1 Using properties of probability laws:

a. $Pr(C \cup P) = Pr((C' \cap P')' = 40\%$

b. $Pr(C \cap P) = Pr(C) + Pr(P) - Pr(C \cup P) = 20\% + 30\% - 40\% = 10\%$

2 Probability calculation:

a. $\Omega = \{1, 2, 3, 4, 5, 6\}$
$Pr(\text{outcome is less than four}) = Pr(1) + Pr(2) + Pr(3),$
$Pr(odds) + Pr(evens) = 1$
$2 * Pr(odds) = Pr(evens)$
$Pr(odds) = \frac{1}{3}$
$Pr(evens) = \frac{2}{3}$
$Pr(\text{outcome is less than four}) = \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$

b. $Pr(\text{odds}) = \frac{1}{3}$,work shown above.

3 Manhattan revisited:

$A = \{(x, y)\Big|\,|x| + |y| \leq a,\ 0 \leq x \leq 1,\ 0 \leq y \leq 1\ 0 \leq a \leq 2\}$
$Pr(A) = \frac{1}{2}a^2$ for $0 \leq a \leq 1$ and $1 - \frac{1}{2}(2 - a)^2$ for $1 < a \leq 2$ This is calculated by dividing the area of the region of choice by the area of the unit square.

4 Partition of a sample space:

a.

$$\Omega = \bigcup_{i=1}^{n} S_i$$

$$Pr(A) = \sum_{i=1}^{n} Pr(A \cap S_i) = Pr \bigcup_{i=1}^{n} (A \cap S_i) = Pr((A \cap S_1) \cup (A \cap S_2) \cup (A \cap S_3)...) =$$

$$Pr(A \cap (S_1 \cup S_2 \cup S_3...)) = Pr(A \cap \Omega) = Pr(A)$$

b. Make a disjoint partition: $\Omega = B \cup C \cup (A \cap B' \cap C') - A \cap B$ (subtract off double counted elements) From this partition, apply result from part a.

5 Drawing names from a hat:

a. There is only one way that every student draws their own name because the order matters. Since the order matters, there are n! total ways to draw. There are n possibilities the first time and for each n possibility there are n-1 possibilities for the next draw and so on. As a result, the probability is given by $\frac{1}{n!}$

b. There is only one way that the first m students will draw their name. For that one way, there are (n-m-1)! ways the remaining students can draw a name. As a result, the probability is given by $\frac{(n-m)!}{n!}$

c. There are m! ways that everyone among the first m students to draw gets a name of the last m students to draw. For these m! ways, there are (n-m-1)! ways the remaining students can draw a name. As a result, the probability is given by $\frac{(n-m)!m!}{n!}$

6 ECE faculty needs more exercise:

a. There are $\binom{16}{6} = 8008$ ways to form rosters.

b. There are $\binom{5}{3}$ ways to choose 2 women, and for each of these ways there are $\binom{11}{4}$ ways to choose 4 men. $\binom{5}{2} * \binom{11}{4} = 10 * 330 = 3300$ ways to have teams with two women and four men.

c. Sum the ways to choose 2 women 4 men, 3 women 3 men, 4 women 2 men, and 5 women 1 man. $\binom{5}{2}*\binom{11}{4}+\binom{5}{3}*\binom{11}{3}+\binom{5}{4}*\binom{11}{2}+\binom{5}{5}*\binom{11}{1} = 10*330+10*165+5*55+1*11 = 5236$ ways to have teams with at least two women.

d. i. Probability randomly selected team has exactly two women = $\frac{3300}{8008}$
ii. Probability randomly selected team has at least two women = $\frac{5236}{8008}$