

The statistical model behind SIAR

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Learning outcomes:

- ▶ Understand the statistical model behind SIAR
- ▶ Know how to run a model in SIAR and check that it works
- ▶ Be able to follow the technical details of the 2010 SIAR Plos ONE paper

Our simple SIMM

- ▶ In the last class we had a simple SIMM defined via:

$$y_i \sim N\left(\sum_{k=1}^2 p_k s_k, \sigma^2\right)$$

with $s_k \sim N(\mu_{s_k}, \sigma_{s_k}^2)$, $p_1 \sim U(0, 1)$ and $\sigma \sim U(0, 100)$

- ▶ Here y_i is the isotope value, s are the source values, p are the dietary proportions, and σ is the residual standard deviation
- ▶ The goal is to estimate the p and its uncertainty. The other parameters can be considered nuisance parameters

Expanding the simple SIMM

- ▶ This SIMM is currently too simplistic. We need to expand it by:
 - ▶ increasing the number of food sources
 - ▶ including trophic enrichment factors (TEFs)
 - ▶ including concentration dependence
 - ▶ allowing for multiple isotopes
 - ▶ allowing for richer source sampling by consumers
- ▶ If we include all of these factors we end up with the SIAR model
- ▶ We will take them in turn and add them into our JAGS code

Reminder: the SIAR geese data

```
data(geese1demo,sourcesdemo)
head(geese1demo,3)
```

```
##          d15NP1 d13CP1
## [1,]    10.22 -11.36
## [2,]    10.37 -11.88
## [3,]    10.44 -10.60
```

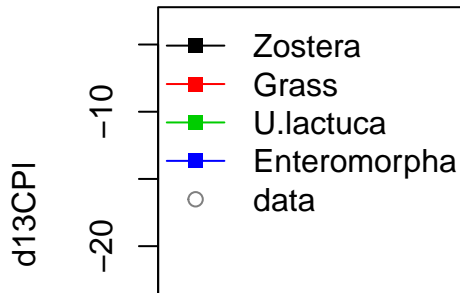
```
sourcesdemo
```

```
##          Sources  Meand15N    SDd15N  Meand13C    SDd13C
## 1      Zostera    6.488984  1.4594632 -11.17023  1.2149562
## 2        Grass    4.432160  2.2680709 -30.87984  0.6413182
## 3   U.lactuca    11.192613  1.1124385 -11.17090  1.9593306
## 4 Enteromorpha    9.816280  0.8271039 -14.05701  1.1724677
```

Plotting the data

A plot in isotope space:

```
data(correctionsdemo,concdepdemo)
out = siarmcmcdirichletv4(geese1demo,sourcesdemo,correction
out$TITLE = 'Geese data'
siarplotdata(out)
```



Including multiple sources

- ▶ Adding in multiple sources to the likelihood is straightforward:

$$y_i \sim N \left(\sum_{k=1}^K p_k s_k, \sigma^2 \right)$$

- ▶ In the above we have K sources and hence K dietary proportions
- ▶ We also now need K source prior distributions
- ▶ The tricky part about adding in multiple proportions is the prior distribution

Priors for constrained dietary proportions

- ▶ We must have $\sum_{k=1}^K p_k = 1$ so any prior distribution we place on the p s must satisfy this restriction
- ▶ (You will often hear values restricted in sum referred to as a *simplex*)
- ▶ Luckily there is a distribution known as the *Dirichlet* which is suitable for restricted sum parameters
- ▶ The Dirichlet has one parameter for each proportion $\alpha_1, \dots, \alpha_K$. The larger the α value the larger prior weight that dietary proportion will be given
- ▶ Setting all the α values to 1 is equivalent to the simplex uniform distribution, i.e. a prior assumption that all sources are consumed equally

JAGS SIMM with a Dirichlet prior

```
modelstring = '
model {
  for(i in 1:N) { y[i] ~ dnorm(inprod(p,s),sigma^-2) }
  p ~ ddirch(alpha)
  for(k in 1:K) { s[k] ~ dnorm(s_mean[k],s_sd[k]^-2) }
  sigma ~ dunif(0,100)
}
'

sources = sourcesdemo[,4:5]
data=list(y=consumers,s_mean=sources[,1],s_sd=sources[,2],
          N=length(consumers),K=nrow(sources),
          alpha=rep(1,nrow(sources)))
model=jags.model(textConnection(modelstring), data=data)
output=coda.samples(model=model,variable.names=c("p"),n.iter=
```

This is now running with all 4 sources

Results

- ▶ We can explore/plot results with `summary(output)`, `plot(output)`, and also run multiple chains, form predictive distributions, check convergence, etc
- ▶ One important thing to note is that the fitting method (MCMC) produces a joint posterior distribution of the dietary proportions. This means that each set of samples will sum to 1:

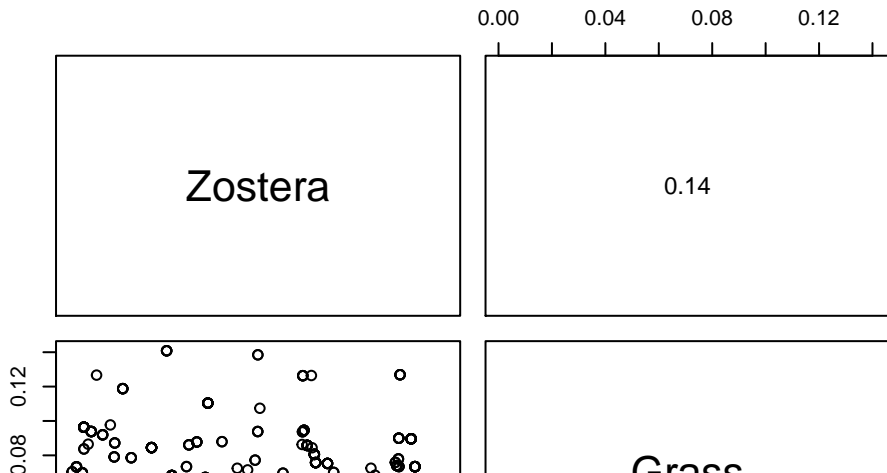
```
head(as.matrix(output[[1]]),2)
```

```
##           p[1]           p[2]           p[3]           p[4]
## [1,] 0.2153654 0.008112022 0.4422421 0.3342805
## [2,] 0.2153654 0.008112022 0.4422421 0.3342805
```

- ▶ The key implication of this is that, aside from exploring the *marginal* posterior distributions (with means, sds, etc) we can explore the *joint* uncertainty of the dietary proportions

A joint plot of the posterior dietary proportions

```
out_2 = as.matrix(output[[1]])  
colnames(out_2) = sourcesdemo[,1]  
pairs(out_2, lower.panel = panel.smooth, upper.panel = panel.cor)
```



Trophic enrichment factors and concentration dependence

- ▶ Trophic enrichment factors (c) and concentration dependence (q) represent adjustments to the source values to account for various measurement effects
- ▶ We can include them by expanding the likelihood:

$$y_i \sim N \left(\frac{\sum_{k=1}^K p_k q_k (s_k + c_k)}{\sum_{k=1}^K p_k q_k}, \sigma^2 \right)$$

- ▶ The extra part on the denominator is needed so that the dietary proportions still sum to 1
- ▶ The prior for c_k comes from external data and are given normal distributions like the source values
- ▶ In SIAR the concentration dependencies must be less than 1 (given as proportions) and are treated as fixed. You could use a strong Dirichlet prior on these instead

Including TEFs and CD - JAGS model

```
modelstring = '  
model {  
  for(i in 1:N) {  
    y[i] ~ dnorm(inprod(p*q,s+c)/inprod(p,q),sigma^-2)  
  }  
  p ~ ddirch(alpha)  
  for(k in 1:K) {  
    s[k] ~ dnorm(s_mean[k],s_sd[k]^-2)  
    c[k] ~ dnorm(c_mean[k],c_sd[k]^-2)  
  }  
  sigma ~ dunif(0,100)  
}  
'  
  
data(concdepdemo)  
data(correctionsdemo)  
data=list(y=consumers,s_mean=sources[,1],s_sd=sources[,2],  
          c_mean=correctionsdemo[,4],c_sd=correctionsdemo[,5],  
          q=concdepdemo[,4],N=length(consumers),K=nrow(sources))
```

Notes on the TEF and CD model

- ▶ If you run this, you'll find that convergence isn't quite as neat and it starts to get a bit slower
- ▶ Although it's a nuisance parameter, saving `sigma` is often a good idea because a large value indicates a poorly fitting model (usually also seen in the iso-space plot)
- ▶ The model will also create posterior distributions for s and c , though these are usually pretty similar to the prior, as there isn't much information about their values in the data

Adding extra isotopes

- ▶ If we have extra isotopes we can just list the likelihood twice, once for each value of the isotope. Only the dietary proportions are 'shared' between the isotopes
- ▶ Now write y_{ij} as the consumer values for observation i on isotope j , where $j = 1, \dots, J$
- ▶ We now have source values s_{jk} , TEF values c_{jk} , concentration dependencies q_{jk} , and each isotope has its own residual standard deviation σ_j
- ▶ The likelihood is now:

$$y_{ij} \sim N \left(\frac{\sum_{k=1}^K p_k q_{jk} (s_{jk} + c_{jk})}{\sum_{k=1}^K p_k q_{jk}}, \sigma_j^2 \right)$$

Richer source sampling

- ▶ The model we've been fitting up to now assumes that all individuals sample the same source value s_k for each source and isotope. This is unrealistic
- ▶ A better model has each individual sampling a different source value from the source prior distribution, i.e. we now have s_{ik} (or s_{ikj} with multiple isotopes)
- ▶ The JAGS code becomes:

```
for(k in 1:K) {  
  for(i in 1:N) {  
    s[i,k] ~ dnorm(s_mean[k], s_sd[k]^-2)  
  }  
}
```

- ▶ We can do the same with the concentration dependence values
- ▶ In fact with a bit of clever maths we can remove (*marginalise over*) the s_{ik} values to get a simpler model with fewer parameters.

The full SIAR model

- ▶ Using the trick mentioned on the last slide, we end up with a full model which looks like this:

$$y_{ij} \sim N \left(\frac{\sum_{k=1}^K p_k q_{jk} (\mu_{s,jk} + \mu_{c,jk})}{\sum_{k=1}^K p_k q_{jk}}, \frac{\sum_{k=1}^K p_k^2 q_{jk}^2 (\sigma_{s,jk}^2 + \sigma_{c,jk}^2)}{(\sum_{k=1}^K p_k q_{jk})^2} + \sigma_j^2 \right)$$

- ▶ This model has a more complicated likelihood, but removes the extra s and c parameters

Full SIAR model: JAGS code

```
modelstring = '  
model {  
  for (i in 1:N) {  
    for (j in 1:J) {  
      y[i,j] ~ dnorm(inprod(p*q[,j], s_mean[,j]+c_mean[,j])  
        inprod(p,q[,j]), var_y[j]^-1)  
    }  
  }  
  p ~ ddirch(alpha)  
  for(j in 1:J) {  
    var_y[j] <- inprod(pow(p*q[,j],2),s_sd[,j]^2+c_sd[,j]^2  
      + pow(sigma[j],2)  
  }  
  for(j in 1:J) { sigma[j] ~ dunif(0,100) }  
}
```

Full SIAR model: R code

```
sources = as.matrix(sourcesdemo[,2:5])
tefs = as.matrix(correctionsdemo[,2:5])
cd = as.matrix(concdepdemo[,c(2,4)])
data=list(y=geese1demo,s_mean=sources[,c(1,3)],s_sd=sources[,c(1,3)],
          c_mean=tefs[,c(1,3)],c_sd=tefs[,c(2,4)],
          q=cd,N=nrow(geese1demo),
          J=ncol(geese1demo),alpha=rep(1,nrow(sources)))
model=jags.model(textConnection(modelstring), data=data)
output=coda.samples(model=model,variable.names=c("p",'sigma'))
```

Summary of posterior dietary proportions

```
out_2 = as.matrix(output[[1]])  
colnames(out_2) = c(as.character(sourcesdemo[,1]), 'SD1', 'SD2')  
t(round(apply(out_2, 2, quantile, probs=c(0.025, 0.5, 0.975)), 2))
```

##	2.5%	50%	97.5%
## Zostera	0.40	0.60	0.80
## Grass	0.03	0.07	0.12
## U.lactuca	0.00	0.11	0.33
## Enteromorpha	0.01	0.19	0.47
## SD1	0.02	0.38	1.51
## SD2	0.06	0.89	2.44

Some of these proportions are quite imprecise: perhaps see better with matrix plot?

Running SIAR

- ▶ The SIAR R package runs exactly this model with a few extra tweaks
- ▶ It contains a slightly optimised algorithm as JAGS sometimes gets a bit stuck on harder data sets. It's also much faster than JAGS for complicated problems
- ▶ It allows for direct plotting of the data in isotope space and p -space (i.e. dietary proportion space - pairs plots)
- ▶ It allows for changing the α values to put in proper prior information
- ▶ It includes convergence checking
- ▶ Most of this covered in the practical this afternoon

Running SIAR 2

- ▶ Running SIAR as simple as giving it the relevant parts:

```
out = siarmcmcdirichletv4(geese1demo,sourcesdemo,correction
```

```
out_2 = out$output  
colnames(out_2) = c(as.character(sourcesdemo[,1]), 'SD1', 'SD2')  
t(round(apply(out_2,2,quantile,probs=c(0.025,0.5,0.975))),2)
```

##	2.5%	50%	97.5%
## Zostera	0.40	0.56	0.77
## Grass	0.03	0.07	0.12
## U.lactuca	0.01	0.12	0.35
## Enteromorpha	0.01	0.22	0.49
## SD1	0.02	0.37	1.48
## SD2	0.07	0.90	2.42

- ▶ Most people get stuck using SIAR just getting their data into the right format

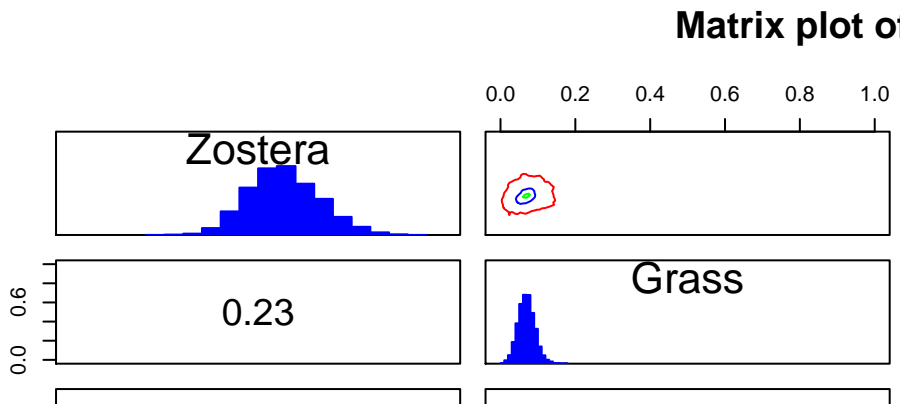
Matrix plot of dietary proportions

```
siarmatrixplot(out)
```

```
## Matrix plot of groups proportions.
```

```
## Producing plot.....
```

```
##
```



Summary

- ▶ The SIAR model is just a complicated regression-type model
- ▶ The response is multivariate and the prior distributions on some of the parameters have to be constrained to sum to 1
- ▶ It used to be the case that JAGS was slow and couldn't run SIMM-type models. This is no longer true. You can fit much richer models in JAGS (and now MixSIAR) than with SIAR
- ▶ More details on running SIAR in the practical this afternoon