Worksheet 8

Slide 5

Brier score $= \frac{1}{N} \sum_i (p_i - y_i)^2$

$$y_i = \begin{cases} 0 & \text{no rain} \\ 1 & \text{rain} \end{cases}$$

Days	Forecast	Obs. y_i	$(p_i - y_i)^2$
1	0.2	0	0.04
2	0.7	1	0.09
3	0.1	0	0.01
4	0.9	1	0.01
5	0.4	0	0.16

$$BS = \frac{0.31}{5} = 0.06$$

You could compare this with another forecast model to see which is better.

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$$LS(\mu, \sigma) = -\log f(y)$$
$$= +\frac{1}{2}\log(2\pi\sigma^2) + \frac{(y-\mu)^2}{2\sigma^2}$$

Suppose observation $y = 12$ and forecast $\mu = 10, \sigma = 3$

$$LS = -\frac{1}{2}\log(2\pi 3^2) - \frac{(12-10)^2}{2 \times 3^2}$$
$$= 2.24$$

Again $\rightarrow$ compare with another forecast.
We would average these over multiple forecasts.

Slide 8

$$CRPS(\mu, \sigma, y) = \sigma\Big[z(2\Phi(z) - 1) + 2\phi(z) - \frac{1}{\sqrt{\pi}}\Big]$$

where Φ is the normal cdf and ϕ is the pdf.

$$z = \frac{x - \mu}{\sigma}$$

Observation $y = 12, \mu = 10, \sigma = 3$

$$z = \frac{12 - 10}{3} = \frac{2}{3}$$

$$\Phi(z) = 0.7475, \quad \phi(z) = 0.3194$$

$$CRPS = 3\Big[\frac{2}{3}(2 \times 0.7475 - 1) + 2 \times 0.3194 - \frac{1}{\sqrt{\pi}}\Big]$$

$$= 1.214$$

from scipy.stats import norm
norm.cdf
norm.pdf

Slide 9

$$\begin{bmatrix} \mu \\ \sigma \end{bmatrix} = \begin{bmatrix} 0.32 \\ -0.242 \end{bmatrix}$$

so $\mu = 0.32$

$\sigma = \text{softplus}(-0.242)$
$= \log(1 + e^{-0.242})$
$= 0.251$

Note $\sigma = \log(1 + e^{s})$
$e^{\sigma} = e^{s} - 1$
$s = \log(e^{\sigma} - 1)$

Suppose $x = \begin{bmatrix} 0.5 & -0.2 \end{bmatrix}$

$$W = \begin{bmatrix} 0.8 & -0.4 \\ 0.3 & 0.6 \end{bmatrix}, \quad b = \begin{bmatrix} 0.1 \\ -0.05 \end{bmatrix}$$

$$z = Wx + b = \begin{bmatrix} 0.8 & -0.4 \\ 0.3 & 0.6 \end{bmatrix} \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix} + \begin{bmatrix} 0.1 \\ -0.05 \end{bmatrix} = \begin{bmatrix} 0.58 \\ -0.02 \end{bmatrix}$$

$$h = \text{ReLU} \begin{bmatrix} 0.58 \\ -0.02 \end{bmatrix} = \begin{bmatrix} 0.58 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \mu \\ \sigma \end{bmatrix} = W_y h + b = \begin{bmatrix} 0.5 & -0.2 \\ 0.1 & 0.3 \end{bmatrix} \begin{bmatrix} 0.58 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.03 \\ -0.3 \end{bmatrix} = \begin{bmatrix} 0.32 \\ -0.242 \end{bmatrix}$$