

Gradient updates for Rezende et al., 2014

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Link to paper: <https://doi.org/10.3389/fncom.2014.00038>

Setup

- $u_i(t)$: membrane potential of neuron i at time t
- w_{ij} : synaptic weight from neuron i to neuron j
- $X_i(t)$: observed spike in neuron i at time t
- $\phi_i(t)$: evoked potential of neuron i at time t
- $\eta_i(t)$: adaptation potential of neuron i at time t
- $\rho_i(t)$: firing rate of neuron i at time t
- $\tau, \tau_{\text{adapt}}, \delta u, v$: neuron constants

Membrane potential update

This update is based on equation 1 in the paper.

$$\begin{aligned}
u_i(t) &= \sum_j w_{ij} \phi_j(t) + \eta_i(t) \\
&= \sum_j w_{ij} \int_0^t \exp\left(-\frac{(t-s)}{\tau}\right) X_j(s) ds - \eta_0 \int_0^t \exp\left(-\frac{(t-s)}{\tau_{\text{adapt}}}\right) X_j(s) ds \\
&\approx \sum_j w_{ij} \sum_{k=0}^t \exp\left(-\frac{(t-k)}{\tau}\right) X_j(k) - \eta_0 \sum_{k=0}^t \exp\left(-\frac{(t-k)}{\tau_{\text{adapt}}}\right) X_j(k)
\end{aligned}$$

M-network gradient updates

This update is based on equation 24 in the paper (and all associated equations).

We take the Poisson link function to be

$$g(u_i(t)) = \exp(u_i(t)).$$

The gradient of the log likelihood wrt w_{ij} is as follows:

$$\begin{aligned}
\dot{w}_{ij}^{\mathcal{M}}(T) &\approx \int_0^T \frac{g'(u_i^{\mathcal{M}}(t))}{g(u_i^{\mathcal{M}}(t))} [X_i(t) - \rho_i^{\mathcal{M}}(t)] \phi_j(t) dt \\
&= \int_0^T \frac{\frac{\partial}{\partial u_i(t)} \left(\rho_0 \exp \left[\frac{u_i(t)-v}{\Delta u} \right] \right)}{\rho_0 \exp \left[\frac{u_i(t)-v}{\Delta u} \right]} [X_i(t) - \rho_i^{\mathcal{M}}(t)] \phi_j(t) dt \\
&= \int_0^T \frac{\rho_0 \exp \left[\frac{u_i(t)-v}{\Delta u} \right]}{\rho_0 \exp \left[\frac{u_i(t)-v}{\Delta u} \right]} [X_i(t) - \rho_i^{\mathcal{M}}(t)] \phi_j(t) dt \tag{1} \\
&= \int_0^T [X_i(t) - \rho_i^{\mathcal{M}}(t)] \phi_j(t) dt \\
&= \int_0^T [X_i(t) - \rho_i^{\mathcal{M}}(t)] \phi_j(t) dt \\
&= \int_0^T \left[X_i(t) - \rho_0 \exp \left[\frac{u_i(t)-v}{\Delta u} \right] \right] \int_0^t \exp \left(-\frac{(t-s)}{\tau} \right) X_j(s) ds dt \\
&\approx \sum_{t=0}^T \left[X_i(t) - \rho_0 \exp \left[\frac{u_i(t)-v}{\Delta u} \right] \right] \sum_{s=0}^t \left[\exp \left(-\frac{(t-s)}{\tau} \right) X_j(s) \Delta s \right] \Delta t \tag{2}
\end{aligned}$$

In (1), we used the gradient of g with respect to $u_i(t)$:

$$\frac{\partial}{\partial u_i(t)} \left(\rho_0 \exp \left[\frac{u_i(t)-v}{\Delta u} \right] \right) = \rho_0 \exp \left[\frac{u_i(t)-v}{\Delta u} \right]$$

Finally the update to w_{ij} is

$$w_{ij}^{\mathcal{M}}(T) = w_{ij}^{\mathcal{M}}(T) + \mu^{\mathcal{M}} \dot{w}_{ij}^{\mathcal{M}}(T)$$