# On Market Makers Performance

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March 15, 2017

## 1 Motivation

The Logarithmic Market Scoring Rule (LMSR), while popular in the theoretical literature, faces several critical barriers to adoption. The first is that the **market maker** is bounded loss, which is a non-starter for most real world applications. The second is that the **market maker** is not sensitive to liquidity. This means that a improperly configured **market maker** might allow any agent with a positive budget to drastically swing the price or at the other extreme, not allow any agent to modify the price much at all. LMSR is well studied, however, because it has Path Independence and Translation Invariance. Many other theoretical **market makers** have been proposed that also have these two properties, but with additional benefits, and some offer tradeoffs in order to gain more desirable properties. This work seeks to quantify these tradeoffs empirically.

## 2 Goals

To test three different market maker mechanisms, including a novel design, for their liquidity sensitivity, profit expectation, and accuracy.

## 3 Model

### 3.1 Definitions

We denote **time** by $t \in \mathbb{R}_+$.

An **event** $e$ has an outcome at time $e_t \in \mathbb{R}_+$ where we restrict our attention to a binary outcome. The outcome is equal to YES in case the event occurs, and NO otherwise. We denote the YES outcome with a 1, and the NO outcome with a 0. We assume there is a way to unambiguously determine the outcome of an event.

$R$ is an **oracle** for mapping an event to an outcome. We denote the outcome of event $e$ under $R$ as $R(e) \in \{0, 1\}$.

An **option** $o$ is a security that yields a return depending on the outcome of an event $o_e$ at time $e_t$. Each option has a direction $o_d \in \{0, 1\}$ and a strike time $o_t = e_t$ when $R(o_e)$ will be evaluated. The option will convert to \$1 at time $o_t$ if $R(o_e)$ equals $o_d$, otherwise it converts to \$0. Two options are said to be **complementary options** if they trade opposite directions in the same event. Given an event $e$, we denote the complementary options as $o_c = \langle o_0, o_1 \rangle$.

Given complementary options $o_0$ and $o_1$, let $A_{o_0}$ be the **set of agents** that acquire option $o_0$, and similarly let $A_{o_1}$ be the set of agents that acquire option $o_1$. Let $A_o = A_{o_0} \cup A_{o_1}$ be the set of all agents trading complementary options.

An **agent** $a \in A_o$ has a **private belief** $a_v \in 0, 1$ and a **budget** $a_b \in \mathbb{R}_+$. The agent's private belief $a_v$ is the subjective probability that the agent assigns to the outcome of event $o_e$ being direction $o_d$ at strike time $o_t$.

An agent's $a \in A_o$ **strategy** $s_a(t) \in \mathbb{R}_+$ specifies the quantity of option $o$ purchased by the agent at time $t$.

A **prediction market** $M$ trades complementary options. Formally, a prediction market is a tuple $M = \langle o_c, A_o \rangle$ where each **agent** $a \in A_o$ purchases either some number of $o_0$ or $o_1$ options paying the price quoted by the market maker.

A **market maker** $p_M$ is a function that maps an option $o$, an agent $a \in A_o$ and a quantity $q \in \mathbb{R}_+$ at time $t \in [0, t_o)$ to a price $p(o, a, q, t) \in \mathbb{R}_+$. By definition $p(o, a, t) = R(o_e)$ if $t \geq t_o$.

### 3.2 Assumptions

In our model we assume without loss of generality that agent $a \in A_o$ has an arrival time $a_t \in \mathbb{R}_+$ where $a_t < o_t$, and execute their strategy only once at time $a_t$.

We assume that agents' private beliefs and budgets are not common knowledge but are drawn from known distributions $v \sim v(\cdot)$ and $b \sim b(\cdot)$.

We assume agents are allowed to observe the true current price of any option.

We assume that the market maker mechanism and all its parameters are common knowledge.

## 4 Equilibria

We will use two classic equilibria concepts from the literature and a third of our design.

### 4.1 RE and PI

[**Luke: Define RE and PI here**]

A major flaw in the current literature, which is reflected in LMSR and the agent behavior theories, is the generalization that agents cannot choose when to enter the market. Although fixing entry time is useful for equilibrium calculations nevertheless it is important to theorize about the value of a certain entry time $t$. Under a model based in RE where the collective signal is truthful, a rational agent would want to be the last decision maker and then have the outcome revealed with certainty.

### 4.2 No Trade Theorem

Under relaxed assumptions from RE and PI, agents wait until $x \in [0, |A|]$ agents have executed their strategies depending on their valuation of the information gained. It is trivial to show that the valuations are monotonically increasing with $x$. When $x \geq \frac{|A|}{2}$, agents can determine with certainty what the outcome will be, incur no risk, and therefore have spot $x$ is worth $E[o]$ Each spot $x \leq \frac{|A|}{2}$ is worth $\frac{x}{|A|}E[o]$. This implies that rational agents would need to be paid the difference between their riskless profits and the value of $x$ in order to select spot $x < \frac{|A|}{2}$: $E[o] - \frac{x}{|A|}$. Therefore, lacking any external payments, no agent will enter the market and no trades will occur.

### 4.3 Prior Information Timing

We present a novel theory called Prior Information Timing (PIT) that takes into account our No Trade Theorem where agents choose valuations based on a linear combination of their signal and the market price, but value deferring this assesment in order to gain more information. Risk neutral agents can price the ability to defer the decision until position $x$ based on the added information of the preceding $x-1$ agents can pay up to the value of the information to defer. In order for markets to clear in PIT then the market maker needs to compensate agents for their spot selection.

### 4.4 Agent Strategy

In order to assess accuracy in situations where agents have variable information, we will consider different types of agent strategies in our experiments. There are two types of agent strategies **myopic** $Y$ and **farsighted** $F$. Prediction markets are known to be myopically incentive compatible, which means that myopic agents bid truthfully. Similarly, **informed** $I$ agents have an exogenous signal about the outcome whereas **uninformed** $U$ agents can only base their decision on the market price.

$s_a^{YI}(t)$ is an informed agent holding exogenous signal $a_v$ who is willing to pay up to $a_b$ in order to move the market price $p_M$ as close as possible to their belief $a_v$.

$s_a^{YU}(t)$ is an uninformed agent holding no exogenous signal $a_v$ who who is willing to pay up to $a_b$ in order to move the market price $p_M$ as close as possible to 1 if at time $t$, $p_{Mt} \geq .5$ otherwise 0.

$s_a^{FI}(t)$ is an informed agent holding exogenous signal $a_v$ who is attempting to maximize the expected value by bidding based on a linear combination of their signal $a_v$ and the current market price $p_M$ accounting for how many agents $x$ have already bid.

$s_a^{FU}(t)$ is an uninformed agent holding exogenous signal $a_v$ who is attempting to maximize the expected value by bidding based on the current market price $p_M$ accounting for how many agents $x$ have already bid.

## 5 Benchmarks

We establish two benchmarks for the market making mechanisms. The first is market maker profit. This is essential because LMSR and our trade incentivizing market makers often operate at no profit or at a loss. The second is accuracy. We will evaluate accuracy according to expected price as determined by our three equilibrium concepts.

## 5.1 Profit

**Definition 5.1.** (Market Maker Revenue) Given a market maker $p_M$ and a set of participating agents $A_o$, the revenue obtained from $p_M$ is defined as

$$R(p_M, A_o) = \sum_{a \in A_o} \left[ \int_{t=0}^{t=o_t} p_M(o_0, a, t, s_a^0(t)) s_a^0(t) dt + \int_{t=0}^{t=o_t} p_M(o_1, a, t, s_a^1(t)) s_a^1(t) dt \right]$$

**Definition 5.2.** (Market Maker Cost) Given a market maker $p_M$ and a set of participating agents $A_o$, the cost to $p_M$ is defined as

$$C(p_M, A_o) = \sum_{a \in A_o} \left[ \int_{t=0}^{t=o_t} R(o_e) s_a^0(t) dt + \int_{t=0}^{t=o_t} R(o_e) s_a^1(t) dt \right]$$

**Definition 5.3.** (Market Maker Proft) Given a market maker $p_M$ and a set of participating agents $A_o$, the profit of $p_M$ is defined as

$$P(p_M, A_o) = R(p_M, A_o) - C(p_M, A_o)$$

**Definition 5.4.** (Profit-Maximizing Market Maker). Among all Market Makers $L$, given a set of participating agents $A_o$ find the one that maximizes Profit:

$$PM(L, A_o) = argmax_{a \in A_o} P(p_M, A_o)$$

## 5.2 Accuracy

[**Luke: Define RE, PI, and PIT equilbrium prices**]

# 6 Market Makers

In these experiments we will test the following three market markers.

## 6.1 Logarithmic Market Scoring Rule

LMSR is a strictly proper scoring rule developed by Robert Hanson. LMSR uses a logarithmic cost function:
$C(q_1, q_2) = b \ln(e^{\frac{q_1}{b}} + e^{\frac{q_2}{b}})$ where $q_1$ and $q_2$ represent the number of shares acquired for each of the binary events: 1 and 2. The cost charged to a trader wanting to buy $q_a$ shares on event 1 and $q_b$ shares on event 1 is: $C(q_1 + q_a, q_2 + q_b) - C(q_1, q_2)$. Traders are charged for their movement in the market prediction. $b$ is the liquidity parameter set ex ante by the market maker. It controls how much the market maker can lose and also adjusts how easily a trader can change the market price. The market maker always loses up to $b \ln(2)$. A large b means that it would cost a lot to move the market price while a small b makes large swings relatively inexpensive.

The instantaneous price of LMSR market x, which is also the market prediction for x, is quoted with: $p_x = \frac{e^{\frac{q_1}{b}}}{e^{\frac{q_1}{b}} + e^{\frac{q_2}{b}}}$.

**Advantages**

1. Path Independence - any way the market moves from one state to another state yields the same payment or cost to the traders in aggregate [Hanson 2003]
2. Translation Invariance - all prices sum to unity. (Direct mapping to a probability.)

**Disadvantages**

1. Liquidity Insensitive - the market cannot adjust to periods with low or high activity. The market maker must set the liquidity parameter based on their prior belief, but has little to no guidance on how to set it.
2. Guaranteed Loss - the market maker cannot profit and has a guaranteed bounded loss.

### 6.2 Practical Liquidity Sensitive Market Maker

The LSMM uses the underlying LMSR mechanism but invokes a novel function for setting the liquidity parameter b. The function is: $b(q) = \alpha \sum_i q_i$. $q$ represents the quantity vector for each option available from the market maker. In a binary prediction market, $q$ has two values which represent the quantity outstanding for shares of YES and NO.

**Advantages**

1. Path Independence - any way the market moves from one state to another state yields the same payment or cost to the traders in aggregate [Hanson 2003]
2. Liquidity Sensitive - the market adjusts to periods with low or high activity. The market maker decreasingly subsudizes the market as activity rises.
3. Guaranteed Profit - the market maker has unbounded profit but bounded loss at near 0.

**Disadvantages**

1. Translation Variance - all prices sum beyond unity. (No direct mapping to a probability though it does provide a tight range.)
2. Market makers are incentivized to raise their commission to 1, which not only hurts traders, but also increases the valid probability range and decreases the number of traders who are willing to trade with the market maker. See $\frac{1}{n} - \alpha(n-1)\ln(n) \leq p(q_i) \leq \frac{1}{n} + \alpha \ln(n)$.

### 6.3 Luke's New MM

**[Luke: It needs to be homogenous degree 1.]**

It needs to have an understanding of time. LSMM scales with price but does not reward traders that insert information when the market has less information to offer them in exchange. It incentivizes you to wait until the end to trade.

It needs to incentivize lowering the commission in a competition. LSMM encourages MMs to ramp up their take in a group setting, which is suboptimal for the market at large since that disincentivizes trading.

# 7 Problem Statements

## 7.1 Optimal MM

Given N traders and M market makers where M < N, which of our three market makers is the dominant strategy to implement for a profit maximizing agent. If none exists then what is the Bayes Nash Equilibrium (BNE).

## 7.2 Social Planner MM

Given N traders and M market makers where M < N, which of our five market makers would a social planner implement to maximize social welfare.

Given N traders and M market makers where M < N, which of our five market makers would a social planner implement to maximize accuracy.

# 8 Setup

## 8.1 Types

A **BasicAgent** follows from the definition of these **market makers** as myopically truthful mechanisms. Each **BasicAgent** reports its belief $v$ and budget $b$ truthfully when requested by the mechanism. Each **BasicAgent** is rational and profit maximizing.

## 8.2 Metrics

This research will use the following metrics to evaluate each of the five mechanisms.

### 8.2.1 Liquidity Sensitivity

This research uses a novel definition of the vague concept of liquidity sensitivity. The need for a unified definiton stems from the known issue that LMSR can provide ẗoo littleör ẗoo muchẅeight to each agent depending on its liquidity parameter and market depth. One extreme example us that an infinitesimally low liquidity parameter will allow an **agent** with a nonzero budget to dictate the market price regardless of how many **agents** have already participated in the market. Our novel definition attempts to quanity consistency across **market depths** without overly constraining implementations.

We define a **market maker** as **liquidity sensitive** if two **agents** that have identical transactions in the **market** will have equal impacts on the **market price** proportional to the **market depth** $D_1$ at time $t_1$ of the first purchase, the **market depth** $D_2$ at time $t_2$ of the second purchase, and the difference in time $t_1 - t_2$.

We formalize **liquidity sensitive** as the function $\Delta = f(D_1, D_2, t_\Delta)$ having a constant $\Delta$.

#### 8.2.2 Market Maker Profit

We define the **market maker profit** on prediction market $M$ at time $t$ as $\sum_{a \in A_o} [\int_t p(o_0, a, t, s_a^0(t)) s_a^0(t) dt + \int_t p(o_1, a, t, s_a^1(t)) s_a^1(t) dt - \int_t R(o_e) s_a^0(t) dt - \int_t R(o_e) s_a^1(t) dt]$

#### 8.2.3 Social Welfare

We define the profit of an agent as the sum of the realized value of its **options** minus the sum of the cost of each of its **options**.

We define **social welfare** as the **market maker profit** plus the sum of each agent's profit.

#### 8.2.4 Accuracy

Since the accuracy of a prediction market depends on the accuracy of each of its participants as well as a usually unknown ground truth value, we utilize several well studied definitions to evaluate each mechanism's accuracy.

**Regret** We define the **regret** of a mechanism as the difference between the mechanism's accuracy and the accuracy of the most accurate agent $a \in A$, who can be thought of as the best expert. The most accurate agent $a \in A$ is the agent $a$ whose belief is most frequently aligned with the outcome over n trials.

**Expectation** We define the **expectation accuracy** as the difference between the expert's prediction and the weighted expected value for each agent $a \in a$. Each agent's belief is weighted based off their budget and then an expected value is calculated.

**Mean Squared Error** We define the **Mean Squared Error** as $\frac{1}{n} \sum_{t=1}^{n} \left( M_{0,1} - O_{0,1} \right)^2$. MSE is computed across n agents $a \in A$.

#### 8.2.5 Precision

**Precision** is the consistency of the **market maker**. **Precision** is computed by taking a **market maker** and a set of agents $A$, and randomizing the order that the agents enter the market. **Precision** is the percentage of predictions across all simulations that are the same as the majority outcome. This means that if 6 of the 10 simulations have the **market** predicting $Outcome_1$ then the **Precision** is 0.6.

## 9 Experimental Design

The following simulations will be run by having an identical set of agents enter each **market** using each of the five **market makers**. After each simulation, the **market makers** will be scored for every metric. After all the simulations, the **market makers** will be ranked for each metric depending on the emperical results. A simulation will be run for the cross product of the following characteristics:

1. Agent Number: 3, 5, 10, 50, 100, 1000, 10000.
2. Agent Belief: Uniform $[0, 1]$, Normal w/ Mean 0.5, Beta(0.5,0.5).
3. Agent Budget: Uniform $[0, 100]$, Uniform $[0, 10]$, Normal w/ Mean 5, 10*Beta(0.5,0.5).

# 10 Conclusion

## 10.1 Future Directions