Derivation reduction of metarules in meta-interpretive learning

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Input	Output
Examples Background knowledge **Bias**	Logic program

Biases

- Mode declarations (Progol, ILASP, Aleph, XHAIL, …)
- **Metarules** (Metagol, MIL-Hex, ∂ILP, ProPPR, Clint, MOBAL …)

Metarules

$\exists PQ \forall AB$	$P(A,B) \leftarrow Q(A,B)$
$\exists PQR \forall AB$	$P(A,B) \leftarrow Q(A),R(A,B)$
$\exists PQR \forall ABC$	$P(A,B) \leftarrow Q(A,C),R(C,B)$

Metarules

```
P(A,B) ← Q(A,B)
P(A,B) ← Q(A),R(A,B)
P(A,B) ← Q(A,C),R(C,B)
```

P,*Q*,*R* are **existentially** quantified **second-order** variables

A,B,*C* are **universally** quantified **first-order** variables

Input	Output

`% background`
`parent(ann,amy)←`
`parent(amy,amelia)←`
 	
`% example`
`grandparent(ann,amelia)←`
 	
`% metarule`
`P(A,B)←Q(A,C),R(C,B)`	

Input	Output

% background
parent(ann,amy)←
parent(amy,amelia)←
 	
% example
grandparent(ann,amelia)←
 	
% metarule
P(A,B)←Q(A,C),R(C,B)
	grandparent(A,B)←
	    parent(A,C),
	    parent(C,B)
	
	{
	 P\granparent,
	 Q\parent,
	 R\parent
	}

Why?

Completeness

cannot learn grandparent/2 with only `P(X)←Q(X)`

Efficiency

more metarules = larger hypothesis space

Remove redundant metarules [ILP14]

The Horn clause C is **entailment redundant** in the Horn theory $T \cup \{C\}$ when $T \models C$

Entailment redundancy

```
C1 = h(A,B)←s(A,B)
C2 = h(A,B)←s(A,B),u(B)
C3 = h(A,B)←s(A,B),u(A,B)
C4 = h(A,B)←s(A,B),u(A,B),v(A,B)
```

Entailment redundancy

```
C1 = h(A,B)←s(A,B)
```
~~C2 = h(A,B)←s(A,B),u(B)~~

~~C3 = h(A,B)←s(A,B),u(A,B)~~

~~C4 = h(A,B)←s(A,B),u(A,B),v(A,B)~~

$$\{C1\} \models \{C2, C3, C4\}$$

Entailment reduction of metarules [ILP14]

$P(A,B) \leftarrow Q(A,B)$
$P(A,B) \leftarrow Q(B,A)$
$P(A,B) \leftarrow Q(A,C),R(B,C)$
$P(A,B) \leftarrow Q(A,C),R(C,B)$
$P(A,B) \leftarrow Q(B,A),R(A,B)$
$P(A,B) \leftarrow Q(B,A),R(B,A)$
$P(A,B) \leftarrow Q(B,C),R(A,C)$
$P(A,B) \leftarrow Q(B,C),R(C,A)$
$P(A,B) \leftarrow Q(C,A),R(B,C)$
$P(A,B) \leftarrow Q(C,A),R(C,B)$
$P(A,B) \leftarrow Q(C,B),R(A,C)$
$P(A,B) \leftarrow Q(C,B),R(C,A)$

$\longrightarrow$?

Entailment reduction of metarules [ILP14]

P(A,B)←Q(A,B)
P(A,B)←Q(B,A)
P(A,B)←Q(A,C),R(B,C)
P(A,B)←Q(A,C),R(C,B)
P(A,B)←Q(B,A),R(A,B)
P(A,B)←Q(B,A),R(B,A)
P(A,B)←Q(B,C),R(A,C)
P(A,B)←Q(B,C),R(C,A)
P(A,B)←Q(C,A),R(B,C)
P(A,B)←Q(C,A),R(C,B)
P(A,B)←Q(C,B),R(A,C)
P(A,B)←Q(C,B),R(C,A)

P(A,B)←Q(B,A)

P(A,B)←Q(A,C),R(C,B)

Entailment redundancy

```
C1 = P(A,B)←Q(A,B)
C2 = P(A,B)←Q(A,B),R(A)
C3 = P(A,B)←Q(A,B),R(A,B)
C4 = P(A,B)←Q(A,B),R(A,B),S(A,B)
```

Entailment redundancy

```
C1 = P(A,B)←Q(A,B)
```

~~`C2 = P(A,B)←Q(A,B),R(A)`~~

~~`C3 = P(A,B)←Q(A,B),R(A,B)`~~

~~`C4 = P(A,B)←Q(A,B),R(A,B),S(A,B)`~~

$$\{C1\} \models \{C2,C3,C4\}$$

Entailment redundancy

```
C1 = P(A,B)←Q(A,B)
```

~~C2 = P(A,B)←Q(A,B),R(A)~~

~~C3 = P(A,B)←Q(A,B),R(A,B)~~

~~C4 = P(A,B)←Q(A,B),R(A,B),S(A,B)~~

$$\{C1\} \models \{C2, C3, C4\}$$

father(A,B)←parent(A,B),male(A) ✖

Derivation redundancy

The Horn clause C is **derivationally redundant** in the Horn theory $T \cup \{C\}$ when $T \vdash C$

SLD-resolution

Derivation redundancy

```
C1 = P(A,B)←Q(A,B)
C2 = P(A,B)←Q(A,B),R(A)
C3 = P(A,B)←Q(A,B),R(A,B)
C4 = P(A,B)←Q(A,B),R(A,B),S(A,B)
```

Derivation redundancy

```
C1 = P(A,B)←Q(A,B)
C2 = P(A,B)←Q(A,B),R(A)
C3 = P(A,B)←Q(A,B),R(A,B)
```
~~C4 = P(A,B)←Q(A,B),R(A,B),S(A,B)~~

{C1,C2,C3} ⊢ {C4}

father(A,B)←parent(A,B),male(A) ✓

Derivation redundancy

While there is a clause in T such that T - {C} $\vdash_k$ C:

Set T = T - {C}

Connected clauses

body literals are connected to the head literal

P(A)←Q(A) ✓

P(A,B)←Q(A,C) ✓

P(A,B)←Q(A,B),R(B,D),S(D,B) ✓

P(A)←**Q(B)** ✖

P(A)←Q(A),**R(B,C)** ✖

P(A,B)←Q(A,B),**S(C)** ✖

H^2_m

restriction on literal arity

```
P(A,B)←Q(A,B) ✓
P(A)←Q(A,B),R(B) ✓

P(A,B,C)←Q(A,B,C) ✖
P(A)←Q(A,B,C),R(B,C) ✖
```

$H^{2=}_{m}$

P(A,B)←Q(A,B) ✓
P(A,B)←Q(A,C),R(C,B) ✓

P(A)←**Q(A)** ✖
P(A,B)←Q(A,B),**R(B)** ✖

H^a_2

restriction on number of body literals

P(A,B)←Q(A,B) ✓

P(A)←Q(A,B,C),R(B,C) ✓

P(A)←**Q(A),R(A),S(A)** ✖

P(A,B)←**Q(A),R(B),S(A,B)** ✖

$H^{a}{}_{2=}$

$P(A) \leftarrow Q(A), R(A)$ ✓

$P(A,B) \leftarrow Q(A,B), R(A,B)$ ✓

$P(A) \leftarrow \mathbf{Q(A)}$ ✖

$P(A,B) \leftarrow \mathbf{Q(A,B), R(B)}$ ✖

Exactly-two connected

each first-order variable appears exactly twice

```
P(A)←Q(A) ✓
P(A,B)←Q(A,B) ✓
P(A,B)←Q(A,C),R(C,B) ✓

P(A,B)←Q(A) ✖
P(A)←Q(A,B) ✖
P(A)←Q(A),R(A) ✖
```

$E^{2}=5$

$E^{2=}{}_{5}$

E-reduction	D-reduction
P(A,B)←Q(B,A)	P(A,B)←Q(B,A)
P(A,B)←Q(A,C),R(C,B)	P(A,B)←Q(A,C),R(C,B)

$\mathbf{E}^{2=}{}_{5}$

E-reduction	D-reduction
`P(A,B)←Q(B,A)`	`P(A,B)←Q(B,A)`
`P(A,B)←Q(A,C),R(C,B)`	`P(A,B)←Q(A,C),R(C,B)`

$$\mathbf{E}^{2=}{}_{2} \vdash \mathbf{E}^{2=}{}_{\infty} \checkmark$$

$E^2{}_5$

E^2_5

E-reduction	D-reduction
P(A)←Q(A)	P(A)←Q(A)
P(A)←Q(A,B),R(B)	P(A)←Q(A,B),R(B)
P(A,B)←Q(B,A)	P(A,B)←Q(B,A)
P(A,B)←Q(A),R(B)	P(A,B)←Q(A),R(B)
P(A,B)←Q(A,C),R(C,B)	P(A,B)←Q(A,C),R(C,B)

$E^2{}_5$

E-reduction	D-reduction
P(A)←Q(A)	P(A)←Q(A)
P(A)←Q(A,B),R(B)	P(A)←Q(A,B),R(B)
P(A,B)←Q(B,A)	P(A,B)←Q(B,A)
P(A,B)←Q(A),R(B)	P(A,B)←Q(A),R(B)
P(A,B)←Q(A,C),R(C,B)	P(A,B)←Q(A,C),R(C,B)

$$\mathbf{E^2{}_2 \vdash E^2{}_\infty}$$ ✔

Two connected

each first-order variable appears at least twice
(i.e. forbids singleton variables)

```
P(A)←Q(A) ✓
P(A)←Q(A),R(A) ✓
P(A,B)←Q(A,B),R(B) ✓
P(A,B)←Q(A,C),R(C,B) ✓

P(A,B)←Q(A) ✖
P(A)←Q(A,B) ✖
P(A)←Q(A),R(A,B) ✖
```

$K^{2=}_{5}$ two connected

$K^{2=}_{5}$ two connected

E-reduction	D-reduction
P(A,B)←Q(B,A)	P(A,B)←Q(B,A)
P(A,B)←Q(A,C),R(C,B)	P(A,B)←Q(A,C),R(C,B)
	P(A,B)←Q(A,B),R(A,B)
	P(A,B)←Q(A,B),R(A,C),S(C,D),T(C,D)
	P(A,B)←Q(A,C),R(A,C),S(B,D),T(B,D)
	P(A,B)←Q(A,C),R(A,D),S(B,C),T(B,D),U(C,D)

$K^{2=}_{5}$ two connected

E-reduction	D-reduction
P(A,B)←Q(B,A)	P(A,B)←Q(B,A)
P(A,B)←Q(A,C),R(C,B)	P(A,B)←Q(A,C),R(C,B)
	P(A,B)←Q(A,B),R(A,B)
	P(A,B)←Q(A,B),R(A,C),S(C,D),T(C,D)
	P(A,B)←Q(A,C),R(A,C),S(B,D),T(B,D)
	P(A,B)←Q(A,C),R(A,D),S(B,C),T(B,D),U(C,D)

$K^{2=}_{2} \nvdash K^{2=}_{\infty}$ ✖

K^2_5 two connected

$K^2{}_5$ two connected

E-reduction	D-reduction
P(A)←Q(A)	P(A)←Q(A)
P(A)←R(A,B),Q(A,B)	P(A)←R(A,B),Q(A,B)
	P(A)←Q(A),R(A)
	P(A)←Q(B),R(A,B)
P(A,B)←Q(B,A)	P(A,B)←Q(B,A)
P(A,B)←Q(A),R(B)	P(A,B)←Q(A),R(B)
P(A,B)←Q(A,C),R(C,B)	P(A,B)←Q(A,C),R(C,B)
	P(A,B)←Q(A,B),R(A,B)
	P(A,B)←Q(A),R(A,B)
	P(A,B)←Q(A,C),R(A,D),S(C,B),T(B,D),U(C,D)

$K^2{}_5$ two connected

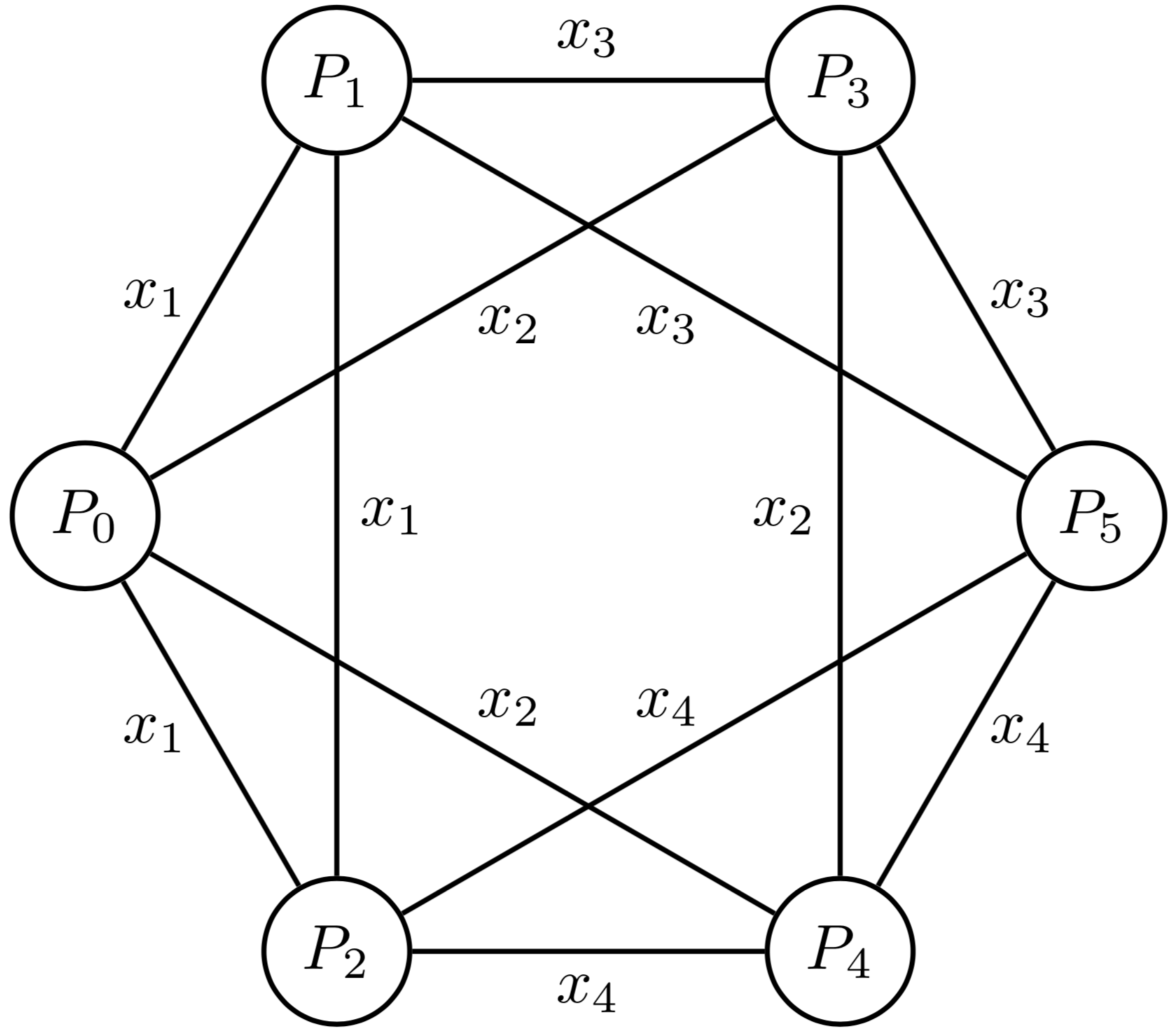
E-reduction	D-reduction
P(A)←Q(A)	P(A)←Q(A)
P(A)←R(A,B),Q(A,B)	P(A)←R(A,B),Q(A,B)
	P(A)←Q(A),R(A)
	P(A)←Q(B),R(A,B)
P(A,B)←Q(B,A)	P(A,B)←Q(B,A)
P(A,B)←Q(A),R(B)	P(A,B)←Q(A),R(B)
P(A,B)←Q(A,C),R(C,B)	P(A,B)←Q(A,C),R(C,B)
	P(A,B)←Q(A,B),R(A,B)
	P(A,B)←Q(A),R(A,B)
	P(A,B)←Q(A,C),R(A,D),S(C,B),T(B,D),U(C,D)

$$K^{2=}{}_2 \nvdash K^{2=}{}_5$$ ✖

$K^2{}_5$ two connected

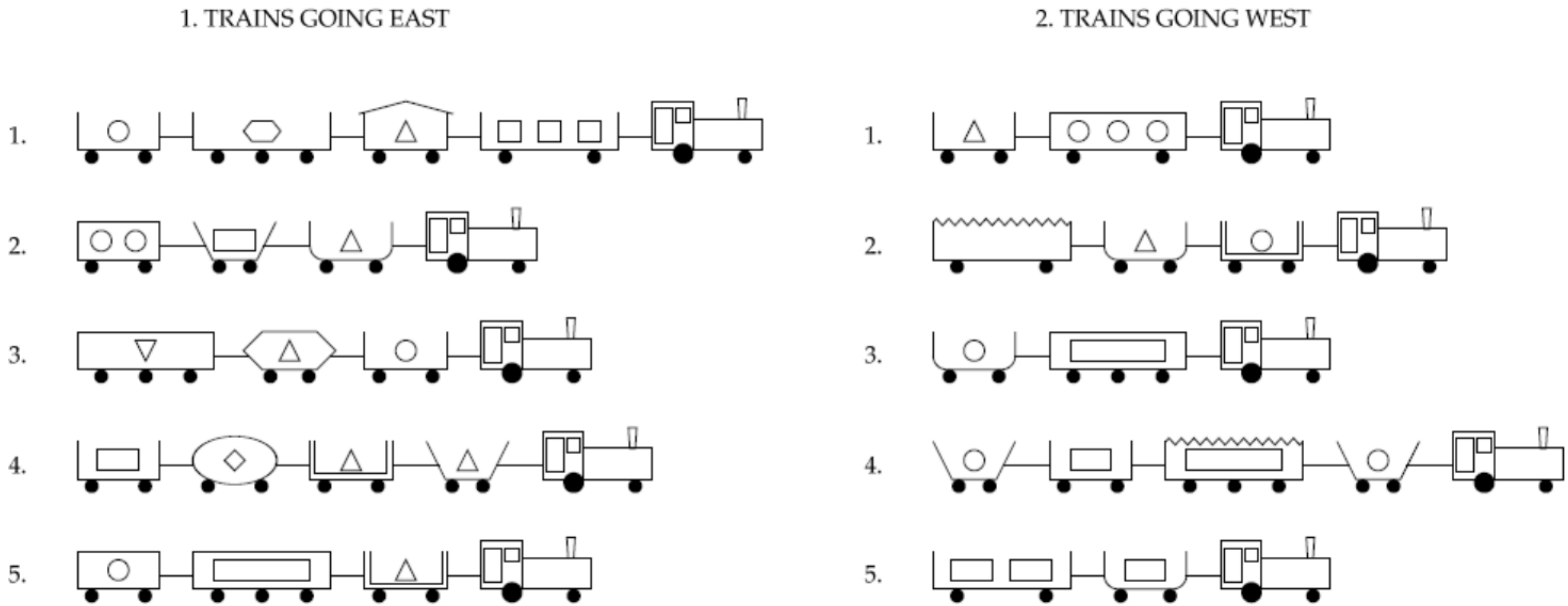
E-reduction	D-reduction
P(A)←Q(A)	P(A)←Q(A)
P(A)←R(A,B),Q(A,B)	P(A)←R(A,B),Q(A,B)
	P(A)←Q(A),R(A)
	P(A)←Q(B),R(A,B)
P(A,B)←Q(B,A)	P(A,B)←Q(B,A)
P(A,B)←Q(A),R(B)	P(A,B)←Q(A),R(B)
P(A,B)←Q(A,C),R(C,B)	P(A,B)←Q(A,C),R(C,B)
	P(A,B)←Q(A,B),R(A,B)
	P(A,B)←Q(A),R(A,B)
	P(A,B)←Q(A,C),R(A,D),S(C,B),T(B,D),U(C,D)

$K^{2=}{}_\infty$ cannot be reduced ✖



$$P_0(x_1, x_2) \leftarrow P_1(x_1, x_3), P_2(x_1, x_4), P_3(x_2, x_3), P_4(x_2, x_4), P_5(x_3, x_4)$$

Does it matter?



Accuracies

Task	E-reduction	D-reduction	D*-reduction
T1	95 ± 1	100 ± 0	100 ± 0
T2	99 ± 1	100 ± 0	100 ± 0
T3	56 ± 3	96 ± 2	96 ± 2
T4	69 ± 4	96 ± 2	96 ± 2
T5	59 ± 3	93 ± 3	93 ± 3
T6	50 ± 1	96 ± 3	96 ± 3
T7	68 ± 4	95 ± 2	95 ± 2
T8	54 ± 3	60 ± 3	90 ± 3

Learning times

Task	E-reduction	D-reduction	D*-reduction
T1	0.01 ± 0	0 ± 0	0 ± 0
T2	0.01 ± 0	0 ± 0	0 ± 0
T3	431 ± 59	0.01 ± 0	0.01 ± 0
T4	300 ± 68	0 ± 0	0.01 ± 0
T5	427 ± 60	1 ± 0.3	1 ± 0.41
T6	600 ± 0	1 ± 0.41	1 ± 0.42
T7	917 ± 535	1 ± 0.27	1 ± 0.36
T8	487 ± 51	360 ± 67	26 ± 5

```
% target program
f(X):-has_car(X,C1),
    long(C1),
    two_wheels(C1),
    has_car(X,C2),
    long(C2),
    three_wheels(C2).
```

```
% E-reduction
f(A):-has_car(A,B),f1(A,B).
f1(A,B):-has_car(A,C),f2(C,B).
f2(A,B):-long(A),three_wheels(B).
```

```
% D-reduction
f(A):-f1(A),f2(A).
f1(A):-has_car(A,B),three_wheels(B).
f2(A):-has_car(A,B),roof_open(B).
```

```
% D*-reduction
f(A):-f1(A),f2(A).
f1(A):-has_car(A,B),three_wheels(B).
f2(A):-has_car(A,B),f3(B).
f3(A):-long(A),two_wheels(A).
```

```
% target program
f(X):-
    has_car(X,C1),
    long(C1),
    two_wheels(C1),
    has_car(X,C2),
    long(C2),
    three_wheels(C2).
```

```
% D*-reduction
f(A):-
    has_car(A,B),
    three_wheels(B),
    has_car(A,C),
    long(C),
    two_wheels(C).
```

Todo

- Study derivation reduction problem
- Other fragments
  - Triadics
  - Connected
- Unconstrained resolution