

Chapter 2 Exercise Homework

Question 15

Suppose that a teacher wishes to distribute 25 identical pencils to Ahmed, Barbara, Casper, and Dieter such that Ahmed and Dieter receive at least one pencil each, Casper receives no more than five pencils, and Barbara receives at least four pencils. In how many ways can such a distribution be made.

Let's give Barbara 3 pencils so now she follows the same rules as Ahmed and Dieter. There are 6 different scenarios here. A case where Casper receives anywhere from 0 to 5 pencils. If we just imagine putting 2 dividers in the spaces between the pencils after Casper has taken his share the number of ways a distribution can be made is...

$$C(21,2) + C(20,2) + C(19,2) + C(18,2) + C(17,2) + C(16,2)$$

Question 17

How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 132$ provided that $x_1 < 0$, and $x_2, x_3, x_4 \geq 0$? What if we add the restriction that $x_4 < 17$?

Let's add 1 to x_2, x_3, x_4 this will make the equality change and give them the same rule as x_1 . With that let's imagine 135 folders with 134 spaces and 3 dividers with this we get

$$C(134,3)$$

When the extra rule is added we can extract the element and add up all the scenarios where x_4 is the value n .

$$C(133,2) + C(132,2) + \dots + C(117,2)$$

Question 19

A teacher has 450 identical pieces of candy. He wants to distribute them to his class of 65 students, although he is willing to take some leftover candy home. (He does not insist on taking any candy home, however.) The student who won a contest in the last class is to receive the last 10 pieces of candy as a reward. Of the remaining students, 34 of them insist on receiving at least one piece of candy, while the remaining 30 students are willing to receive no candy.

a. In how many ways can he distribute the candy?

Let's just give the student his candy now we have 450 pieces of candy 34 students who want at least 1 and 32 who want at least 0 (32 adds in the student who won and the teacher who will take candy as well but doesn't need to). Using previous techniques we know that the number of ways the teacher can distribute the candy is

$$C(482,65)$$

b. In how many ways can he distribute the candy if, in addition to the conditions above, one of his students is diabetic and can receive at most 7 pieces of candy?

Let's take this student out of the equation and sum up all the different scenarios that could happen

$$C(481,64) + C(480,64) + \dots + C(475,64)$$

Question 29

Determine the coefficient on $x^{15} y^{120} z^{25}$ in $(2x + 3y^2 + z)^{100}$.

$$\binom{100}{k_1, k_2, k_3} (2x)^{k_1} (3y^2)^{k_2} z^{k_3}$$

=

$$\binom{100}{k_1, k_2, k_3} 2^{k_1} x^{k_1} 3^{k_2} y^{2(k_2)} z^{k_3}$$

$x^{15} y^{120} z^{25}$ arises when $k_1=15, k_2=60, k_3=25$ so the coefficient is...

$$\binom{100}{15, 60, 25} 2^{15} 3^{60}$$

Question 31

For each word below, determine the number of rearrangements of the word in which all letters must be used.

$$\frac{\text{num of letters!}}{\text{letter1! letter2! ...}}$$

a. OVERNUMEROUSNESSES

$$\frac{18!}{2! 1! 4! 2! 2! 2! 1! 4!}$$

b. OPHTHALMOOTORRHINOLARYNGOLOGY

$$\frac{28!}{7! 1! 3! 2! 2! 3! 1! 2! 1! 2! 2! 2!}$$

c. HONORIFICABILITUDINITATIBUS

$$\frac{27!}{1! 2! 2! 1! 7! 1! 1! 2! 2! 1! 3! 2! 1! 1!}$$