# Class 3: Integrated models and ARIMA

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PRESS RECORD

## Learning outcomes

- Understand how differencing works to help make data stationary
- ► Know the basics of the ARIMA(p, d, q) framework
- ▶ Understand how to fit an ARIMA(p, d, q) model in a realistic setting

## Reminder: stationarity

- ▶ A time series is said to be weakly stationary if:
  - ► The mean is stable
  - ► The variance is stable
  - ▶ The autocorrelation doesn't depend on where you are in the series

### Reminder: ARMA models

- ▶ Combine the autoregressive and the moving average framework into one
- ► The general equation for an ARMA(p, q) model is:

$$y_t = \alpha + \sum_{i=1}^{p} \beta_i y_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t$$

# Combining ARMA with the random walk to produce ARIMA

▶ There is one other time series model we have already met, that of the random walk:

$$y_t = y_{t-1} + \epsilon_t$$

where  $\epsilon_t \sim N(0, \sigma^2)$ 

► We could re-write this as:

$$y_t - y_{t-1} = \epsilon_t$$

i.e. the differences are random normally-distributed noise

## Differencing

- Differencing is a great way of getting rid of a trend
- ▶ If  $y_t \approx y_{t-1} + b$  then there will be an increasing linear slope in the time series
- ightharpoonup Creating  $y_t y_{t-1}$  will remove it and all values will hover around the value b
- Even when the trend is non-linear differencing might help
- ▶ Differencing twice will remove a quadratic trend for the same reasons
- ▶ You can do even higher levels of differencing but this starts to cause problems
- ► The twice differenced series is:

$$(y_t - y_{t-1}) - (y_{t-1} - y_{t-2}) = y_t - 2y_{t-1} + y_{t-2}$$

# Idea: combine differencing into the ARMA framework

- ► We can combine these ideas into the ARMA framework to produce an ARIMA model (the I stands for integrated, i.e. differenced)
- ► An ARIMA model isn't really stationary as the differences are actually removing part of the trend
- ► The ARIMA model is written as ARIMA(p,d,q) where p and q are as before and d is the number of differences

# Example: the ARIMA(1,1,1) model

If we want to fit an ARIMA(1,1,1) model we first let  $z_t = y_t - y_{t-1}$  then fit the model:

$$z_t \sim N(\alpha + \beta z_{t-1} + \theta \epsilon_{t-1}, \sigma^2)$$

- ► This is equivalent to an ARMA model on the first differences
- Note that by default forecast does not include the term  $\alpha$  in the model. You need to add include.drift = TRUE

## Fitting an ARIMA(1, 1, 1) model to the wheat data

- ▶ Recall that the ARMA(2,1) fit wasn't very good to the wheat data
- ▶ Instead try an ARIMA(1, 1, 0) model (i.e. AR(1) on the first differences)

```
wheat = read.csv('../../data/wheat.csv')
Arima(wheat$wheat, order = c(1, 1, 0),
    include.drift = TRUE)
```

```
## Series: wheat$wheat
## ARIMA(1.1.0) with drift
##
## Coefficients:
##
            ar1
                    drift
## -0.0728 529.4904
## s.e. 0.1503 401.5639
##
## sigma^2 estimated as 9945763: log likelihood=-491.7
## ATC=989.39 ATCc=989.89 BTC=995.25
```

# General format: the ARIMA(p,d,q) model

- $\triangleright$  First take the dth difference of the series  $y_t$ , and call this  $z_t$
- ► If you want to do this by hand in R you can use the diff function, e.g. diff(y, differences = 2)
- ► Then fit the model:

$$z_t \sim N \left( \alpha + \sum_{i=1}^p \beta_i z_{t-i} + \sum_{j=1}^q \theta_j \epsilon_{t-j}, \sigma^2 \right)$$

# Choosing p, d and q

- $\triangleright$  It's much harder to have an initial guess at all of p, d and q in one go
- We can usually guess at the number of differences d from the time series and ACF plots. If there is a very high degree of autocorrelation it's usually a good idea to try a model with d=1 or 2
- ► I've never met a model where you needed to difference more than twice. Beware of over-differencing

Revisiting the real-world example

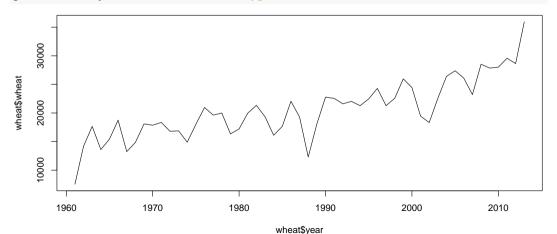
# Steps in an ARIMA time series analysis

- 1. Plot the data and the ACF/PACF
- 2. Decide if the data look stationary or not. If not, perform a suitable transformation and return to 1. If the data has a strong trend or there is a high degree of autocorrelation try 1 or 2 differences
- 3. Guess at values of p, d, and q for an ARIMA(p, d, q) model
- 4. Fit the model
- 5. Try a few models around it by increasing/decreasing p, d and q and checking the AIC (or others)
- 6. Check the residuals
- 7. Forecast into the future

### A real example: wheat data

▶ Plot reminder

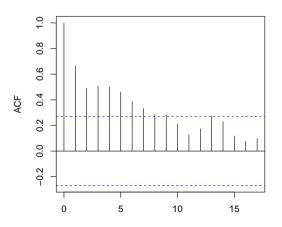
```
wheat = read.csv('../../data/wheat.csv')
plot(wheat$year, wheat$wheat, type = '1')
```



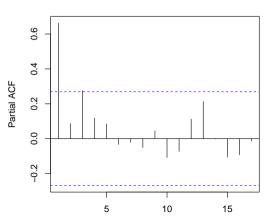
### ACF and PACF

```
par(mfrow = c(1, 2))
acf(wheat$wheat)
pacf(wheat$wheat)
```

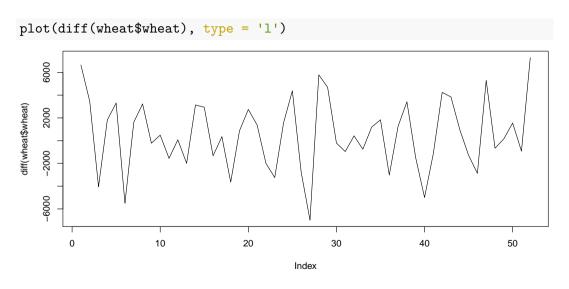
#### Series wheat\$wheat



#### Series wheat\$wheat



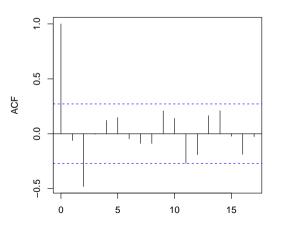
### Plot of first differences



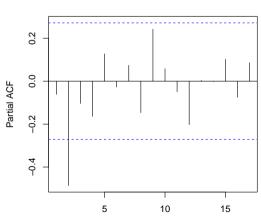
## ACF/PACF of first differences

```
par(mfrow = c(1, 2))
acf(diff(wheat$wheat))
pacf(diff(wheat$wheat))
```

### Series diff(wheat\$wheat)



#### Series diff(wheat\$wheat)



### First model

```
Arima(wheat$wheat, order = c(0, 1, 0),
      include.drift = TRUE)
## Series: wheat$wheat
## ARIMA(0,1,0) with drift
##
## Coefficients:
            drift.
##
## 546,4265
## s.e. 429.8333
##
```

► This is just a random walk model. Can also get these from forecast with the function naive

## sigma^2 estimated as 9795708: log likelihood=-491.81

## AIC=987.63 AICc=987.87 BIC=991.53

### Next models

Try ARIMA(1, 1, 1), ARIMA(1, 1, 0), ARIMA(0, 1, 1)

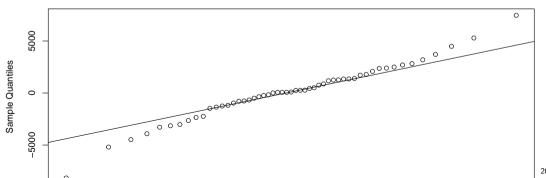
```
Arima(wheat$wheat, order = c(1, 1, 1),
      include.drift = TRUE)$aic
## [1] 979.1519
Arima(wheat$wheat, order = c(1, 1, 0),
      include.drift = TRUE)$aic
## [1] 989.3936
Arima(wheat$wheat, order = c(0, 1, 1),
      include.drift = TRUE)$aic
## [1] 981.2407
```

▶ Best one seems to be ARIMA(1, 1, 1). (though BIC suggests others)

### Check residuals

Check the residuals of this model

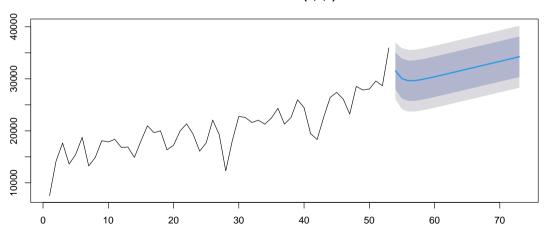
#### Normal Q-Q Plot



### Forecast into the future

plot(forecast(my\_model\_ARIMA111, h = 20))

#### Forecasts from ARIMA(1,1,1) with drift



## Why do we need to the drift term?

- Without the drift term the forecast will stabilise at or near the first few values of the series
- ► The MA part of the model is obviously flat (as previously discussed) because there are no further errors to correct
- The AR part of the model reverts back to the estimated mean of the last data point because the  $\beta$  parameter is less than 1 it dampens out the future predictions and stops them from going crazy
- ▶ The drift keeps the values going up into the future
- forecast doesn't seem to like including the drift/mean when there are multiple differences and AR terms too (not sure why)

## Summary

- ARIMA models extend the ARMA framework to further add in differencing
- ightharpoonup ARIMA models are no longer stationary as soon as d>0
- ▶ A single difference will remove a linear trends, a second difference quadratic trends
- Can spot the need for differencing from the time series plot and the ACF
- Do not over-difference your data!