubic metre	mol/m <sup>3</sup>	molarity, amount of substance concentration	mol/m <sup>3</sup>
cubic metre per mole	m <sup>3</sup> /mol	molar volume	m <sup>3</sup> /mol
joule per kelvin mole	J/(Kmol)	molar heat capacity, molar entropy	J/(Kmol)
joule per mole	J/mol	molar energy	J/mol
siemens square metre per mole	S m <sup>2</sup> /mol	molar conductivity	S m <sup>2</sup> /mol
mole per kilogram	mol/kg	molality	mol/kg
kilogram per mole	kg/mol	molar mass	kg/mol
cubic metre per mole second	m <sup>3</sup> /(mol s)	catalytic efficiency	m <sup>3</sup> /(mol s)

## Electromagnetic SI derived units

coulomb per square metre	C/m <sup>2</sup>	electric displacement field, polarization density	m <sup>2</sup> sA
coulomb per cubic metre	C/m <sup>3</sup>	electric charge density	m <sup>3</sup> sA
ampere per square metre	A/m <sup>2</sup>	electric current density	m <sup>2</sup> A
siemens per metre	S/m	electrical conductivity	m <sup>3</sup> kg <sup>1</sup> s <sup>3</sup> A <sup>2</sup>
farad per metre	F/m	permittivity	m <sup>3</sup> kg <sup>1</sup> s <sup>4</sup> A <sup>2</sup>
henry per metre	H/m	magnetic permeability	mkgs <sup>2</sup> A <sup>2</sup>
volt per metre	V/m	electric field strength	mkgs <sup>3</sup> A <sup>1</sup>
ampere per metre	A/m	magnetization, magnetic field strength	m <sup>1</sup> A
coulomb per kilogram	C/kg	exposure (X and gamma rays)	kg <sup>1</sup> sA
ohm metre	m	resistivity	m <sup>3</sup> kgs <sup>3</sup> A <sup>2</sup>
coulomb per metre	C/m	linear charge density	m <sup>1</sup> sA
joule per tesla	J/T	magnetic dipole moment	m <sup>2</sup> A
square metre per volt second	m <sup>2</sup> /(Vs)	electron mobility	kg <sup>1</sup> s <sup>2</sup> A
reciprocal henry	H <sup>-1</sup>	magnetic reluctance	m <sup>2</sup> kg <sup>1</sup> s <sup>2</sup> A <sup>2</sup>
weber per metre	Wb/m	magnetic vector potential	mkgs <sup>2</sup> A <sup>1</sup>
weber metre	Wbm	magnetic moment	m <sup>3</sup> kgs <sup>2</sup> A <sup>1</sup>
tesla metre	Tm	magnetic rigidity	mkgs <sup>2</sup> A <sup>1</sup>
ampere radian	Arad	magnetomotive force	A
metre per henry	m/H	magnetic susceptibility	m <sup>1</sup> kg <sup>1</sup> s <sup>2</sup> A <sup>2</sup>

### Photometric SI derived units

lumen second	lms	luminous energy	scd
lux second	lx s	luminous exposure	m <sup>2</sup> scd
candela per square metre	cd/m <sup>2</sup>	luminance	m <sup>2</sup> cd
lumen per watt	lm/W	luminous efficacy	m <sup>2</sup> kg <sup>1</sup> s <sup>3</sup> cd

## Thermodynamic SI derived units

joule per kelvin	J/K	heat capacity, entropy	$\text{m}^2\text{kg s}^{-2}\text{K}^{-1}$
joule per kilogram kelvin	J/(Kkg)	specific heat capacity, specific entropy	$\text{m}^2\text{s}^{-2}\text{K}^{-1}$
watt per metre kelvin	W/(mK)	thermal conductivity	$\text{m kg s}^{-3}\text{K}^{-1}$
kelvin per watt	K/W	thermal resistance	$\text{m}^2\text{kg}^{-1}\text{s}^3\text{K}^{-1}$
reciprocal kelvin	K <sup>-1</sup>	thermal expansion coefficient	K <sup>-1</sup>
kelvin per metre	K/m	temperature gradient	$\text{m}^{-1}\text{K}$

## Newton's Laws of Motion

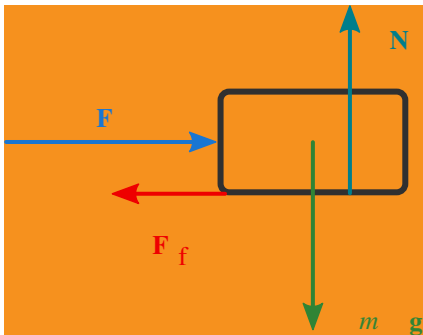
### First law

An object at rest remains at rest unless a net force acts on it.

An object moving with constant velocity continues to move with same speed and in the same direction unless a net force acts on it.

What is a Free-body diagram? Or Force diagram A diagram that shows all the forces acting On the object only and not by the object. To draw FBD - Represent the object of interest as a dot on the origin of a co-ordinate system. Isolate the object and draw all the forces acting on the object. Find the resultant force acting on the object and apply Newton's law.

<https://www.phyley.com/freebody-diagram>



force is in newtons

$$\Sigma F = N - W = 0$$

$$\Sigma F = N - ma = N - mg = 0$$

sum of all forces = normal force - weight = 0

## Second Law

When a net force acts on an object, it experiences acceleration.

$$\Sigma F = ma \Rightarrow a = \frac{\Sigma F}{m}$$

A 50 kg crate is being pulled with a constant force of 200N on a frictionless surface. Find its acceleration.

$$\Sigma F_x = ma_x \quad \Sigma F_y = ma_y$$

$$200 = 50a \quad N - (50 \times 9.8) = 0$$

$$a = 4 \text{ m/s}^2 \quad N = 490 \text{ N}$$

Exercise. What force is needed to stop a 1000 kg car in 6 s if it is traveling at 90 km/h? (Assume constant acceleration) Ans. 4170 N

$$v = \frac{90 \times 10^3 \text{ m}}{60 \times 60} = 25 \text{ m/s}$$

$$v = v_0 + at$$

$$\frac{v - v_0}{t} = a$$



$$\frac{25\text{m}\cdot\text{s}^{-1}}{6\text{s}} = 4.166666\text{m}\cdot\text{s}^{-2}$$

$$N - (1000 \times 4.166666) = 0$$

$$N = 4166$$

### Third Law

Whenever one body exerts a force on a second body, the second body exerts an equal and opposite force on the first body.

These are called Action-Reaction forces.

Note: Action and Reaction Forces act on different objects.

force by hand on desk (action)

force by desk on hand (reaction)

$$F_{HonD} = -F_{DonH}$$

$$F_s \leq \mu_s N \quad \text{static friction}$$

$$F_k = \mu_k N \quad \text{kinetic friction}$$

$$\mu_s > \mu_k$$

### Electricity

Electric current is the amount of charge flowing per unit time at any point in the circuit.

$$\text{Current} \quad I = \frac{Q}{t}$$

$Q$  = Charge in Coulomb

$t$  = time in second

Unit is ampere (A)

$$1\text{A} = 1\text{C}/\text{s}$$

Smaller units are :

$$\text{mA} = 10^{-3}\text{A}$$

$$\mu\text{A} = 10^{-6}\text{A}$$

Ohm's Law

Experiment shows that the current in a conductor is proportional to the potential difference (PD) between its ends, and inversely proportional to its resistance

$$I \propto V \quad \text{and} \quad I \propto \frac{1}{R}$$

$$I = \frac{V}{R} \Rightarrow V = IR \quad [\text{Ohm's Law}]$$

Unit of Resistance is ohm (V/A)

$$1 \text{ ohm} = \frac{1V}{A}$$

#### 18.4 - Resistivity

Experiment shows that R of any material is directly proportional to its length L and inversely proportional to its cross-sectional area A.

$$R \propto L$$

$$R \propto \frac{1}{A}$$

$$R = \rho \frac{L}{A} \Rightarrow \rho = \frac{RA}{L}$$

$\rho$  = Resistivity of the material ( $\Omega m$ )

#### Effect of Temperature on Resistivity

The resistivity of metals increases linearly with temperature

$$\rho_T = \rho_o(1 + \alpha \Delta T) \dots (1)$$

where,  $\alpha$  = Temperature coefficient of resistivity

$\rho_o$  = Resistivity at some reference temperature  $T_o$ . eg.  $0^\circ$  Cor  $20^\circ C$

$$\Delta T = T - T_o$$

The above expression can also be written in terms of R.

$$R_T = R_o(1 + \alpha \Delta T) \dots (2)$$

Exercise. A 100 W light globe has a resistance of 12  $\Omega$  at 20 C and 140  $\Omega$  when switched on. What is the temperature of the filament? Given  $\alpha = 0.0060 \text{ C}^{-1}$

[Ans. 1800 C]

Hint: Use equation (2)

$$R = R_o(1 + \alpha \Delta T)$$

$$\Delta T = \frac{(R - R_o)}{\alpha R_o}$$

$$= 1778$$

$$\Delta T = T_h - T_c = 1778$$

$$T_h = 1778 + 20 = 1798$$

$$\approx 1800^\circ\text{C}$$

### 18.5 Electric Power

Electric energy can be transformed into other forms of energy such as mechanical, heat and light energy.

Power is the energy transformed by a device per unit time.

$$P = \frac{\text{energy transformed}}{\text{time}} = \frac{QV}{t}$$

$$P = IV \dots (18.5)$$

$$\text{Unit: watt} \Rightarrow 1\text{W} = 1\text{J/s}$$

$$P = IV$$

$$= I(IR) = I^2R$$

$$= (V/R)V = V^2/R$$

Energy = power consumption  $\times$  time So far we have been measuring energy in joules, but the electric company measures it in kilowatt - hours, kWh.

$$1\text{kWh} = (1000\text{W})(3600\text{s})$$

$$= 3.60 \times 10^6 \text{ J}$$

Example. The element of an electric oven is designed to produce 3.3 kW of heat when connected to 240 V source. How much will it cost to run the oven for 2 hour, if electricity costs 15 c/kWh?

$$\text{Power} = \frac{\text{Energy}}{\text{time}}$$

$$E = P \times t$$

$$= (3.3\text{kW}) \times 2\text{h} \times \frac{15\text{c}}{\text{kWh}}$$

$$= 99\text{c}$$

## 18.7 Alternating Current

The electrons in a wire first move in one direction and then in the other. Thus polarity changes every half a cycle

Hint: Power dissipated in a radiator is given by

Power consumed in an AC circuit

The current and voltage both have average values of zero, so we square them, take the average, then take the square root, yielding the root mean square (rms) value.

For DC

$$P = VI$$

For AC

$$I_{rms} = \frac{I_o}{\sqrt{2}}$$

$$V_{rms} = \frac{V_o}{\sqrt{2}}$$

$$\text{Ohm's law} \Rightarrow V_{rms} = I_{rms}R$$

$$\text{Average Power} = V_{rms}I_{rms}$$

Summary

$$\text{Current, } I = \frac{Q}{t}$$

$$V = IR \quad [\text{Ohm's Law}]$$

$$R = \rho \frac{L}{A}$$

$$R_T = R_o(1 + \alpha \Delta T)$$

$$P = VI$$

$$P = \frac{\text{Energy}}{t}$$

For AC

$$\text{Average Power} = V_{rms}I_{rms}$$

$$\text{Max power, } P_o = V_o I_o$$

$$I_{rms} = \frac{I_o}{\sqrt{2}}, \quad V_{rms} = \frac{V_o}{\sqrt{2}}$$

## Chapter 16. Electric Charge and Electric Field

### Static Electricity

When two unlike materials are rubbed together, they become charged.

Atom has +ve nucleus and

-vely charged electrons

Unit of charge is coulomb (C)

micro ( $\mu\text{C}$ ) =  $10^{-6}\text{C}$  nano (nC) =  $10^{-9}\text{C}$

Charge on an electron,

$$e = 1.6 \times 10^{-19}\text{C}$$

Example When you comb your hair, about  $1\text{ }\mu\text{C}$  of -ve charge is acquired by the comb.

How many electrons are transferred to the comb?

$$\begin{aligned} n &= \frac{Q}{e} \\ &= \frac{1 \times 10^{-6}\text{C}}{1.60 \times 10^{-19}\text{C}} \\ &= 6.25 \times 10^{12} \text{ electrons} \end{aligned}$$

So  $1\text{ }\mu\text{C}$  needs  $6.25 \times 10^{12}$  electrons or protons

Coulomb's Law (1780)

Electric Force between two point charges is given by Coulomb's law

$Q_1$  and  $Q_2$  are both point charges

$r$  is the distance between them

$$\begin{aligned} F &= k \frac{Q_1 \times Q_2}{r^2} \\ k &= 8.99 \times 10^9 \frac{\text{N.m}^2}{\text{C}^2} \end{aligned}$$

Example. What is the force between two spheres carrying a charge of  $+4.0\text{ }\mu\text{C}$  and  $-6.0\text{ }\mu\text{C}$  separated by  $0.25\text{ m}$ ?

$$\begin{aligned} F &= k \frac{Q_1 \times Q_2}{r^2} \\ F &= 9 \times 10^9 \frac{(4.0 \times 10^{-6})(6.0 \times 10^{-6})}{0.25^2} = 3.5\text{N} \end{aligned}$$

## 16.7 Electric Field

- Michael Faraday (1791 – 1867) introduced the idea of electric fields to provide understanding of forces, which act at a distance.
- Person applying a contact force to push the box
- Charges apply force without any physical contact to push or pull the other charges.

16.7 The Electric Field The electric field at a point is the force on a small charge, divided by the charge:

$$\vec{E} = \frac{\vec{F}}{q}$$

Example. What is the magnitude and direction of electric field at point P due to a charge  $Q = +3 \mu\text{C}$ ?

$$r = 30\text{cm}$$

$$E = \frac{F}{q}$$

$$F = k \frac{Q \times q}{r^2}$$

$$E = \left( k \frac{Q \times q}{r^2} \right) \frac{1}{q}$$

$$\text{or } E = 9 \times 10^9 \frac{Q}{r^2}$$

$$E = \frac{9 \times 10^9 (3 \times 10^{-6})}{0.3^2} = 3 \times 10^5 \text{N/C}$$

## Electric Potential

### Electric Potential Energy

When a charge  $q$  moves from point  $a$  to point  $b$  in an electric field, the change in its PE is equal to the work done on it by the electric force.

$$W_{ab} = PE_b - PE_a = -\Delta PE$$

$$W_{ab} = F \cdot d = -qEd$$

Analogy between gravitational and electrical potential energy: See Fig 17.3

Giancoli

(a) Large rock has more GPE =  $m g h$

(b) Large charge has more EPE =  $q E d$

Electric Potential  $V$

Potential at any point "a" is defined as

$$V_a = \frac{PE_a}{q}$$

Unit: Volt =  $J/C$

$$1V = 1J/C$$

since only changes in PD can be measured. PD between b and a

$$\Delta V = V_b - V_a = -\frac{W}{q}$$

$$W = -q\Delta V$$

Example. How many electrons will flow through the filament of a 60.0 W car headlight in one hour when connected to a 12 V car battery? Charge on

a single electron =  $1.6 \times 10^{-19} C$

$$\text{Power} = \frac{\text{Energy}}{\text{time}}$$

$$\text{Energy} = 60.0W \times 3600s = 2.2 \times 10^5 J$$

$$\Delta V = \frac{W}{q} = \frac{\Delta PE}{q}$$

$$q = \frac{2.2 \times 10^5 J}{12V} = 1.8 \times 10^4 C$$

$$n = \frac{1.8 \times 10^4 C}{1.6 \times 10^{-19} C/e} = 1.1 \times 10^{23} \text{ electrons}$$

Example. How much KE an electron will gain if it accelerates through a PD of 23,000 V in TV picture tube? (pr 3.17)

Solution. The KE gained is equal to the work done on the electron by the electric field.

$$W = -q\Delta V$$

$$W = -(-1.60 \times 10^{-19} C)(+23,000V) \\ = 3.7 \times 10^{-15} J$$

$$\text{But } W = -\Delta PE$$

$$\text{But Loss in PE of electron} = \text{Gain in KE}$$

$$\Delta KE = -\Delta PE$$

$$\Delta KE = -\left(-3.7 \times 10^{-15} J\right) = 3.7 \times 10^{-15} J$$

## 17.2 Relation Between V and E

$$V = \frac{W}{q} \Rightarrow W = -qV \dots (1)$$

$$E = \frac{F}{q} \text{ and } W = Fd \dots (2)$$

$$W = qEd \dots (3)$$

$$-qV = qEd$$

$$E = -\frac{V}{d}$$

Exercise. How strong is the E-Field between two parallel plates 5.8 mm apart if the PD between them is 220 V. (Ans.  $3.8 \times 10^4$  V/m) (prob 17.5 Giancoli)

Magnitude of E is

$$\begin{aligned} E &= -\frac{V}{d} \\ &= \frac{-(220-0)V}{5.8 \times 10^{-3} \text{ m}} \\ &= 3.8 \times 10^4 \text{ V/m} \end{aligned}$$

## 17.4 Electron Volt

Joule is a very large unit for dealing with energies of electrons, atoms or molecules, hence we use a unit called “electron volt”.

1 eV is the energy an electron gain or lose when accelerating through a PD = 1 volt

since change in  $PE$  is

$$\begin{aligned} \Delta PE &= qV \\ 1eV &= (1.60 \times 10^{-19} \text{ C}) \times 1V \\ 1eV &= 1.60 \times 10^{-19} \text{ J} \end{aligned}$$

Exercise. An electron accelerates through a potential difference of 200V. What is its KE in joules and in eV? [Ans.  $3.2 \times 10^{-17} \text{ J}, 200 \text{ eV}$ ]

Loss in  $PE$  = Gain in  $KE$

$$\Delta PE = qV \Rightarrow (1.60 \times 10^{-19} \text{ C}) 200V$$



$$KE = 3.2 \times 10^{-17} \text{ J}$$

$$\text{since } 1\text{eV} = 1.60 \times 10^{-19} \text{ J}$$

$$KE = \frac{3.2 \times 10^{-17} \text{ J}}{1.60 \times 10^{-19} \text{ J/eV}} = 200\text{eV}$$

See Example 17.2 Giancoli

Capacitor

A device used to store electric charge

It consists of two conductors separated by an insulator (called dielectric)

$$Q \propto V$$

$$Q = CV \Rightarrow C = \frac{Q}{V}$$

Unit : farad ( F )

$$1F = \frac{1C}{V}$$

Capacitance of a Parallel Plate Capacitor

Consider two plates each of area A separated by distance d, connected to the battery as shown.

$$C \propto A$$

$$C \propto \frac{1}{d}$$

$$C = \epsilon_o \frac{A}{d}$$

$\epsilon_o$  = permittivity of free space

$$= 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$$

Q. What is the effect of dielectric material in capacitor?

Dielectric are electrically insulating materials. When inserted between the plates of an empty capacitor it would increase its capacitance by a factor k, called the dielectric constant of the material. Hence more charge can be stored in the capacitor.

$$C_o = \epsilon_o \frac{A}{d}$$

$$C = k \left( \epsilon_o \frac{A}{d} \right)$$

$$\text{Capacitance} = \text{dielectric constant} \left( \text{permittivity of free space} \frac{\text{Area}}{\text{distance}} \right)$$

Exercise. A capacitor is made from two 1.1cm diameter circular plates separated by a 0.15mm thick piece of paper of dielectric constant  $K = 3.7$ . A 12V battery is connected to the capacitor. How much charge is on each plate? [Ans.  $2.5 \times 10^{-10} \text{C}$ ]

$$Q = CV, \text{ and } C = \epsilon_o \frac{A}{d}$$

$$Q = K \epsilon_o \frac{A}{d} V$$

$$Q = 3.7 (8.85 \times 10^{-12}) \frac{\pi (0.55 \times 10^{-2} \text{m})^2}{(0.15 \times 10^{-3} \text{m})} 12 \text{V}$$

$$= 2.5 \times 10^{-10} \text{C}$$

17-9. Storage of Electric Energy In charging up capacitor the battery does work in transferring the charge from one plate to the other. This energy is stored in the capacitor as electric PE.

Since  $V$  is not constant during the charging process, we take average value of  $V$

$$W = Q \left( \frac{V_f - 0}{2} \right) \Rightarrow U = \frac{1}{2} QV$$

$$U = \frac{1}{2} (CV) V \Rightarrow \frac{1}{2} CV^2 \dots (1)$$

or

$$Q = CV$$

$$U = \frac{1}{2} Q \left( \frac{Q}{C} \right) \Rightarrow \frac{1}{2} \frac{Q^2}{C} \dots (2)$$

What is  $U$ ? What is  $U$ ? What is  $U$ ? What is  $U$ ?

## DC Circuits

## Latex tools

[https://www.tablesgenerator.com/latex\\_tables](https://www.tablesgenerator.com/latex_tables)

<http://w2.syronex.com/jmr/latex-symbols-converter>