$\tilde{A} := \frac{A_t}{G_t}$. Combining $A_t$ and $G_t$ within one variable reduces the number of computations that must be performed
$x_0$ := the age of the policyholder when purchasing the Variable Annuity
$\omega$ := the age at which the policyholder is assumed to be dead
$r^{gua}$ := The interest rate guaranteed by the insurance company in case of annuitizing the value of the policyholder's account
$r^{act}$ := The risk-free interest rate

Applying a PINN to the GMIB variable annuity follows an similar algorithm to the one described in sections 3.1 - 3.3. Discrepancies exist in the terminal/boundary conditions and constants used, the way in which the residuals function is expressed, and the addition of a jump condition. The PDE framework utilized in solving for the optimal withdrawals schedule was obtained from [3].

While the American option PDE had an initial condition (due to the fact that it was solved forward in time), the GMIB variable annuity's PDE has a terminal condition (because it is solved backwards in time). The terminal/boundary conditions are defined for the GMIB variable annuity as follows

Let $a_t^{\cdots} = \int_t^{\omega - x_0} e^{-r^{\cdots}(t-t_N)} dt$. Then

$$\phi(t, \tilde{A}) = \frac{a_t^{gua}}{a_t^{act}} \tag{10}$$

$$\phi(t, \tilde{A_{max}}) = \tilde{A_{max}} \tag{11}$$

$$\phi(t_N, \tilde{A}) = max(\tilde{A}, \frac{a_t^{gua}}{a_t^{act}}) \tag{12}$$

Although many of the constants involved in determining the optimal withdrawal strategy of the GMIB VA are similar or identical to those used within the American option PDE, there are a few that differ, namely

$\alpha^A$ := a fee charged by the insurance company which is proportional to the account value
$\alpha_G$ := a fee charged by the insurance company which is proportional to the benefit base

We can now define the residuals function to be used in the GMIB VA's PINN. The equation is nearly identical to the Black-Scholes equation, with the exceptions being the differing constants and the addition of a component incorporating aspects of the benefit base. Assuming $\phi_\theta$ is the approximated solution,

$$\frac{\partial \phi_\theta}{\partial t} + \frac{1}{2}\sigma^2 \tilde{A}^2 \frac{\partial^2 \phi_\theta}{\partial \tilde{A}^2} + (r - \alpha^A)\tilde{A}\frac{\partial \phi_\theta}{\partial \tilde{A}} - \alpha^G \frac{\partial \phi_\theta}{\partial \tilde{A}} - r\phi_\theta = residual \tag{13}$$

Finally, we must include the jump condition that is present within the GMIB product. Let $\gamma$ be a withdrawal amount and let $\tilde{\gamma} = \frac{\gamma}{G}$
Additionally, let

$$h_1(\tilde{\gamma}) = max(\tilde{A} - \tilde{\gamma}, 1 - \frac{\tilde{\gamma}}{\tilde{A}}) \tag{14}$$