Gradient updates for Rezende et al., 2014

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Link to paper: `https://doi.org/10.3389/fncom.2014.00038`

Membrane potential update

This update is based on equation 1 in the paper.

$$
\begin{aligned}
u_i(t) &= \sum_j w_{ij}\phi_j(t) + \eta_i(t) \\
&= \sum_j w_{ij} \int_0^t \exp\left(-\frac{(t-s)}{\tau}\right) X_j(s) ds - \eta_0 \int_0^t \exp\left(-\frac{(t-s)}{\tau_{\text{adapt}}}\right) X_j(s) ds \\
&\approx \sum_j w_{ij} \sum_{k=0}^{t} \exp\left(-\frac{(t-k)}{\tau}\right) X_j(k) - \eta_0 \sum_{k=0}^{t} \exp\left(-\frac{(t-k)}{\tau_{\text{adapt}}}\right) X_j(k)
\end{aligned}
$$

M-network gradient updates

This update is based on equation 24 in the paper (and all associated equations).

We take the Poisson link function to be

$$g(u_i(t)) = \exp(u_i(t)).$$

$$
\begin{aligned}
\dot{w}_{ij}^{\mathcal{M}}(T) &\approx \mu^{\mathcal{M}} \int_0^T \frac{g'(u_i^{\mathcal{M}}(t))}{g(u_i^{\mathcal{M}}(t))} \left[X_i(t) - \rho_i^{\mathcal{M}}(t)\right] \phi_j(t) dt \\
&= \mu^{\mathcal{M}} \int_0^T \frac{\frac{\partial}{\partial w_{ij}} \left(\rho_0 \exp\left[\frac{u_i(t)-v}{\Delta u}\right]\right)}{\rho_0 \exp\left[\frac{u_i(t)-v}{\Delta u}\right]} \left[X_i(t) - \rho_i^{\mathcal{M}}(t)\right] \phi_j(t) dt \\
&= \mu^{\mathcal{M}} \int_0^T \frac{\phi_j(t) \rho_0 \exp\left[\frac{u_i(t)-v}{\Delta u}\right]}{\rho_0 \exp\left[\frac{u_i(t)-v}{\Delta u}\right]} \left[X_i(t) - \rho_i^{\mathcal{M}}(t)\right] \phi_j(t) dt \qquad (1) \\
&= \mu^{\mathcal{M}} \int_0^T \phi_j(t) \left[X_i(t) - \rho_i^{\mathcal{M}}(t)\right] \phi_j(t) dt \\
&= \mu^{\mathcal{M}} \int_0^T \phi_j(t)^2 \left[X_i(t) - \rho_i^{\mathcal{M}}(t)\right] dt \\
&= \mu^{\mathcal{M}} \int_0^T \left[\int_0^t \exp\left(-\frac{(t-s)}{\tau}\right) X_j(s) ds\right]^2 \left[X_i(t) - \rho_0 \exp\left[\frac{u_i(t)-v}{\Delta u}\right]\right] dt \\
&\approx \mu^{\mathcal{M}} \sum_{t=0}^{T} \left[\sum_{s=0}^{t} \exp\left(-\frac{(t-s)}{\tau}\right) X_j(s)\right]^2 \left[X_i(t) - \rho_0 \exp\left[\frac{u_i(t)-v}{\Delta u}\right]\right] \qquad (2)
\end{aligned}
$$

In (1), we used the gradient of g with respect to w_{ij}:

$$
\begin{aligned}
\frac{\partial}{\partial w_{ij}} \left(\rho_0 \exp\left[\frac{u_i(t)-v}{\Delta u}\right]\right) &= \frac{\partial}{\partial w_{ij}} \left(\rho_0 \exp\left[\frac{\sum_k w_{ik}\phi_k(t) + \eta_i(t) - v}{\Delta u}\right]\right) \\
&= \phi_j(t) \rho_0 \exp\left[\frac{\sum_k w_{ik}\phi_k(t) + \eta_i(t) - v}{\Delta u}\right] \\
&= \phi_j(t) \rho_0 \exp\left[\frac{u_i(t)-v}{\Delta u}\right]
\end{aligned}
$$