On Market Makers Performance

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December 15, 2016

1 Motivation

The Logarithmic Market Scoring Rule (LMSR), while popular in the theoretical literature, faces several critical barriers to adoption. The first is that the **market maker** is bounded loss, which is a non-starter for most real world applications. The second is that the **market maker** is not sensitive to liquidity. This means that a improperly configured **market maker** might allow any agent with a positive budget to drastically swing the price or at the other extreme, not allow any agent to modify the price much at all. LMSR is well studied, however, because it has Path Independence and Translation Invariance. Many other theoretical **market makers** have been proposed that also have these two properties, but with additional benefits, and some offer tradeoffs in order to gain more desirable properties. This work seeks to quantify these tradeoffs empirically.

2 Goals

To test five different market maker mechanisms for their liquidity sensitivity, profit expectation, and accuracy.

3 Model

3.1 Definitions

We denote **time** by $t \in \mathbb{R}_+$.

An **event** e has an outcome at time $e_t \in \mathbb{R}_+$ where we restrict our attention to a binary outcome. The outcome is equal to YES in case the event occurs, and NO otherwise. We denote the YES outcome with a 1, and the NO outcome with a 0. We assume there is a way to unambiguously determine the outcome of an event.

R is an **oracle** for mapping an event to an outcome. We denote the outcome of event e under R as $R(e) \in \{0, 1\}$.

An **option** o is a security that yields a return depending on the outcome of an event o_e at time e_t. Each option has a direction $o_d \in \{0, 1\}$ and a strike time $o_t = e_t$ when $R(o_e)$ will be evaluated. The option will convert to \$1 at time o_t if $R(o_e)$ equals o_d, otherwise it converts to \$0. Two options are said to be **complementary options** if they trade opposite directions in the same event. Given an event e, we denote the complementary options as $o_c = \langle o_0, o_1 \rangle$.

Given complementary options o_0 and o_1, let A_{o_0} be the **set of agents** that acquire option o_0, and similarly let A_{o_1} be the set of agents that acquire option o_1. Let $A_o = A_{o_0} \cup A_{o_1}$ be the set of all agents trading complementary options.

An **agent** $a \in A_o$ has a **private belief** $a_v \in [0, 1]$ and a **budget** $a_b \in \mathbb{R}_+$. The agent's private belief a_v is the subjective probability that the agent assigns to the outcome of event o_e being direction o_d at strike time o_t.

An agent's $a \in A_o$ **strategy** $s_a(t) \in \mathbb{R}_+$ specifies the quantity of option o purchased by the agent at time t.

A **prediction market** M trades complementary options. Formally, a prediction market is a tuple $M = \langle o_c, A_o \rangle$ where each **agent** $a \in A_o$ purchases either some number of o_0 or o_1 options paying the price quoted by the market maker.

A **market maker** p_M is a function that maps an option o, an agent $a \in A_o$ and a quantity $q \in \mathbb{R}_+$ at time $t \in [0, t_o)$ to a price $p(o, a, q, t) \in \mathbb{R}_+$. By definition $p(o, a, t) = R(o_e)$ if $t \geq t_o$.

3.2 Assumptions

[Enrique: WITHOUT LOSS OF GENERALITY???] In our model we assume that agent $a \in A_o$ has an arrival time $a_t \in \mathbb{R}_+$ where $a_t < o_t$, and execute their strategy only once at time a_t.

We assume that agents' private beliefs and budgets are not common knowledge but are drawn from known distributions $v \sim v(\cdot)$ and $b \sim b(\cdot)$.

We assume agents are allow to observe the true current price of any option.

We assume that the market maker mechanism and all its parameters are common knowledge.

3.3 Problems

Definition 3.1. (Market Maker Revenue) Given a market maker p_M and a set of participating agents A_o, the revenue obtained from p_M is defined as

$$R(p_M, A_o) = \sum_{a \in A_o} \left[\int_{t=0}^{t=o_t} p_M(o_0, a, t, s_a^0(t)) s_a^0(t) dt + \int_{t=0}^{t=o_t} p_M(o_1, a, t, s_a^1(t)) s_a^1(t) dt \right]$$

Definition 3.2. (Market Maker Cost) Given a market maker p_M and a set of participating agents A_o, the cost to p_M is defined as

$$C(p_M, A_o) = \sum_{a \in A_o} \left[\int_{t=0}^{t=o_t} R(o_e) s_a^0(t) dt + \int_{t=0}^{t=o_t} R(o_e) s_a^1(t) dt \right]$$

Definition 3.3. (Market Maker Proft) Given a market maker p_M and a set of participating agents A_o, the profit of p_M is defined as

$$P(p_M, A_o) = R(p_M, A_o) - C(p_M, A_o)$$

Definition 3.4. (Profit-Maximizing Market Maker). Among all Market Makers, find the one that maximizes Profit.

4 Market Makers

In these experiments we will test the following five market markers.

4.1 Logarithmic Market Scoring Rule

4.2 Luke's Online Budget Weighted Average

4.3 Yiling and Jen's Expert Weighted Majority

4.4 Luke's Weighted Majority

4.5 Practical Liquidity Sensitive Market Maker

5 Setup

5.1 Types

A **BasicAgent** follows from the definition of these **market makers** as truthful mechanisms. Each **BasicAgent** reports its belief v and budget b truthfully when requested by the mechanism. Each **BasicAgent** is rational and profit maximizing.

5.2 Metrics

This research will use the following metrics to evaluate each of the five mechanisms.

5.2.1 Liquidity Sensitivity

This research uses a novel definition of the vague concept of liquidity sensitivity. The need for a unified definiton stems from the known issue that LMSR can provide ẗoo littleör ẗoo muchẅeight to each agent depending on its liquidity parameter and market depth. One extreme example us that an infinitesimally low liquidity parameter will allow an **agent** with a nonzero budget to dictate the market price regardless of how many **agents** have already participated in the market. Our novel definition attempts to quanity consistency across **market depths** without overly constraining implementations.

We define a **market maker** as **liquidity sensitive** if two **agents** that have identical transactions in the **market** will have equal impacts on the **market price** proportional to the **market depth** D_1 at time t_1 of the first purchase, the **market depth** D_2 at time t_2 of the second purchase, and the difference in time $t_1 - t_2$.

We formalize **liquidity sensitive** as the function $\Delta = f(D_1, D_2, t_\Delta)$ having a constant Δ.

5.2.2 Market Maker Profit

We define the **market maker profit** on prediction market M at time t as
$\sum_{a \in A_o}[\int_t p(o_0, a, t, s_a^0(t))s_a^0(t)dt + \int_t p(o_1, a, t, s_a^1(t))s_a^1(t)dt - \int_t R(o_e)s_a^0(t)dt - \int_t R(o_e)s_a^1(t)dt]$

5.2.3 Social Welfare

We define the profit of an agent as the sum of the realized value of its **options** minus the sum of the cost of each of its **options**.

We define **social welfare** as the **market maker profit** plus the sum of each agent's profit.

5.2.4 Accuracy

Since the accuracy of a prediction market depends on the accuracy of each of its participants as well as a usually unknown ground truth value, we utilize several well studied definitions to evaluate each mechanism's accuracy.

Regret We define the **regret** of a mechanism as the difference between the mechanism's accuracy and the accuracy of the most accurate agent $a \in A$, who can be thought of as the best expert. The most accurate agent $a \in A$ is the agent a whose belief is most frequently aligned with the outcome over n trials.

Expectation We define the **expectation accuracy** as the difference between the expert's prediction and the weighted expected value for each agent $a \in a$. Each agent's belief is weighted based off their budget and then an expected value is calculated.

Mean Squared Error We define the **Mean Squared Error** as $\frac{1}{n}\sum_{t=1}^{n}\left(M_{0,1} - O_{0,1}\right)^2$. MSE is computed across n agents $a \in A$.

5.2.5 Precision

Precision is the consistency of the **market maker**. **Precision** is computed by taking a **market maker** and a set of agents A, and randomizing the order that the agents enter the market. **Precision** is the percentage of predictions across all simulations that are the same as the majority outcome. This means that if 6 of the 10 simulations have the **market** predicting $Outcome_1$ then the **Precision** is 0.6.

6 Experimental Design

The following simulations will be run by having an identical set of agents enter each **market** using each of the five **market makers**. After each simulation, the **market makers** will be scored for every metric. After all the simulations, the **market makers** will be ranked for each metric depending on the emperical results. A simulation will be run for the cross product of the following characteristics:

1. Agent Number: 3, 5, 10, 50, 100, 1000, 10000.

2. Agent Belief: Uniform $[0, 1]$, Normal w/ Mean 0.5, Beta(0.5,0.5).

3. Agent Budget: Uniform $[0, 100]$, Uniform $[0, 10]$, Normal w/ Mean 5, 10*Beta(0.5,0.5).

7 Conclusion

7.1 Future Directions