5. Discussion

The feasibility of using PINNs for pricing financial and insurance products is obvious. However, the current limitations of this type of approach are equally obvious. Though we were successfully able to implement PINNs that priced products with acceptable levels of error, the computation times of each of these methods were unacceptable high. However, these limitations can likely be circumvented using different methodologies and packages. At the beginning of the research process, we came across a library called DeepXDE that was created specifically for solving PDEs using PINNs. Though we later decided to move forward with the more mainstream library Tensorflow, DeepXDE had a specialized class meant to compute partial derivatives much more quickly than Tensorflow's Gradient Tape method. In the future, we might consider incorporating the DeepXDE method for computing derivatives while still using Tensorflow for our neural network architecture and training process. In addition, a huge bottleneck in determining the optimal withdrawal strategy for the GMIB VA was in the implementation of the jump condition. Finding a value of $\tilde{\gamma}$ that maximizes our value function iteratively proved to take just as long if not longer than training our PINN. Creating a separate function within the PINN or enforcing this constraint within the loss function could prove to be a far more efficient method than implementing the condition at end of the training steps. In addition, the inability of the PINN to properly form the corner at $\tau = 0$ for Puts or Calls is concerning, and a point to look into. We may need to make use of a different activation function, or focus even more collocation/boundary points towards this section.

6. Conclusion

We have utilized a PINN for solving the HJB formulation of the American options model. We have also implemented a PINN for solving the GMIB VA. We made use of the PINN framework first established by [1], and repurposed it to solve for both of these products. Though this methodology failed to beat out the established benchmark methods in terms of speed or accuracy, it proved that there may be space for PINNs in the near future as technology continues to advance. Future work will consist of the points made in the Discussion section, but we would also be interested in adding dimensions to our PDEs in the form of stochastic interest rates and volatility. PINNs have been mentioned to perform incredibly well in solving PDEs with high dimensions, so increasing the dimensionality of the PDEs we aim to solve could lead to interesting results.

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