

1 Using properties of probability laws:

- a. $Pr(C \cup P) = Pr((C' \cap P')') = 40\%$
- b. $Pr(C \cap P) = Pr(C) + Pr(P) - Pr(C \cup P) = 20\% + 30\% - 40\% = 10\%$

2 Probability calculation:

- a. $\Omega = \{1, 2, 3, 4, 5, 6\}$
 $Pr(\text{outcome is less than four}) = Pr(1) + Pr(2) + Pr(3),$
 $Pr(\text{odds}) + Pr(\text{evens}) = 1$
 $2 * Pr(\text{odds}) = Pr(\text{evens})$
 $Pr(\text{odds}) = \frac{1}{3}$
 $Pr(\text{evens}) = \frac{2}{3}$
 $Pr(\text{outcome is less than four}) = \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$
- b. $Pr(\text{odds}) = \frac{1}{3}$, work shown above.

3 Manhattan revisited:

$$A = \{(x, y) \mid |x| + |y| \leq a, 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq a \leq 2\}$$

$Pr(A) = \frac{1}{2}a^2$ for $0 \leq a \leq 1$ and $1 - \frac{1}{2}(2-a)^2$ for $1 < a \leq 2$. This is calculated by dividing the area of the region of choice by the area of the unit square.

4 Partition of a sample space:

a.

$$\Omega = \bigcup_{i=1}^n S_i$$

$$Pr(A) = \sum_{i=1}^n Pr(A \cap S_i) = Pr \bigcup_{i=1}^n (A \cap S_i) = Pr((A \cap S_1) \cup (A \cap S_2) \cup (A \cap S_3) \dots) = \\ Pr(A \cap (S_1 \cup S_2 \cup S_3 \dots)) = Pr(A \cap \Omega) = Pr(A)$$

- b. Make a disjoint partition: $\Omega = B \cup C \cup (A \cap B' \cap C') - A \cap B$ (subtract off double counted elements)
From this partition, apply result from part a.

5 Drawing names from a hat:

- a. There is only one way that every student draws their own name because the order matters. Since the order matters, there are $n!$ total ways to draw. There are n possibilities the first time and for each n possibility there are $n-1$ possibilities for the next draw and so on. As a result, the probability is given by $\frac{1}{n!}$

- b. There is only one way that the first m students will draw their name. For that one way, there are $(n-m)!$ ways the remaining students can draw a name. As a result, the probability is given by $\frac{(n-m)!}{n!}$
- c. There are $m!$ ways that everyone among the first m students to draw gets a name of the last m students to draw. For these $m!$ ways, there are $(n-m)!$ ways the remaining students can draw a name. As a result, the probability is given by $\frac{(n-m)!m!}{n!}$

6 ECE faculty needs more exercise:

- a. There are $\binom{16}{6} = 8008$ ways to form rosters.
- b. There are $\binom{5}{3}$ ways to choose 2 women, and for each of these ways there are $\binom{11}{4}$ ways to choose 4 men. $\binom{5}{2} * \binom{11}{4} = 10 * 330 = 3300$ ways to have teams with two women and four men.
- c. Sum the ways to choose 2 women 4 men, 3 women 3 men, 4 women 2 men, and 5 women 1 man. $\binom{5}{2} * \binom{11}{4} + \binom{5}{3} * \binom{11}{3} + \binom{5}{4} * \binom{11}{2} + \binom{5}{5} * \binom{11}{1} = 10 * 330 + 10 * 165 + 5 * 55 + 1 * 11 = 5236$ ways to have teams with at least two women.
- d. i. Probability randomly selected team has exactly two women = $\frac{3300}{8008}$
ii. Probability randomly selected team has at least two women = $\frac{5236}{8008}$