Week 3: Data types, summary statistics, & transformations

ANTH 674: Research Design & Analysis in Anthropology

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Lecture outline

1. Different types of data ("**qualitative**" vs. **quantitative**)
   1. How are they described & summarized?
   2. How are they plotted (visualizing the distribution)?
2. Different types of data transformations
   1. What are they & what are they used for?
3. Plotting two data types against each other

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Different data types

How are they described, summarized, and plotted?

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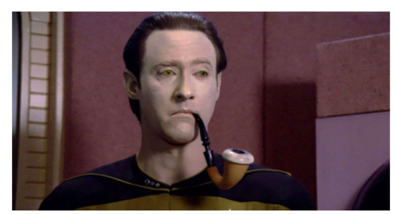
What is data?

- Wikipedia: **Data** are characteristics or information, usually numerical, that are collected through observation
- Want to learn something from data through analysis and/or visualization (plotting)



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Summary statistics

- Used to summarize distribution of data w/ one number (except for multivariate distributions)

1. Location or central tendency
2. Spread or variation

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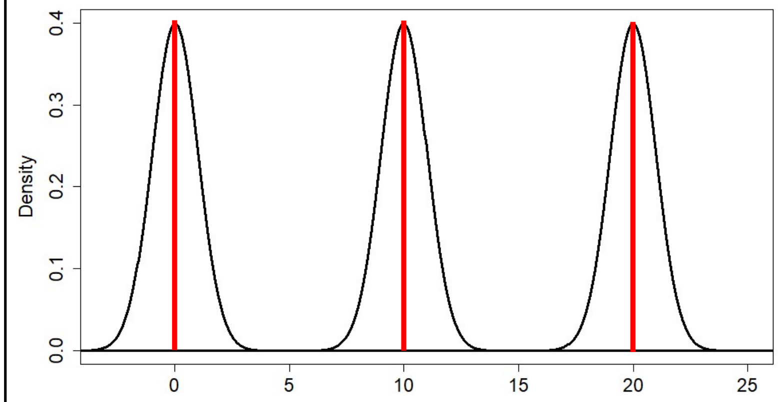
Location/central tendency



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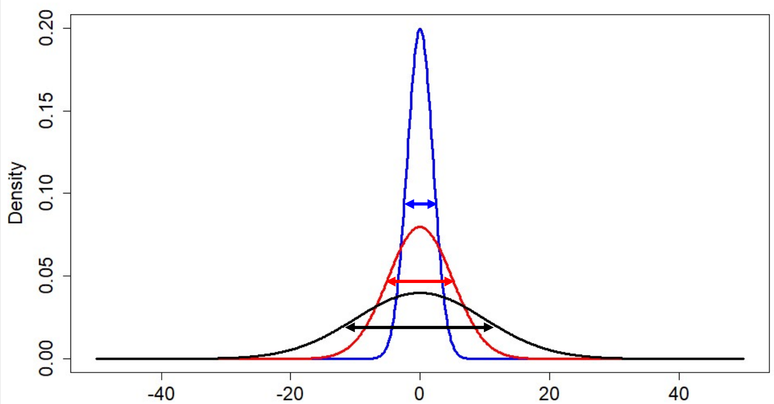
Spread/variation



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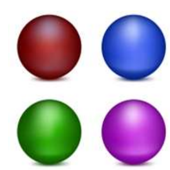
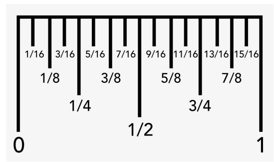
Different types of data

<u>"Qualitative"</u>
- Categorical/nominal
- Ordinal



<u>Quantitative</u>
- Discrete
- Continuous



- Data type tells you which summary statistics, plots, and analyses to use!

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Questions?

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“Qualitative” data

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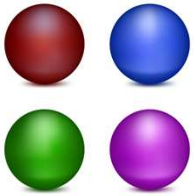
What is “qualitative” data?

- Data assigned to groups, usually based on some qualitative property

1. Categorical/nominal data (unordered)
2. Ordinal data (ordered)



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Categorical/nominal data

- Unordered qualitative data
- E.g., head/tail (binomial); basalt/chert/quartz (multinomial)
- Quantified as counts or proportions
- = factors in R

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Plotting categorical data

black	brown	red	blond
108	286	71	127

- Bar plot

*Order along x-axis is arbitrary

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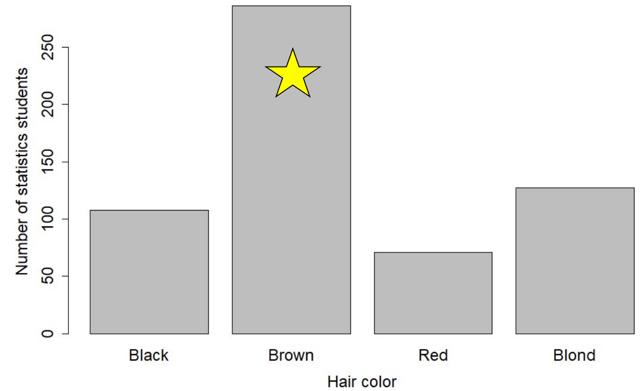
Central tendency

- Mode: most common category



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Spread

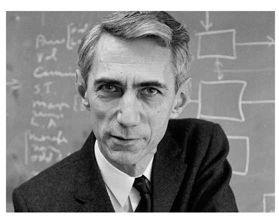
	black	brown	red	blond
#	108	286	71	127
prop.	0.18	0.48	0.12	0.21

- Information theory measures (most common is Shannon's index)
- Not commonly used

$$-\sum p_i \log(p_i)$$

p_i = proportion of i^{th} category

$$-[0.18 \times \log(0.18) + 0.48 \times \log(0.48) + 0.12 \times \log(0.12) + 0.21 \times \log(0.21)] = \mathbf{1.25}$$



Claude Shannon

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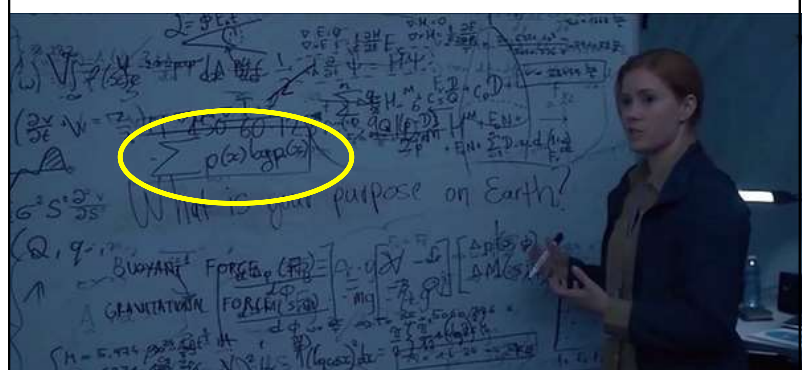
From the movie *Arrival*



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Ordinal data

- Ordered qualitative data
- Distance between categories not known
- E.g., small/medium/large; juvenile/adult
- Quantified as counts or proportions
- = ordered factor levels in R

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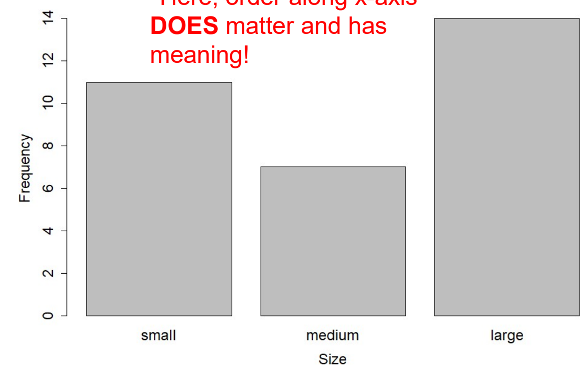
Plotting ordinal data

- Bar plot

small	med.	large
11	7	14

*Here, order along x-axis **DOES** matter and has meaning!



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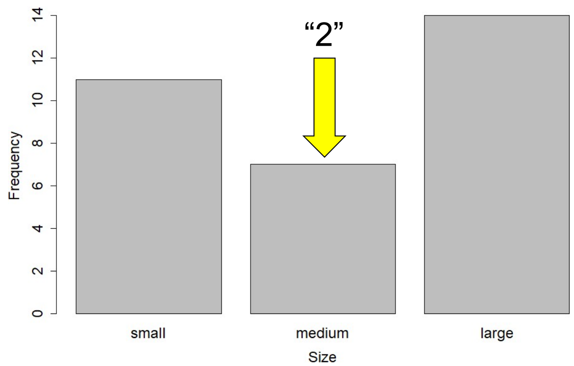
Central tendency

- Median: middle value in ordered data (first convert to ranks: 1, 2, 3)



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Central tendency

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Spread

small	med.	large
11	7	14

- Interquartile range (IQR): difference between 75th and 25th percentiles (AKA 3rd and 1st quartiles; median is the 2nd quartile)
- Middle 50% of data

After converting sizes to ranks

1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3

25% of data

1

$\mathrm{IQR} = 3 - 1 = \mathbf{2}$

3

75% of data

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Questions?

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Quantitative data

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What is quantitative data?

- Each data point is a number, and distances have meaning (e.g., 1 vs. 3)

Discrete

- Finite or countable values (e.g., integers)

Continuous

- Any value within a *continuous* interval (e.g., 2.4575)

- In practice, all continuous numbers are discrete due to limited precision of measurements (e.g., 1.21, 1.22, 1.23)

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Discrete data

- Finite or countable values
- E.g., count data, anything measured in integers
- Treated as numeric class in R

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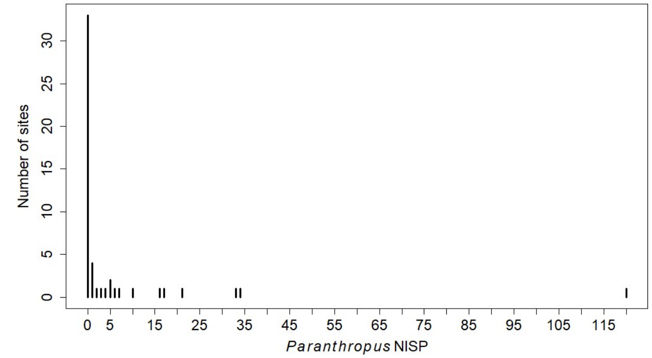
Plotting discrete data

- Line plot (like a bar plot w/ more categories)



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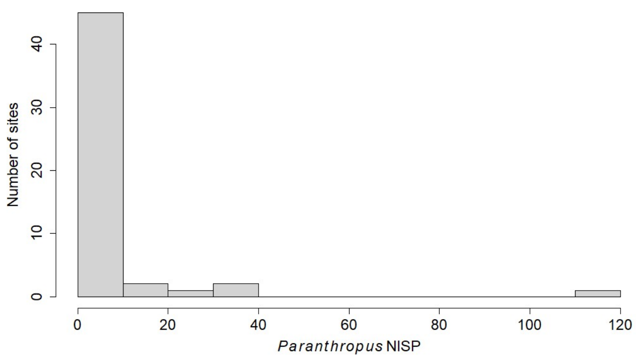
Plotting discrete data

- Histogram



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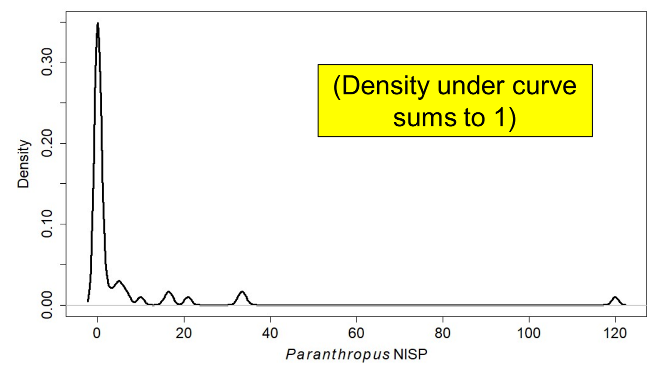
Plotting discrete data

- Density plot ("smoothed histogram")



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Central tendency & spread

- Same as with continuous data

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Continuous data

- Any value within a *continuous* interval (e.g., 2.4575)
- E.g., stone tool mass, hominin femur length
- Treated as numeric class in R

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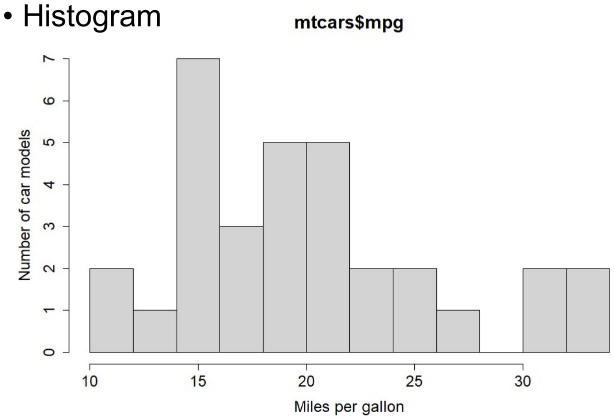
Plotting continuous data

- Histogram



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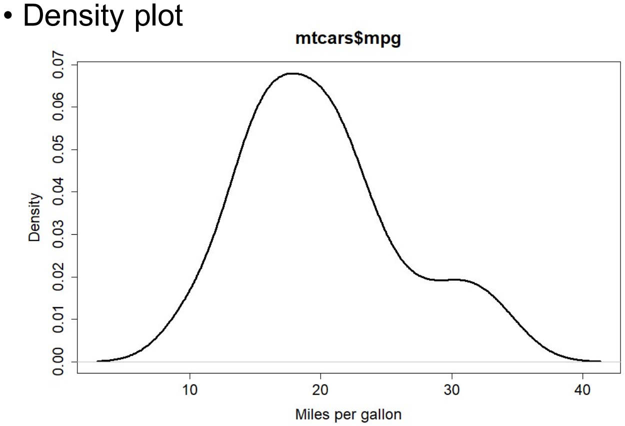
Plotting continuous data

- Density plot



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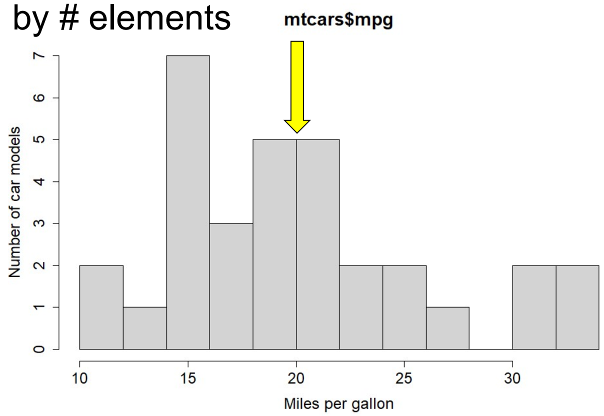
Central tendency

elements $\frac{\sum x_i}{n}$ ith element

- Arithmetic mean: sum elements & divide by # elements



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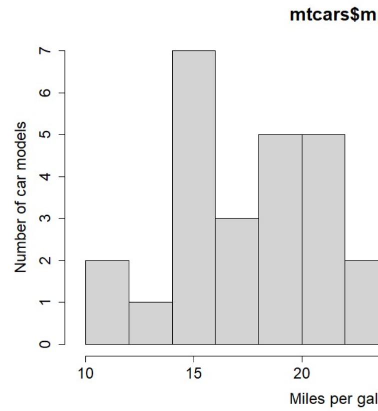
Central tendency



Arithmetic mean = 20.1
Median = 19.2

Arithmetic mean & median identical if distribution is symmetrical

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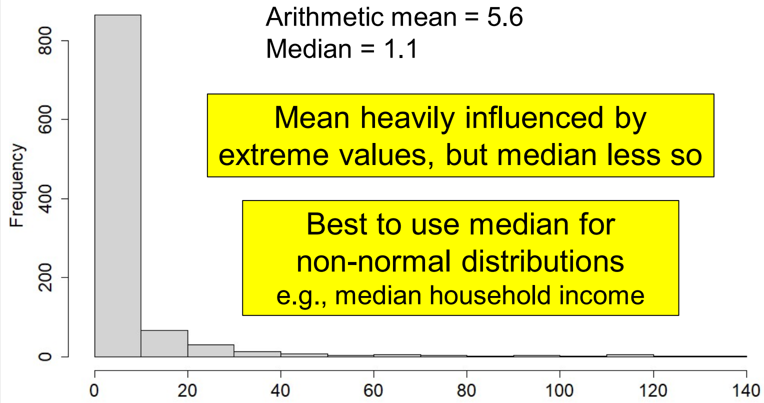
What if data are non-normal?



Arithmetic mean = 5.6
Median = 1.1

Mean heavily influenced by extreme values, but median less so

Best to use median for non-normal distributions
e.g., median household income

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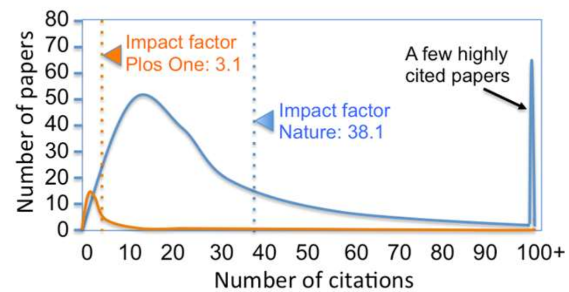
An example

90% of *Nature*'s 2004 impact factor was based on only 25% of its publications

- Journal impact factor: average # of times articles from journal published in the past two years have been cited in year of consideration



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Central tendency

ith element

$$\left(\prod x_i\right)^{\frac{1}{n}}$$

\# elem.

- Geometric mean: nth root of elements multiplied together

  E.g., `x <- c(1, 2, 3)`

  $GM = \sqrt[3]{(1 \times 2 \times 3)}$
- Same as taking arithmetic mean of log-transformed values & calculating antilog

$$GM = \exp\left(\frac{\sum \log(x_i)}{n}\right)$$

E.g., $GM = \exp\left(\frac{\log(1)+\mathrm{l}\quad(2)+\log(3)}{3}\right)$

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Central tendency (geometric mean)

- Used when dealing with data produced by multiplicative processes (e.g., % increases) → lognormal distributions
- E.g., population size, body size, household income, citation numbers
- Can't use when you have zeros or negative numbers

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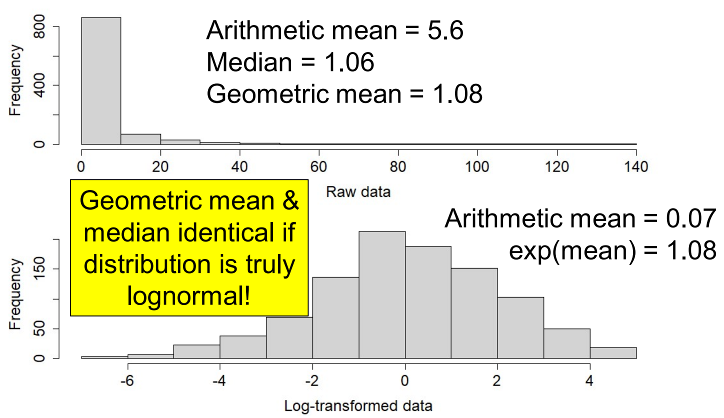
Geometric mean (on lognormal distribution)



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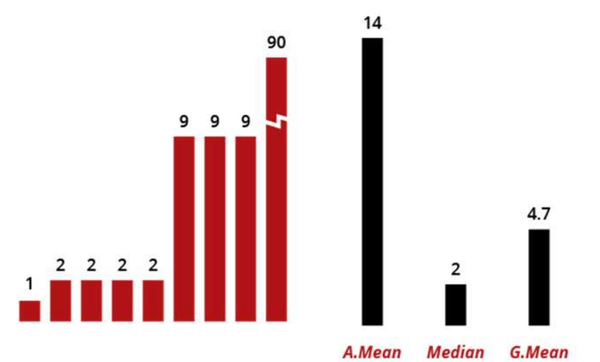
Comparing all three



Order always the same!

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Questions?

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Spread

- Variance: measures how far values deviate from the arithmetic mean
- Subtract the mean from each element, square the results, add them up, and divide by number of elements minus 1

ith element, # elements, Arithmetic mean

$$\frac{\sum(x_i - \bar{x})^2}{n-1}$$

- Square-root of variance = standard deviation

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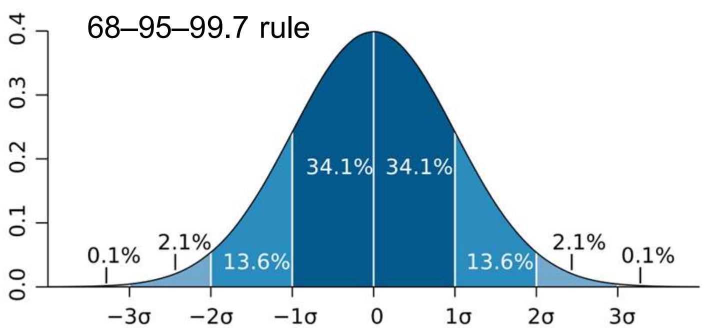
Spread (standard deviation)



- As with the arithmetic mean, variance and SD are most interpretable for normal distributions

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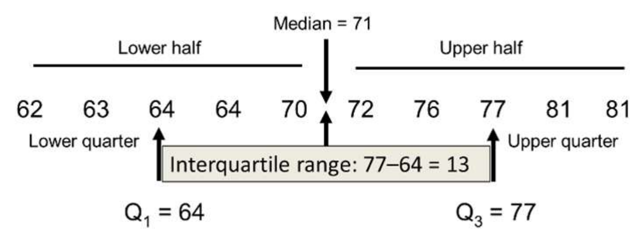
What if data are non-normal?

- Interquartile range
- Doesn't depend on arithmetic mean or type of distribution



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Geometric standard deviation

- Used when you would use the geometric mean (e.g., lognormal data)
- Calculate std. dev. of log-transformed values and take the antilog

ith element

$$\exp\left(\sqrt{\frac{\sum\left[\log\left(\frac{x_i}{\mathrm{GM}}\right)\right]^2}{n}}\right)$$

Geometric mean

elements

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Summary: which plot to make to visualize distribution?

Data type	**Plot**
Categorical	Bar plot
Ordinal	Bar plot
Discrete	Line plot Histogram Density plot
Continuous	Histogram Density plot

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Summary: which statistic to use?

Data type	**Location**	**Spread**
Categorical	Mode	Information measures
Ordinal	Median	Interquartile range
Discrete/Continuous		
Normal	Arithmetic mean	- Variance - Standard deviation
Non-normal	- Median - Geometric mean	- Interquartile range - Geometric SD

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Questions?

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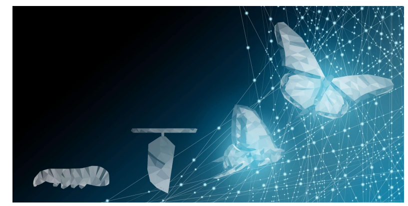
Data transformations

What are they & what are they used for?

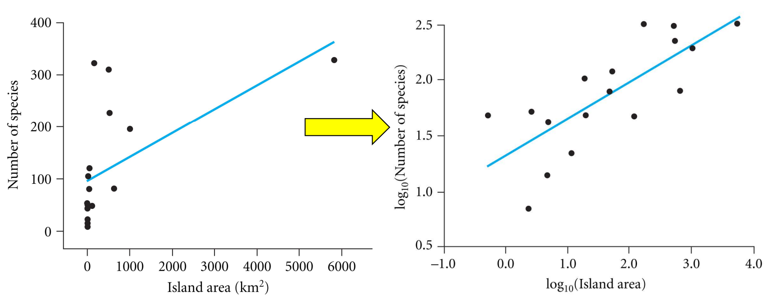
What is data transformation?

- Applying a mathematical function to data to change its distribution
- Rank order of data maintained (monotonic transformation)



Why transform data?

- To make data & results easier to understand and visualize



Why transform data?

- To make data & results easier to understand and visualize
- To make sure assumptions of statistical methods are not violated

Types of data transformations

1. Centering and scaling
2. Log transformations
3. Square-root transformations
4. Arcsine & logit transformations

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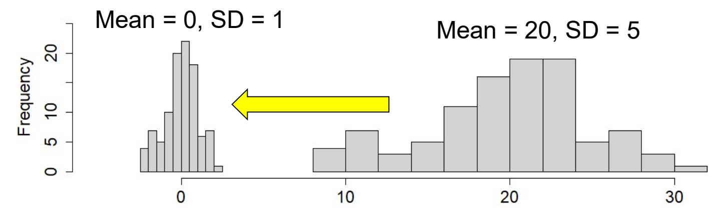
Centering and scaling

- Transforms data to have mean = 0 & standard deviation = 1 (i.e., Z-scores)
- Subtract the mean & divide by SD



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Centering and scaling

- Transforms data to have mean = 0 & standard deviation = 1 (i.e., Z-scores)
- Subtract the mean & divide by SD
- Converts data to units of standard deviation, so variables of different units can be compared (e.g., mass & inches)
- Can be used on non-normal data!

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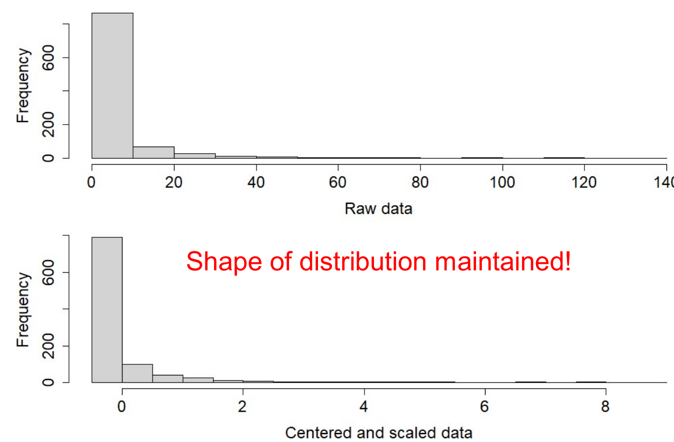
Centering and scaling



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Log transformations

- Replaces data with their logarithms
- $b^a = x;\ \log_b x = a$
- Base *e* (analyses) and 10 (plotting) most common
- Cannot transform zeros and negative numbers

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Log transformations

- Many statistical & plotting methods deal with additive/absolute/linear change
- E.g., linear regression: $y = a + bx$

- 1 → 2: +1
- 100 → 200: +100

Treated differently w/ linear methods

- 1 → 2: x2
- 100 → 200: x2

But multiplicative/ relative change the same!

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Log transformations

- What if you are interested in multiplicative/relative/proportional/percent change?
- Log-transformations: multiplicative → additive
- $\log\left(\frac{2}{1}\right) = \log(2) - \log(1) = 0.69$
- $\log\left(\frac{200}{100}\right) = \log(200) - \log(100) = 0.69$
- Doubling from 1 to 2 now treated the same as doubling from 100 to 200!
- Can now use linear methods to investigate multiplicative change

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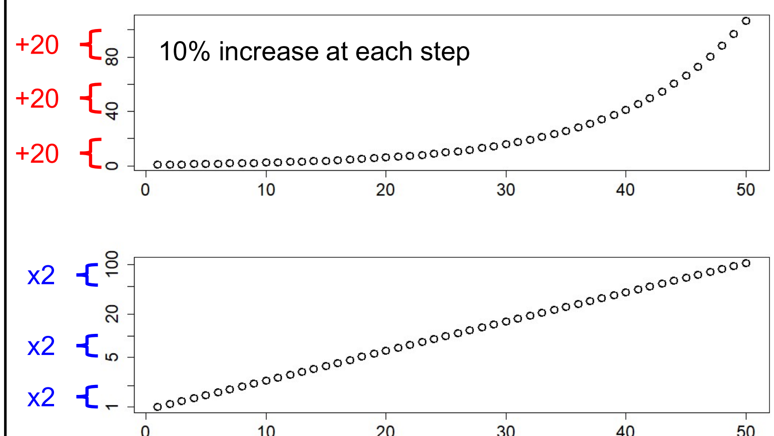
Log transformations



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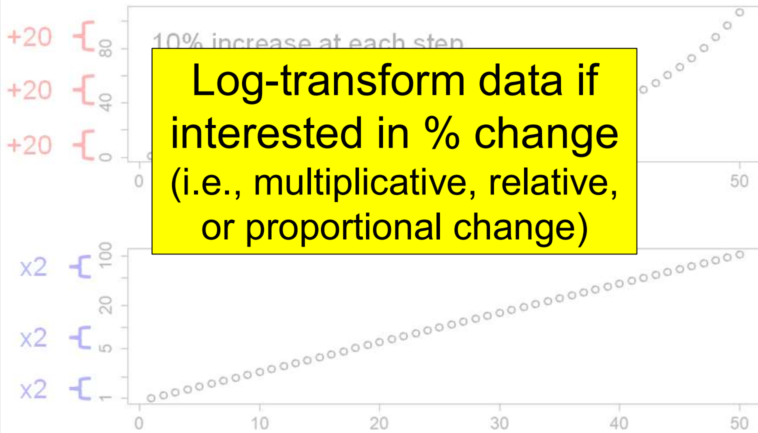
Log transformations



Log-transform data if interested in % change (i.e., multiplicative, relative, or proportional change)

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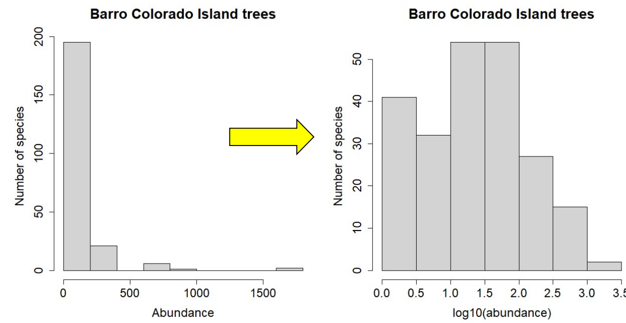
Log transformations

- Can aid visualization if data vary over orders of magnitude (spreads out small numbers and squeezes large ones)



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Other transformations

- Removes dependence between mean & variance of a variable

1. Square-root: $\sqrt{x}$
   - Used for count data
2. Arcsine: $\text{arcsine}(\sqrt{x})$
   - Used for proportions
3. Logit: $\log(\frac{x}{1-x})$
   - Used for proportions

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Questions?

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Plotting two data types against each other

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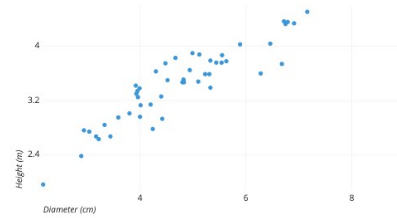
Why do this?

- Want to see how two variables are related to each other
- Good way to visually describe two variables



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First, a note on ordinal & discrete data when plotting

- Ordinal treated as categorical (maintaining order of categories)
- Discrete treated as continuous
- When ordinal data have many categories, can mimic discrete data (e.g., ranks of all countries by GDP)
- When discrete data are too few, can mimic ordinal data (e.g., 3, 4, & 5 number of forward gears in `mtcars`)

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Which plot to make

X-axis

Y-axis	Categorical	Continuous
Categorical	Bar plot	Box plot Violin plot
Continuous	Box plot Violin plot	Scatter plot

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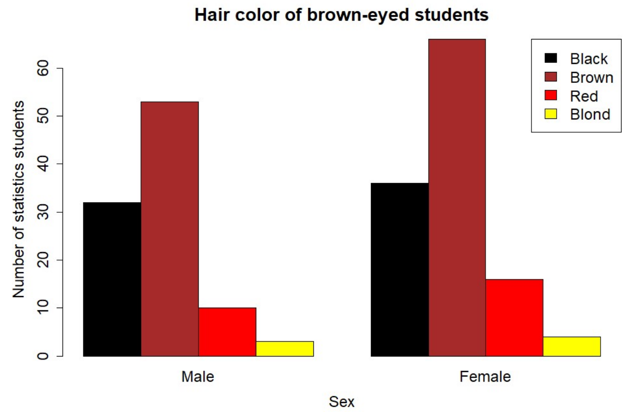
Bar plots
(categorical vs. categorical)



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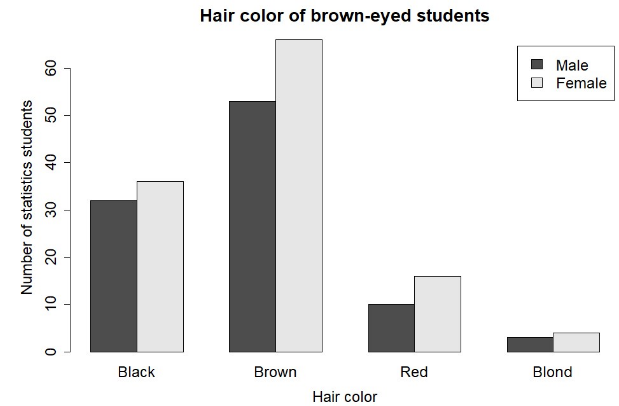
Bar plots
(categorical vs. categorical)



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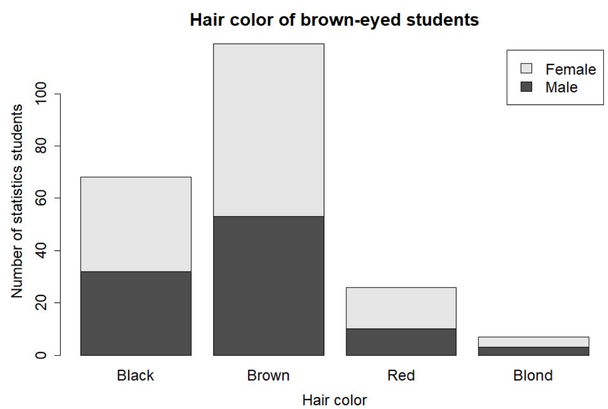
Bar plots (stacked)
(categorical vs. categorical)



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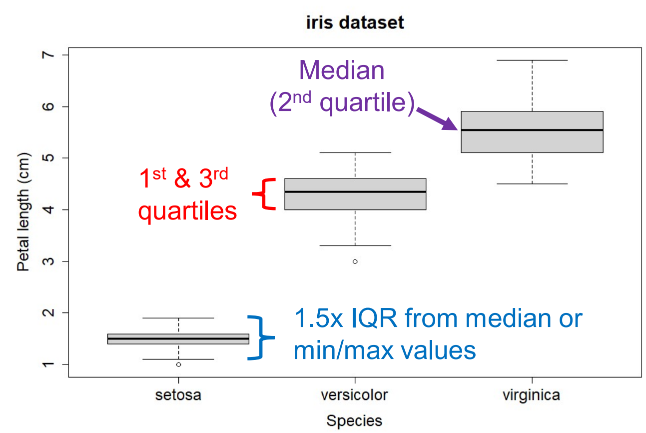
Box plots
(categorical vs. continuous)



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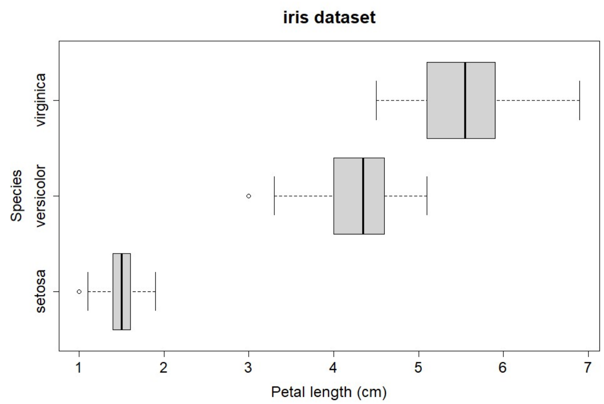
Box plots
(continuous vs. categorical)



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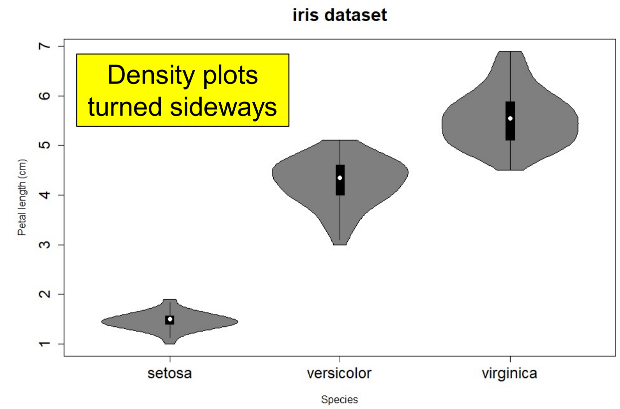
Violin plots
(categorical vs. continuous)



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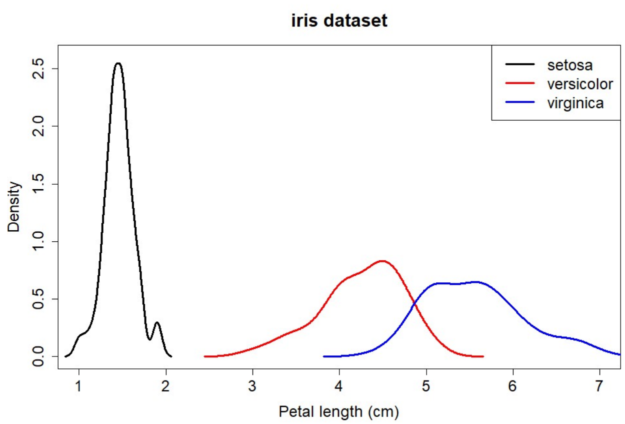
Density plot works too!
(categorical vs. continuous)



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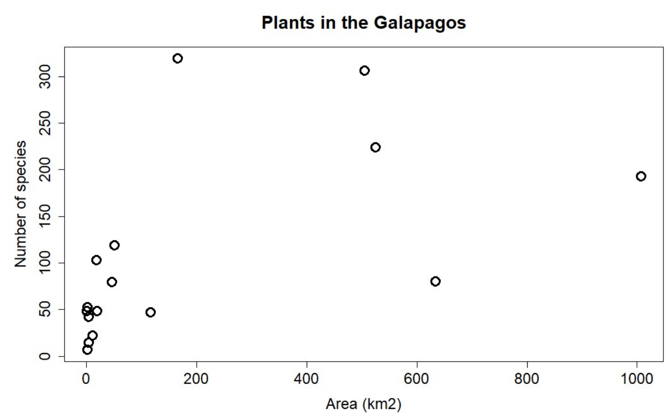
Scatter plots
(continuous vs. continuous)



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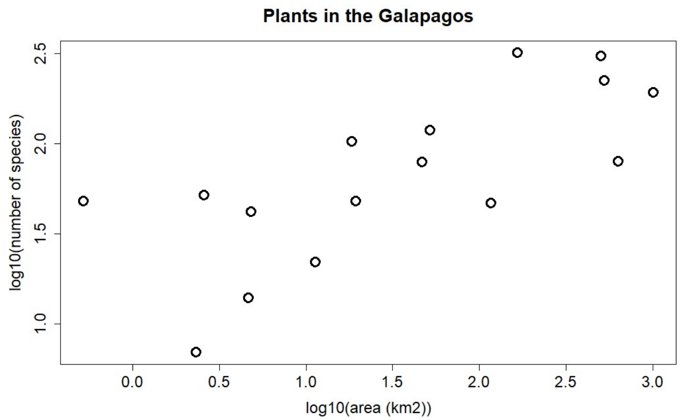
Scatter plots (log-transformed variables) (continuous vs. continuous)



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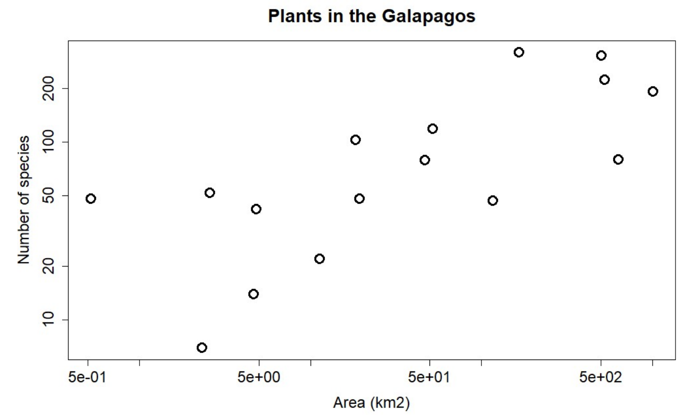
Scatter plots (log-transformed axes) (continuous vs. continuous)



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Plotting summary

- These are the general rules of plotting
  (R will actually automatically make these plots according to your variable type)
- BUT, use best judgment for showing what YOU *want* to show (<u>according to your research question</u>)
- Data visualization is very important: want to convey your data and results as clearly & effectively as possible!

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Questions?

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Summary

- There are four main data types: categorical, ordinal, discrete, & continuous
- Data type tells you which summary statistics and plots to use
- Data transformations aid visualization and interpretation & help data satisfy statistical assumptions
- How to plot & compare two variables depends on data type

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Statistics vignette

- What is Euler's number & where does it come from?
- $e \approx 2.71828$
- Used as base of natural logarithm
- Fundamental to continuous growth & rate of change


Leonhard Euler

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Derivation using compound interest

- You have \$1 in your bank, which offers 100% interest every year
  - After one year, \$1 → \$2
- 50% interest twice a year
  - After one year, \$1 → \$1.50 → \$2.25
- 1/12th interest every month
  - After one year, \$1 → \$1.08 → … → \$2.61
- <u>General formula</u>: $N_0\left(1+\frac{1}{n}\right)^n$

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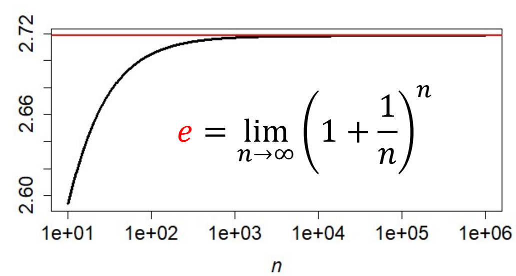
Derivation using compound interest

- <u>Every day</u>: $1\left(1+\frac{1}{365}\right)^{365} = 2.715$
- <u>Every hour</u>: $1\left(1+\frac{1}{8760}\right)^{8760} = 2.718$



85

A probability interpretation

- Probability every dropped chocolate is placed in wrong position = $\mathbf{1/e}$ (as # chocolates → ∞)

86

Widely considered the most beautiful equation in math

- Euler's identity

$$e^{i\pi} + 1 = 0$$

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