Lecture Notes: Neural Networks Basics

Slide 7: Dot Product Quiz

Given

$$x = [1, 2, 3], \quad y = [2, -1, 0, 1]$$

$$x \cdot y = 1 \cdot 2 + 2 \cdot 0 + 3 \cdot 1 = 5$$

Now take

$$x = [1, 1, 2], \quad \omega = [-2, -0.5, 0], \quad b = [2].$$

$$x \cdot \omega + b = (1)(-2) + (1)(-0.5) + (2)(0) + 2 = -1.5$$

Slide 8: Matrix Revision

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3}, \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3\times 2}$$

Then

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}$$

So AB is a 2×2 matrix.
In general, if

$$A = [a_{ij}]_{n\times p}, \quad B = [b_{ij}]_{p\times r},$$

then

$$[AB]_{sr} = \sum_i a_{si}b_{it}.$$

Slide 10: Exercise

$$W = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ -1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\hat{y} = Wx + b = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 0 \end{bmatrix}$$

So there are 2 input values and 3 output values.

Slide 15: Two Hidden Layers

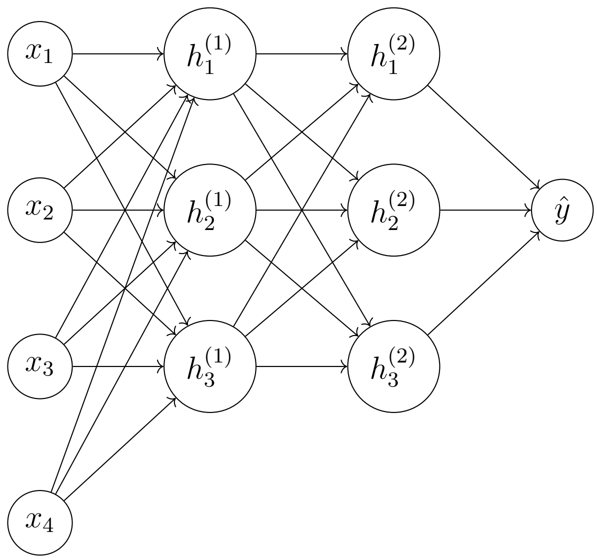
We have 2 inputs, 2 hidden layers, and 1 output.

Input:

$$x = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad W^{(1)} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 2 & 2 \end{bmatrix}, \quad b^{(1)} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}.$$

Slide 22: Weight Counting Exercise

Consider the following architecture:



There are 12 weights from inputs to the first hidden layer, 9 weights from the first to the second hidden layer, and 3 weights from the second hidden layer to the output, totalling $12 + 9 + 3 = 24$ weights.