Learning efficient logic programs

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Input	Output
[s,h,e,e,p]	e
[a,l,p,a,c,a]	a
[c,h,i,c,k,e,n]	?

Input	Output
`[s,h,e,e,p]`	`e`
`[a,l,p,a,c,a]`	`a`
`[c,h,i,c,k,e,n]`	**c**

```
%% metagol
f(A,B):-head(A,B),tail(A,C),element(C,B).
f(A,B):-tail(A,C),f(C,B).
```

```
%% alternative
f(A,B):-mergesort(A,C),f1(C,B).
f1(A,B):-head(A,B),tail(A,C),head(C,B).
f1(A,B):-tail(A,C),f1(C,B).
```

Idea

1. Given examples **E**, learn any program **H**
2. Repeat whilst possible:
    A. Learn program **H’** such that cost(**H’,E**) < cost(**H,E**)
    B. Set **H**=**H’**
3. Return **H**

Metagol

```
prove([],P,P).

prove([Atom|Atoms],P1,P2):-
    prove_aux(Atom,P1,P3),
    prove(Atoms,P3,P2).

prove_aux(Atom,P,P):-
    call(Atom).

prove_aux(Atom,P1,P2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    prove(Body,P3,P2).
```

Metagol

```
prove([],P,P).

prove([Atom|Atoms],P1,P2):-
    prove_aux(Atom,P1,P3),
    prove(Atoms,P3,P2).

prove_aux(Atom,P,P):-
    call(Atom).

prove_aux(Atom,P1,P2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    prove(Body,P3,P2).
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Metagol

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    prove_aux(Atom,P1,P3),
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prove_aux(Atom,P,P):-
    call(Atom).

prove_aux(Atom,P1,P2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    prove(Body,P3,P2).
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Metagol

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prove([],P,P).

prove([Atom|Atoms],P1,P2):-
    prove_aux(Atom,P1,P3),
    prove(Atoms,P3,P2).

prove_aux(Atom,P,P):-
    call(Atom).

prove_aux(Atom,P1,P2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    prove(Body,P3,P2).
```

Metaopt

```
prove([],P,P,C,C).

prove([Atom|Atoms],P1,P2,C1,C2):-
    prove_aux(Atom,P1,P3,C1,C3),
    prove(Atoms,P3,P2,C3,C2).

prove_aux(Atom,P,P,C1,C2):-
    pos_cost(Atom,Cost).
    C2 is C1+Cost,
    max_cost(MaxCost),
    C2 < MaxCost.

prove_aux(Atom,P1,P2,C1,C2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    C3 is C1+1,
    prove(Body,P3,P2,C3,C2).
```

Metaopt

```
prove([],P,P,C,C).

prove([Atom|Atoms],P1,P2,C1,C2):-
    prove_aux(Atom,P1,P3,C1,C3),
    prove(Atoms,P3,P2,C3,C2).

prove_aux(Atom,P,P,C1,C2):-
    pos_cost(Atom,Cost).
    C2 is C1+Cost,
    max_cost(MaxCost),
    C2 < MaxCost.

prove_aux(Atom,P1,P2,C1,C2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    C3 is C1+1,
    prove(Body,P3,P2,C3,C2).
```

Metaopt

```
prove([],P,P,C,C).

prove([Atom|Atoms],P1,P2,C1,C2):-
    prove_aux(Atom,P1,P3,C1,C3),
    prove(Atoms,P3,P2,C3,C2).

prove_aux(Atom,P,P,C1,C2):-
    pos_cost(Atom,Cost).
    C2 is C1+Cost,
    max_cost(MaxCost),
    C2 < MaxCost.

prove_aux(Atom,P1,P2,C1,C2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    C3 is C1+1,
    prove(Body,P3,P2,C3,C2).
```

Iterative descent

1. Given examples **E**, learn program **H** with minimal textual complexity
2. Repeat whilst possible:
   A. Learn program **H’** such that cost(**H’,E**) < cost(**H,E**)
   B. Set **H**=**H’**
3. Return **H**

Metaopt prunes as it learns

Tree cost

Positive examples: size of the leftmost successful branch

Tree cost

Positive examples: size of the leftmost successful branch

```
pos_cost(Atom,Cost):-
    statistics(inferences,I1),
    call(Atom),
    statistics(inferences,I2),
    Cost is I2-I1.
```

Tree cost

Negative examples: size of the finitely-failed SLD-tree

Tree cost

Negative examples: size of the finitely-failed SLD-tree

```
neg_cost(Atom,Cost):-
    statistics(inferences,I1),
    \+ call(Atom),
    statistics(inferences,I2),
    Cost is I2-I1.
```

Tree cost

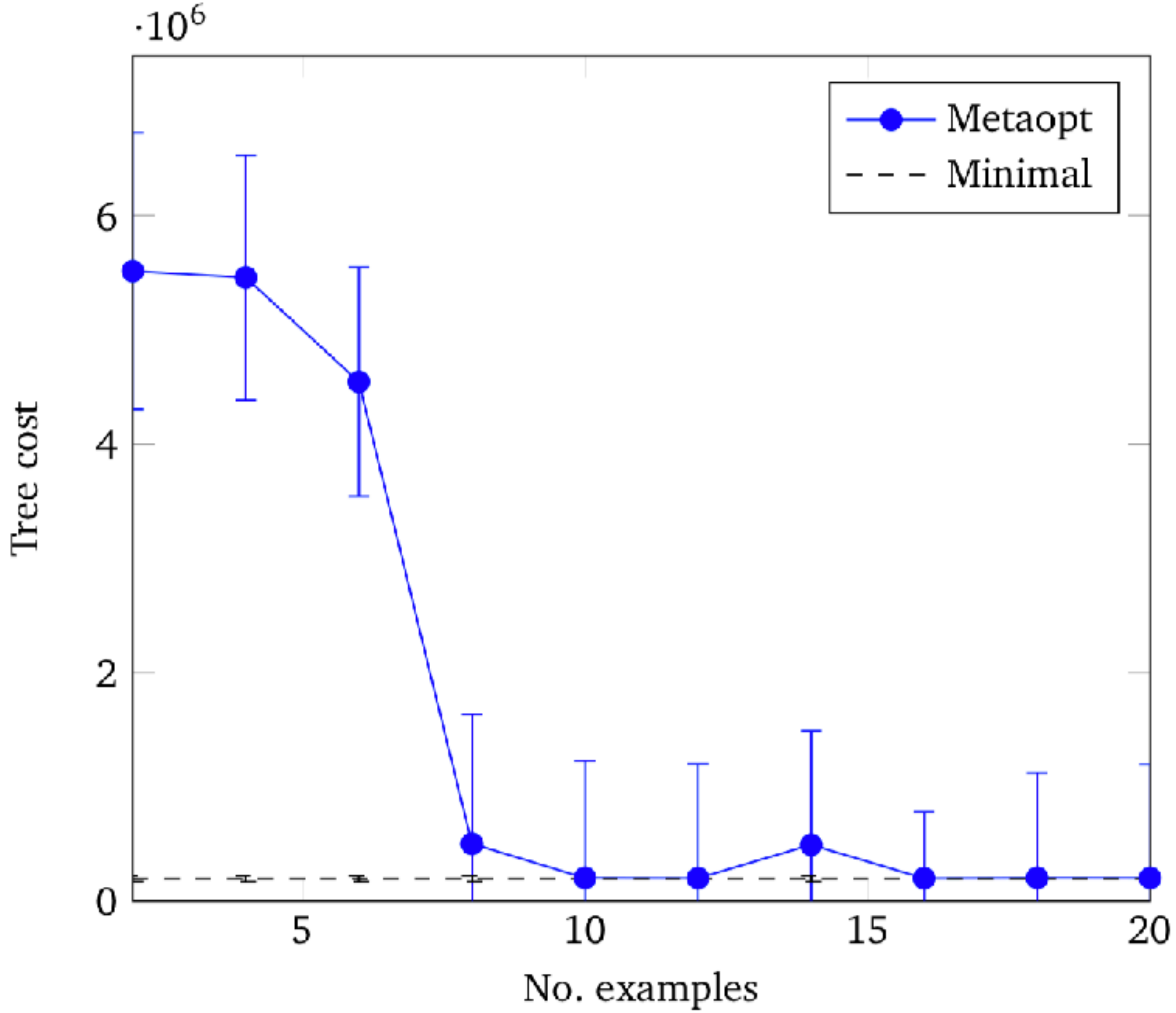
- any arity logics
- no user-supplied costs
- backtracking and non-determinism

Input	Output
`[s,h,e,e,p]`	`e`
`[a,l,p,a,c,a]`	`a`
`[c,h,i,c,k,e,n]`	`c`

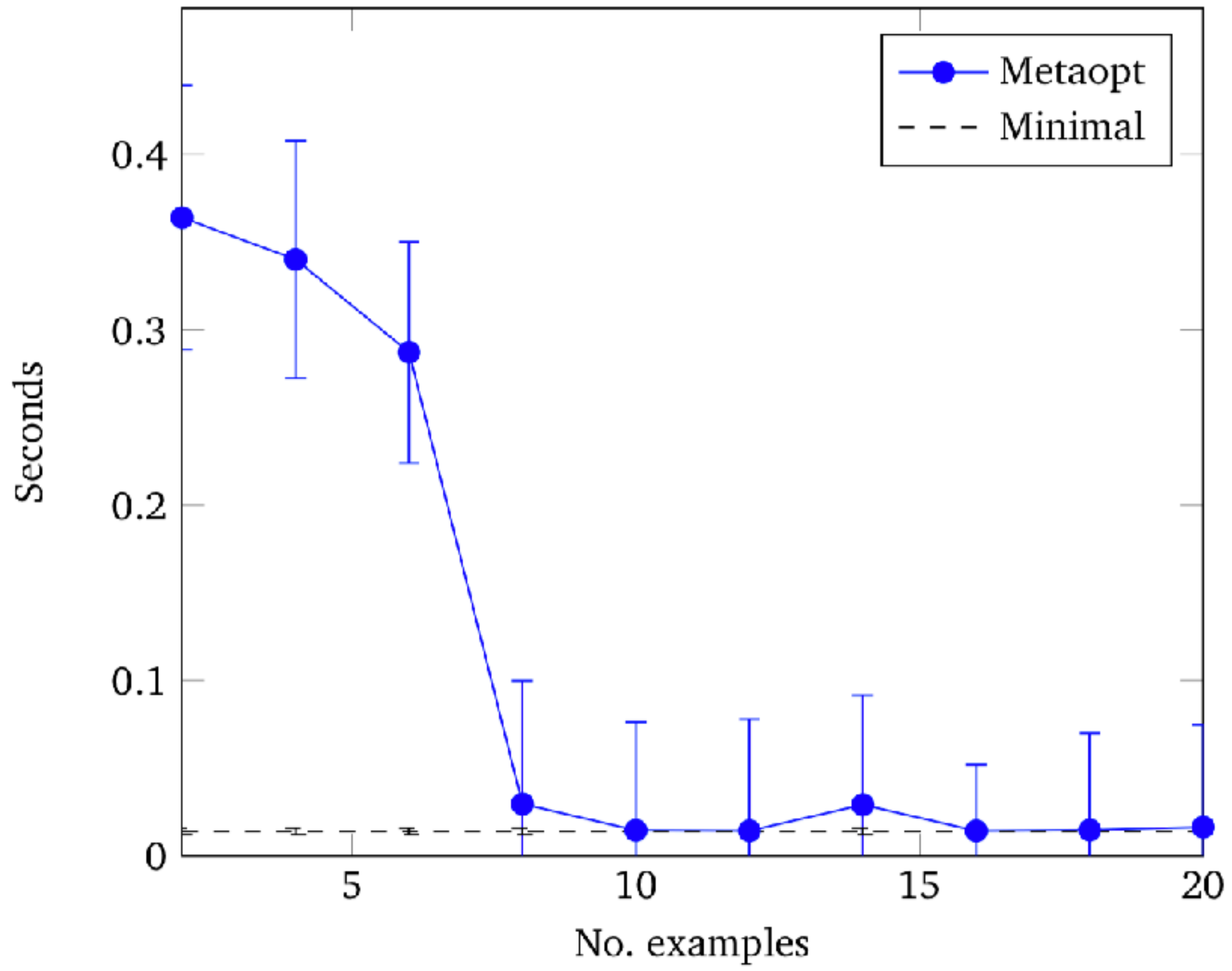
Input	Output
[s,h,e,e,p]	e
[a,l,p,a,c,a]	a
[c,h,i,c,k,e,n]	c

```
f(A,B):-mergesort(A,C),f1(C,B).
f1(A,B):-head(A,B),tail(A,C),head(C,B).
f1(A,B):-tail(A,C),f1(C,B).
```

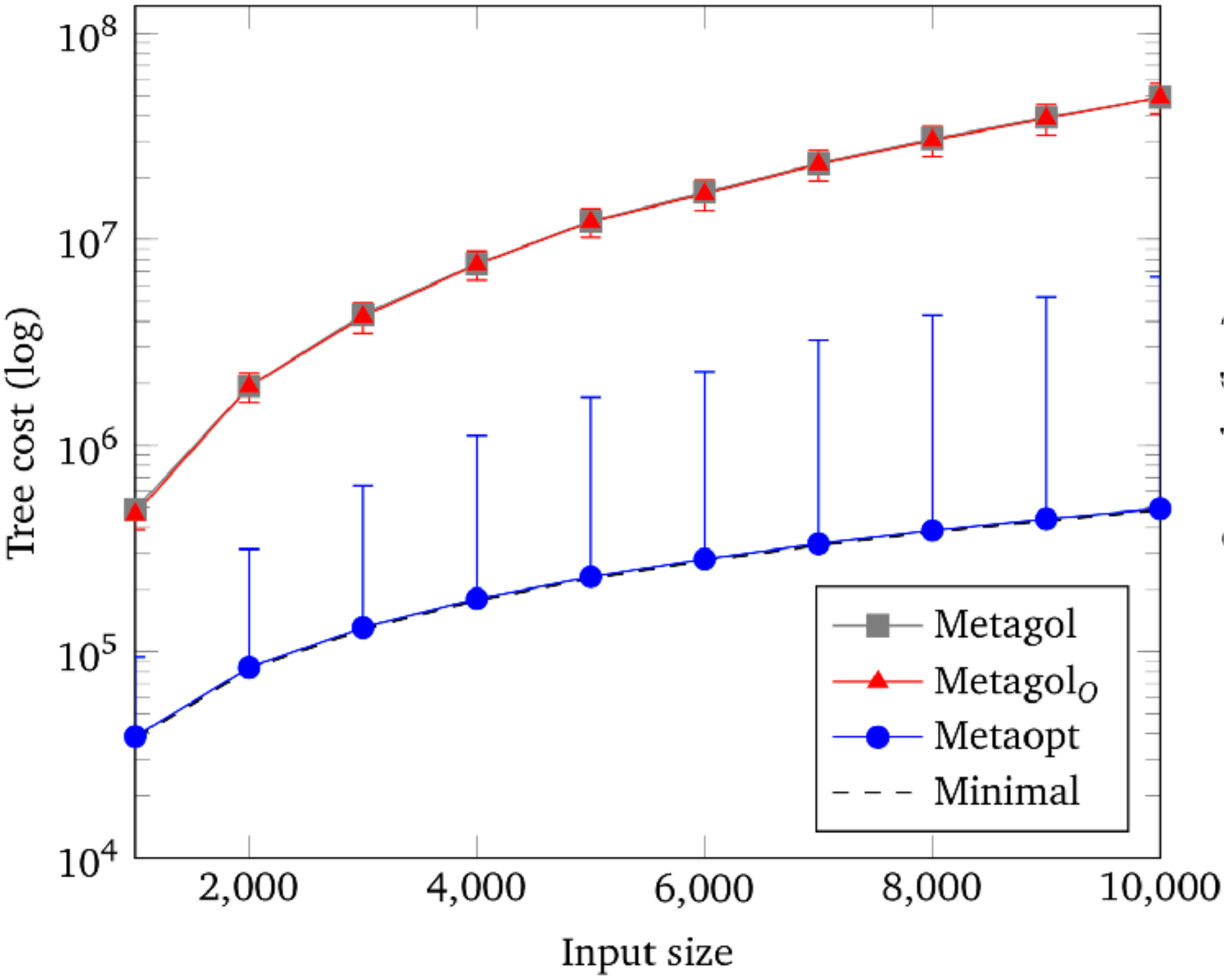
Convergence



Convergence

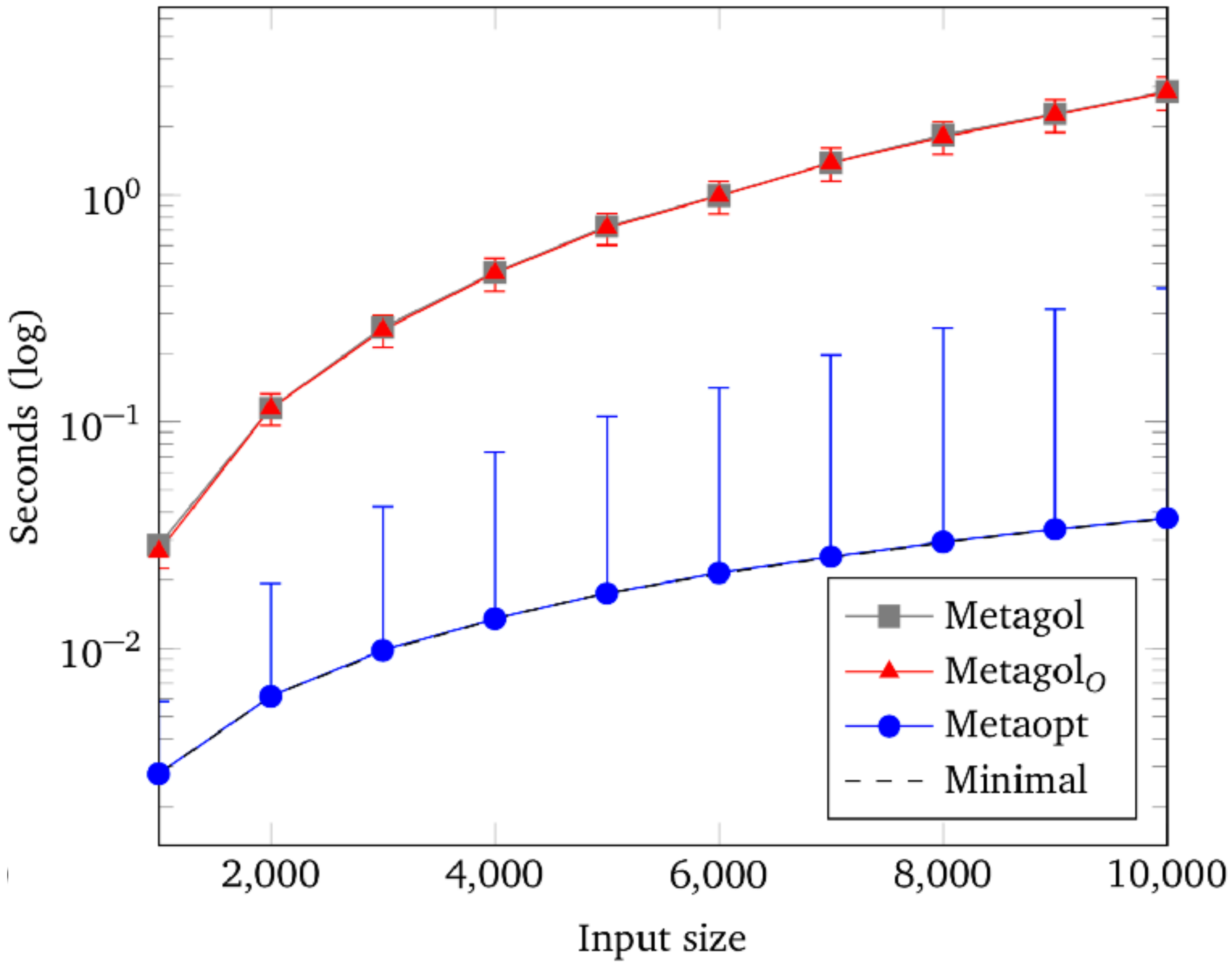


Performance



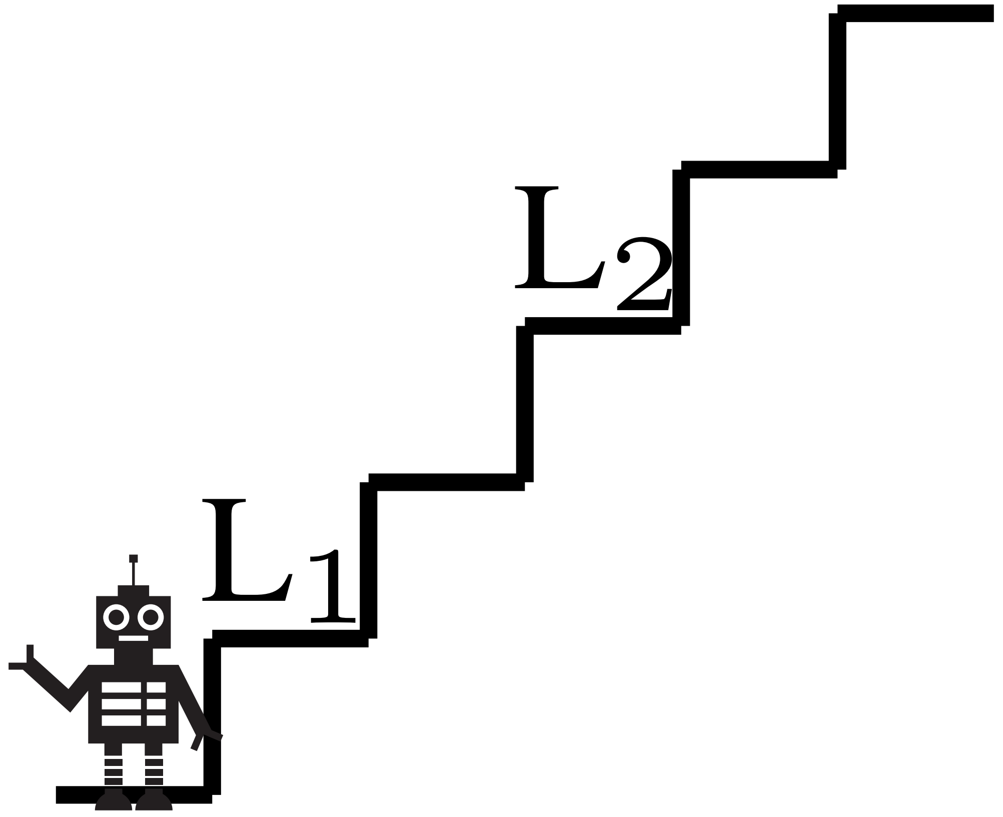
(a) Tree costs

Performance

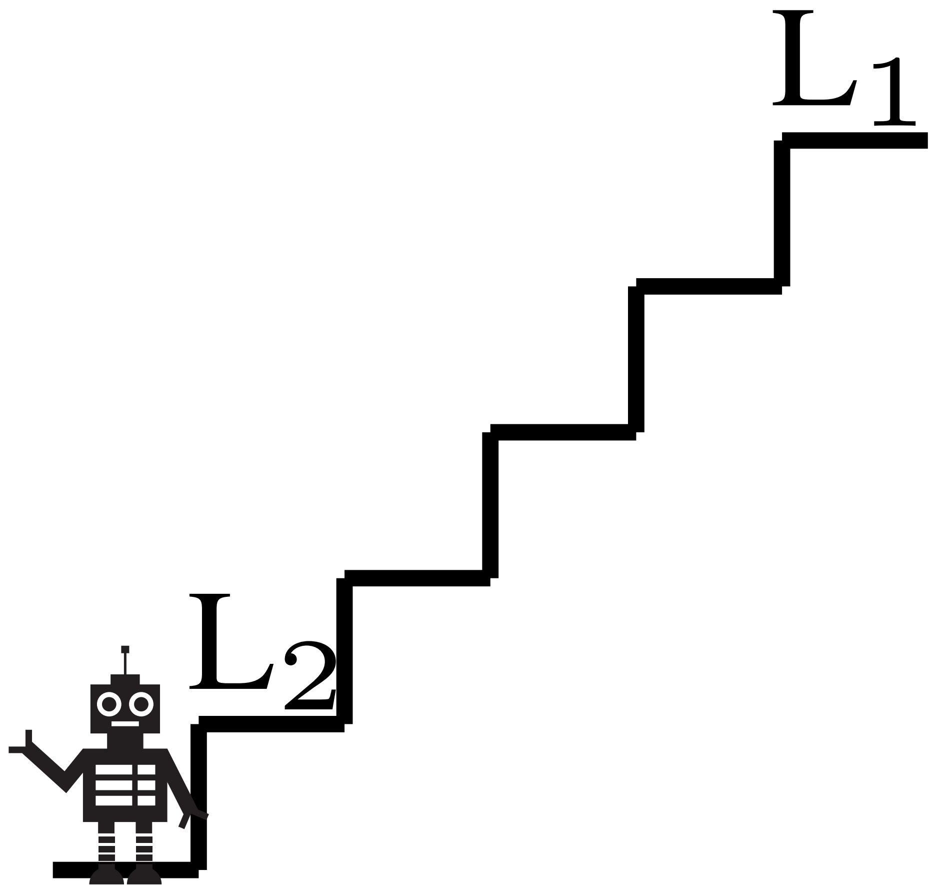


(b) Program runtimes

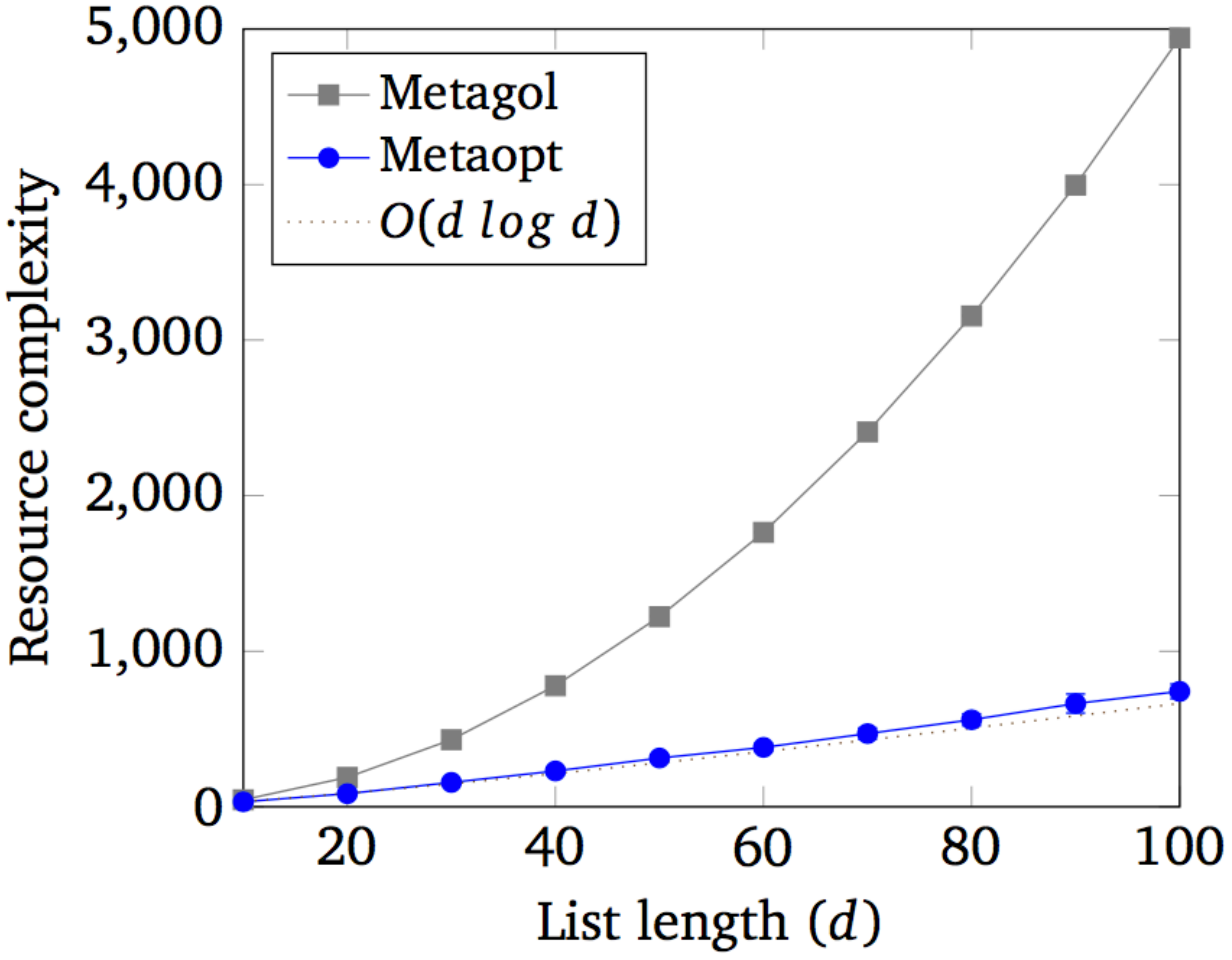
Resource complexity



Initial state

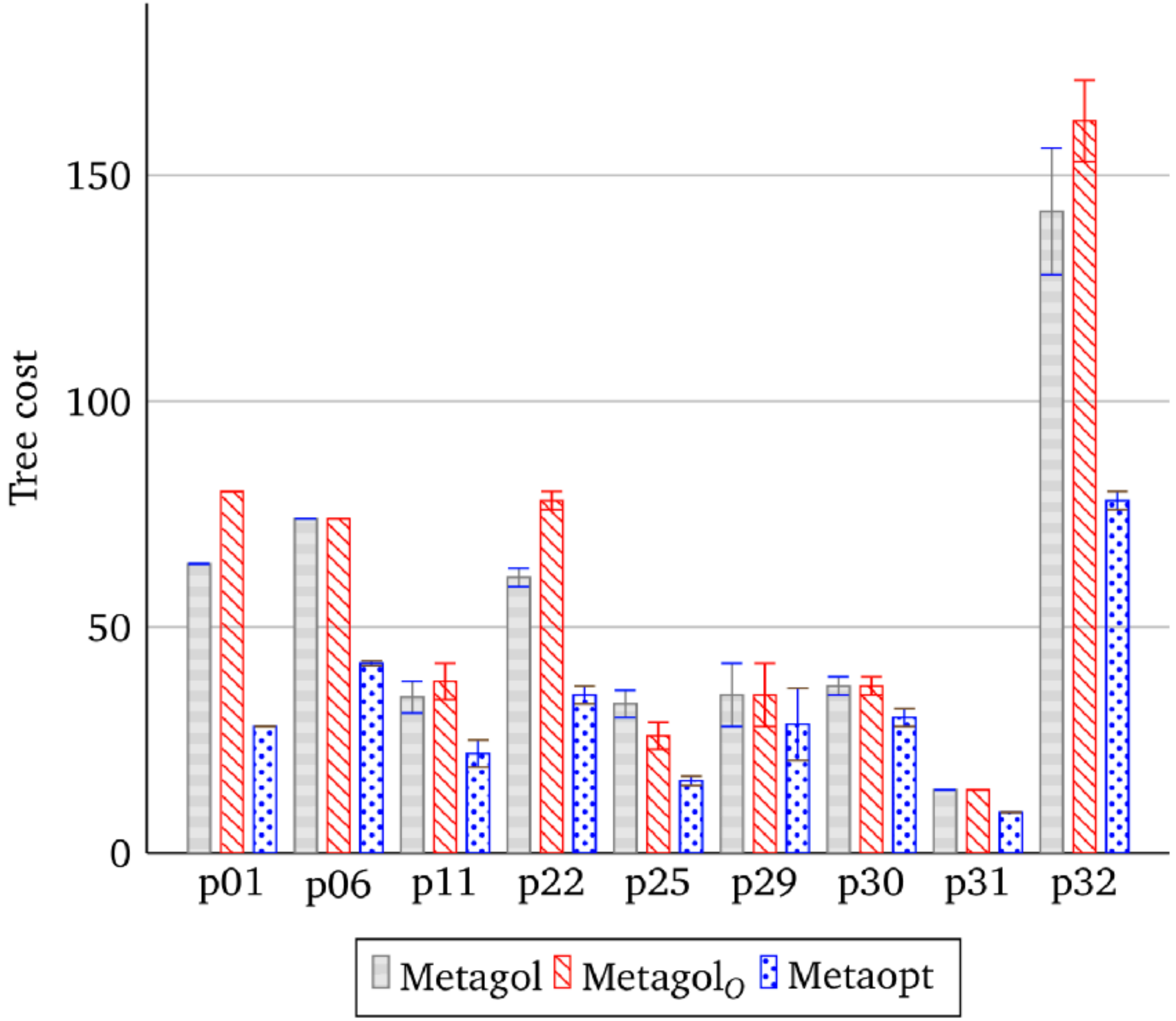


Final state



5,000
4,000
3,000
2,000
1,000
0
Resource complexity
20
40
60
80
100
List length (d)
Metagol
Metaopt
$O(d\ log\ d)$

Input	**Output**
My name is John.	John
My name is Bill.	Bill
My name is Josh.	Josh
My name is Albert.	Albert
My name is Richard.	Richard


Tree cost
150
100
50
0
p01
p06
p11
p22
p25
p29
p30
p31
p32
Metagol
Metagol$_O$
Metaopt

```
%% metagol
f(A,B):-tail(A,C),f1(C,B).
f1(A,B):-dropLast(A,C),f2(C,B).
f2(A,B):-dropWhile(A,B,not_uppercase).
```

```
%% metagol unfolded
f(A,B):-
    tail(A,C),
    dropLast(C,D),
    dropWhile(D,B,not_uppercase).
```

```
% metagol0
f(A,B):-f1(A,C),f4(C,B).
f1(A,B):-f2(A,C),f3(C,B).
f2(A,B):-filter(A,B,is_letter).
f3(A,B):-dropWhile(A,B,is_uppercase).
f4(A,B):-dropWhile(A,B,not_uppercase).
```

```
% metagol0 unfolded
f(A,B):-
    filter(A,C,is_letter).
    dropWhile(C,D,is_uppercase),
    dropWhile(D,B,not_uppercase).
```

```prolog
% metaopt
f(A,B):-tail(A,C),f1(C,B).
f1(A,B):-f2(A,C),dropLast(C,B).
f2(A,B):-f3(A,C),f3(C,B).
f3(A,B):-tail(A,C),f4(C,B).
f4(A,B):-f5(A,C),f5(C,B).
f5(A,B):-tail(A,C),tail(C,B).
```

```prolog
% metaopt unfolded
f(A,B):-
    tail(A,C),
    tail(C,D),
    tail(D,E),
    tail(E,F),
    tail(F,G),
    tail(G,H),
    tail(H,I),
    tail(I,J),
    tail(J,K),
    tail(K,L),
    tail(L,M),
    dropLast(M,B).
```

```
% metaopt unfolded
f(A,B):-
    tail(A,C),
    tail(C,D),
    tail(D,E),
    tail(E,F),
    tail(F,G),
    tail(G,H),
    tail(H,I),
    tail(I,J),
    tail(J,K),
    tail(K,L),
    tail(L,M),
    dropLast(M,B).
```

does this last

Todo

- Characterise complexity of learned program
- Study complexity of Metaopt variants
- Discover new efficient algorithms