Learning efficient logic programs

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Input	Output
[s,h,e,e,p]	e
[a,l,p,a,c,a]	a
[c,h,i,c,k,e,n]	?

Input	Output
[s,h,e,e,p]	e
[a,l,p,a,c,a]	a
[c,h,i,c,k,e,n]	**c**

```
%% metagol
f(A,B):-head(A,B),tail(A,C),element(C,B).
f(A,B):-tail(A,C),f(C,B).
```

```
%% alternative
f(A,B):-mergesort(A,C),f1(C,B).
f1(A,B):-head(A,B),tail(A,C),head(C,B).
f1(A,B):-tail(A,C),f1(C,B).
```

Idea

1. Given examples **E**, learn any program **H**
2. Repeat whilst possible:
   A. Learn program **H'** such that cost(**H',E**) < cost(**H,E**)
   B. Set **H**=**H'**
3. Return **H**

Metagol

```
prove([],P,P).

prove([Atom|Atoms],P1,P2):-
    prove_aux(Atom,P1,P3),
    prove(Atoms,P3,P2).

prove_aux(Atom,P,P):-
    call(Atom).

prove_aux(Atom,P1,P2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    prove(Body,P3,P2).
```

Metagol

```
prove([],P,P).

prove([Atom|Atoms],P1,P2):-
    prove_aux(Atom,P1,P3),
    prove(Atoms,P3,P2).

prove_aux(Atom,P,P):-
    call(Atom).

prove_aux(Atom,P1,P2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    prove(Body,P3,P2).
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Metagol

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prove([],P,P).

prove([Atom|Atoms],P1,P2):-
    prove_aux(Atom,P1,P3),
    prove(Atoms,P3,P2).

prove_aux(Atom,P,P):-
    call(Atom).

prove_aux(Atom,P1,P2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    prove(Body,P3,P2).
```

Metagol

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prove([],P,P).

prove([Atom|Atoms],P1,P2):-
    prove_aux(Atom,P1,P3),
    prove(Atoms,P3,P2).

prove_aux(Atom,P,P):-
    call(Atom).

prove_aux(Atom,P1,P2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    prove(Body,P3,P2).
```

Metaopt

```
prove([],P,P,C,C).

prove([Atom|Atoms],P1,P2,C1,C2):-
    prove_aux(Atom,P1,P3,C1,C3),
    prove(Atoms,P3,P2,C3,C2).

prove_aux(Atom,P,P,C1,C2):-
    pos_cost(Atom,Cost).
    C2 is C1+Cost,
    max_cost(MaxCost),
    C2 < MaxCost.

prove_aux(Atom,P1,P2,C1,C2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    C3 is C1+1,
    prove(Body,P3,P2,C3,C2).
```

```
prove([],P,P,C,C).

prove([Atom|Atoms],P1,P2,C1,C2):-
    prove_aux(Atom,P1,P3,C1,C3),
    prove(Atoms,P3,P2,C3,C2).

prove_aux(Atom,P,P,C1,C2):-
    pos_cost(Atom,Cost).
    C2 is C1+Cost,
    max_cost(MaxCost),
    C2 < MaxCost.

prove_aux(Atom,P1,P2,C1,C2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    C3 is C1+1,
    prove(Body,P3,P2,C3,C2).
```

Metaopt

```
prove([],P,P,C,C).

prove([Atom|Atoms],P1,P2,C1,C2):-
    prove_aux(Atom,P1,P3,C1,C3),
    prove(Atoms,P3,P2,C3,C2).

prove_aux(Atom,P,P,C1,C2):-
    pos_cost(Atom,Cost).
    C2 is C1+Cost,
    max_cost(MaxCost),
    C2 < MaxCost.

prove_aux(Atom,P1,P2,C1,C2):-
    metarule(Atom,Body,Subs),
    save(Subs,P1,P3),
    C3 is C1+1,
    prove(Body,P3,P2,C3,C2).
```

Iterative descent

1. Given examples **E**, learn program **H** with minimal textual complexity
2. Repeat whilst possible:
    A. Learn program **H'** such that cost(**H',E**) < cost(**H,E**)
    B. Set **H**=**H'**
3. Return **H**

Metaopt prunes as it learns

Tree cost

Positive examples: size of the leftmost successful branch

Tree cost

Positive examples: size of the leftmost successful branch

```
pos_cost(Atom,Cost):-
statistics(inferences,I1),
call(Atom),
statistics(inferences,I2),
Cost is I2-I1.
```

Tree cost

Negative examples: size of the finitely-failed SLD-tree

Tree cost

Negative examples: size of the finitely-failed SLD-tree

```
neg_cost(Atom,Cost):-
statistics(inferences,I1),
\+ call(Atom),
statistics(inferences,I2),
Cost is I2-I1.
```

Tree cost

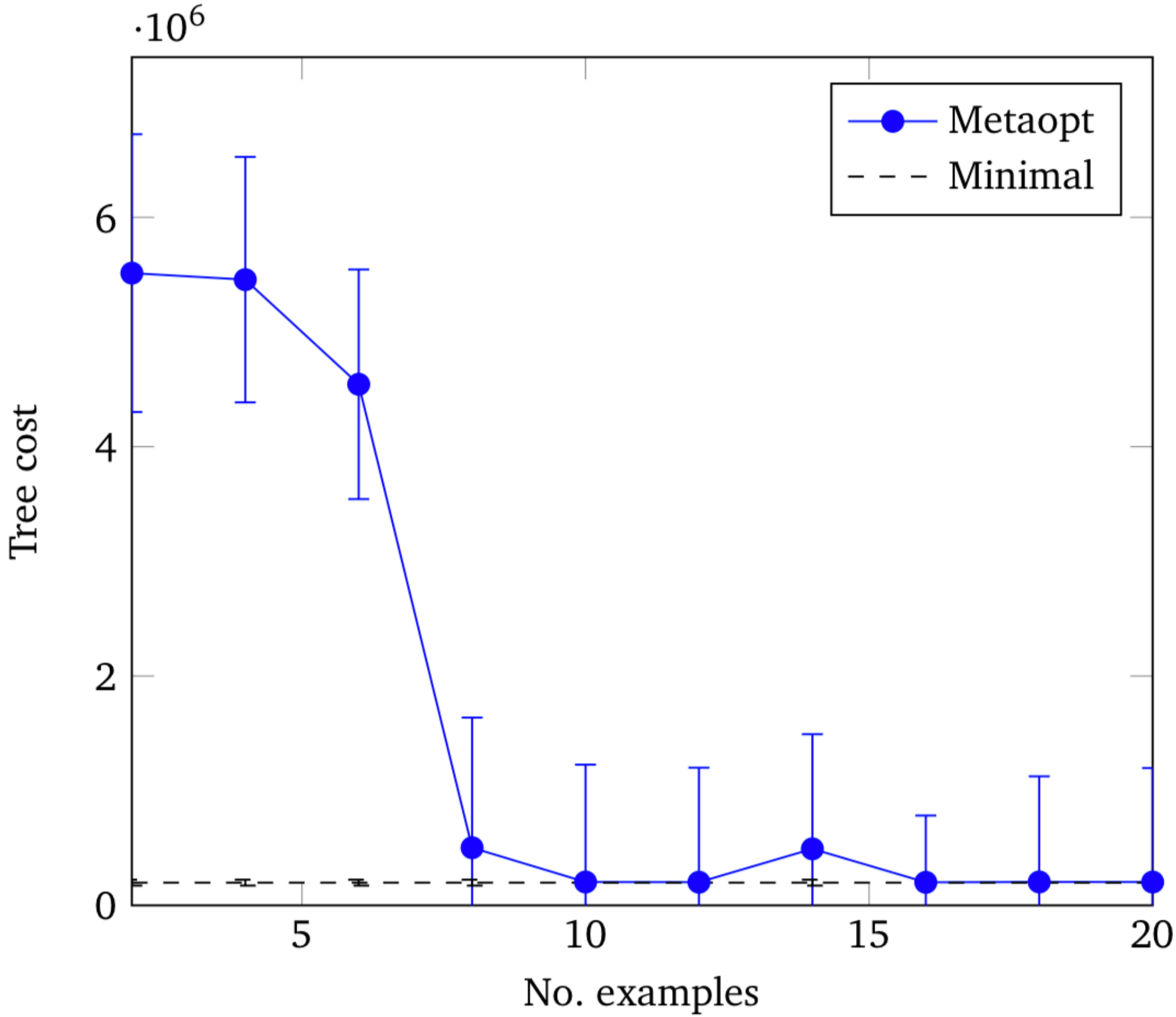
- any arity logics
- no user-supplied costs
- backtracking and non-determinism

Input	Output
[s,h,e,e,p]	e
[a,l,p,a,c,a]	a
[c,h,i,c,k,e,n]	c

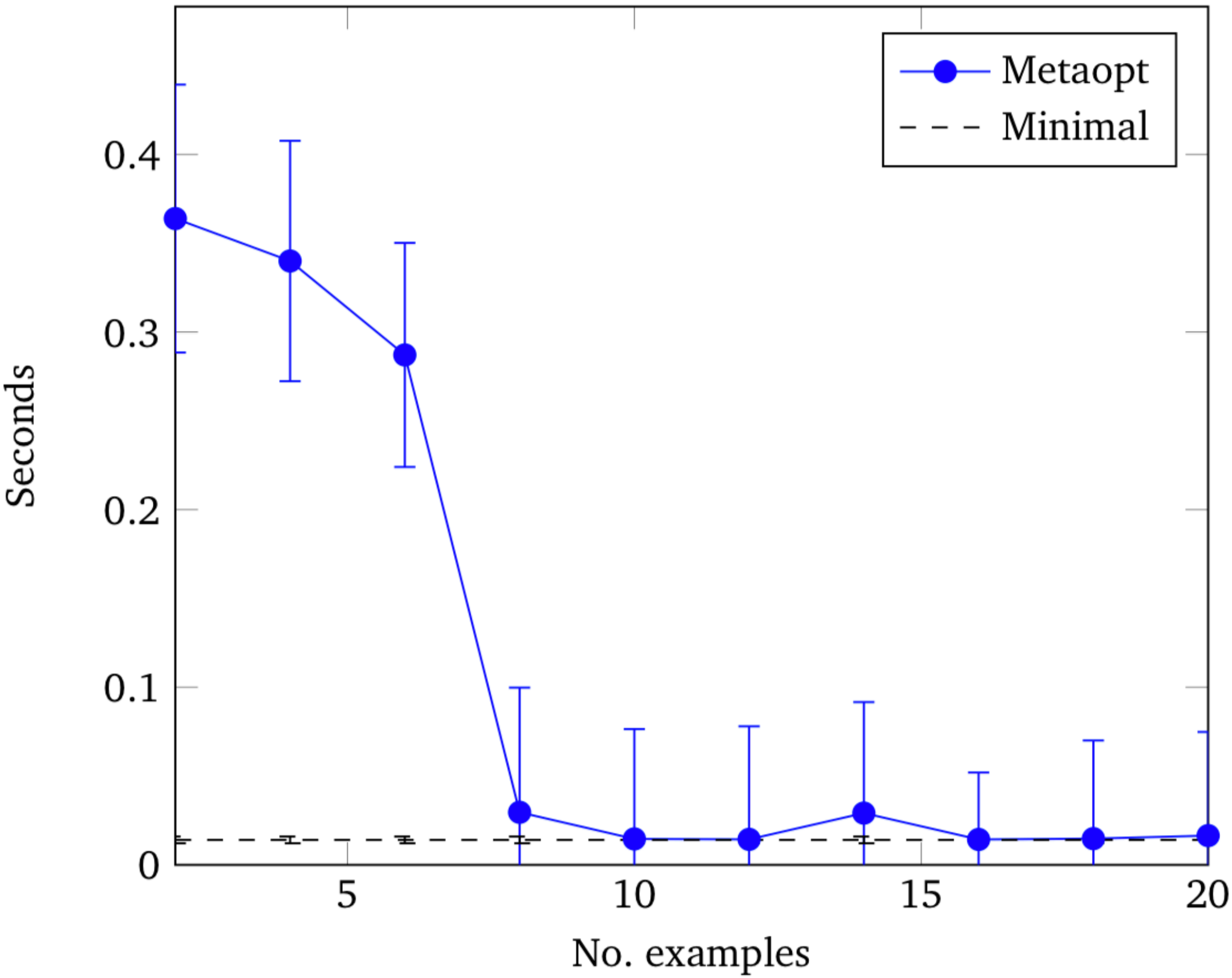
Input	Output
[s,h,e,e,p]	e
[a,l,p,a,c,a]	a
[c,h,i,c,k,e,n]	c

```
f(A,B):-mergesort(A,C),f1(C,B).
f1(A,B):-head(A,B),tail(A,C),head(C,B).
f1(A,B):-tail(A,C),f1(C,B).
```

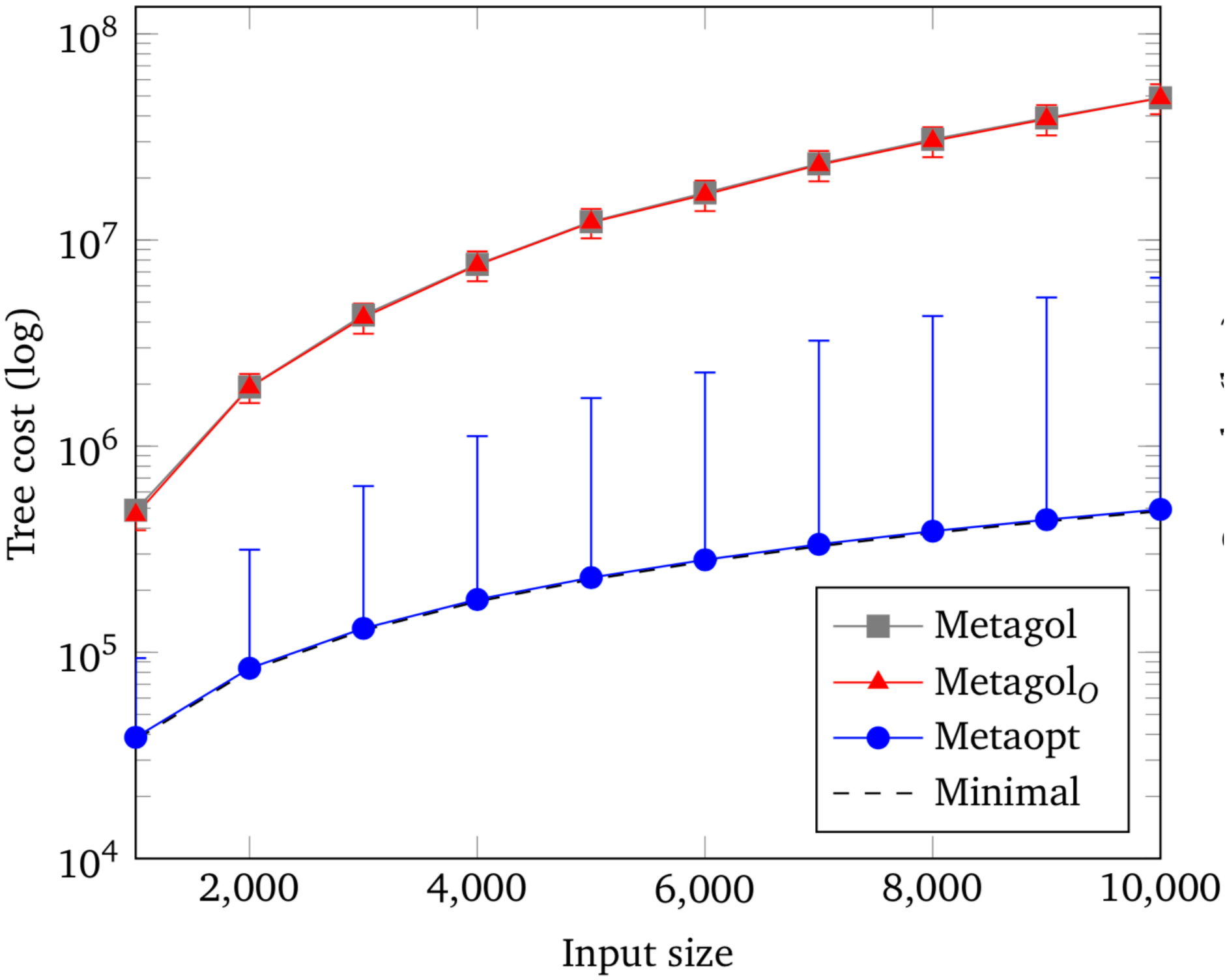
Convergence



Convergence

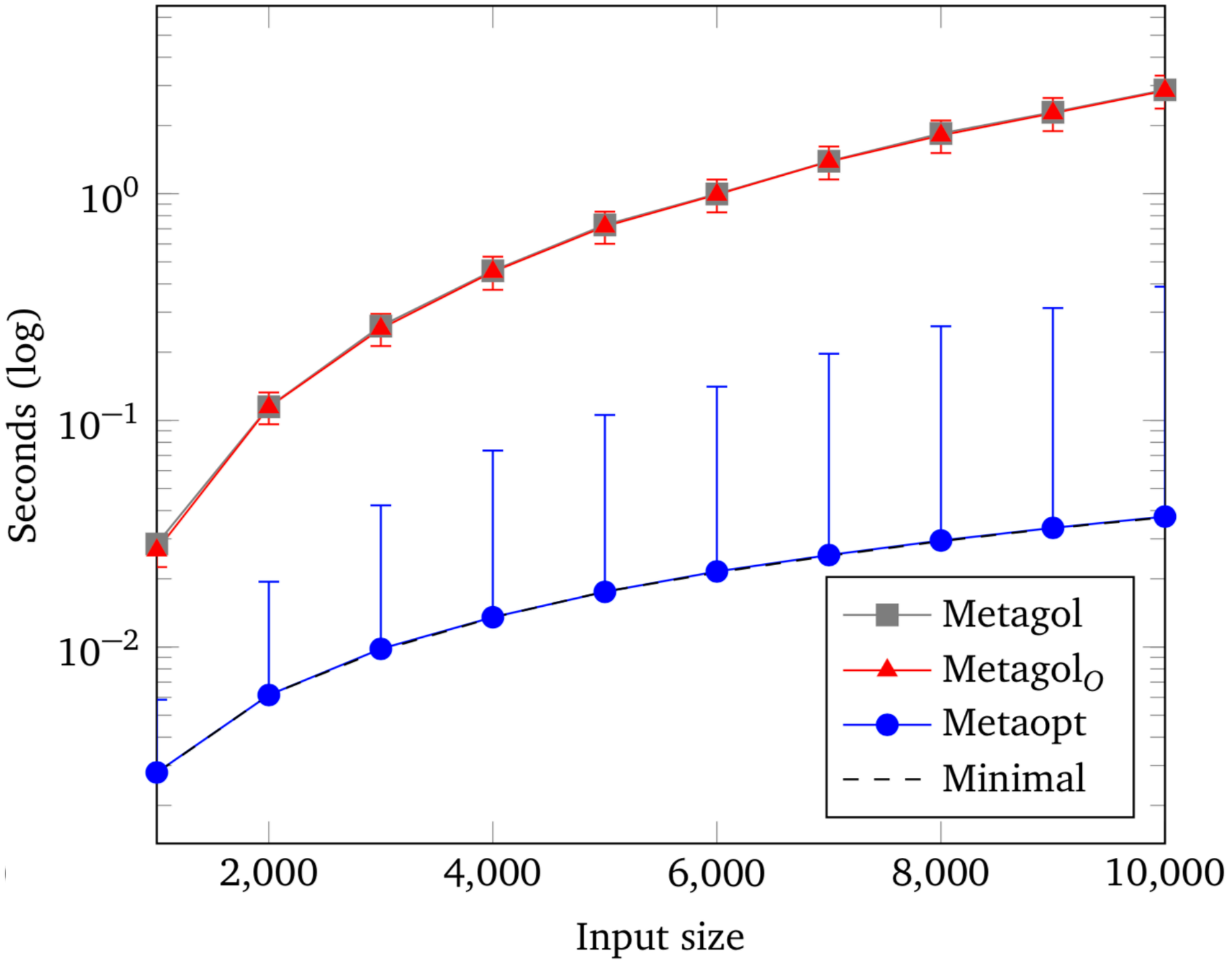


Performance



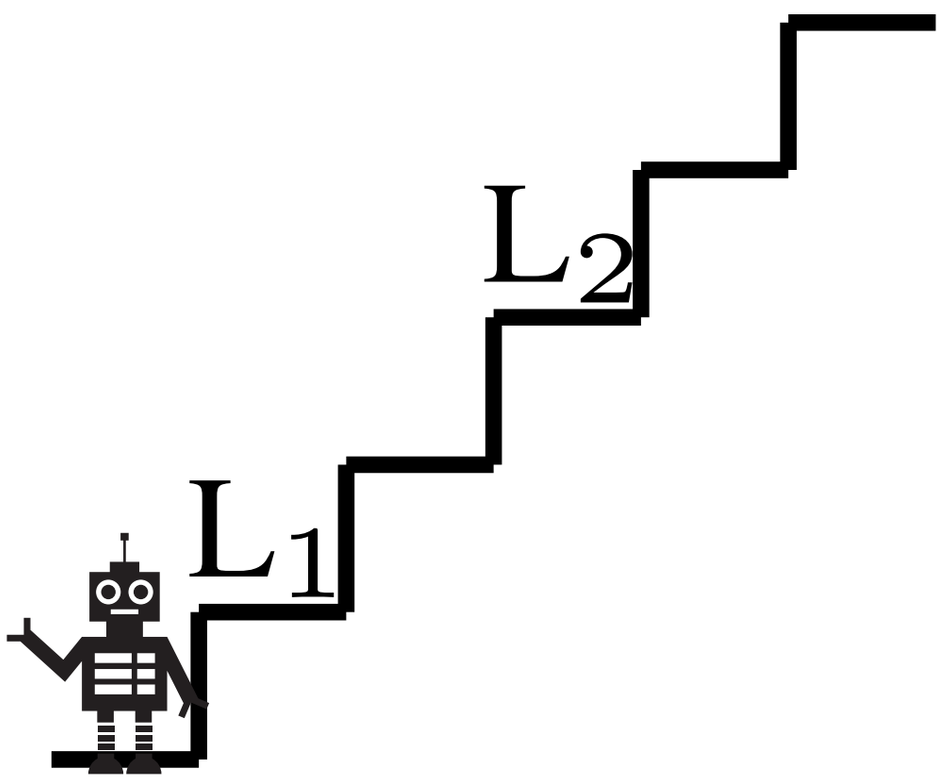
(a) Tree costs

Performance

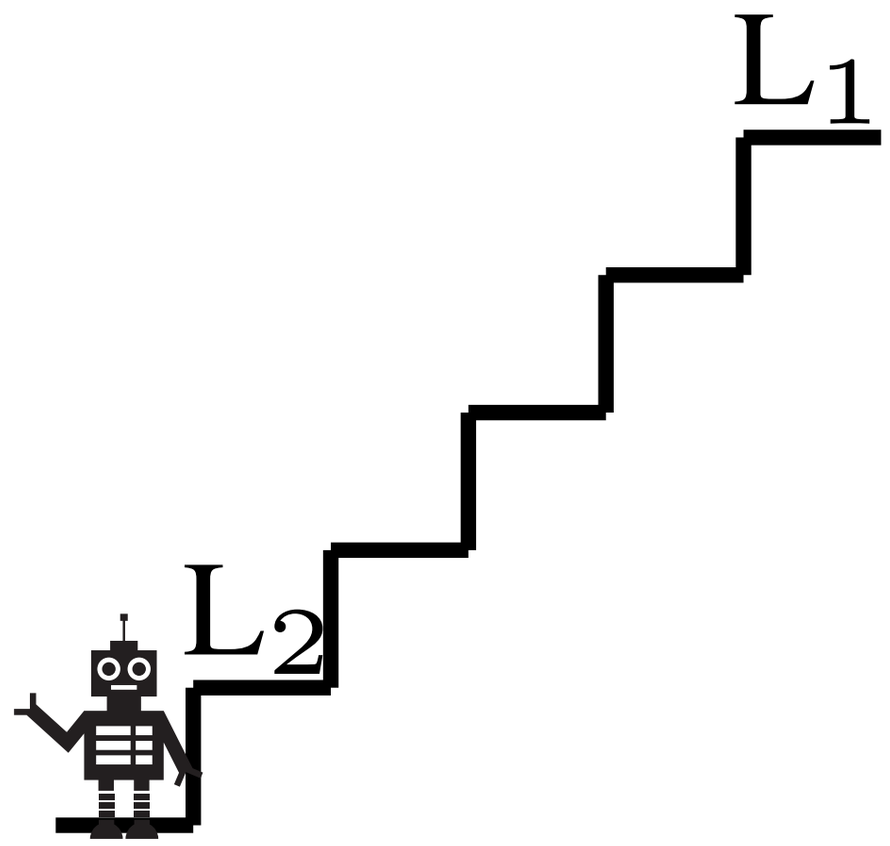


(b) Program runtimes

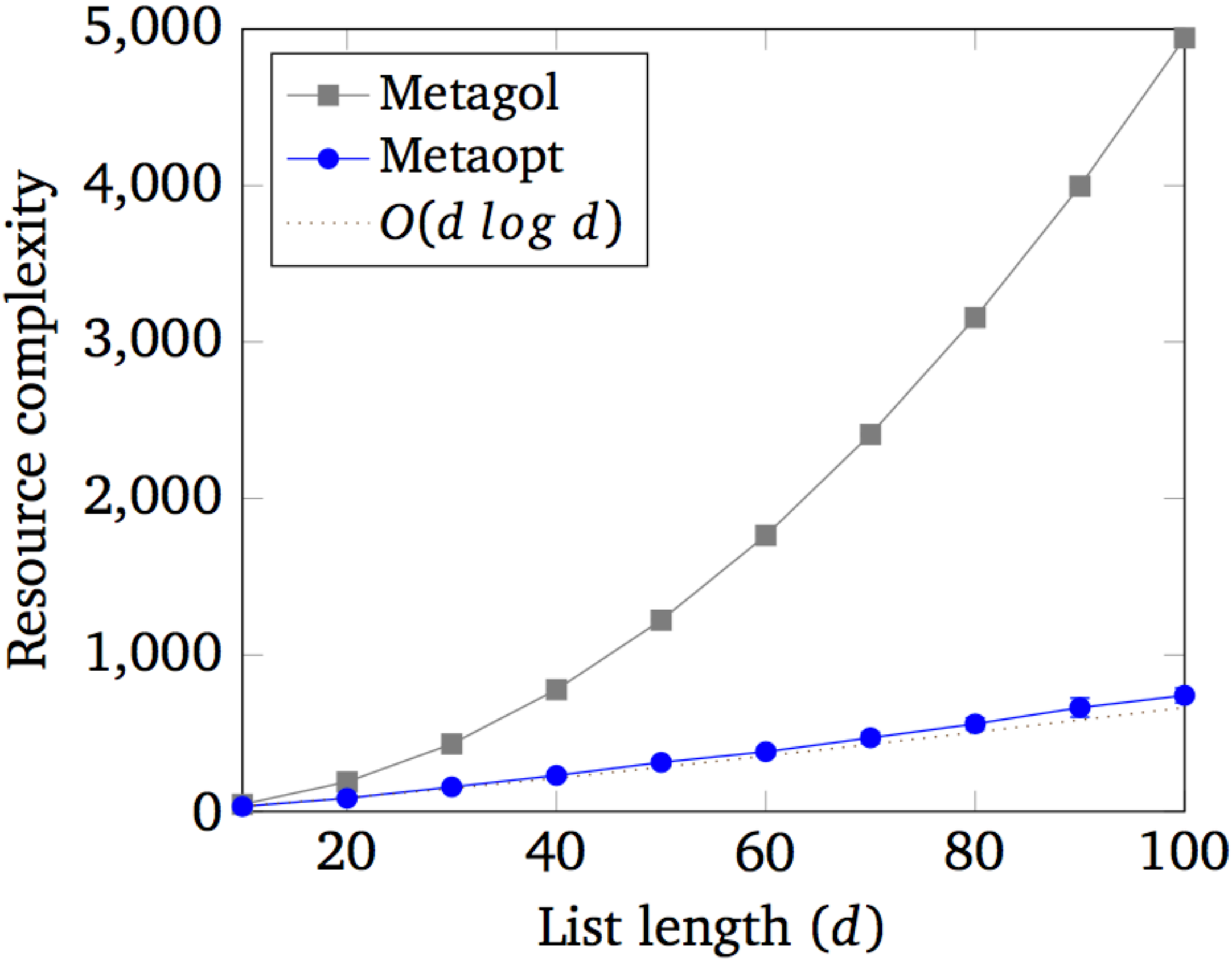
Resource complexity



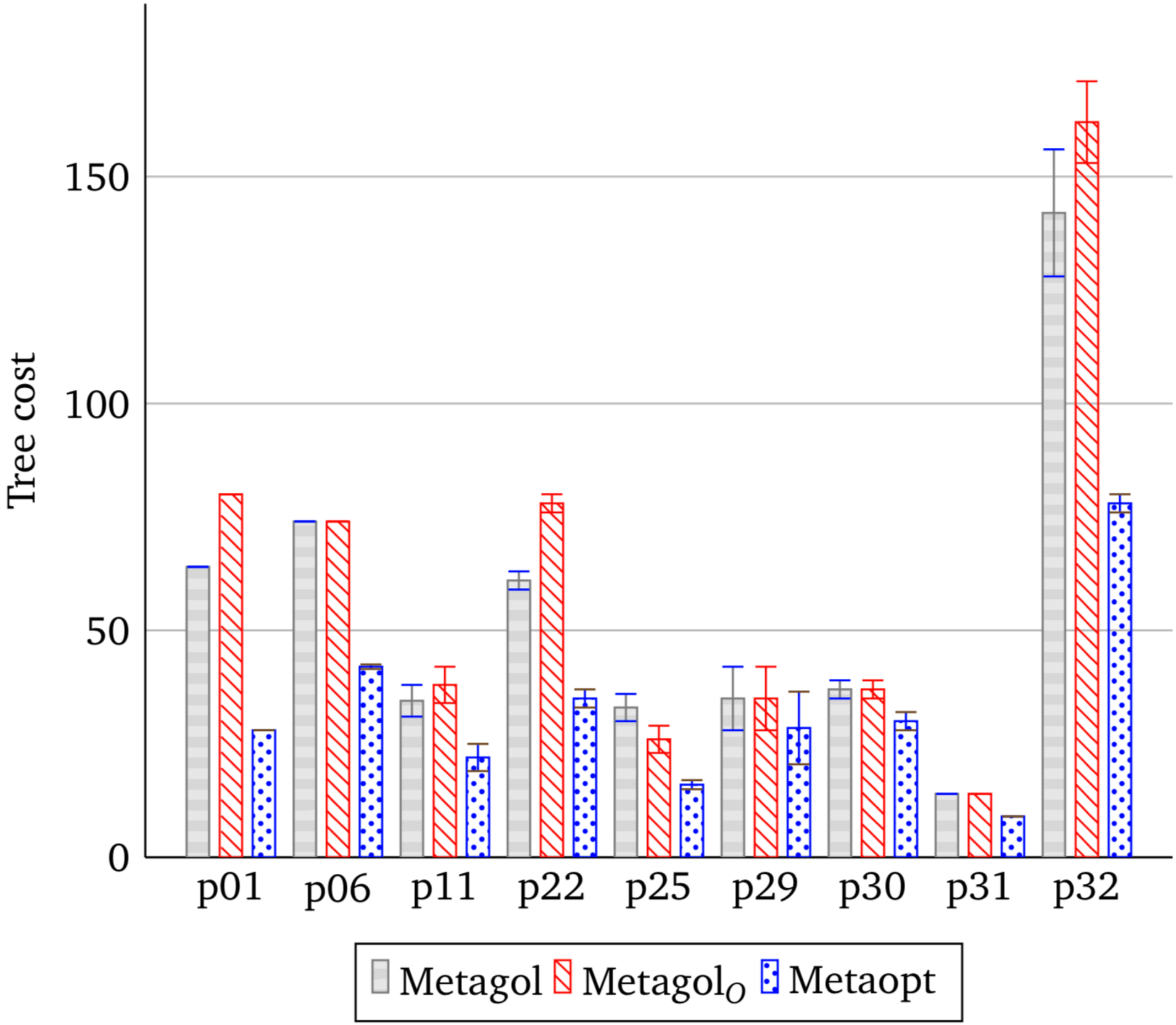
Initial state



Final state


Metagol
Metaopt
$O(d\ log\ d)$
Resource complexity
5,000
4,000
3,000
2,000
1,000
0
20
40
60
80
100
List length (d)

Input	Output
My name is John.	John
My name is Bill.	Bill
My name is Josh.	Josh
My name is Albert.	Albert
My name is Richard.	Richard


Tree cost
150
100
50
0
p01
p06
p11
p22
p25
p29
p30
p31
p32
Metagol
Metagol$_O$
Metaopt

```
%% metagol
f(A,B):-tail(A,C),f1(C,B).
f1(A,B):-dropLast(A,C),f2(C,B).
f2(A,B):-dropWhile(A,B,not_uppercase).
```

```
%% metagol unfolded
f(A,B):-
  tail(A,C),
  dropLast(C,D),
  dropWhile(D,B,not_uppercase).
```

```
% metagol0
f(A,B):-f1(A,C),f4(C,B).
f1(A,B):-f2(A,C),f3(C,B).
f2(A,B):-filter(A,B,is_letter).
f3(A,B):-dropWhile(A,B,is_uppercase).
f4(A,B):-dropWhile(A,B,not_uppercase).
```

```
% metagol0 unfolded
f(A,B):-
  filter(A,C,is_letter).
  dropWhile(C,D,is_uppercase),
  dropWhile(D,B,not_uppercase).
```

```
% metaopt
f(A,B):-tail(A,C),f1(C,B).
f1(A,B):-f2(A,C),dropLast(C,B).
f2(A,B):-f3(A,C),f3(C,B).
f3(A,B):-tail(A,C),f4(C,B).
f4(A,B):-f5(A,C),f5(C,B).
f5(A,B):-tail(A,C),tail(C,B).
```

```
% metaopt unfolded
f(A,B):-
    tail(A,C),
    tail(C,D),
    tail(D,E),
    tail(E,F),
    tail(F,G),
    tail(G,H),
    tail(H,I),
    tail(I,J),
    tail(J,K),
    tail(K,L),
    tail(L,M),
    dropLast(M,B).
```

```
% metaopt unfolded
f(A,B):-
    tail(A,C),
    tail(C,D),
    tail(D,E),
    tail(E,F),
    tail(F,G),
    tail(G,H),
    tail(H,I),
    tail(I,J),
    tail(J,K),
    tail(K,L),
    tail(L,M),
    dropLast(M,B).
```

does this last

Todo

- Characterise complexity of learned program
- Study complexity of Metaopt variants
- Discover new efficient algorithms