1. Introduction

In their simplest form, financial options allow an option holder to exercise the right to buy or sell a stock. The type of option that is purchased determines the rights that the holder is entitled to, as well as when the holder is allowed to exercise their option. A Put option allows the holder to sell the underlying stock at a predetermined price, commonly referred to as the "strike". However, if the option holder decides that selling the stock is unprofitable, they may opt out of exercising their Put. A Call option works completely opposite, allowing an option holder to buy a stock at the strike. Similar to a Put option, the holder can choose to avoid exercising if their option becomes worthless. A European option allows a holder to exercise only at the expiration of their contract, whereas an American option allows a holder to exercise their option at any point between the inception of their contract and its expiration. Option holders can generate revenue by purchasing a Put or Call based on whether they believe the underlying stock will move up or down in price before expiration. For example, an American Call holder would likely hope that the underlying stock's price increases enough to where they can exercise their option, buy the stock at the fixed strike price, and produce revenue by then selling the stock at the spot price. The difference between the spot price and strike price, along with the option premium cost, is considered profit by the option holder.

Determining the price at which to set the premium of an option has lead to what is known as the Black-Scholes equation, a PDE whose solution returns the theoretical price of a European option with respect to the stock's spot price, the volatility of the stock, the current risk-free interest rate, and the time to expiration. The Black-Scholes model has been proven to have a closed form analytical solution, but computationally efficient and accurate numerical methods are more often utilized to solve the associated PDE. Pricing an American option is more complicated, as the ability to exercise at any point in time gives rise to what is known as the free-boundary problem. Solving for this free-boundary returns a border which separates the times and stock prices at which it is optimal to exercise the option, and the times and stock prices at which it is optimal to wait. Though a wide class of methods exist for pricing American options, we utilized a Crank-Nicolson discretization scheme, and solved the resulting matrix equations using a basic matrix equation solver. FDMs are commonly used for pricing American options, and therefore make an appropriate benchmark to compare our PINN against.

VAs are insurance products frequently sold by insurance companies which can be indexed on the financial market for potential upsides in value. The GMIB rider guarantees the policy holder the ability to opt for regular, minimum payments after annuitization for the rest of their lives. Similar to American options, VA owners can "exercise", or withdraw, their investments before the expiration of their annuity. However, withdrawals can only occur on specific dates, typically the first of each month. Additionally, annuitants can also choose to withdraw a percentage of their account balance, complicating the process of pricing the product. A policy holder who happens to follow an optimal withdrawal strategy can generate losses for an insurance company, so it's crucial to track the optimal withdrawal schedule of a hypothetical policy holder