

# Bi-Objective Optimization of Pollution Management Policies in Shallow and Deep Lake Systems

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## 1 Objective Formulation

We simulate the lake model for  $T$  years on  $N$  Monte-Carlo samples.

### 1.1 Economic Benefit

The economic benefit objective represents the net present value of a given  $P$  release policy. We define a constant  $\alpha$  that represents the economic value of a unit of pollution.  $\alpha$  is a (somewhat) arbitrary scaling factor; we assume a value of 0.4 from Quinn et. al. We also define  $\delta$ , the discount factor to adjust for net present value. For  $\delta = 1$ , we would say the value of a dollar today is equivalent to its value at any point in the future. We assume a discount factor of  $\delta = 0.98$ . With this, we define the objective  $O_1$  as

$$\max O_1 = \frac{1}{N} \sum_{i=1}^N \left( \sum_{t=0}^{T-1} \alpha a_{t,i} \delta^t \right) \quad (1)$$

### 1.2 Environmental Stability

We quantify the environmental objective associated with the health of the ecosystem as the maximum average  $P$  concentration achieved in the lake, across all Monte Carlo simulations. By taking the maximum (instead of the mean, for example) we take a *risk averse* approach to defining policies. Naturally, we want to *minimize* this quantity, as pollution is bad for the system health. We define this objective  $O_2$  as

$$\min O_2 = \max_t \frac{1}{N} \sum_{i=1}^N X_{t,i} \quad (2)$$

## 2 Optimization Method

We use the Borg Multi-Objective Evolutionary Algorithm (Hadka and Reed, 2013) as the optimization component of our simulation-optimization experiment. This algorithm uses heuristic stochastic search to explore the decision space and find an optimal set of solutions. Our model is highly nonlinear, so the optimization modes learned in class (linear programming, mixed integer programming, etc.) are not suitable. We ran our model for 10 random seeds and 75,000 function evaluations. Based on previous literature, we determined that convergence is highly likely after this many evaluations.

## 3 Model Assumptions

We assume a distribution of natural P inflows  $Y_t \sim LN(\mu, \sigma^2)$  with values  $\mu = 0.03$  and  $\sigma^2 = 10^{-5}$ , as defined in Quinn et. al. 2017. We assume  $q = 2.5$  for both lakes, as defined in Carpenter et. al. 1999, based on empirical evidence from various lake studies. Similarly, we define  $b_{shallow} = 0.42$  and  $b_{cayuga} = 0.62$  based on the studies on shallow/deep lake dynamics analyzed in Carpenter et. al. 1999.

## 4 Key Takeaways

1. Modeling seemingly simple systems (the original lake mass-balance) gets exponentially more complex with the addition of uncertainties and Monte Carlo simulation.
2. Multi-objective solutions can be more informative about the nature of the problem space than traditional, single-objective ones.
3. Larger natural systems may provide a "buffer" for controlling environmental degradation than smaller, sensitive systems.

## 5 References

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