to hedge from large losses. Determining the optimal behavior strategy of a GMIB VA holder using a PINN will follow a similar procedure as pricing an American option. The main distinction lies in the fact that American option holders only ever have two decisions available, whereas VA holders can withdraw percentage amounts.

# 2. Background

## 2.1 The Neural Network

In order to begin understanding how PINNs work, it's essential to first understand how neural networks work. A neural network can be thought of almost as an artificial human brain, consisting of layers and layers of nodes, or "neurons". These neurons receive information in the form of input data and activate depending on whether some threshold is met. Each individual neuron is made up of two components: weights and a bias. Information that is received by a neuron is multiplied by the weights, and is then added to the bias. The newly computed number is then input into an activation function, and depending on whether the output is large enough, the neuron will either activate or lay dormant. If the neuron activates, then the output of the activation function will be used as an input in the next layer of neurons. Upon reaching the last layer of neurons and being fed through the activation functions, the neural network will produce an output which acts as its prediction.

For a neural network to begin making accurate predictions, it must optimize its weights and biases. Each neural network comes equipped with what is known as a loss function, a function which represents the error produced by a neural network. Minimizing the loss function iteratively by modifying each weight and bias is what leads to the high levels of accuracy that neural networks are famous for. The process of tuning a neural networks trainable parameters is made possible through a technique called gradient descent. The gradient descent algorithm works by determining the partial derivatives of the loss function with respect to the weights and biases of the neural network, and determining the direction in which to move in order to minimize error. Once the neural network knows which direction in which to move, it begins taking tiny steps towards a local minimum. The size of these steps must be manually chosen through trial and error. Upon reaching the local minimum of the loss function, we collect the weights and biases that have lead to this minimum, and save them to be used for the next neural network iteration. This process of feeding input data through a neural network and finding the local minimum of the associated loss function is often repeated hundreds of times until the error produced by the neural network is deemed acceptable low.

## 2.2 The Physics-Informed Neural Network

A physics-informed neural network operates similarly to the mentioned neural network in the sense that it utilizes a structure similar to the human brain. However, there are some major differences between the two models. While a typical neural network's loss function is simply the sum of the squared error between the predicted value and the true value, a PINN's loss function has a PDE's residuals embedded within