Statistical palaeoclimate reconstruction: recent results and opportunities for collaboration

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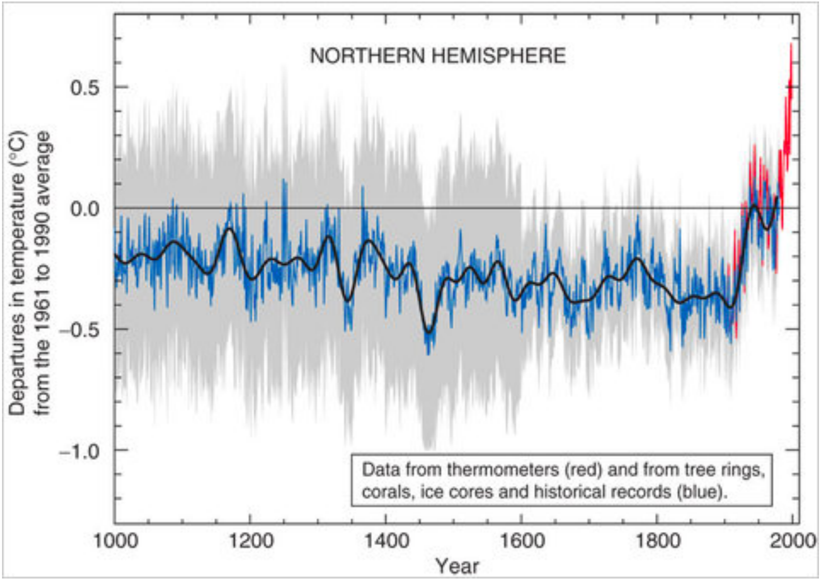
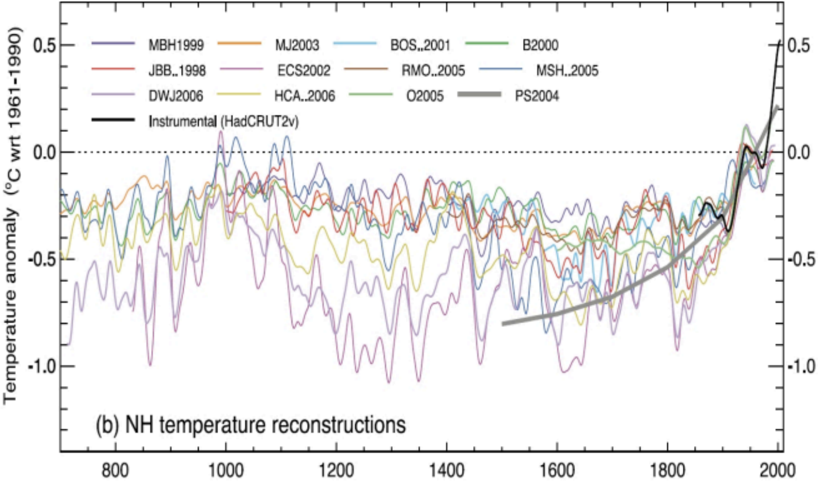
Some history...



More versions ...



How are these created?

Year	Temperature	Proxy Data (p variables)			
2014	$Temp\_record_{2014}$	$Proxy_{2014,1}$	$Proxy_{2014,2}$	$\cdots$	$Proxy_{2014,p}$
2013	$Temp\_record_{2013}$	$Proxy_{2013,1}$	$Proxy_{2013,2}$	$\cdots$	$Proxy_{2013,p}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Year n	$Temp\_record_{n}$	$Proxy_{n,1}$	$Proxy_{n,2}$	$\cdots$	$Proxy_{n,p}$
Year n-1	$Temp\_estimate_{n-1}$	$Proxy_{n-1,1}$	$Proxy_{n-1,2}$	$\cdots$	$Proxy_{n-1,p}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Year m+1	$Temp\_estimate_{m+1}$	$Proxy_{m+1,1}$	$Proxy_{m+1,2}$	$\cdots$	$Proxy_{m+1,p}$
Year m	$Temp\_estimate_{m}$	$Proxy_{m,1}$	$Proxy_{m,2}$	$\cdots$	$Proxy_{m,p}$

Table 1.1: *Climate reconstruction layout*

Some notation

Let:

- y be the ancient proxy data. Time indexed and usually multivariate
- c be ancient 'climate'. Time indexed and occasionally multivariate. Sometimes spatial too
- y^{cal} be the proxy data for the calibration period
- c^{cal} be the climate data for the calibration period

Main aim is to find $c|y, y^{\text{cal}}, c^{\text{cal}}$

The regression version

Write:

$$c^{\text{cal}} = f(y^{\text{cal}}) + \epsilon$$

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Then create:

$$\hat{c} = \hat{f}(y)$$

Problem solved!

Problems with this approach

Statistical:

- Hard to do model checking on f due to the size and nature of the calibration data
- The calibration period is autocorrelated, leading to many spurious relationships
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Biological:

- The causation is the wrong way round. **Changes in climate cause changes in proxy values**
- The uncertainty in the proxies is usually substantial and not included
- The proxies might not be sensitive to northern hemisphere temperature, or other chosen aspects of climate

A better Bayesian way

Instead write:

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Now **use Bayes**:

$$p(c|y, y^{\text{cal}}, c^{\text{cal}}) \propto p(y^{\text{cal}}|c^{\text{cal}})p(y|c)p(c)$$

We have the extra advantage that we can include a prior distribution $p(c)$ on the climate process

Bayesian palaeoclimate reconstruction in more detail

$$p(c, \theta, \phi | y, y^{\text{cal}}, c^{\text{cal}}) \propto p(y^{\text{cal}} | c^{\text{cal}}, \theta) p(y | c, \theta) p(c | \phi) p(\theta, \phi)$$

Bayesian palaeoclimate reconstruction in more detail

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- $p(\theta, \phi)$ is a prior on the parameters that control the proxy/climate relationship, and climate dynamics respectively
- $p(c|\phi)$ is a prior distribution on climate dynamics. This might be a simple statistical time series model (e.g. a random walk) all the way up to a full general circulation model
- $p(y|c, \theta)$ is the forward model again, but this time applied to the missing ancient climates
- $p(y^{\text{cal}}|c^{\text{cal}}, \theta)$ is the forward model applied to the calibration data.

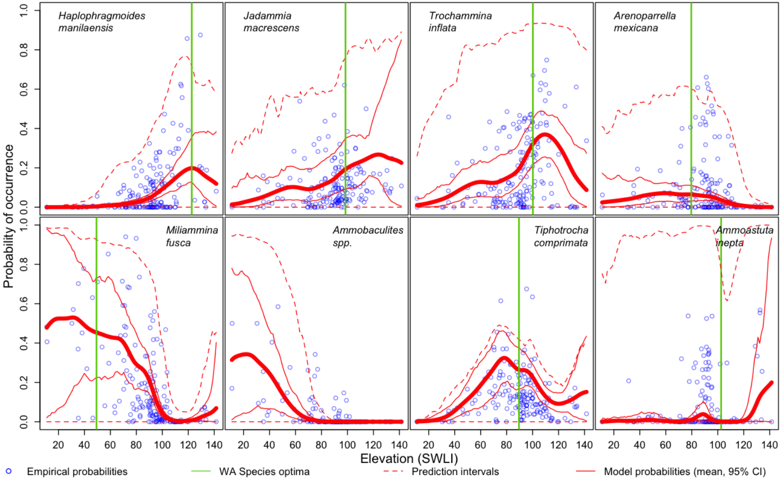
Why is this not the standard way people do this?

1. Building forward models is hard because you need a good calibration data set, some statistical modelling knowledge (especially with multivariate data), and some knowledge of the pollen/climate relationship
2. People want to avoid testing their models (out of sample evaluation etc)
3. Finding a good prior for climate dynamics is hard, especially if you have timing uncertainty
4. Bayes is still not common in climate science

Example: sea level rise in East Coast USA

- **Foramnifera** (or forams) live in the tidal range along coastal marshes
- There are lots of different species, and they all like slightly different bits of the tidal range
- If you take a sediment core on the marsh you can count lots of fossilised forams (which can also be dated) and produce a history of sea level height at that site
- We also take a number of surface samples from the local region to build up a calibration data set of which forams like which aspect of the tidal range

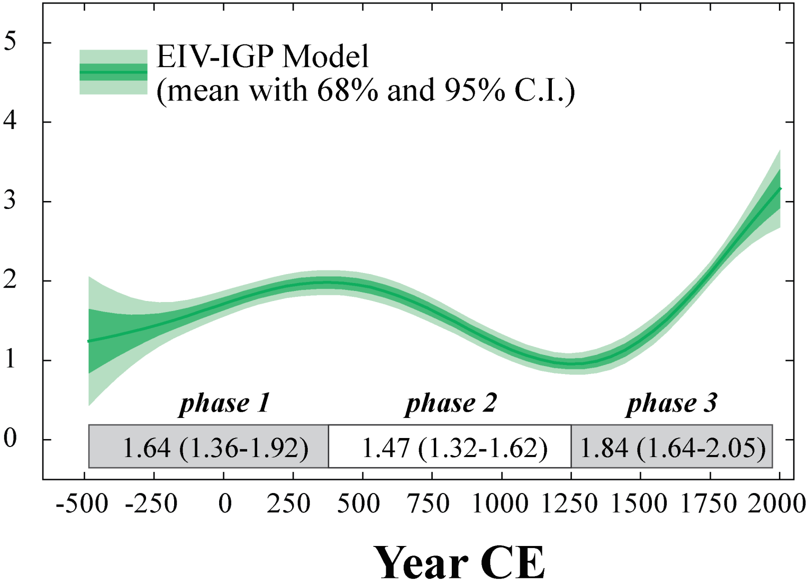
The forward model



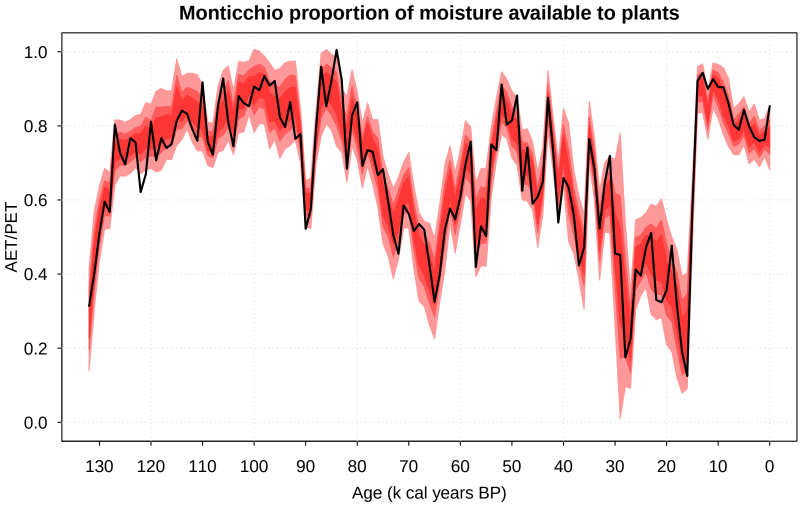
Model description

- Our forward model for the forams uses multinomial counts and splines
- We have a second proxy (called $\delta^{13}C$) that gives further information on the position in the tidal frame at that depth in the core
- Our prior on climate dynamics (here height of sea level over time) uses a fancy Gaussian process

Rate of sea level rise (mm/yr) for New Jersey, USA



Example 2: multivariate climate in Italy



Other examples:

- Parnell, A. C., Sweeney, J., Doan, T. K., Salter-Townshend, M., Allen, J. R. M., Huntley, B., & Haslett, J. (2015). Bayesian inference for palaeoclimate with time uncertainty and stochastic volatility. Journal of the Royal Statistical Society: Series C (Applied Statistics), 64(1), 115–138.
- Tolwinski-Ward, S. E., Tingley, M. P., Evans, M. N., Hughes, M. K., & Nychka, D. W. (2014). Probabilistic reconstructions of local temperature and soil moisture from tree-ring data with potentially time-varying climatic response. Climate Dynamics, 44(3-4), 791–806.
- Holmström, L., Ilvonen, L., Seppä, H., & Veski, S. (2015). A Bayesian spatiotemporal model for reconstructing climate from multiple pollen records. The Annals of Applied Statistics, 9(3), 1194–1225.

The grand challenge

Fit a Bayesian model to:

- Reconstruct spatio-temporal palaeoclimate ...

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The resulting output should be a large sample of spatio-temporal climate histories

Challenges 1: fitting state space models to large and complex data sets

What we really have is a state-space model in continuous time:

$$\begin{aligned} y(t) &= f(c(t)) + \epsilon \\ c(t) &= c(t-\Delta) + \gamma \end{aligned}$$

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What we really have is a state-space model in continuous time:

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- Fitting these models is hard when all the quantities are multivariate and f is a complex function
- Pseudo-marginal partical approaches seem to be the way to go for single-site models
- No obvious method yet for multi-site models. Perhaps SPDE-INLA?

Challenges 2: Incorporating mechanistic models

A new version

$$
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c(t) &= g(c(t_-)
\end{aligned}
$$

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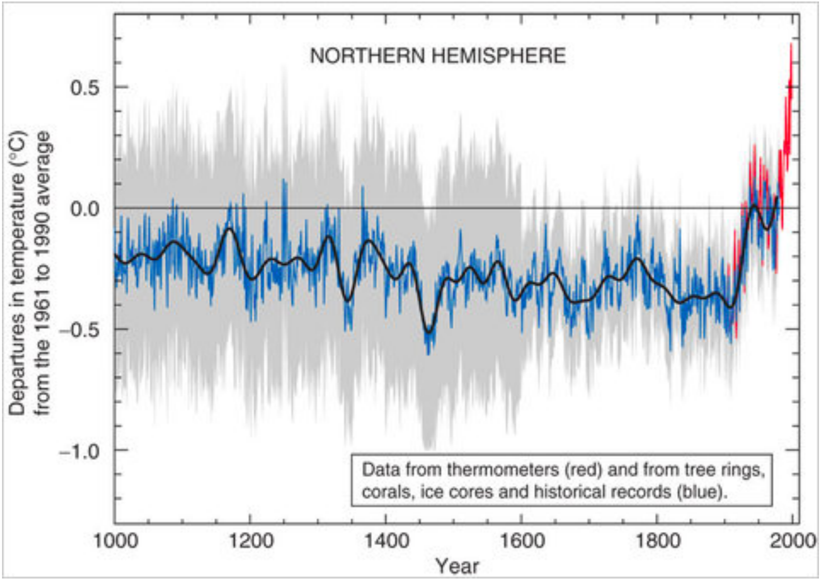
- The problem gets trickier if f and g above are deterministic models
- Some quite complex deterministic models have been suggested for pollen/climate. Not many for other proxies
- Quite a few simple climate models that might work over the palaeoclimate period:

$$
\begin{aligned}
dX_{(1)} &= -\left(X_{(1)} + X_{(2)} + vX_{(3)} + F(\gamma_P, \gamma_C, \gamma_E)\right) dt + \sigma_1 dW_{(1)} \\
dX_{(2)} &= \left(rX_{(2)} - pX_{(3)} - sX_{(2)}^2 - X_{(2)}^3\right) dt + \sigma_2 dW_{(2)} \\
dX_{(3)} &= -q\left(X_{(1)} + X_{(3)}\right) dt + \sigma_3 dW_{(3)}
\end{aligned}
$$

Back to the start: can we do better than this?



Summary

- A Bayesian version model with good forward models which produces climate histories seems like the best way to go for this work
- We need help with Bayesian computation for large multivariate non-linear non-Gaussian state space models
- We need help with combining deterministic/stochastic elements in forward models and climate models
- We can do better than the Hockey Stick!