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# A FUZZY TIME SERIES-MARKOV CHAIN MODEL WITH AN APPLICATION TO FORECAST THE EXCHANGE RATE BETWEEN THE TAIWAN AND US DOLLAR

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Abstract. *In this study, a fuzzy time series-Markov chain approach for analyzing the linguistic or a small sample time series data is proposed to further enhance the predictive accuracy. By transferring fuzzy time series data to the fuzzy logic group, and using the obtained fuzzy logic group to derive a Markov chain transition matrix, a set of adjusted enrollment forecasting values can be obtained with the smallest forecasting error of various fuzzy time series methods. Finally, an illustrated example for exchange rate forecasting is used to verify the effectiveness of the proposed model and confirms the potential benefits of the proposed approach with a very small MAPE.*
**Keywords:** Fuzzy time series model, Markov chain, Fuzzy logic group, Exchange rate

1. **Introduction.** Forecasting methodology is most important and relevant in the field of management, including that for financial forecasting, production demand and supply forecasting, technology forecasting, and so on. In international economics forecasting, explicating the behavior of nominal exchange rates has been a central theme in economists' work when executing the notoriously challenging task of modeling exchange rates, since the celebrated work of Meese and Rogoff [1] who found that the fundamentals-based exchange rate models systematically fail to deliver better forecasts than a simple random walk at horizons of up to one year. Subsequent studies by Engel and Hamilton [2], who modeled exchange rates alternating between appreciation and depreciation regimes in a Markovian fashion, while considering more recent data, led to a model that no longer beats the random walk. From the above analysis, it is evident that, normally, we cannot directly use the established model for forecasting because there may be some additional causes that are not considered in the collected historical data. That is, if we applied the collected data in a Group $A$ to construct a forecasting model for extrapolation, then, because of its similar structure, and when all conditions remain the same, we could use Group $A$ for forecasting. However, once the trend of future changes in Group $B$ is determined, the derived model cannot be used for forecasting because we have not collected sufficient factors to be incorporated in the forecasting model. For such an insufficient factors problem, fuzzy forecasting models such as the fuzzy regression model and fuzzy time series model are considered a solution. The fuzzy time series model is applied as a valid approach for forecasting the future value in a situation where neither a trend is viewed nor a pattern in variations of time series is visualized and, moreover, the information is incomplete and ambiguous [3].

The fuzzy time series model was first proposed by Song and Chissom [4,5], who applied the concept of fuzzy logic to develop the foundation of fuzzy time series using time invariant and time variant models. Thereafter, the fuzzy time series model had drawn much attention to the researchers. For model modifications, Chen [6] focused on the operator used in the model and simplified the arithmetic calculations to improve the composition operations and then introduced fuzzy logical groups to improve the forecast; Huarng [7] made a study of the effective length of intervals to improve the forecasting; Tsaur et al. [8] made an analysis of fuzzy relations in fuzzy time series on the basis of entropy and used it to determine the minimum value of an invariant time index $t$ to minimize errors in the enrollments forecasting; Cheng et al. [9] introduced a novel multiple-attribute fuzzy time series method based on fuzzy clustering in which fuzzy clustering was integrated in the processes of fuzzy time series to partition datasets objectively and enable processing of multiple attributes. For forecasting with applications, Yu [10] proposed a weighted method for forecasting the TAIEX to tackle two issues, recurrence and weighting, in fuzzy time-series forecasting; Huarng and Yu [11] applied a back propagation neural network to handle nonlinear forecasting problems. Chen et al. [12] presented high-order fuzzy time series based on a multi-period adaptation model for forecasting stock markets. Chen and Hwang [13], Wang and Chen [14], and Lee et al. [15] proposed methods for temperature prediction and TAIFEX forecasting. Since the developing trend of the exchange rate is affected by variant or unknown factors, it is not realistic to establish a single forecasting model that can take all the unknown factors into account. As we know, the Markov process has better performance in exchange rates forecasting [16]. We inquire further into the advantage of connecting the Markov process with the fuzzy time series model, following which we can derive the fuzzy time series-Markov model to induce the characteristics of the exchange rate in international economics. Thus, with the hybrid model, the more the information pertaining to the system dynamics is induced, the better the forecasting will be. We exploit the advantage of the fuzzy logic relationship to group the collected time data so as to reduce the effect of fluctuated values, and we incorporate the advantage of the Markov chain [17] stochastic analysis process to derive the outcomes with the largest probability. Furthermore, the statistics of the exchange rate from Jan. 2006 to Aug. 2009 is used to verify the effectiveness of the proposed model. The experimental results show that the proposed model has proved an effective tool in the prediction of the trend of the exchange rate.

This paper is organized as follows. Section 2 introduces the concept of the fuzzy time series model. Section 3 proposes a fuzzy time series-Markov chain model, for which we take an illustration forecasting for the enrollment at the University of Alabama with the smallest forecasting error when compared with the other models. Section 4 presents the forecasting for the exchange rate using the proposed model, and Section 5 summarizes the conclusion.

2. **Basic Concept of Fuzzy Time Series.** Song and Chissom first proposed the definitions of fuzzy time series in 1993 [4]. Let $U$ be the universe of discourse with $U = \{u_1, u_2, \ldots, u_n\}$ in which a fuzzy set $A_i$ $(i = 1, 2, \ldots, n)$ is defined as follows.

$$A_1 = f_{Ai}(u_1)/u_1 + f_{Ai}(u_2)/u_1 + \cdots + f_{Ai}(u_n)/u_n \tag{1}$$

where $f_{Ai}$ is the membership function of the fuzzy set $A_i$, $u_k$ is an element of fuzzy set $A_i$, and $f_{Ai}(u_k)$ is the membership degree of $u_k$ belonging to $A_i$, $k = 1, 2, \ldots, n$.

**Definition 2.1.** *Let the universe of discourse $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots, n, \ldots)$ be a subset of $R$ defined by the fuzzy set $A_i$. If $F(t)$ consists of $A_i$ $(i = 1, 2, \ldots, n)$, $F(t)$ is defined as a fuzzy time series on $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots, n, \ldots)$.*

**Definition 2.2.** *Suppose that $F(t)$ is caused by $F(t-1)$, then the relation of the first-order model of $F(t)$ can be expressed as $F(t) = F(t-1) \circ R(t, t-1)$, where $R(t, t-1)$ is the relation matrix to describe the fuzzy relationship between $F(t-1)$ and $F(t)$, and '$\circ$' is the max-min operator.*

Let the relationship between $F(t)$ and $F(t-1)$ be denoted by $F(t-1) \to F(t)$, $(t = \ldots, 0, 1, 2, \ldots, n, \ldots)$. Then, the fuzzy logical relationship between $F(t)$ and $F(t-1)$ is defined as follows.

**Definition 2.3.** *Suppose $F(t) = A_i$ is caused by $F(t-1) = A_j$, then the fuzzy logical relationship is defined as $A_i \to A_j$.*

If there are fuzzy logical relationships obtained from state $A_2$, then a transition is made to another state $A_j$, $j$= 1, 2,..., $n$, as $A_2 \to A_3, A_2 \to A_2, \ldots, A_2 \to A_1$; hence, the fuzzy logical relationships are grouped into a fuzzy logical relationship group [4] as

$$A_2 \to A_1, A_2, A_3, \tag{2}$$

Although, various models have been proposed to establish fuzzy relationships, Chen's fuzzy logical relationship group [6] approach is easy to work with and is being used in our proposed model. Therefore, Song and Chissom [4] have proposed the following procedure for solving the fuzzy time series model:

**Step 1.** Define the universe of discourse $U$ for the historical data. When defining the universe of discourse, the minimum data and the maximum data of given historical data are obtained as $D_{\min}$ and $D_{\max}$, respectively. On the basis of $D_{\min}$ and $D_{\max}$, we can define the universal discourse $U$ as $[D_{\min} - D_1,\ D_{\max} + D_2]$ where $D_1$ and $D_2$ are proper positive numbers.

**Step 2.** Partition universal discourse $U$ into several equal intervals. Let the universal discourse $U$ be partitioned into $n$ equal intervals; the difference between two successive intervals can be defined as $\ell$ as follows:

$$\ell = [(D_{\max} + D_2) - (D_{\min} - D_1)]/n \tag{3}$$

Each interval is obtained as $u_1 = [D_{\min} - D_1, D_{\min} - D_1 + \ell]$, $u_2 = [D_{\min} - D_1 + \ell, D_{\min} - D_1 + 2\ell]$, ..., $u_n = [D_{\min} - D_1 + (n-1)\ell, D_{\min} - D_1 + n\ell]$.

**Step 3.** Define fuzzy sets on the universe of discourse $U$. There is no restriction on determining how many linguistic variables can be fuzzy sets. Thus, the "enrollment" can be described by the fuzzy sets of $A_1$ = (not many), $A_2$ = (not too many), $A_3$ = (many), $A_4$ = (many many), $A_5$ = (very many), $A_6$ = (too many), $A_7$ = (too many many). For simplicity, each fuzzy set $A_i$ $(i = 1, 2, \ldots, 7)$ is defined on 7 intervals, which are $u_1 = [d_1, d_2]$, $u_2 = [d_2, d_3]$, $u_3 = [d_3, d_4]$, $u_4 = [d_4, d_5]$, ..., $u_7 = [d_7, d_8]$; thus, the fuzzy sets $A_1$, $A_2$, ..., $A_7$ are defined as follows:

$A_1 = \{1/u_1,\ 0.5/u_2,\ 0/u_3,\ 0/u_4,\ 0/u_5,\ 0/u_6,\ 0/u_7\}$,
$A_2 = \{0.5/u_1,\ 1/u_2,\ 0.5/u_3,\ 0/u_4,\ 0/u_5,\ 0/u_6,\ 0/u_7\}$,
$A_3 = \{0/u_1,\ 0.5/u_2,\ 1/u_3,\ 0.5/u_4,\ 0/u_5,\ 0/u_6,\ 0/u_7\}$,
$A_4 = \{0/u_1,\ 0/u_2,\ 0.5/u_3,\ 1/u_4,\ 0.5/u_5,\ 0/u_6,\ 0/u_7\}$,
$A_5 = \{0/u_1,\ 0/u_2,\ 0/u_3,\ 0.5/u_4,\ 1/u_5,\ 0.5/u_6,\ 0/u_7\}$,
$A_6 = \{0/u_1,\ 0/u_2,\ 0/u_3,\ 0/u_4,\ 0.5/u_5,\ 1/u_6,\ 0.5/u_7\}$,
$A_7 = \{0/u_1,\ 0/u_2,\ 0/u_3,\ 0/u_4,\ 0/u_5,\ 0.5/u_6,\ 1/u_7\}$.

**Step 4.** Fuzzify the historical data. This step aims to find an equivalent fuzzy set for each input data. The used method is to define a cut set for each $A_i$ $(i = 1, \ldots, 7)$. If the collected time series data belongs to an interval $u_i$, then it is fuzzified to the fuzzy set $A_i$.

**Step 5.** Determine fuzzy logical relationship group. By the Definition 2.3, the fuzzy logical relationship group can be easily obtained.

**Step 6.** Calculate the forecasted outputs. If $F(t-1) = A_j$, the forecasting of $F(t)$ is conducted on the basis of the following rules.

*Rule* 1: If the fuzzy logical relationship group of $A_j$ is empty (i.e., $A_j \to \emptyset$), then the forecasting of $F(t)$ is $m_j$, which is the midpoint of interval $u_j$:

$$F(t) = m_j. \tag{4}$$

*Rule* 2: If the fuzzy logical relationship group of $A_j$ is one-to-one (i.e., $A_j \to A_k$, $j, k = 1, 2, \ldots, 7$), then the forecasting of $F(t)$ is $m_k$, the midpoint of interval $u_k$:

$$F(t) = m_k. \tag{5}$$

*Rule* 3: If the fuzzy logical relationship group of $A_j$ is one-to-many (i.e., $A_j \to A_1$, $A_3$, $A_5$, $j = 1, 2, \ldots, 7$), then the forecasting of $F(t)$ is equal to the arithmetic average of $m_1$, $m_3$, $m_5$, the midpoint of $u_1$, $u_3$, $u_5$:

$$F(t) = (m_1 + m_2 + m_3)/3. \tag{6}$$

Fuzzy time series models have been applied and designed to forecast when the collected data is linguistic or a smaller sample of data. However, it is still a developing method, so that any innovation in improving the forecasting performance of the fuzzy time series model is important. The more the information that relates to the system dynamics is considered, the better the prediction will be. Therefore, the Markov chain using the statistical method is incorporated with the fuzzy time series model to further enhance the predicted accuracy.

3. **Fuzzy Time Series-Markov Chain Model.** The fuzzy time series-Markov chain model is introduced by application and comparisons among previous research methods.

3.1. **Fuzzy time series-Markov chain model.** The forecasting procedure from Step 1 to Step 4 is the same as the conventional fuzzy time series model, and some descriptions of my proposed method are defined from Step 5 to Step 7 below.

**Step 5.** Calculate the forecasted outputs. For a time series data, using the fuzzy logical relationship group, we can induce some regular information and try to find out what is the probability for the next state. Therefore, we can establish Markov state transition matrices; $n$ states are defined for each time step for the $n$ fuzzy sets; thus the dimension of the transition matrix is $n \times n$. If state $A_i$ makes a transition into state $A_j$ and passes another state $A_k$, $i$, $j = 1, 2, \ldots, n$, then we can obtain the fuzzy logical relationship group. The transition probability of state [17] is written as

$$P_{ij} = (M_{ij})/M_i, \quad i,\ j = 1, 2, \ldots, n \tag{7}$$

where $P_{ij}$ is the probability of transition from state $A_i$ to $A_j$ by one step, $M_{ij}$ is the transition times from state $A_i$ to $A_j$ by one step, and $M_i$ is the amount of data belonging to the $A_i$ state. Then, the transition probability matrix $\mathbf{R}$ of the state can be written as

$$\mathbf{R} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix} \tag{8}$$

For the matrix $\mathbf{R}$, some definitions are described as follows [17]:

**Definition 3.1.** *If $P_{ij} \geq 0$, then state $A_j$ is accessible from state $A_i$.*

**Definition 3.2.** *If states $A_i$ and $A_j$ are accessible to each other, then $A_i$ communicates with $A_j$.*

The transition probability matrix $\mathbf{R}$ reflects the transition rules of the system. For example, if the original data is located in the state $A_1$, and makes a transition into state $A_j$ with probability $P_{1j} \geq 0$, $j = 1, 2, \ldots, n$, then $P_{11} + P_{12} + \ldots + P_{1n} = 1$.

If $F(t-1) = A_i$, the process is defined to be in state $A_i$ at time $t-1$; then forecasting of $F(t)$ is conducted using the row vector $[P_{i1}, P_{i2}, \ldots, P_{in}]$. The forecasting of $F(t)$ is equal to the weighted average of $m_1$, $m_2$, $\ldots$, $m_n$, the midpoint of $u_1, u_2, \ldots, u_n$. The expected forecasting values are obtained by the following *Rules*:

*Rule* 1: If the fuzzy logical relationship group of $A_i$ is one-to-one (i.e., $A_i \to A_k$, with $P_{ik} = 1$ and $P_{ij} = 0$, $j \neq k$), then the forecasting of $F(t)$ is $m_k$, the midpoint of $u_k$, according to the equation $F(t) = m_k\ P_{ik} = m_k$.

*Rule* 2: If the fuzzy logical relationship group of $A_j$ is one-to-many (i.e., $A_j \to A_1$, $A_2$, $\ldots$, $A_n$, $j = 1, 2, \ldots, n$), when collected data $Y(t-1)$ at time $t-1$ is in the state $A_j$, then the forecasting of $F(t)$ is equal as $F(t) = m_1 + P_{j1} + m_2 P_{j2} + \ldots + m_{j-1} P_{j(j-1)} + Y(t-1) P_{jj} + m_{j+1} P_{j(j+1)} + \ldots + m_n P_{jn}$, where $m_1$, $m_2$, $\ldots$, $m_{j-1}$, $m_{j+1}$, $\ldots$, $m_n$ are the midpoint of $u_1$, $u_2$, $\ldots$, $u_{j-1}$, $u_{j+1}$, $\ldots$, $u_n$, and $m_j$ is substituted for $Y(t-1)$ in order to take more information from the state $A_j$ at time $t-1$.

**Step 6.** Adjust the tendency of the forecasting values. For any time series experiment, a large sample size is always necessary. Therefore, under a smaller sample size when modeling a fuzzy time series-Markov chain model, the derived Markov chain matrix is usually biased, and some adjustments for the forecasting values are suggested to revise the forecasting error. First, in a fuzzy logical relationship group where $A_i$ communicates with $A_i$ and is one-to-many, if a larger state $A_j$ is accessible from state $A_i$, $i, j = 1, 2, \ldots, n$, then the forecasting value for $A_j$ is usually underestimated because the lower state values are used for forecasting the value of $A_j$. On the other hand, an overestimated value should be adjusted for the forecasting value $A_j$ because a smaller state $A_j$ is accessible from $A_i$, $i, j = 1, 2, \ldots, n$. Second, any transition that jumps more than two steps from one state to another state will derive a change-point forecasting value, so that it is necessary to make an adjustment to the forecasting value in order to obtain a smoother value. That is, if the data happens in the state $A_i$, and then jumps forward to state $A_{i+k}$ ($k \geq 2$) or jumps backward to state $A_{i-k}$ ($k \geq 2$), then it is necessary to adjust the trend of the pre-obtained forecasting value in order to reduce the estimated error. The adjusting rule for the forecasting value is described below.

*Rule* 1. If state $A_i$ communicates with $A_i$, starting in state $A_i$ at time $t-1$ as $F(t-1) = A_i$, and makes an increasing transition into state $A_j$ at time $t$, $(i < j)$, then the adjusting trend value $D_t$ is defined as $D_{t1} = (\ell/2)$.

*Rule* 2. If state $A_i$ communicates with $A_i$, starting in state $A_i$ at time $t-1$ as $F(t-1) = A_i$, and makes an increasing transition into state $A_j$ at time $t$, $(i < j)$, then the adjusting trend value $D_t$ is defined as $D_{t1} = -(\ell/2)$.

*Rule* 3. If the current state is in state $A_i$ at time $t-1$ as $F(t-1) = A_i$, and makes a jump-forward transition into state $A_{i+s}$ at time $t$, $(1 \leq s \leq n-i)$, then the adjusting trend value $D_t$ is defined as $D_{t2} = (\ell/2)s$, $(1 \leq s \leq n-i)$, where $\ell$ is the length that the universal discourse $U$ must be partitioned into as $n$ equal intervals.

*Rule* 4. If the process is defined to be in state $A_i$ at time $t-1$ as $F(t-1) = A_i$, then makes a jump-backward transition into state $A_{i-v}$ at time $t$, $1 \leq v \leq i$, the adjusting trend value $D_t$ is defined as $D_{t2} = -(\ell/2)v$, $1 \leq v \leq i$.

**Step 7.** Obtain adjusted forecasting result. If the fuzzy logical relationship group of $A_i$ is one-to-many, and state $A_{i+1}$ is accessible from state $A_i$ in which state $A_i$ communicates with $A_i$, then adjusted forecasting result $F'(t)$ can be obtained as $F'(t) = F(t) + D_{t1} + D_{t2} = F(t) + (\ell/2) + (\ell/2)$. If the fuzzy logical relationship group of $A_i$ is one-to-many, and state $A_{i+1}$ is accessible from state $A_i$ but state $A_i$ does not communicate with $A_i$, then adjusted forecasting result $F'(t)$ can be obtained as $F'(t) = F(t) + D_{t2} = F(t) + (\ell/2)$. If the fuzzy logical relationship group of $A_i$ is one-to-many, and state $A_{i-2}$ is accessible from state $A_i$ but state $A_i$ does not communicate with $A_i$, then adjusted forecasting result $F'(t)$ can be obtained as $F'(t) = F(t) - D_{t2} = F(t) - (\ell/2) \times 2 = F(t) - \ell$.

When $v$ is the jump step, the general form for forecasting result $F'(t)$ can be obtained as

$$F'(t) = F(t) \pm D_{t1} \pm D_{t2} = F(t) \pm (\ell/2) \pm (\ell/2)v. \tag{9}$$

Finally, the MAPE is used to measure the accuracy as a percentage as follows.

$$\text{MAPE} = \frac{1}{n} \sum\nolimits_{t=1}^{n} \frac{|Y(t) - F'(t)|}{Y(t)} \times 100\% \tag{10}$$

3.2. **Enrollment forecasting.** The proposed model for forecasting the enrollment at the University of Alabama [4] is described in this subsection by the following steps.

**Step 1.** Define universe of discourse $U$ and partition it into several equal-length intervals. The collected data is shown in the second column of Table 1; we have the enrollments of the university from 1971 to 1992 with $D_{\min} = 13055$ and $D_{\max} = 19337$. We choose $D_1 = 55$ and $D_2 = 663$. Thus, $U = [13000, 20000]$. $U$ is divided into 7 intervals with $u_1 = [13000, 14000]$, $u_2 = [14000, 15000]$, $u_3 = [15000, 16000]$, $u_4 = [16000, 17000]$, $u_5 = [17000, 18000]$, $u_6 = [18000, 19000]$ and $u_7 = [19000, 20000]$.

**Step 2.** Define fuzzy sets on the universe $U$. The step has the same defined fuzzy sets as in Section 2 proposed by S&C's model.

**Step 3.** Fuzzify the historical data. The equivalent fuzzy sets to each year's enrollment are shown in Table 1 and each fuzzy set has 7 elements.

**Step 4.** Determine the fuzzy logical relationship group. The fuzzy logical relationship group is obtained as shown in Table 2.

TABLE 1. The forecasted values

| Year | Enrollment data | Fuzzy enrollment | Year | Enrollment data | Fuzzy enrollment | Year | Enrollment data | Fuzzy enrollment |
|---|---|---|---|---|---|---|---|---|
| 1971 | 13055 | $A_1$ | 1979 | 16807 | $A_4$ | 1987 | 16859 | $A_4$ |
| 1972 | 13563 | $A_1$ | 1980 | 16919 | $A_4$ | 1988 | 18150 | $A_6$ |
| 1973 | 13867 | $A_1$ | 1981 | 16388 | $A_4$ | 1989 | 18970 | $A_6$ |
| 1974 | 14696 | $A_2$ | 1982 | 15433 | $A_3$ | 1990 | 19328 | $A_7$ |
| 1975 | 15460 | $A_3$ | 1983 | 15497 | $A_3$ | 1991 | 19337 | $A_7$ |
| 1976 | 15311 | $A_3$ | 1984 | 15145 | $A_3$ | 1992 | 18876 | $A_6$ |
| 1977 | 15603 | $A_3$ | 1985 | 15163 | $A_3$ | | | |
| 1978 | 15861 | $A_3$ | 1986 | 15984 | $A_3$ | | | |

TABLE 2. Fuzzy logical relationship group

| | | | |
|---|---|---|---|
| $A_1 \to A_1, A_1, A_2$ | $A_3 \to A_3, A_3, A_3, A_4$ | $A_3 \to A_3, A_3, A_3, A_3, A_4$ | $A_6 \to A_6, A_7$ |
| $A_2 \to A_3$ | $A_4 \to A_4, A_4, A_3$ | $A_4 \to A_6$ | $A_7 \to A_7, A_6$ |

Thus, using the fuzzy logical relationship group in Table 2, the transition probability matrix $\mathbf{R}$ may be obtained.

**Step 5.** Calculate the forecasted outputs. According to the proposed rules in Step 5, the forecasting values are obtained as in the third column of Table 3. The forecasting value of 1972 is $F(1972) = (2/3) * Y(1971) + (1/3) * (m_2) = (2/3) * (13055) + (1/3) * (14500) = 13537$.

**Step 6.** Adjust the tendency of the forecasting values. The relationships between the states are analyzed in Figure 1. It is clear that state 3 and 4 communicate with each other, thus an adjusted value should be considered, or vice versa. By contrast, state 6 and state 7 also communicate with each other, but in the end these states' uncertainty in relation to the future trend is larger and unknown; thus, we do not adjust the value of state 7 to state 6. According to the proposed rules in Step 6, the adjusted values are obtained as in the fourth column of Table 3.

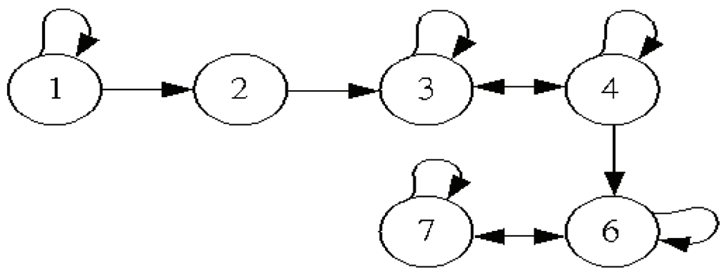


FIGURE 1. Transition process for enrollment forecasting

**Step 7.** Obtain adjusted forecasting values. The adjusted forecasting values are obtained in the last column of Table 3. The adjusted forecasting value for 1974 is $F'(1974) = F(1974) + 500 = 14578$.

Following the above steps, a comparison among actual enrollment, some revised fuzzy time series methods, and the proposed model are shown in Figure 2. It is obvious that these revised methods also have plots similar to the proposed model. Therefore, an estimated error method, MAPE, is used for comparing the methods as shown in Table 4. It is obvious that the forecasting error of MAPE in regard to the proposed method is 1.4042%, which is better than that of the other methods. Using the fuzzy time series-Markov model, a better forecasting result can be derived.

TABLE 3. Enrollment forecasting

| **Year** | **Historical data** | **Forecasting value** | **Adjusted value** | **Adjusted forecasting value** | **Year** | **Historical data** | **Forecasting value** | **Adjusted value** | **Adjusted forecasting value** |
|---|---|---|---|---|---|---|---|---|---|
| 1971 | 13055 | | | | 1981 | 16388 | 16960 | 0 | 16960 |
| 1972 | 13563 | 13537 | 0 | 13537 | 1982 | 15433 | 16694 | −1000 | 15694 |
| 1973 | 13867 | 13875 | 0 | 13875 | 1983 | 15497 | 15670 | 0 | 15670 |
| 1974 | 14696 | 14078 | 500 | 14578 | 1984 | 15145 | 15720 | 0 | 15720 |
| 1975 | 15460 | 15500 | 0 | 15500 | 1985 | 15163 | 15446 | 0 | 15446 |
| 1976 | 15311 | 15691 | 0 | 15691 | 1986 | 15984 | 15460 | 0 | 15460 |
| 1977 | 15603 | 15575 | 0 | 15575 | 1987 | 16859 | 16099 | 1000 | 17099 |
| 1978 | 15861 | 15802 | 0 | 15802 | 1988 | 18150 | 16930 | 1000 | 17930 |
| 1979 | 16807 | 16003 | 1000 | 17003 | 1989 | 18970 | 18600 | 0 | 18600 |
| 1980 | 16919 | 16904 | 0 | 16904 | 1990 | 19328 | 19147 | 500 | 19647 |
| 1991 | 19337 | 18914 | 0 | 18914 | 1991 | 19337 | 18914 | 0 | 18914 |
| 1992 | 18876 | 18919 | 0 | 18919 | 1992 | 18876 | 18919 | 0 | 18919 |

TABLE 4. Comparison of forecasting errors for six types of methods

| **Method** | **S&C Method [4]** | **Tsaur et al. [8]** | **Cheng et al. [18]** | **Singh [19]** | **Li and Cheng [20]** | **Proposed model** |
|---|---|---|---|---|---|---|
| MAPE | 3.22% | 1.86% | 1.7236% | 1.5587% | 1.53% | 1.4042% |

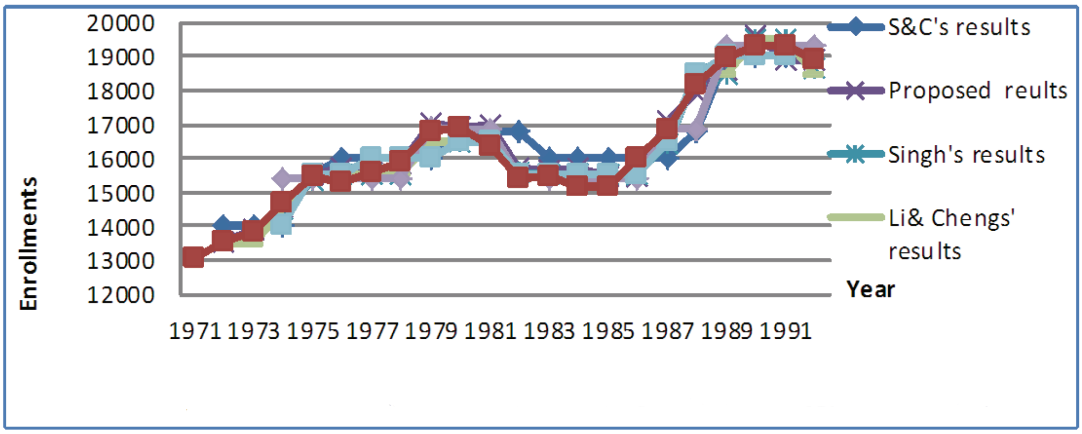


FIGURE 2. Comparisons among the fuzzy time series methods

4. **An Illustrated Example for Exchange Rate Forecasting.** In international economics, the volatility of the New Taiwan Dollar (NTD) against the US Dollar (USD) may significantly affect both exporters and importers in Taiwan, which has a typical island-style economic system that is very open to international trade and investment. Because the NTD/USD relationship plays a crucial role and may influence Taiwan's economy, the forecasting analysis for exchange rates is an important topic. Especially, during the global financial crisis, there was a tremendous change in the exchange rate of the NTD against the USD from Jan.-2008 to Aug.-2009. In forecasting analysis, the time series model is a commonly used tool, but it has been more recently suggested that linear conventional time series methodologies fail to consider limited time series data. This leads to inefficient estimation and therefore lower testing power. Because the dynamic system behavior is often uncertain and complicated, this section illustrates an efficient estimation with smaller forecasting error using the proposed fuzzy time series-Markov method below. Table 5 lists the collected time series data of the exchange rate of NTD/USD (from 2006 to Aug.-2009).

**Step 1.** Define universe of discourse $U$ and partition into equal-length intervals. In Table 5, we set $D_{\min} = 30.35$ and $D_{\max} = 34.34$ with $D_1 = 0.35$ and $D_2 = 0.56$; $U = [30, 34.9]$. $U$ is divided into 7 intervals with $u_1 = [30, 30.7]$, $u_2 = [30.7, 31.4]$, $u_3 = [31.4, 32.1]$, $u_4 = [32.1, 32.8]$, $u_5 = [32.8, 33.5]$, $u_6 = [33.5, 34.2]$ and $u_7 = [34.2, 34.9]$.

**Step 2.** Define fuzzy sets on the universe discourse $U$. This step has the same defined fuzzy sets as in Section 2 proposed by S&C's model.

**Step 3.** Fuzzify the historical data. The fuzzy sets equivalent to each month's exchange rate are shown in Table 5 where each fuzzy set has 7 elements.

**Step 4.** Determine fuzzy logical relationship group. By Definition 2.3, the fuzzy logical relationship group can be easily obtained as shown in Table 6. Clearly, some are one-to-one groups and the others are one-to-many groups. Thus, using the fuzzy logical relationship groups in Table 6, the transition probability matrix $\mathbf{R}$ can be obtained.

**Step 5.** Calculate the forecasted outputs. According to the proposed rules in Step 5, the forecasting values are obtained as in the third and eighth columns of Table 7. For example, the forecasting value of May-2006 can be obtained as $F(\text{May-2006}) = (2/16) * 31.75 + (9/16) * 32.311 + (5/16) * 33.15 = 32.50306$.

**Step 6.** Adjust the tendency of the forecasting values. The relation between the states are plotted in Figure 3; it is clear that state 3 and state 4, state 2 and state 4, state 5 and state 6, and state 6 and state 7 communicate with each other, and an adjusted value should be considered when there is a transition from one state to another state or any

Table 5. The forecasted values

| Month/ year | NTD/ USD | Fuzzy Value | Month/ year | NTD/ USD | Fuzzy Value | Month/ year | NTD/ USD | Fuzzy Value | Month/ year | NTD/ USD | Fuzzy Value |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan.-2006 | 32.107 | $A_4$ | Dec. | 32.523 | $A_4$ | Nov. | 32.332 | $A_4$ | Oct. | 32.689 | $A_4$ |
| Feb. | 32.371 | $A_4$ | Jan.-2007 | 32.768 | $A_4$ | Dec. | 32.417 | $A_4$ | Nov. | 33.116 | $A_5$ |
| Mar. | 32.489 | $A_4$ | Feb. | 32.969 | $A_5$ | Jan.-2008 | 32.368 | $A_4$ | Dec. | 33.146 | $A_5$ |
| Apr. | 32.311 | $A_4$ | Mar. | 33.012 | $A_5$ | Feb. | 31.614 | $A_3$ | Jan.-2009 | 33.33 | $A_5$ |
| May | 31.762 | $A_3$ | Apr. | 33.145 | $A_5$ | Mar. | 30.604 | $A_1$ | Feb. | 34.277 | $A_7$ |
| Jun. | 32.48 | $A_4$ | May | 33.26 | $A_5$ | Apr. | 30.35 | $A_1$ | Mar. | 34.34 | $A_7$ |
| Jul. | 32.632 | $A_4$ | Jun. | 32.932 | $A_5$ | May | 30.602 | $A_1$ | Apr. | 33.695 | $A_6$ |
| Agu. | 32.79 | $A_4$ | Jul. | 32.789 | $A_4$ | Jun. | 30.366 | $A_1$ | May | 32.907 | $A_5$ |
| Sep. | 32.907 | $A_5$ | Agu. | 32.952 | $A_5$ | Jul. | 30.407 | $A_1$ | Jun. | 32.792 | $A_4$ |
| Oct. | 33.206 | $A_5$ | Sep. | 32.984 | $A_5$ | Agu. | 31.191 | $A_1$ | Jul. | 32.92 | $A_5$ |
| Nov. | 32.824 | $A_5$ | Oct. | 32.552 | $A_4$ | Sep. | 31.957 | $A_3$ | Agu. | 32.883 | $A_5$ |

Table 6. Fuzzy logical relationship group

| | | | | |
|---|---|---|---|---|
| $A_4 \to A_4$, $A_4$, $A_4$, $A_3$ | $A_4 \to A_4$ | $A_5 \to A_5$, $A_4$ | $A_3 \to A_4$ | $A_6 \to A_5$ |
| $A_3 \to A_4$ | $A_4 \to A_5$ | $A_4 \to A_4$, $A_4$, $A_4$, $A_3$ | $A_4 \to A_5$ | $A_5 \to A_4$ |
| $A_4 \to A_4$, $A_4$, $A_5$ | $A_5 \to A_5$, $A_5$, $A_5$, $A_5$, $A_4$ | $A_3 \to A_1$ | $A_5 \to A_5$, $A_5$, $A_7$ | $A_4 \to A_5$ |
| $A_5 \to A_5$, $A_5$, $A_4$ | $A_4 \to A_5$ | $A_1 \to A_3$ | $A_7 \to A_7$, $A_7$, $A_6$ | |

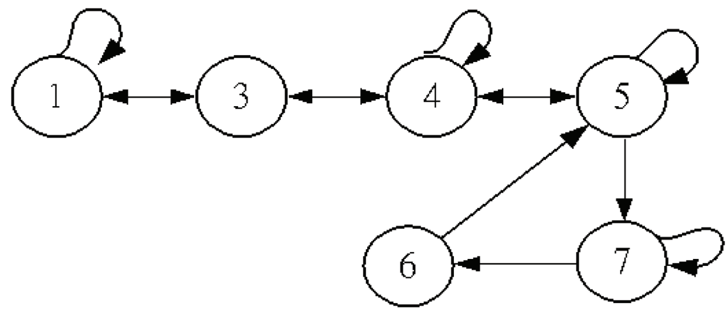


Figure 3. Transition process for exchange rate forecasting

transition among the communicating states. According to the proposed rules in Step 6, the adjusted values are obtained in the fourth and ninth columns of Table 7.

**Step 7.** Adjust forecasting values. According to the proposed rules in Step 6, the adjusted values are obtained as in the fifth and last column of Table 7.

To compare the proposed model with the conventional time series one, an ARIMA-GARCH model using the software (E-Views) and the grey method for different periods of exchange rates from 2006 to Aug.-2009, as well as the forecasting results are shown in Figure 4 and Table 8. It is obvious that the proposed method is better than the other two methods with the smallest forecasting error according to MAPE; thus, the proposed model is the most accurate of the approaches used. Forecasting is done by obtaining the relation between already-known data to analyze the future trend of the exchange rate price, which can be regarded as exhibiting uncertain system behavior because of the relation between the exchange rate price and economic development. Therefore, the fuzzy time series-Markov chain method can be used to establish the forecasting model with relative ease and accurate forecasting performance. However, as previous researches indicate, there are still some criticisms of the fuzzy time series method that have not been overcome, such as the optimum lengths of the interval $u_i$, membership function of the

TABLE 7. Exchange rate forecasting

| Year | Forecasting value | Adjusted value | Adjusted forecasting value | Year | Forecasting value | Adjusted value | Adjusted forecasting value | Year | Forecasting value | Adjusted value | Adjusted forecasting value |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan./2006 | | | | 4 | 32.96467 | 0 | 32.96467 | 7 | 30.59667 | 0 | 30.5967 |
| 2 | 32.38831 | 0 | 32.38831 | 5 | 33.05333 | 0 | 33.05333 | 8 | 30.63083 | 0 | 30.63083 |
| 3 | 32.53681 | 0 | 32.53681 | 6 | 33.13 | 0 | 33.13 | 9 | 31.28417 | 0.7 | 31.98417 |
| 4 | 32.60319 | 0 | 32.60319 | 7 | 32.91133 | −0.35 | 32.56133 | 10 | 31.75 | 0.35 | 32.1 |
| 5 | 32.50306 | −0.7 | 31.80306 | 8 | 32.77194 | 0.35 | 33.12194 | 11 | 32.71569 | 0.35 | 33.06569 |
| 6 | 31.75 | 0.7 | 32.45 | 9 | 32.92467 | 0 | 32.92467 | 12 | 33.03413 | 0 | 33.03413 |
| 7 | 32.59813 | 0 | 32.59813 | 10 | 32.946 | −0.35 | 32.596 | Jan/2009 | 33.054 | 0 | 33.054 |
| 8 | 32.55863 | 0 | 32.55863 | 11 | 32.63863 | 0 | 32.63863 | 2 | 33.1767 | 0.7 | 33.8767 |
| 9 | 32.7725 | 0.35 | 33.1225 | 12 | 32.51488 | 0 | 32.51488 | 3 | 34.0635 | 0 | 34.0635 |
| 10 | 32.89467 | 0 | 32.89467 | Jan./2008 | 32.53681 | 0 | 32.53681 | 4 | 34.095 | −0.7 | 33.395 |
| 11 | 33.094 | 0 | 33.094 | 2 | 32.53513 | −0.7 | 31.83513 | 5 | 33.15 | 0 | 33.15 |
| 12 | 32.83933 | −0.35 | 32.48933 | 3 | 31.75 | −0.7 | 31.05 | 6 | 32.89467 | −0.35 | 32.54467 |
| Jan/2007 | 32.62231 | 0 | 32.62231 | 4 | 30.795 | 0 | 30.795 | 7 | 32.77363 | 0.35 | 33.12363 |
| 2 | 32.63513 | 0.35 | 32.98513 | 5 | 30.58333 | 0 | 30.58333 | 8 | 32.90333 | 0 | 32.90333 |
| 3 | 32.936 | 0 | 32.936 | 6 | 30.79333 | 0 | 30.79333 | | | | |

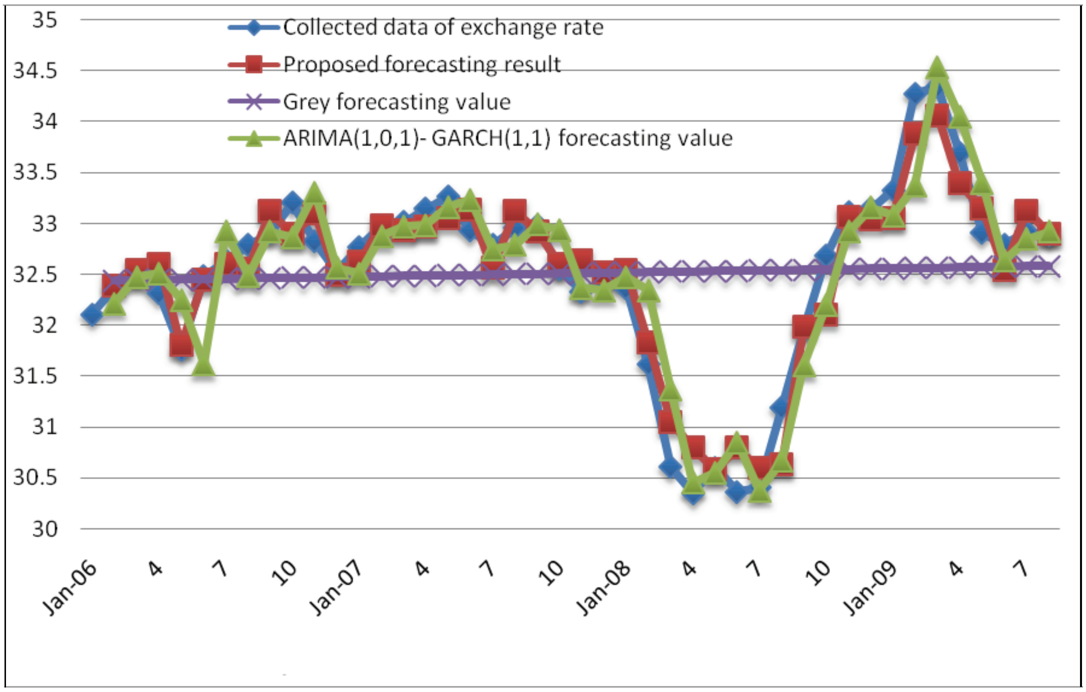


FIGURE 4. The comparisons in exchange rate forecasting

TABLE 8. Comparison of forecasting errors for three types of methods

| Method | ARIMA $(1,0,1)$-GARCH $(1,1)$ | Grey model GM $(1,1)$ | Proposed model |
|---|---|---|---|
| **MAPE** | 0.7983% | 2.1038% | 0.6092% |

defined fuzzy set $A_i$. Besides, in our proposed method, if the collected data set is too limited, we might not derive the transition probability matrix R and fail to model the fuzzy time series-Markov chain method.

5. **Conclusions.** In this study, a fuzzy time series-Markov approach for analyzing the linguistic or smaller size time series data has been proposed. The results indicated considerable forecasting value by transferring fuzzy time series data to the fuzzy logic group, and using the obtained fuzzy logic group to derive a Markov chain transition matrix. Both the enrollment forecasting and the analytical exchange rate forecasting confirm the potential benefits of the new approach in terms of the proposed model. Most importantly, the illustrated experiments were archived with a very small MAPE. If the fuzzy time series-Markov chain model meets expectations, then this approach will be an important tool in forecasting. However, this work only examines forecasting models to determine which has the better performance in forecasting, including some revised fuzzy time series methods, ARIMA-GARCH, and the grey forecasting model.

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