

the dividend yield percentage (δ), and the spot price of the stock (S). We also defined the initial and boundary conditions of our PDE:

$$V(S, 0) = \max(K - S, 0) \quad (3)$$

$$\lim_{S \rightarrow \infty} V(S, \tau) = 0, \quad (4)$$

$$V(0, \tau) = K \quad (5)$$

for Puts, and

$$V(S, 0) = \max(S - K, 0) \quad (6)$$

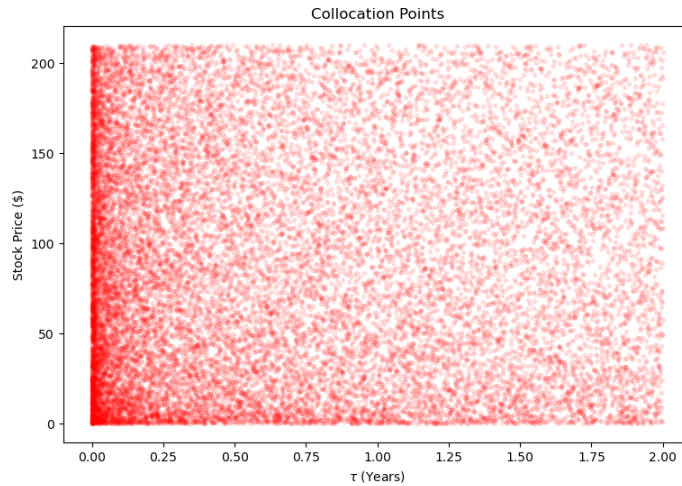
$$\lim_{S \rightarrow \infty} V(S, \tau) = \infty, \quad (7)$$

$$V(0, \tau) = 0 \quad (8)$$

for Calls. We also defined a function which returns the residuals of our PDE. This equation was developed using an HJB formulation of the Black-Scholes equation with a built-in maximum constraint making this PDE appropriate for pricing American options. Letting V_θ be the approximated PINN solution, we get that

$$\min\left[\frac{\partial V_\theta}{\partial \tau} - \frac{\sigma^2}{2} S^2 \frac{\partial^2 V_\theta}{\partial S^2} - rS \frac{\partial V_\theta}{\partial S} + rV_\theta, V_\theta - V^*\right] = \text{residual} \quad (9)$$

Collocation points for determining residuals, boundary and initial training points, and boundaries for maximum and minimum stock price and time values were also set. When we first began numerical testing of our PINN for pricing American options, we noticed that our model struggled to exhibit the undifferentiable point present when there are 0 years left to expiration. We attempted to focus our PINNs training efforts towards modeling this corner by using stretched grid sampling as described in [2]. The basic idea is to randomly sample points in a 2D space, then raise the x coordinate to some power $p > 1$, and the y coordinate to the power of \sqrt{p}



This method of sampling ensures that a large portion of our sampled collocation points are located towards the left of the graph, which enables our PINN more opportunities to learn to model the discontinuities present at/near $\tau = 0$.